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OLIY VA O‘RTA MAXSUS TA’LIM VAZIRLIGI
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HISOBLASH USULLARI FANIDAN AMALIY MASHG’ULOTLAR

(na’muna uchun yechimlar ko‘rsatilgan)

O‘quv qo‘llanma

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Ushbu qo‘llanma hisoblash usullari fanidan laboratoriya-amaliy ishlarni bajarishga mo‘ljallangan. Qo‘llanma beshta bobdan iborat bo‘lib, hisoblash matematikasining taqribiy sonlarning xatoliklarini hisoblash, algebraik va transtsendent tenglamalarni taqribiy yechish, matritsaning xos son va xos vektorlarini topish, teskari matritsani topish, chiziqli algebraik tenglamalar sistemasini yechish, funksiyani interpolyatsiyalash kabi bo‘limlarini o‘z ichiga olgan. Har bir bobda mavzularga mos nazariy qismi yoritilgan, ko‘plab misollar ishlab ko‘rsatilgan va mustaqil ishlash uchun ko‘plab misollar berilgan.

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Zamonaviy hisoblash texnikasi texnik va injenerlardan hisoblash matematikasining asosini bilishni va uni turli xil amaliy masalalarni yechishga tadbiq qila bilishini talab qiladi. Xalq xo‘jaligining turli sohasida ishlaydigan mutaxasislarni tayyorlashda asosiy fanlardan biri hisoblash matematikasi hisoblanadi.

Ushbu qo‘llanma hisoblash matematikasidan laboratoriya-amaliy ishlarni bajarishga mo‘ljallangan. Qo‘llanma beshta bobdan iborat bo‘lib, hisoblash matematikasining taqribiy sonlarning xatoliklarini hisoblash, algebraik va transtsendent tenglamalarni taqribiy yechish, matritsalar algebrasi, chiziqli algebraik tenglamalar sistemasini yechish, funksiyani interpolyatsiyalash kabi bo‘limlarini o‘z ichiga olgan. Har bir bobda mavzularga mos nazariy qismi qisqacha berilgan, ko‘plab misollar ishlab ko‘rsatilgan va mustaqil ishlash uchun ko‘plab misollar berilgan.

Qo‘llanmani yozishda quyidagi adabiyotlardan foydalanildi:

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I BOB. TAQRIBIY SONLARNING XATOLARINI HISOBLASH

1.1-§. TAQRIBIY SONLARNING XATOLARINI HISOBLASH

Bizga A aniq son va uning taqribiy qiymati a son berilgan bo‘lsin. A aniq sonni uning taqribiy qiymati a son bilan almashtirganda hosil bo‘ladigan xatoliklarni ko‘rib chiqaylik.

A aniq sonni a taqribiy qiymati bilan almashtirilganda ma’lum xatolikka yo‘l qo‘yiladi. Bu xatolik A-a ga teng [1].

Agar $A - a > 0$ bo‘lsa, a soni kami bilan (yoki chapdan) yaqinlashadi; agar $A - a < 0$ bo‘lsa, a soni ortig‘i bilan (yoki o‘ngdan) yaqinlashadi deyiladi. Masalan, 1,73 soni $\sqrt{3}$ ga chapdan yaqinlashadi, 1,74 soni $\sqrt{3}$ ga o‘ngdan yaqinlashadi. Boshqacha aytganda, 1,73 soni $\sqrt{3}$ ga kami bilan taqribiy qiymat, 1,74 soni esa $\sqrt{3}$ ga ortig‘i bilan taqribiy qiymat. Chunki 1,73$< \sqrt{3} = 1{,}7320508 ... < 1{,}74$. Xuddi shunday 3,14 soni π uchun kami bilan taqribiy qiymat, chunki $\pi > 3.14$; 2,72 soni e uchun ortig‘i bilan taqribiy qiymat chunki $e < 2{,}72$.

1-ta’rif: a taqribiy sonning absolyut xatosi deb $\Delta(a) = |A - a|$ ga aytiladi.

A ning aniq qiymati noma’lum bo‘lgani uchun $|A - a|$ absolyut xatolik ham noma’lum bo‘ladi. Shuning uchun chegaraviy absolyut xatolik tushunchasi kiritamiz.

2-ta’rif: a taqribiy sonning chegaraviy absolyut xatoligi deb, bu sonning absolyut xatoligidan kichik bo‘lmagan Δ songa aytiladi:

$$|A - a| \leq \Delta.$$

Bundan $a - \Delta \leq A \leq a + \Delta$.

1-misol. π sonini almashtiradigan $a = 3{,}14$ sonining chegaraviy absolyut xatoligini toping.

Yechish: $3{,}14 < \pi < 3{,}15$ bo‘lgani uchun $|\pi - a| < 0{,}01$. Demak, chegaraviy absolyut xatolik $\Delta = 0{,}01$ ga teng.

Taqribiy sonning absolyut xatoligi A aniq sonni uning a taqribiy qiymati bilan yaqinlashish aniqligini yetarlicha tavsiflay olmaydi. Masalan, $\Delta = 0{,}5\ m$ absolyut xatolik xonaning bo'yini o'lchashda haddan tashqari katta xatolik hisoblanadi, uy qurish uchun yer maydonini ajratishda yo'l qo'yish mumkin bo'lgan xatolik, shaharlar orasidagi masofani o'lchashda esa sezilmaydigan xatolik.

O'lchash yoki hisoblash natijasi aniqligining haqiqiy ko'rsatgichi uning nisbiy xatoligidir.

3-ta'rif: a taqribiy sonning nisbiy xatosi deb

$$\delta(a) = \frac{|A-a|}{|a|}$$

ga aytiladi.

4-ta'rif: a taqribiy sonning chegaraviy nisbiy xatoligi deb, bu sonning nisbiy xatoligidan kichik bo'lmagan δ songa aytiladi:

$$\frac{|A-a|}{|a|} \le \delta \quad \text{yoki } \delta = \frac{\Delta}{|a|}.$$

Bundan $\Delta = |a|\delta$.

Absolyut xato ismli miqdor bo'lib, nisbiy xato ismsiz miqdordir. Nisbiy xato ko'pincha foiz(%) va promillya (‰)larda ifodalanadi (bir promillya foizning o'ndan bir qismiga teng).

Masalan, 3,14 soni π sonining taqribiy qiymati. Uning absolyut xatoligi 0,00159 ga teng; chegaraviy absolyut xatolik $\Delta = 0{,}0016$ ga teng, chegaraviy nisbiy xatolik esa

$$\delta = \frac{\Delta}{3,,14} = 0{,}00051 = 0{,}051\%$$

ga teng deb hisoblash mumkin.

1.2-§. ABSOLYUT VA NISBIY XATOLIKLARNING XOSSALARI

Faraz qilaylik a va b lar ikkita taqribiy sonlar bo'lsin. U holda quyidagilar o'rinli:

Absolyut xatoliklarning xossalari:

$$\Delta(a+b) = \Delta a + \Delta b\ ,$$
$$\Delta(a-b) = \Delta a + \Delta b\ ,$$
$$\Delta(a \cdot b) = a\Delta b + b\Delta a\ ,$$

$$\Delta\left(\frac{a}{b}\right) = \frac{a\Delta b + b\Delta a}{b^2}.$$

Nisbiy xatoliklar xossalari:

$$\delta(a+b) = \frac{\Delta(a+b)}{|a+b|} = \frac{\Delta a + \Delta b}{|a+b|} = \frac{|a|}{|a+b|}\cdot\frac{\Delta a}{|a|} + \frac{|b|}{|a+b|}\cdot\frac{\Delta b}{|b|}$$

$$= \frac{|a|}{|a+b|}\delta a + \frac{|b|}{|a+b|}\delta b,$$

$$\delta(a-b) = \frac{\Delta(a-b)}{|a-b|} = \frac{\Delta a + \Delta b}{|a-b|} = \frac{|a|}{|a-b|}\cdot\frac{\Delta a}{|a|} + \frac{|b|}{|a-b|}\cdot\frac{\Delta b}{|b|}$$

$$= \frac{|a|}{|a-b|}\delta a + \frac{|b|}{|a-b|}\delta b, \delta(a\cdot b) = \delta\left(\frac{a}{b}\right) = \delta a + \delta b,$$

$$\delta(a^k) = k\delta a.$$

5-ta'rif: Sonning yozilishidagi , chapdan birinchi noldan farqli raqamidan boshlab, hamma raqamlari ***ma'noli raqamlar*** deyiladi.

1-misol. 0,00015 sonida ikkita *ma'noli raqam*, 12,150 sonining hamma raqamlari *ma'noli raqam* bo'ladi .

Sonning kasr qismining oxirgi xonalariga qo'shimcha nollar yozib yoki nollarni tashlab sonning ma'noli raqamlarini ko'paytirish yoki kamaytirish mumkin. Bu bilan berilgan son o'zgarmaydi.

6-ta'rif: a sonini *yaxlitlash* deb, uni kamroq ma'noli raqamga ega bo'lgan b soni bilan almashtirishga aytiladi.

2-misol. $a = 0{,}9445$ sonini verguldan keyin ikki xonagacha yaxlitlasak $a \approx 0{,}94$ son hosil bo'ladi.

7-ta'rif: Yaxlitlash natijasida olingan a taqribiy sonning Δ *chegaraviy absolyut xatoligi* a taqribiy sonning yozuvidagi oxirgi xona birligining yarmiga teng.

Misollar: 1) $a = 0{,}9445$ taqribiy sonning chegaraviy absolyut xatoligi $\Delta = 0{,}00005$.

2) $a = 0{,}817$ taqribiy sonning chegaraviy absolyut xatoligi $\Delta = 0{,}0005$.

8-ta'rif: Agarda a sonning absolyut xatosi shu ma'noli raqam turgan xonaning bir birligadan ortiq bo'lmasa, a taqribiy sonning shu ma'noli raqami keng ma'noda ishonchli raqam deyiladi (agarda a sonning absolyut xatosi shu ma'noli raqam turgan xonaning bir birligining yarmidan ortiq bo'lmasa, a taqribiy sonning shu ma'noli raqam tor ma'noda ishonchli raqam deyiladi).

Umuman olganda: Agar $\Delta(a) \le \omega q^{n-r+1}$ tengsizlik bajarilsa, u holda taqribiy

$$a = \alpha_1 q^n + \alpha_2 q^{n-1} + \cdots + \alpha_m + \cdots \quad \left(\frac{1}{2} \le \omega \le 1, \qquad q - \text{sanoq sistema asosi.}\right)$$

sonda α_r raqam *ishonchli raqam* deyiladi, aks holda α_r *shubhali raqam* deyiladi.

Ko'rinib turubdiki, α_r raqam ishonchli raqam bo'lsa, undan oldingi raqamlarning barchasi ham ishonchli raqam bo'ladi. Demak, ishonchli raqamlar orasida har doim oxirgisi mavjud.

1-qoida: Agar a soni n ta ishonchli raqamga ega bo'lsa, u xolda uning δ nisbiy xatoligi

$$\delta(a) \le \frac{1}{k}\left(\frac{1}{10}\right)^{n-1} \qquad (1.1)$$

tengsizlikni qanoatlantiradi, bu yerda k shu a sonning birinchi ma'noli raqami.

2-qoida: Nisbiy xatoligi δ bo'lgan a sonning n ta raqami ishonchli bo'lsa, u holda n ushbu

$$(1+k)\delta \le \left(\frac{1}{10}\right)^{n-1} \qquad (1.2)$$

tengsizlikni qanoatlantiradigan eng katta tub sondir.

3-qoida: Agar taqribiy a sonning ishonchli raqamlari soni ikkitadan ko'p bo'lsa, hisoblash ishlarida

$$\delta(a) = \frac{1}{2k}\left(\frac{1}{10}\right)^{n-1} \qquad (1.3)$$

deb olish mumkin.

1.3-§. ARIFMETIK AMALLAR VA LOGARFMLASHNING XATOSI

1-teorema: Bir xil ishorali qo'shiluvchilar yig'indisining absolyut xatosi qo'shiluvchilar absolyut xatolarining yig'indisiga teng.

2-teorema: Bir xil ishorali taqribiy sonlarni qo'shish natijasida hosil bo'lgan yig'indining nisbiy xatosi qo'shiluvchilarning eng katta va eng kichik nisbiy xatolari orasida yotadi.

1-qoida: Har xil aniqlikdagi sonlarni qo‘shish uchun:

a) O‘nli raqamlari boshqalaridagiga nisbatan eng kam bo‘lgani ajratilib, ularni o‘zgarishsiz qoldirish kerak;

b) Qolgan sonlarda esa bitta yoki ikkita ortiqcha raqamlar qoldirib, ajratilgan sonlarga nisbatan yaxlitlash kerak;

c) Hamma saqlangan xonalarni hisobga olgan holda berilgan sonlarni qo‘shish kerak;

d) Hosil bo‘lgan sonlarni bitta yoki ikkita xonaga yaxlitlash keak.

3-teorema: Taqribiy sonlar ko‘paytmasining nisbiy xatosi ko‘paytuvchilar nisbiy xatolarining yig‘indisiga teng.

2-qoida: Taqribiy sonlarni ko‘paytirish va bo‘lish uchun:

1) ko‘paytirish va bo‘lish kerak bo‘lgan sonlar ichidan ma’noli raqamlari soni eng kami ajratib olinadi;

2) qolgan sonlarni ajratilgan sondagi ma’noli raqamlar sonidan bitta ko‘p ma’noli raqamgacha yaxlitlanadi;

3) saqlangan ma’noli raqamni hisobga olgan holda ko‘paytirish va bo‘lish amali bajariladi;

4) ajratib olingan eng kam ma’noli raqamlari soniga teng miqdorda ma’noli raqam olinadi.

Misol. Hamma raqamlari ishonchli bo‘lgan $a = 3{,}5$ va $b = 83{,}368$ taqribiy sonlarning ko‘paytmasini toping.

Yechish: Birinchi sonda ikkita ma’noli raqam, ikkinchisida beshta ikkita ma’noli raqam bor. Ikkinchi sonni uchta ma’noli raqamgacha yaxlitlaymiz: $b \to 83{,}4$. Yaxlitlagandan keyin ko‘paytirishni bajaramiz: $ab = 3{,}5 \cdot 83{,}4 = 291{,}9 = 2{,}9 \cdot 10^2$.

4-teorema: O‘nli logarifmning absolyut xatosi argument nisbiy xatosining yarmiga teng.

1.4-§. FUNKSIYANI HISOBLASHDAGI XATOLIKLARNI BAHOLASH

Bizga $y = f(x)$ funksiya berilgan bo‘lib, argumentning taqribiy qiymati a bo‘lsin va uning absolyut xatosi Δa bo‘lsin. U holda funksiyaning absolyut xatosi sifatida uning orttirmasini yoki differensialini olish mumkin

$$\Delta y \approx dy, \Delta y = |f'(x)| \cdot \Delta a \,. \qquad (1.4)$$

n o‘zgaruvchili funksiyalar uchun quyidagicha bo‘ladi:

$$\Delta y = \left| f'_{x_1}(x_1, \dots, x_n) \right| \cdot \Delta x_1 + \dots + \left| f'_{x_n}(x_1, \dots, x_n) \right| \cdot \Delta x_n \,,$$

bunda $\Delta x_1, \dots, \Delta x_n$ lar $x_1, \dots, x_n$ larning mos absolyut xatoliklari.

$$\delta y = \frac{\Delta y}{|f(x_1, \dots, x_n)|} \ - nisbiy\ xatolik.$$

Misol. $y = \sin x$, a $-$argument x ning absolyut xatoligi.

Yechish: (1.4) formuladan

$$\Delta y = \Delta(\sin x) = |\cos a| \cdot \Delta a$$

bo‘ladi.

1.5-§. TIPIK MISOLLAR YECHISH

Yuqorida aytilgan qoidalardan foydalanib misollar yechamiz [2].

1-misol. 0,307 taqribiy soni yozilishiga ko‘ra 10^{-3} gacha (ya’ni uchta ma’noli raqamgacha) aniqlikka ega. Shu son 10^{-4} gacha aniqlik bilan berilganda, uni 0,3070 ko‘rinishda, 10^{-2} gacha aniqlik bilan berilganda esa oxirgi «7» raqami ishonchsiz bo‘lib qoladi va shu sabab 0,307 son 0,31 ko‘rinishda yoziladi.

2-misol. $\alpha = (5{,}7 \pm 0{,}08) - (5{,}6 \pm 0{,}06)$ ni hisoblaylik.

Yechish: $5{,}7 - 5{,}6 = 0{,}1$, $\Delta = 0{,}08 + 0{,}06 = 0{,}14$, ya’ni ayirma 0,1 ga teng bo‘lgani holda, uning absolyut xatosi o‘zidan ham katta (0,14) bo‘lmoqda. Javob qoniqarsiz. Komponentalar aniqroq olinish kerak. Masalan, ularning absolyut xatosi $\pm 0{,}025$ ga teng bo‘lganda, ayirmaning xatosi $\Delta = 0{,}05$ bo‘lib, unda bitta ishonchli raqam, xato $\pm 0{,}0001$ bo‘lganda, ayirmaning xatosi $\Delta = 0{,}0002$ bo‘lib, unda uchta ishonchli raqam mavjud bo‘lardi.

3-misol. *x*=7,3344 sonning hamma raqamlari ishonli raqam uni uchta ma’noli raqamgacha yaxlitlang. Topilgan $x_1 \approx x$ sonning chegaraviy absolyut va chegaraviy nisbiy xatosini aniqlang. x_1 sonning yozuvidagi ishonchli raqamlarni ko‘rsating(tor va keng ma’noda)

Yechish:

a) berilgan sonni uchta ma'noli raqamgacha yaxlitlasak $x_1 = 7{,}33$ son hosil bo'ladi,

b) absolyut xatosi: $\Delta x_1 = |x - x_1| = |7{,}3344 - 7{,}33| = 0{,}0044$,

c) chegaraviy absolyut xato:$\Delta_{x_1} = 0{,}005$,

d) chegaraviy nisbiy xato: $\delta_{x_1} = \frac{\Delta_{x_1}}{|x_1|} = \frac{0{,}005}{7{,}33} = 0{,}0007 = 0{,}07\%$

e) endi $x_1 = 7{,}33$ sonning tor va keng ma'nodagi ishonchli raqamlarini ko'rsatamiz:

$\Delta x_1 = 0{,}005 \leq 0{,}005$ bo'lgani uchun x_1 sonning barcha raqamlari tor ma'noda ishonchli raqam bo'ladi;

$\Delta x_1 = 0{,}005 \leq 0{,}01$ bo'lgani uchun x_1 sonning barcha raqamlari keng ma'noda ishonchli raqam bo'ladi.

4-misol. Agar $\alpha = 47{,}542$ taqribiy sonning chegaraviy nisbiy xatoligi $\delta = 0{,}1\ \%$ bo'lsa, uning ishonchli raqamlari sonini aniqlang.

Yechish: (1.1) tengsizlikni tuzamiz. αsondagi birinchi raqam 4 ga teng, demak $k = 4$. $\delta = 0{,}1\ \% = \frac{1}{10} \cdot \frac{1}{100} = \frac{1}{1000}$.

Bundan ko'rinadiki

$$(1+k)\,\delta = 5 \cdot \frac{1}{1000} = 5 \cdot \left(\frac{1}{10}\right)^3 \leq \left(\frac{1}{10}\right)^{n-1}.$$

Yuqoridagi tengsizlik to'g'ri bo'lishi uchun $n - 1 = 2$ yoki $n = 3$ bo'lishi kerak. Demak, $\alpha = 47{,}542$ soni uchta ishonchli raqam: 4, 7 va 5 ga ega.

5-misol. $y = lnx$ funksiyaning chegaraviy absolyut xatoligini toping.

Yechish: $y = lnx$ funksiya uchun (1.4) formuladan foydalanamiz:

$$y' = \frac{1}{x}.$$

U holda

$$\Delta y = \Delta x |y'| = \Delta x \left|\frac{1}{x}\right| = \frac{\Delta x}{|x|} = \delta_x \ .$$

Demak, logarifmning chegaraviy absolyut xatoligi argumentning chegaraviy nisbiy xatoligiga teng.

6-misol. $y = x^n$ funksiyaning chegaraviy nisbiy xatoligini toping.

Yechish:: $y' = nx^{n-1}$ bo‘lganligi uchun (1.3) formulaga asosan quyidagini hosil qilamiz:

$$\delta_y = \left| x \cdot \frac{nx^{n-1}}{x^n} \right| \delta_x = |n| \delta_x \, .$$

Demak darajaning chegaraviy nisbiy xatoligi δ_y asosning chegaraviy nisbiy xatoligi δ_x ni daraja ko‘rsatkichning absolyut qiymatiga ko‘paytirilganiga teng.

7-misol. $a = 3{,}5$ va $b = 83{,}368$ taqribiy sonlarning ko‘paytmasini barcha ishonchli raqamlarigacha toping.

Yechish: Birinchi sonda ikkita ma’noli raqam, ikkinchisida esa beshta. Ikkinchi sonni uchta ma’noli raqamgacha yaxlitlaymiz: $b \to 83{,}4$. Keyin sonlarni ko‘paytiramiz: $ab = 3{,}5 \cdot 83{,}4 = 291{,}9 = 2{,}9 \cdot 10^2$. Eng kam ma’noli ramga ega bo‘lgan ko‘paytuvchida nechta ma’noli raqam bo‘lsa, oxirgi natijaga ham o‘shancha ma’noli raqam olish kerak.

8-misol. $a = 8{,}6;\ b = 8{,}60;\ c = 3200;\ d = 3{,}2 \cdot 10^3$ taqribiy sonlari berilgan. Xar bir sonning chegaraviy absolyut xatoligini ko‘rsating.

Yechish: a soni uchun xatolik $\Delta_a^* \leq 0{,}1$, b soni uchun xatolik $\Delta_b^* \leq 0{,}01$, c soni uchun xatolik $\Delta_c^* \leq 1$, d soni uchun xatolik $\Delta_d^* \leq 0{,}1 \cdot 10^3$.

«Odatdagi» matematika nuqtai nazaridan a va b, c va d sonlari bir-biriga teng, «hisoblash» matematikasi nuqtai nazaridan a va b, c va d sonlar turlicha: absolyut xatolikdan ko‘rinadiki b soni a soniga nisbatan aniqroq c soni esa d dan aniqroq.

9-misol. Quyidagi taqribiy sonlarning yig’indisini toping: $a = 414{,}8;\ b = 0{,}025; c = 24{,}17; d = 0{,}000326.$ Bu yerda barcha raqamlar ishochli.

Yechish: a sonida verguldan keyin bitta raqam bo‘lgani uchun, qolgan sonlarni verguldan keyin ikkita raqamgacha yaxlitlaymiz: $b \to 0{,}02;\ c \to 24{,}17;\ d \to 0{,}00.$ Endi yaxlitlangan sonlarni qo‘shib chiqamiz: 414,8 + 0,02 + 24,17 + 0,00 = 438,99. Hosil bo‘lgan natijani verguldan keyin birinchi raqamgacha yaxlitlab oxirgi javobni olamiz. Javob: 439,0.

10-misol. O‘zgarmaslarning ta’rifi bo‘yicha Xalqaro Komitetining fan va texnologiyalar uchun oxirgi (2015y.) ta’rifiga asosan gravitasion doimiy

$$\gamma = (6{,}67259 \pm 0{,}00085) \cdot 10^{-11} m^3 \cdot kg^{-1} \cdot s^{-2} \, ,$$

electron zaryadi esa

$$e = (1{,}60217733 \pm 0{,}00000049) \cdot 10^{-19} Kl.$$

Bu fundamental fizik doimiylarni aniqlik darajasini taqqoslang.

Yechish: Gravitasion doimiy uchun chegaraviy nisbiy xatolik

$$\delta_{\gamma}^{*} = \frac{0{,}00085}{6{,}67259} = 1{,}27 \cdot 10^{-4} \, ,$$

ga teng, electron zaryadning nisbiy xatoligi esa

$$\delta_{e}^{*} = \frac{0{,}00000049}{1{,}60217733} = 3{,}1 \cdot 10^{-7} \, .$$

Shunday qilib, oxirgi nisbiy xatolik oldingisidan uch tartibga kam, ya’ni electron zaryad gravitasion doimiyga nisbatan ancha aniqroq aniqlangan.

11-misol. Agar π soni o‘rniga 3,14 desak, chegaraviy nisbiy xatolik qanday bo‘ladi?

Yechish: (1.3) formuladan $k = 3 \; va \; n = 3$ bo‘lgani uchun

$$\delta(\pi) = \frac{1}{2 \cdot 3}\left(\frac{1}{10}\right)^{3-1} = \frac{1}{600} = 0{,}00166666 \ldots$$

bo‘ladi. Demak $\delta = 0{,}002$ desak bo‘ladi.

12-misol. $a = \sqrt{22}$ da nechta raqam olsak, nisbiy xatolik $\delta(a) = 0{,}001$ bo‘ladi?

Yechish: $\delta(a) \leq \frac{1}{k}\left(\frac{1}{10}\right)^{n-1}$ formuladan foydalanamiz. $a = \sqrt{22}$ ning birinchi ma’noli raqami $k = 4$ va $\delta(a) = 0{,}001$ bo‘lgani uchun

$$0{,}001 \leq \frac{1}{4 \cdot 10^{n-1}}$$

bo‘lib, bundan $10^{n-1} \leq 250, n \leq 4$ bo‘ladi. Agar $\sqrt{22}$ da 4 ta raqam olsak, uning nisbiy xatoligi 0,001 bo‘ladi.

13-misol. $a = 15342$ taqribiy sonning nisbiy xatoligi 0,1% bo‘lsa, uning nechta raqami ishonchli bo‘ladi?

Yechish: $\delta(a) = \frac{0,1\%}{100\%} = 0,001,\ \Delta(a) = a \cdot \delta(a) = 15342 \cdot 0,001 \cong 15,3 = 1,53 \cdot 10,$

$\Delta(a) \leq \frac{1}{2} 10^{m-n+1}$ bo‘lsa, a sonining birinchi n ta ma’noli raqami ishonchli bo‘lar edi. Bu yerda $m = 4$ bo‘lganligi uchun $1,53 \cdot 10 \leq \frac{1}{2} 10^{4-n+1}$ dan $n = 3$ desak bo‘ladi. Demak, $a = 15342$ ning 3 ta raqami ishonchli, ya’ni berilgan sonni $a = 153 \cdot 10^2$ deb yozish maqsadga muvofiq bo‘ladi.

14-misol. Agar $x_1 = 23,3$ va $x_2 = 56,73$ bo‘lib, ulardagi barcha raqamlar ishonchli bo‘lsa, $U = x_1 \cdot x_2$ ko‘paytmada nechta ishonchli raqam bo‘ladi?

Yechish: $\Delta(x_1) = \frac{1}{2} 10^{1-3+1} = 0,05$, $\Delta(x_2) = \frac{1}{2} 10^{1-4+1} = 0,005.$

Bundan, $\delta(u) = \frac{\Delta(x_1)}{x_1} + \frac{\Delta(x_2)}{x_2} = \frac{0,05}{12,2} + \frac{0,005}{73,56} = 0,0042, u = 897,432$ bo‘lib, $\Delta(u) = u \cdot \delta(u) = 897,432 \cdot 0,0042 = 3,6$ bo‘ladi. $\Delta(u) \leq \frac{1}{2} 10^{m-n+1}$ tengsizlikda $m = 2$ bo‘lib, $n = 2$ ekanligi kelib chiqadi. Demak u kamida 2 ta ishonchli raqamga ega va uni $u = 874 \pm 4$ yozish maqsadga muvofiqdir.

15-misol. Agar shar diametri $d = 3,7\ sm\ \pm 0,5 sm$ bo‘lsa, shar hajmi $V = \frac{1}{6}\pi d^3$ ning chegaraviy absolyut va chegaraviy nisbiy xatoligi qanday bo‘ladi?

Yechish: $\frac{\delta V}{\delta \pi} = \frac{1}{6} d^3 = 8,44;\ \frac{\delta V}{\delta d} = \frac{1}{2}\pi d^2 = 21,5.$

$$\Delta(V) = \frac{\delta V}{\delta \pi} \cdot |\Delta \pi| + \frac{\delta V}{\delta d} \cdot |\Delta d| = 8,44 \cdot 0,0016 + 21,5 \cdot 0,05$$
$$= 0,013 + 1,075 == 1,088 .$$

$\Delta(V) \cong 1,1\ sm^3$.

Demak, $V = \frac{1}{6}\pi d^3 = 27,4\ sm^3,$

$\delta(V) = \frac{1,088}{27,4} = 0,0397 \cong 0,04$; $\delta(V) = 4\%.$

$\Delta(V) = 1,1\ sm^3$; $\delta(V) = 4\%.$

Misol 16. To‘g‘ri to‘rtburchakning tomonlari $a \cong 5m,\ b \cong 200m$ bo‘lsin. $\Delta(S) = 1\ m^2$ dan oshmasligi uchun $\Delta(a) = \Delta(b)$ qanday bo‘lishi kerak?

Yechish: $S = a \cdot b,\ \Delta(S) = b \cdot \Delta(a) + a \cdot \Delta(b) = \Delta(a)(a + \mathrm{b})$.

$$\Delta(a) = \frac{\Delta(S)}{a + \mathrm{b}} = \frac{1}{205} < 0{,}005\,,\ \ \Delta(a) = 5\ mm.$$

Javob : $\Delta(a) = \Delta(b) = 5\ mm$.

Misol 17. Berilganlar asosida $Z = \frac{ab-4c}{\ln a+b}$ ni hisoblang bunda $a = 12{,}762\,, b = 0{,}4534\,,\ s = 0{,}290$.

Yechish: Berilganlar $\Delta a = 0{,}0005, \Delta b = 0{,}00005\,,\ \Delta s = 0{,}0005$ (ya’ni $a, b\ va\ s$ larning barcha raqamlari tor ma’noda ishonchli) chegaraviy absolyut xatoga ega.

a) $a \cdot b$ ni hisoblaymiz. $a \cdot b = 12{,}762 \cdot 0{,}290 = 5{,}786\,290\,8$. Endi chegaraviy absolyut xatosini hisoblaymiz:

$\Delta(a \cdot b) = b \cdot \Delta a + a \cdot \Delta b = 0{,}4534 \cdot 0{,}0005 + 12{,}762\ \cdot 0{,}00005 = 0{,}000865 \approx 0{,}00087$.

Topilganlarga ko‘ra , $a \cdot b = 5{,}786\,290\,8$ ko‘paytma verguldan keyin tor ma’noda ikkita ishonchli raqamga ega. Bu qiymatni bitta ortiqcha xona bilan yaxlitlaymiz 5,78$\underline{6}$ (ortiqcha xona raqami tagiga chizib ajratilgan).

b) $4 \cdot c = 4 \cdot 0{,}290 = 0{,}160$ ni hisoblaymiz. Endi chegaraviy absolyut xatosini hisoblaymiz: $\Delta(4 \cdot c) = |(4c)'| \cdot \Delta c = 4 \cdot 0{,}0005 = 0{,}002$.

Topilganlarga ko‘ra , $4 \cdot c = 0{,}160$ ko‘paytma verguldan keyin tor ma’noda ikkita ishonchli raqamga ega. Bu qiymatni bitta ortiqcha xona bilan yaxlitlaymiz 0,16$\underline{0}$ (ortiqcha xona raqami tagiga chizib ajratilgan).

c) $a \cdot b - 4 \cdot c = 5{,}786\underline{3} + 0{,}16\underline{0} = 4{,}6263$. Endi chegaraviy absolyut xatosini hisoblaymiz:

Taqribiy sonlarni qo‘shganda va ayirganda yig‘indidagi o‘nli xonalar sonini qo‘shiluvchilarda bor bo‘lgan o‘nli xonalar soning eng kamiga teng qilib olinadi.

Bu qoidaga asosan: $5{,}786\underline{3}$ sonda uchta o‘nli xona (verguldan keyin) raqam; $1{,}16\underline{0}$ sonda ikkita o‘nli xona (verguldan keyin) raqam bor.

Demak, $a \cdot b - 4 \cdot c = 4{,}6263$ natijani ikkita o‘nli raqamgacha yaxlitlab olish kerak (ya’ni 4,63). Bitta ortiqcha raqami bilan olsak $a \cdot b - 4 \cdot c =$4,62$\underline{6}$.

d) $\ln 12{,}762 = 2{,}546472005446$. Endi chegaraviy absolyut xatosini hisoblaymiz:

$\Delta(\ln a) = |(\ln a)'| \cdot \Delta a = \frac{1}{12{,}762} \cdot 0{,}0005 = 0{,}000039178\,81 \approx 0{,}000040.$

Hosil qilingan natija verguldan keyin tor ma’noda to‘rtta ishonchli raqamga ega. Bu qiymatni bitta ortiqcha xona bilan yaxlitlaymiz $2{,}5464\underline{7}$ (ortiqcha xona raqami tagiga chizib ajratilgan).

e) $\ln a + b = 2{,}54647 + 0{,}4534 = 2{,}99987$. Endi chegaraviy absolyut xatosini hisoblaymiz: $\Delta(\ln a + b) = \Delta(\ln a) + \Delta b = 0{,}000040 + 0{,}00005 = 0{,}00009$

Hosil qilingan natija verguldan keyin tor ma’noda uchta ishonchli raqamga ega. Bu qiymatni bitta ortiqcha xona bilan yaxlitlaymiz 2,999 $\underline{9}$(ortiqcha xona raqami tagiga chizib ajratilgan).

f) Z ni hisoblaymiz.

$$Z = \frac{ab - 4c}{\ln a \ + b} = \frac{4{,}626}{2{,}9999} = 1{,}5420514.$$

Endi chegaraviy absolyut xatosini hisoblaymiz:

$$\Delta Z = \frac{(ab - 4c)\Delta(\ln a + b) + \ln\,(\,a + b)\Delta(ab - 4c)}{(\ln a + b)^2} = \frac{4{,}626 \cdot 0{,}00009 + 2{,}9999 \cdot 0{,}0029}{2{,}9999^2} =$$

$= 0{,}00101296 \approx 0{,}0011.$

Mustaqil yechish uchun misollar

I. Quyida berilgan (1-17) misollarda quyida aytilgan topshiriqlarni bajaring:

a) Qaysi tengliklar aniqroq.

b) Ishonchli raqamlarni qoldirib, shubxali raqamlarni yaxlitlang. Natijaning absolyut xatoligini toping.

c) Hamma raqami ishonchli bo‘lgan taqribiy sonlarning chegaraviy absolyut va nisbiy xatoligini toping.

1. $a)\ 14/17 = 0.824, \sqrt{53} = 7.28;\ b)\ 23.3748, \delta = 0.27\%;\ d) = 0.645.$
2. $a)\ 7/3 = 2.33, \sqrt{58} = 7.62;\ b)\ 13.5726 \pm 0.0072;\ d)\ 4.8556.$
3. $a)\ 27/31 = 0.871, \sqrt{42} = 6.48;\ b)\ 0.088748, \delta = 0.56\%;\ d)\ 71.385.$
4. $a)\ 23/9 = 2.56, \sqrt{87} = 9.33;\ b)\ 4.57633 \pm 0.00042;\ d)\ 6.8346.$
5. $a)\ 6/7 = 0.857, \sqrt{41} = 6.40;\ b)\ 46.7843, \delta = 0.32\%;\ d)\ 7{,}38.$
6. $a)\ 12/7 = 1.71, \sqrt{47} = 6.86;\ b)\ 0.38725 \pm 0.00112;\ d)\ 0.00646.$
7. a) 13/17 = 0,765, $\sqrt{31}$= 5,57; b) 3,6878 $\pm$ 0,0013; B) 8,74.
8. a) 16/7 = 2,29, $\sqrt{11}$ = 3,32; b) 0,75244 $\pm$ 0,00013; B) 5,374 .
9. a) 18/7 = 2,57, $\sqrt{22}$ = 4,69 ; b) 46,453, δ = 0,15 %; B) 6,125.
10. a) 17/9 = 1,89, $\sqrt{17}$ = 4,12; b) 0,66385 $\pm$ 0,000 42; B) 24,6.
11. a) 51/11 = 4,64, $\sqrt{35}$ = 5,92; b) 0,66385, δ = 0,34 %; B) 0,543.
12. a) 19/12 = 1,58, $\sqrt{12}$ = 3,46; b) 4,884 45 $\pm$ 0,000 52; B) 4,633.
13. a) 13/7 = 1,857, $\sqrt{7}$ = 2,65; b) 2,8867, δ = 0,43%; B) 63,749.
14. a) 49/13 = 3,77, $\sqrt{14}$ = 3,74 ; b) 5,6483 $\pm$ 0,0017; B) 0,00858.
15. a) 5/3 = 1,667, $\sqrt{38}$ = 6,16 ; b) 3,7542, δ = 0,32 %; B) 0,389.
16. a) 17/11 = 1,545, $\sqrt{18}$ = 4,243; b) 0,8647 $\pm$ 0,0013; B) 0,864.
17. a) 7/22 = 0,318, $\sqrt{13}$ = 3,61 ; b) 0,3944, δ = 0,15%; B) 21,7.

II. 17,00675 aniq sonni 0,1 gacha , $0{,}5 \cdot 10^{-3}$ gacha aniqlik bilan yaxlitlash xatoligini toping.

2. O‘lchash natijasida detalning uzunligi 36,0 sm ekani aniqlangan. Topilgan qiymatning nisbiy xatosi 0,8% dan oshmaydi. Detal uzunligining chegaraviy qiymatlarini toping.

3. Ikki kesmadan birining uzunligi 1 m $\pm$ 1 mm, ikkinchisining uzunligi 20sm $\pm$ 0,5 mm. Qaysi kesmaning uzunligi aniqroq topilgan va nima uchun?

4. 34,5867 taqribiy sonning nisbiy xatosi $\pm$ 0,1%. Ishonchsiz raqamlari aniqlangan sonni yaxlitlab, xatosini toping?

5. Natural logarifmning e=2,71828183... asosini: a) yettita ishonchli raqamgacha aniqlik bilan yozing; b) 0,5 $*10^{-3}$ gacha aniqlik bilan yaxlitlang. Natijalarning absolyut va nisbiy xatolarini toping.

6. Nisbiy xatosi 0,02% dan oshmasligi uchun $\sqrt{34}$, $\sqrt[3]{128}$, ln56 ni nechta o'nli raqam bilan olish kerak?

7. (11.) va (1.2) formulalardan foydalanib, a^x (a$> 0, a \neq 1)$ x^k (k-haqiqiy son), $\sin x$, $\cos x$, $\tan x$,$\cot x$ (x – radianlarda) funsiyalarning absolyut va nisbiy xatolarini topish uchun formulalar chiqaring. Ixtiyoriy k va x= =1.2$\pm 0.04n$ $(n = 1,2, \ldots ,10)$ uchun 2^x, x^k,$\sin x$, $\cos x$, $\tan x$, ctgx ning qiymatini toping.

8. Berilgan: a=35$\pm$0,1$, b = 2{,}34 \pm 0{,}02, c = 0{,}55($ 1 $\pm$ 3%). Quyidagilar topilsin: 1) x = 2a-2b; 2) y =a+5b; 3) t=ac; 4)u=c\b; 5) v= $\sqrt[4]{ac}$; 6) z=lgb.

9. Radiusi 2,4$\pm$0,2 sm, yasovchi 56$\pm$0,8 sm bo'lgan silindrning asosi, yon sirti va hajmini toping.

10. Ushbu x=$\frac{(a+b)c}{k-e}$ ifodaning son qiymatini toping. Bunda a=50, 34,6$< b < 34{,}7$. 19$< c < 20,\ 3{,}2 < k < 3{,}3,\ e = 12$.

11. 10-misoda a=50, b$\approx 46{,}00$, c≈21, k$\approx 4{,}842$, e≈1,87 da yechilsin.

12. Silindr asosining radiusi R≈ 4 m, balandligi $H \approx 70dm$. S yon sirti va V hajmini toping. S ni 0,01 m^3 gacha aniqlik bilan topish uchun R va H larni qanday aniqlanganda olinishi kerak? V ni 0.1 m^3 gacha aniqlik bilan topish uchun-chi?

13. $0 < x < \frac{\pi}{2}$ oraliq uchun y=lg$\sin x$, y=lg$\tan x$, y= 10^x funksiyalarning taqribiy qiymatlari buyicha argumentning taqribiy qiymatlarining xatosini aniqlash formulalarini chiqaring.

14. Ushbu lg$\sin x$ va lg$\tan x$ funksiyalar qiymatlari to'rt xonali jadvalidan foydalanib, x≈ 60℃ uchun topilgan. Bu qiymatning absolyut xatoligi baxolansin.

15. 10^x qiymatlarining besh xonali jadvalidan x≈ 5 aniqlangan. Bu topilgan qiymatning absolyut va nisbiy xatosini toping.

16. Aylana uzunligi, doira yuzi, konus va silindrning yon sirti uchun xatoni xisoblash formulalarini chiqaring.

17. Ishonchli raqamlarni sanash qoidalaridan foydalanib, taqribiy sonlar ustida quyidagi amallarni bajaring:

1) 418,66781+12.4266+6,102+3,902

2) $\frac{1}{7}+\frac{1}{6}-\frac{2}{9}+\frac{4}{11}$ (surat va maxrajdan aniq sonlar turibdi)

3) 17,906 - 6,5408

4) 37,523 + 0,60 – 6,5408

5) $\sqrt{12}-\sqrt{8}+\sqrt{14}$

6) $0{,}2\sqrt{200}-5\sqrt{7.08}+7\sqrt{10}+0{,}4\sqrt{60}$

III. X argumentli Δx absolyut yoki δx nisbiy xato qiymatlari ma'lum, $y=f(x)$ funksiya ega bo'lishi mumkin bo'lgan $\Delta(y)\ va$ $\delta(y)$ xato chegaralarini hisoblang.

variant	Δx	$\delta\ x$	$f(x)$	variant	Δx	δx	$f(x)$
1		1,2%	$(x+1)^3$-2	14	0.002		$\sqrt{tgx}$
2		1,4%	$\frac{x^2+1}{x-1}$	15	0,003		$\sqrt{2^x}$
3	0,001		$\sqrt{x^2-4}$	16		1%	$\cos^2 2x$
4	0,002		-$\sqrt{10-x^2}$	17		1%	arctg$\frac{1}{x}$
5		1%	$\log_{\frac{1}{3}}(x+1)$	18		2%	$\cos^2 3x$
6	0,002		$\frac{1}{1+x+x^3}$	19	0,002		4^{3x}
7		2%	$\sin(x+\frac{\pi}{3})$	20	0,002		$\sin^3 2x$
8	0,001		$-2+\cos^2 x$	21		1%	4^{tg2x}
9		1,5%	2tg$\sqrt{x}$	22		2%	$\sqrt{(x+1)(6-x)}$
10	0,001		lg$cosx$	23		1%	x^2-4x-45
11	0,001		$arctgx2$	24	0,002		$Log2x$
12		1%	2^{tgx}	25		2%	$log_3(x+1)$
13	0,002		$\sqrt{cosx}$	26			

1.1-turdagi topshiriqlar [3]

Topshiriqlarni bajarish uchun na'muna:

1-misol. Qaysi tenglik aniqroq ekanini aniqlang.

9/11=0.818; $\sqrt{18}$=4.24.

Yechish: Berilgan ifodaning o'nli raqamlari sonini ko'proq qilib olamiz: $\alpha_1 = 9/11 = 0{,}81818\ldots$, $\alpha_2 = \sqrt{18} = 4.2426$. Keyin chegaraviy absolyut xatoni hisoblab, ularni ortig'i bilan yaxlitlaymiz:

$\alpha_{\alpha_1} = |0.81818 - 0.818\,| \le 0.00019$, $\alpha_{\alpha_2} = |4.2426 - 4.24| \le$ 0.0027.

Chegaraviy nisbiq xatolik

$$\delta_{\alpha_1} = \frac{\alpha_{\alpha_1}}{\alpha_1} = \frac{0.00019}{0.818} = 0.00024 = 0.024\%;$$

$$\delta_{\alpha_2} = \frac{\alpha_{\alpha_2}}{\alpha_2} = \frac{0.0027}{4.24} = 0.00064 = 0.064\%.$$

lardan iborat. $\delta_{\alpha_1} < \delta_{\alpha_2}$ bo'lgani uchun, 9/l l=0.818 tenglik aniqroq bo'ladi.

2-misol. Ishonchli raqamlarni qoldirib, shubhali raqamlarni yaxlitlang: a) tor ma'noda; b) keng ma'noda. Natijaning absolyut xatoligini aniqlang.

a) 72.353 (± 0.026); b) 2.3544; δ=0.2%;

Yechish:

a) $72{,}353(\pm\, 0{,}026\,) = \;a$ bo'lsin. Shartga ko'ra, xatolik $\alpha_a = 0{,}026 < 0{,}05$, bu esa 72,353 sonda 7, 2, 3 raqamlar tor ma'noda ishonchli ekanini ko'rsatadi. Yaxlitlash qoidasiga asosan o'ndan birlar xonasini saqlab qolib berilgan sonning taqribiy qiymatini topamiz:

$$\alpha_1 = 72.4;\ \alpha_{\alpha_1} = \alpha_\alpha + \Delta_{okr} = 0{,}026 + 0.047 = 0.073.$$

Topilgan xatolik 0,05 dan katta. Demak , taqribiy sondagi raqamlar sonini ikkitagacha kamaytirish kerak:

$\alpha_2 = 72;\quad \alpha_{\alpha_2} = \alpha_\alpha + \Delta_{okr} = 0{,}026 + 0{,}353 = 0{,}379.$

$\alpha_{\alpha_2} < 0{,}5$ bo'lgani uchun, qolgan raqamlarning ikkalasi ham tor ma'noda ishonchli bo'ladi.

b) a =2,3544; $\delta_\alpha = 0.2\%$ bo‘lsin, u holda $\alpha_\alpha = \alpha \cdot \delta_\alpha$= 0,00471 bo‘ladi. Berilgan sonda uchta raqam keng ma’noda ishonchli bo‘ladi, shuning uchun shu uchta raqamni saqlab uni yaxlitlaymiz:

$$\alpha_1 = 2.35;\ \alpha_{\alpha_1} = 0.0044 + 0.00471 = 0.00911 < 0.01.$$

Demak, 2,35 yaxlitlangan sondagi uchala raqam ham keng ma’noda ishonchli bo‘ladi.

3-misol. Agar sonlar faqat: a) tor ma’noda; b) keng ma’noda ishonchli

raqamga ega bo‘lsa, u holda sonning chegaraviy absolyut va chegaraviy nisbiy xatoliklarini toping.

a) 0.4357; b) 12,348.

Yechish: a) $a = 0{,}4357$ sondagi to‘rtala raqam ham tor ma’noda ishonchli bo‘lgani uchun , absolyut xatolik α =0,00005, nisbiy xatolik esa

$$\delta_\alpha = \frac{1}{2 \cdot 4 \cdot 10^3} = 0.000125 = 0.0125\%\ \ bo‘ladi.$$

b) $a = 12{,}384$ sonning beshta raqami ham keng ma’noda ishonchli bo‘lgani uchun α_α = 0,001; $\delta_\alpha = 1/(1 \cdot 10^4)$ = 0,0001=0.01 %. Topshiriq to‘liq bajarildi.

Mustaqil yechish uchun:

Quyida berilgan misollar uchun 1), 2), 3) talablarni bajaring:

1) Qaysi tenglik aniqroq ekanini aniqlang.

2) Ishonchli raqamlarni qoldirib, shubhali raqamlarni yaxlitlang: a) tor ma’noda; b) keng ma’noda. Natijaning absolyut xatoligini aniqlang.

3) Agar sonlar faqat: a) tor ma’noda; b) keng ma’noda ishonchli raqamga ega bo‘lsa, u holda sonning chegaraviy absoyut va chegaraviy nisbiy xatoliklarini toping.

№ 1.

1) $\sqrt{44}$=6.63; 19/41=0.463.

2) a) 22.553 ($\pm$0.016);

b) 2,8546; $\delta = 0{,}3\%$,

3) a) 0,2387; b) 42,884,

№ 2

1) 7/15=0.467; $\sqrt{30}$=5.48.

2) a) 17,2834; δ=0,3%.

b) 6,4257 ($\pm$0,0024),

3) a) 3,751; b) 0,537.

№ 3.

1) $\sqrt{10.5}$ =3.24; 4/17=0.235.
2) a) 34,834; $\delta = 0,1\%$;
b) 0,5748 ($\pm$0.0034).
3) a) 11,445; b) 2,043.

№ 4.

1) 15/7=2.14; $\sqrt{10}$=3.16.
2) a) 2,3485 ($\pm$0,0042);
b) 0,34844; δ=0.745.
3) a) 20,43; b)0,745.

№ 5.

1) 6/7=0.857; $\sqrt{4.8}$=2.19.
2) a) 5.435 ($\pm$0.0028);
b) 10,8441; $\delta = 0.5\%$.
3) a) 8,345; b) 0.288.

№ 6.

1) 12/11=1.091; $\sqrt{6.8}$=2.61.
2) a) 8,24163; δ=0.2%;
b) 0,12356 ($\pm$0,00036),
3) a) 12,45; b) 3.4453.

№ 7.

1) 2.21=0,095; $\sqrt{22}$=4.69.
2) a) 2,4543; ($\pm$0.0032);
b) 24.5643; δ=0.1%.
3) a) 0.374; b) 0,678

№ 8.

1) 23/15=1.53; $\sqrt{9.8}$=3.13.
2) a) 23,574; δ=0,2%.
b) 8,3445 ($\pm$0,0022),
3) a) 20,43; b)0,576.

№ 9.

1) 6/11=0.545; $\sqrt{83}$=9.11.
2) a) 21,68563; δ=0.3%.
b) 3,7834 ($\pm$0,0041),
3) a) 41,72; b)0.678.

№ 10.

1) 17/19=0.895; $\sqrt{52}$=7.21.
2) a) 13,537 ($\pm 0{,}0026$).
b) 7,521; $\delta = 0.12\%$.
3) a) 5,634; b) 0,0748.

№ 11.

1) 21/29=0.723; $\sqrt{44}$=6.63.
2) a) 0,3567; δ=0.042%.
b) 13,6253 ($\pm$0,0021).
3) a) 18,357; b) 2,16.

№ 12.

1) 50/19=2.63; $\sqrt{27}$5.19.
2) a) 1,784 ($\pm 0{,}0063$);
b) 0,85637; $\delta = 0.21\%$.
3) a) 0,5746; b) 236,58.

№ 13.

1) 13/17=0.764; $\sqrt{31}$=5.56.
2) a) 3,6878 ($\pm$0,0013);
b) 15,873; δ=0.42%.
3) a) 14,862; b) 8,73.

№ 14.

1) 7/22 =0.318; $\sqrt{13}$=3.60.
2) a) 27,1548 ($\pm$0.0016);
b) 0,3945; $\delta = 0{,}16\%$.
3) a) 0,3648; b)21,7.

№ 15.

1) 17/11=1.545; $\sqrt{18} = 4.24$.

2) a) 0,8647 ($\pm$0,0013);

b)24.3618; $\delta = 0.22\%$.

3) a) 2.4516; b)0,863,

№16.

1) 5/3=1.667; $\sqrt{38} = 6.16$.

2) a) 3.7542; $\delta = 0.32\%$;

b) 0,98351 ($\pm$0,00042),

3) a) 62,74; b) 0,389,

№17.

1) 49/13=3.77; $\sqrt{14} = 3.74$.

2) a) 83.736; $\delta = 0.085\%$;

b) 5.6483 ($\pm$0.0017).

3)a)5.6432; b)0,00858,

№18.

1)13/7=1.857; $\sqrt{7} = 2.64$

2) a) 2.8867; $\delta = 0.43\%$;

b) 32,7486 b) 63,745,

3) a) 0,0384; b) 63,745,

№19.

1) 19/12=1.58;$\sqrt{12} = 3.46$

2) a) 4.88445 ($\pm$0.00052);

b) 0,096835; $\delta = 0.32\%$

3) a)12.688; b) 4,636

№20.

1) 51/11=4.64; $\sqrt{354} = 5.91$.

2) a) 38.4258 ($\pm$0.0014);

b) 0,66385; $\delta = 0.34\%$.

3) a) 6.743; b) 0,543,.

№21.

1)18/7=2.57; $\sqrt{22} = 4.69$.

2) a) 0.39642 ($\pm$0.00022);

b) 46,453; $\delta = 0.15\%$.

3) a)15.644; b) 6,125,

№22.

1)19/9=2.11 $\sqrt{17} = 4.12$.

2) a) 5.8425; $\delta = 0.23\%$.

b) 0,66385 ($\pm$0,00042),

3) a) 0,3825; b) 24.6.

№23.

1) 16/7=2.28; $\sqrt{11} = 3.32$.

2) a) 24.3872; $\delta = 0.34\%$;

b) 0,75244 ($\pm$0,00013).

3) a) 16.383; b) 5,734,

№24.

1)20/13=1.54; $\sqrt{63} = 7.94$.

2) a) 2.3684 ($\pm$0.00017);

b) 45,7832;$\delta = 0{,}18\%$,

3) a) .0,573; b)3,6761.

№25.

1) 12/7=1.71; $\sqrt{47} = 6.86$.

2) a) 72.354; $\delta = 0.24\%$;

b) 0,38725 ($\pm$0,00112),

3) a) 18,275; b)0,00644.

№26.

1)6/7=0.857; $\sqrt{41}6.40$.

2) a) 0.36127 ($\pm$0.00034);

b) 46,7843; $\delta = 0{,}32$.

3) a) 3,425; b) 7,38.

№27.

1)23/9=2.56; $\sqrt{87} = 9.33$.

2) a) 23.7564; $\delta = 0.44\%$;

b) 4.57633 (± 0.00042).

3) a) 26.3; b)6,8343,

№28.

1) 27/31=0.872; $\sqrt{42} = 6.48$.

2) a) 15.8372 (± 0.0026);

b) 0,08874; $\delta = 0{,}56\%$.

3) a)3,643; b)72,385.

№29.

1) 7/3=2.33; $\sqrt{58} = 7.61$.

2) a) 3.87683; $\delta = 0.33\%$;

b)13,5726 ($\pm 0{,}0072$),

3) a) 26,3; b) 4,8556,

№30.

1)14/17=0.823; $\sqrt{53} = 7.28$.

2) a) 0.66835 (± 0.00115);

b) 23,3748; $\delta = 0{,}27\%$.

3) a) 43,813; b) 0,645.

1.2-turdagi topshiriqlar

Topshiriqlarni bajarish uchun na'muna:

1-misol. Hisoblang va natijaning xatoligini aniqlang.

$$X = \frac{m^2 n^3}{\sqrt{k}}, bunda\ \ m = 28{,}3(\pm 0{,}02),$$
$$n = 7{,}45(\pm 0{,}01), k = 0{,}678(\pm 0.003).$$

Yechish: Quyidagilarni hisoblaymiz $m^2 = 800.9$; $n^2 = 413.5$; $\sqrt{k} = 0.8234$;

$$\text{X} = \frac{800.9\ 413.5}{0.8234} = 402200 = 4.02\ \cdot 10^5.$$

Keyin,

$\delta_m = \frac{0.02}{28.3} = 0.00071; \delta_m = \frac{0.01}{7.45}\ = 0.00135;\ \delta_k = \frac{0.003}{0.678} = 0.00443,$

Bundan

$$\delta_x = 2\delta_m + 3\delta_n + 0.5\delta_k = 0.00142 + 0.00405 + 0.00222$$
$$= 0.00769 = 0.77\%;$$

$\alpha_x = 4.02 \cdot 0.0077 = 3.110^3$.

Javob: X=4.02· $10^5(\pm 3.1 \cdot 10^3)$; $\delta_x = 0.77\%$.

$2-$ misol. Hisoblang va natijaning xatoligini aniqlang.

$$N = \frac{(n-1)(m+n)}{(m-n)^2}\ , bunda\ n = 3{,}0567(\pm 0{,}0001), m$$
$$= 5{,}72(\pm 0{,}02).$$

Yechish: Quyidagiga egamiz

$$n-1=2.0567(\pm 0.0001);\ m+n=3.057(\pm 0.0004)+5.72(\pm 0.02)=$$
$$=\ 8.777(\pm 0.0204);\ m-n=5.72(\pm 0.02)-3.057(\pm 0.0004)=$$
$$=2.663(\pm 0.0204);$$

$$\mathrm{N}=\frac{2.0567*8.777}{2.663^2}=\frac{2.0567*8.777}{7.092}=2{,}545;\approx 2.55;$$
$$\delta_N=\frac{0.0001}{2.0567}+\frac{0.0204}{8.777}+2\frac{0.0204}{2.663}=0.000049+0.00233+2\cdot 0.00766=$$
$$=0.00238+0.01532=0.0177=1.77\%;\ \alpha_N=2.55\cdot 0.0177=0.046.$$

Javob: N$\approx 2.55(\pm 0.046);\ \delta_N=1.77\%$.

$3-$misol. Raqamlarni hisoblash usulidan foydalanib hisoblang.

$$V=\pi h^2\left(R-\frac{h}{3}\right), bunda\ \ \mathrm{h}=11.8;\ \mathrm{R}=23.67.$$

Yechish: Hisoblaymiz

V=3.142$\cdot 11.8^2(23.67-3.933)=3.142\cdot 11.8^2\cdot 19.737=$

=3.142$\cdot 139.2\cdot 19.737=437.37\cdot 19.737=8630\approx 8.63\cdot 10^3$.

Javob: V$\approx 8.63\cdot 10^3$. Topshiriq to‘liq bajarildi.

Mustaqil yechish uchun:

№ 1.

1) Hisoblang va natijaning xatoligini aniqlang. $\mathrm{X}=\frac{ab}{\sqrt[3]{c}}$

	A	b	v
a	3.85 (0.01)	4.16 (0.005)	7,27 (0,01)
b	2.0435 (0.0004)	12.162 (0.002)	5,205 (0,002)
c	962.6(0.1)	55.18 (0.01)	87,32 (0,03)

2) Hisoblang va natijaning xatoligini aniqlang. $X=\dfrac{(a-b)c}{\sqrt{m+n}}$

	a	b	V
a b c m n	4.3 (0.05) 17.21 (0.02) 8.2 (0.05) 12.417 (0.003) 8.37 (0.005)	5.2 (0.004) 15.32 (0.01) 7.5 (0.05) 21.823 (0.002) 7.56 (0.003)	2.13 (0.01) 22.16 (0.003) 6.3 (0.04) 16.825 (0.004) 8.13 (0.002)

3) Raqamlarni hisoblash usulidan foydalanib hisoblang.

$$S = \frac{b^2}{18} : \frac{a^2 + 4ab + b^2}{(a+b)^2}$$

	a	b	V
a b c	1,141 3,156 1,14	2,234 4,518 4,48	5,813 1,315 2,56

№ 2.

1) Hisoblang va natijaning xatoligini aniqlang. $X = \frac{\sqrt{a} \cdot b}{c}$

	a	b	V
a b c	228,6 (±0,06) 86,4 (±0,02) 68,7 (±0,05)	315,6 (±0,05) 72,5 (±0,03) 53,8 (±0,04)	186,7 (±0,04) 66,6 (±0,02) 72,3 (=±0,03)

2) Hisoblang va natijaning xatoligini aniqlang.

$$X = \frac{m^3(a+b)}{c-d}$$

	a	b	v
a b m c d	13,5 (±0,02) 3,7 (±0,02) 4,22 (±0,004) 34,5 (±0,02) 23,725 (±0,005)	18,5 (±0,03) 5,6 (±0,02) 3,42 (±0,003) 26,3 (±0,01) 14,782 (±0,006)	11,8 (±0,02) 7,4 (±0,03) 5,82 (±0,005) 26,7 (±0,03) 11,234 (±0,004)

3) Raqamlarni hisoblash usulidan foydalanib hisoblang.

$$X = \frac{(a+b)h^3}{4} + \frac{(a+b)h}{12}$$

	a	b	v
a b h	8,53 6,275 12,48	6,44 5,323 15,44	9,05 3,244 20,18

№ 3.

1) Hisoblang va natijaning xatoligini aniqlang. $X = \frac{\sqrt{ab}}{c}$

	a	b	v
a b c	3,845(±0,004) 16,2(±0,05) 10,8 (±0,1)	4,632 (±0,003) 23,3 (±0,04) 11,3 (±0,06)	7,312 (±0,004) 18,4 (±0,03) 20,2 (±0,08)

2) Hisoblang va natijaning xatoligini aniqlang. $X = \frac{(a+b)m}{(c-d)}$

	a	b	v
a b m c d	2,754 (±0,001) 11,7 (±0,04) 0,56 (±0,005) 10,536 (±0,002) 6,32 (±0,008)	3,236 (±0,002) 15,8 (±0,03) 0,64 (±0,004) 12,415 (±0,003) 7,18 (±0,006)	4,523 (±0,003) 10,8 (±0,02) 0,85 (±0,003) 9,318 (±0,002) 4,17 (±0,004)

3) Raqamlarni hisoblash usulidan foydalanib hisoblang.

$$N = \frac{(a+b)^2}{2h} + \frac{(a^2+b^2)h}{5}$$

	a	b	V
a b h	0,562 0,2518 0,68	0,834 0,3523 0,74	0,445 0,4834 0,87

№ 4.

1) Hisoblang va natijaning xatoligini aniqlang. $X = \frac{a^2 b}{c}$

	a	b	v
a	3,456 (±0,002)	1,245 (±0,001)	0,327 (±0,005)
b	0,642 (±0,0005)	0,121 (±0,0002)	3,147 (±0,0001)
c	7,12 (±0,004)	2,34 (±0,003)	1,78 (±0,001)

3) Hisoblang va natijaning xatoligini aniqlang. $X = \frac{(a+b)m}{\sqrt{c-d}}$

	a	b	v
a	23,16 (±0,02)	17,41 (±0,01)	32,37 (±0,03)
b	8,23 (±0,005)	1,27 (±0,002)	2,35 (0,001)
c	145,5 (±0,08)	342,3 (±0,04)	128,7 (±0,02)
d	28,6 (±0,1)	11,7 (±0,1)	27,3 (±0,04)
m	0,28 (±0,006)	0,71 (±0,003)	0,93 (±0,001)

3) Raqamlarni hisoblash usulidan foydalanib hisoblang.

$$X = \frac{h}{3} \cdot S\left(1 + \frac{a}{A} + \frac{a^2}{A^2}\right)$$

	a	b	v
a	8,51	5,71	7,28
A	23,42	32,17	11,71
S	45,8	51,7	21,8
h	3,81	2,42	5,31

№ 5. 1) Hisoblang va natijaning xatoligini aniqlang. $X = \frac{ab^3}{c}$

	a	b	v
a	0,643 (±0,005)	0,142 (±0,003)	0,258 (±0,004)
b	2,17 (±0,002)	1,71 (±0,002)	3,45 (±0,001)
c	75,843 (±0,001)	3,727 (±0,001)	17,221 (±0,003)

2) Hisoblang va natijaning xatoligini aniqlang. $X = \frac{(a-b)c}{\sqrt{m+n}}$

	a	b	v
a	27,16 (±0,006)	15,71 (±0,005)	12,31 (±0,004)
b	5,03 (±0,01)	3,28 (±0,02)	1,73 (0,03)
c	3,6 (±0,02)	7,2 (±0,01)	3,7 (±0,02)
m	12,375 (±0,004)	13,752 (±0,001)	17,428 (±0,003)
n	86,2 (±0,05)	33,7 (±0,03)	41,7 (±0,01)

3) Raqamlarni hisoblash usulidan foydalanib hisoblang.

$$S = \frac{h^2}{18} \cdot \frac{a^2 + 4ab + b^2}{(a+b)^2}$$

	a	b	v
h a b	21,1 22,08 31,11	17,8 32,47 11,42	32,5 27,51 21,78

№ 6.

1) Hisoblang va natijaning xatoligini aniqlang. $X = \frac{ab}{c^2}$

	a	b	v
a b c	0,3575 (±0,0002) 2,63 (±0,01) 0,854 (±0,005)	0,1756(±0,0001) 3,71 (±0,03) 0,285 (±0,0002)	0,2731 (±0,0003) 5,12 (±0,02) 0,374 (±0,0001)

2) Hisoblang va natijaning xatoligini aniqlang. $X = \frac{a+b}{\sqrt{(c-d)m}}$

	a	b	v
a b c d m	16,342 (±0,001) 2,5 (±0,03) 38,17 (±0,002) 9,14 (±0,005) 3,6 (±0,04)	12,751 (±0,001) 3,7 (±0,02) 23,76 (±0,003) 8,12 (±0,004) 1,7 (±0,01)	31,456 (±0,002) 7,3 (0,01) 33,28 (±0,003) 6,71 (±0,001) 5,8 (±0,02)

3) Raqamlarni hisoblash usulidan foydalanib hisoblang.

$$V = \frac{1}{6}\pi h(3a^2 + h^2)$$

	a	b	v
a h	2,456 1,76	7,751 3,35	5,441 6,17

№ 7.

1) Hisoblang va natijaning xatoligini aniqlang. $V = \frac{\pi^2}{4} D d^2$

	a	b	v
π D d	3,14 54 (±0,5) 8,235 (±0,001)	3,14 72 (±0,3) 3,274 (±0,002)	3,14 31 (±0,01) 7,345 (±0,001)

2) Hisoblang va natijaning xatoligini aniqlang.

$$S = \frac{1}{64} \pi \sqrt{D^2 - d^2}$$

	a	b	v
D d π	36,5 (±0,1) 26,35 (±0,005) 3,14	41,4(±0,2) 31,75(±0,003) 3,14	52,6 (±0,01) 48,39 (±0,001) 3,14

3) Raqamlarni hisoblash usulidan foydalanib hisoblang.

$$a = c^2 \left(1 + \frac{2\beta}{c} + \frac{\gamma^2}{c^2}\right)$$

	a	b	v
c β γ	2,435 0,15 1,27	7,834 0,21 3,71	4,539 0,34 5,93

№ 8.

1) Hisoblang va natijaning xatoligini aniqlang. $= \frac{m^2 n}{c^3}$

	a	b	v
m n c	1,6531 (±0,0003) 3,78 (±0,001) 0,158 (±0,0005)	2,348(±0,002) 4,37 (±0,004) 0,235 (±0,0003)	3,804 (±0,003) 4,05 (±0,003) 0,318 (±0,0002)

2) Hisoblang va natijaning xatoligini aniqlang. $X = \frac{m\sqrt{a - b}}{c + d}$

	a	b	v
a b	9,542 (±0,001) 3,128 (±0,002)	8,357(±0,003) 2,48 (0,004)	4,218 (±0,001) 1,57 (±0,006)
m c d	2.8(±0.03) 0.172(±0.001) 5.4(±0.02)	3.17(±0.01) 1.315(±0.0004) 2.4(±0.02)	2.32(±0.02) 2.418(±0.004) 1.8(±0.01)

3) Raqamlarni hisoblash usulidan foydalanib hisoblang.

$$V = \frac{1}{15}\pi h(2D^2 + Dd + 0.75d^2)$$

	a	b	v
h *D* *d*	84.2 28.3 42.08	76 17.2 9.344	45 48.3 32.14

№9.

1) Hisoblang va natijaning xatoligini aniqlang. $X = \sqrt{\frac{cd}{b}}$

	a	b	v
c *d* *b*	0.7568(±0.0002) 21.7(±0.02) 2.65(±0.01)	0.8345(±0.0004) 13.8(±0.03) 1.84(±0.006)	0.6384(±0.0002) 32.7(±0.04) 4.88(±0.03)

2) Hisoblang va natijaning xatoligini aniqlang. $y = \frac{\sqrt[3]{a-b}}{m(n-a)}$

	a	b	v
a *b* *m* *n*	10.82(±0.03) 2.786(±0.0006) 0.28(±0.006) 14.7(±0.06)	9.37(±0.004) 3.108(±0.0003) 0.46(±0.002) 15.2(±0.04)	11.45(±0.01) 4.431(±0.002) 0.75(±0.003) 16.7(±0.05)

3) Raqamlarni hisoblash usulidan foydalanib hisoblang.

$$S = \sqrt{p(p-a)(p-b)(p-c)}\ , bunda\ p = (a+b+c)/2$$

	a	b	v
a	46.3	10.5	2.48
b	29.72	34.18	5.344
c	37.654	27.327	6.0218

№10.

1) Hisoblang va natijaning xatoligini aniqlang. $f = \frac{Qe^3}{48E}$

	a	b	v
Q	54.8($\pm$0.02)	38.5($\pm$0.01)	17.3($\pm$0.03)
e	2.45($\pm$0.01)	3.35($\pm$0.02)	5.73($\pm$0.01)
E	0.863($\pm$0.004)	0.734($\pm$0.001)	0.956($\pm$0.004)

2) Hisoblang va natijaning xatoligini aniqlang.

$$Q = \frac{(2n-1)^2(x+y)}{x-y}$$

	a	b	v
n	2.0435($\pm$0.0001)	1.1753($\pm$0.0002)	4.5681($\pm$0.0001)
x	4.2($\pm$0.05)	5.8($\pm$0.01)	6.3($\pm$0.02)
y	0.82($\pm$0.01)	0.65($\pm$0.02)	0.42($\pm$0.03)

2) Raqamlarni hisoblash usulidan foydalanib hisoblang.

$$\gamma = \frac{\alpha b - \beta}{b^2} - \frac{\beta(ab - \beta a)}{b^2(b+\beta)}$$

	a	b	v
α	5.27	7.31	3.28
β	0.0562	0.0761	0.0545
a	158.35	234.36	341.17
b	61.21	81.26	52.34

II BOB. ALGEBRAIK VA TRANSSENDENT TENGLAMALARNI TAQRIBIY YECHISH

2.1-§. ALGEBRAIK VA TRANSSENDENT TENGLAMALAR

Umumiy holda ixtiyoriy tenglamani

$$f(x) = 0 \qquad (2.1)$$

ko‘rinishda tasvirlash mumkin.

Chiziqli bo‘lmagan tenglamalar ikki sinfga ajratiladi – algebraik va transsendent.

Faqat algebraik funksiyalar (butun, ratsional, irratsional) dan iborat tenglamalar *algebraik tenglamalar* deyiladi. Algebraik tenglamalarni haqiqiy koeffitsientli n - darajali ko‘pxad ko‘rinishida ifodalash mumkin:

$$f(x) = a_0x^n + a_1x^{n-1} + \cdots + a_n = 0. \qquad (2.2)$$

Masalan, $2x^3 + 3x^2 + x = 0.$

Boshqa funksiyalar (trigonometrik, ko‘rsatgichli, logarifmik va h.k.) dan iborat tenglamalar *transsendent tenglamalar* deyiladi.

Masalan,

$$2x + \cos x - sinx = 0. \qquad (2.3)$$

Ta’rif: (2.1) tenglamani yechish deb, undagi x ning (2.2) va (2.3) larni ayniyatga aylantiradigan, ya’ni chap qismini nolga aylantiradigan qiymatlarini topishga aytiladi. Boshqacha aytganda $x = \xi$ bo‘lib,

$$f(\xi) = 0$$

bo‘lsa, ξ -tenglama ildizi buladi.

Chiziqli bo‘lmagan tenglamalarni yechish usuli ikkiga bo‘linadi: aniq (to‘g‘ri) usul va iteratsion usul. Aniq usul ildizni formula ko‘rinishda yozishga imkon beradi. Lekin hamma vaqt ham tenglamani aniq usul bilan yechib bo‘lmaydi. Shunday hollarda tenglama taqribiy yechiladi, ya’ni iteratsion usul qo‘llaniladi. (2.1) tenglamani taqribiy yechishda odatda aniqlanish sohasi kichik oraliqlarga ajratiladi va bu kichik oralaqlarda tenglamaning yechimi qidiriladi. (2.1) tenglamaning ildizlarini taqribiy hisoblash ikki qismdan iborat:

a) Ildizlarni ajratish.

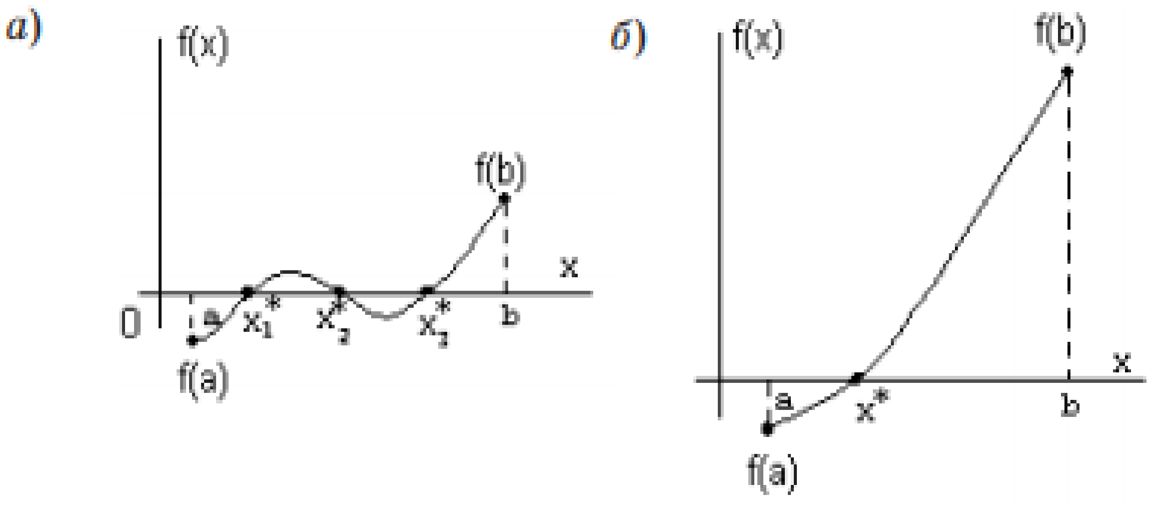
b) Dastlabki yaqinlashish ma’lum bo‘lsa, ildizlarni berilgan aniqlik bilan hisoblash.

2.2-§. TENGLAMALARNING HAQIQIY ILDIZLARINI AJRATISH

Ildizlarni ajratish grafik yoki analitik usullarda bajarilishi mumkin.

- ***Ildizlarni grafik usulda ajratish.*** Maqsadimiz (2.1) tenglamaning bittadan ildizlari joylashgan kichik oraliqlarni ajratish(topish).

$y = f(x)$ funksiyani qaraylik. Bilamizki $f(x)$ funksiya grafigining Ox o‘qi bilan kesishish nuqtalari $f(x) = 0$ tenglamaning taqribiy ildizlari bo‘ladi. $f(x) = 0$ tenglamaning aniq ildizlari uning grafik usulda topilgan x_i^* taqribiy ildizlarining biror bir atrofida yotadi. Shunday qilib, berilgan tenglamaning ildizlari ajratildi.



a) chizmada uchta ildiz ajralib turibdi, *b)* chizmada esa bitta ildiz.

Ildizlarni grafik usulda ajratish uchun (2.1) tenglamani

$$f_1(x) = f_2(x) \qquad (2.4)$$

ko‘rinishda yozib olamiz va $y = f_1(x)$; $y = f_2(x)$ funksiyalarning garafiklarini chizamiz. Bu chizmalar kesishish nuqtalarining abssissalari

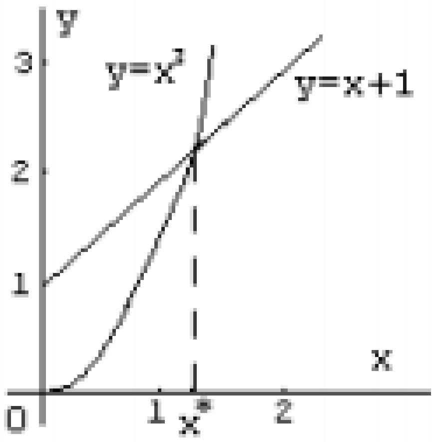
$$f(x) = f_1(x) - f_2(x) = 0$$

tenglama aniq ildizining taqribiy qiymati bo‘ladi. (2.1) tenglamani bir necha usul bilan (2.4) ko‘rinishda yozish mumkin. Bu usullarning ichidan shundayini tanlaymizki, unda $y = f_1(x)$; $y = f_2(x)$ funksiyalarning grafiklarini chizish yengil bo‘lsin.

Chizmadan (2.1) tenglamaning bitta ildizi yotadigan oraliqlarni ajratib olish kerak.

2.1-misol. $x^3 - x - 1 = 0$ tenglamaning musbat ildizlarini grafik usulda ajrating.

Yechish: Berilgan tenglamani $x^3 = x + 1$ ko‘rinishda yozamiz va $y = f_1(x) = x^3$; $y = f_2(x) = x + 1$ funksiyalarning grafiklarini chizamiz. Chizmadan ko‘rinib turibdiki berilgan tenglama $[1.0; 1.5]$ oraliqda yagona musbat ildizga ega bo‘ladi (nima uchun 1.5 nuqtani oldik, chunki $f_1(1.5) = 3.375 \neq f_2(1.5) = 2.5$). Bunda dastlabki yaqinlashish sifatida $x_0 = 1.25$ ni olsak bo‘ladi.



(2.1) tenglamaning manfiy ildizlarini topish uchun (2.1) tenglamada $Z = -x$ almashtirish bajarib, yangi $f(Z) = 0$ tenglamani hosil qilamiz. Dastlabki (2.1) tenglamaning har bir manfiy ildiziga hosil qilingan $f(Z) = 0$ tenglamaning musbat ildizi to‘g‘ri keladi. Oxirida $x^* = -Z^*$ almashtirish bajarish kerak.

- ***Ildizlarni analitik usullarda ajratish***

Quyidagi teoremalarni eslaylik (Matematik analiz kursidan ma’lum).

2.1-teorema (Bolsano Koshi). Agar uzluksiz $f(x)$ funksiya biror $[a, b]$ oraliqning chetki nuqtalarida har xil ishorali qiymatlarni qabul qilsa ($ya'ni\ f(a) \cdot f(b) < 0$), u vaqtda bu oraliqda (2.1) tenglamaning hech bo'lmaganda bitta ildizi mavjuddir.

2.2-teorema. $f(x)$ funksiya $[a, b]$ oraliqda analitik (*ya'ni shu oraliqning har qaysi nuqtasi atrofida yaqinlashuvchi darajali qatorga yoyilsa*) funksiya bo'lsin. Agar $[a, b]$ oraliqning chetki nuqtalarida har xil ishorali qiymatlarni qabul qilsa, u vaqtda (2.1) tenglamaning $a\ va\ b$ nuqtalar orasida yotadigan ildizlarining soni toqdir.

2.3-teorema. Faraz qilaylik $f \in C^1[a; b]$ bo'lsin, agar $f'(x)$ hosila $[a, b]$ kesmada o'z ishorasini saqlasa, u holda (2.1) tenglama $[a, b]$ oraliqda yagona ildizga ega bo'lish uchun $f(a) \cdot f(x) < 0$ shart bajarilishi zarur va yetarlidir.

Agar $f(x)$ funksiya $[a, b]$ oraliqning chetki nuqtalarida bir xil ishorali qiymatlarni qabul qilsa, u vaqtda (2.1) tenglamaning ildizlari yo $[a, b]$ oraliqda yotmaydi yoki ularning soni juftdir.

Algebraik

$$f(x) \equiv a_0 x^n + a_1 x^{n-1} + \cdots + a_{n-1} x + a_n = 0 \qquad (2.5)$$

tenglamaning ildizlarini ajratish talab etilsin $\left(a_j - haqiqiy, a_0 \neq 0,\ a_n \neq 0\right)$.

2.4-teorema. Agar

$$A = \max_{1 \le k \le n} \left|\frac{a_k}{a_0}\right|, \qquad A_1 = \max_{1 \le k \le n-1} \left|\frac{a_k}{a_n}\right|$$

bo'lsa, u holda (2.5) tenglamaning barcha ildizlari

$$r = \frac{1}{1 + A_1} < |x| < 1 + A = R$$

xalqa ichida yotadi.

r va R sonlar (2.5) tenglama musbat ildizlarining quyi va yuqori chegaralari bo'ladi. Manfiy ildizlarining quyi va yuqori chegaralari esa mos ravishda – R va – r sonlar bo'ladi.

2.2-misol: Quyidagi tenglama haqiqiy ildizlarining chegarasi topilsin.

$$f(x) = 2x^4 - 2{,}5x^2 + 8x - 8 = 0 \qquad (2.6)$$

Yechish: 3-teoremadan foydalanamiz, bu yerda $a_0 = 2, A = 4, A_1 = 1.$

Bundan R= $1+4=5$, $r=\frac{1}{1+1}=0{,}5$. Demak, berilgan tenglamaning ildizlari $(0{,}5, 5)$ va $(-5, -0{,}5)$ oraliqda yotar ekan.

2.5-teorema . (Lagranj teoremasi)**.** Agar (2.5) tenglamaning manfiy koeffitsientlaridan eng birinchisi (chapdan o‘ngga) a_k bo‘lib, B manfiy koeffitsientlarning absolyut qiymatlari bo‘yicha eng kattasi bo‘lsa, u holda musbat ildizlarning yuqori chegarasi

$$R=1+\sqrt[k]{\frac{B}{a_0}} \qquad (2.7)$$

son bilan ifodalanadi.

2.3-misol: Yuqoridagi 2.2-misolni Lagranj teoremasidan foydalanib yeching.

Yechish: Teorema 4 dan foydalanamiz. $f(x)=2x^4-2{,}5x^2+8x-8=0$,bu yerda $a_0=2,\ k=2, B=8.$ Bularni (2.7) formulaga qo‘yib, musbat ildizlarning yuqori chegarasi uchun

$$R=1+\sqrt{\frac{8}{2}}=1+2=3$$

ni hosil qilamiz. Keyin (2.6) tenglamada x ni $-x$ bilan almashtirsak,

$$f(x)=2x^4-2{,}5x^2-8x-8=0$$

tenglama hosil bo‘ladi. Bu tenglama musbat ildizlarining yuqori chegarasi uchun ham $R=3$ tengsizlik kelib chiqadi. YA’ni Lagranj teoremasiga ko‘ra (2.1.2) tenglamaning ildizlari $(-3, 3)$ oraliqda joylashgan ekan.

2.6-teorema (Nyuton teoremasi)**.** Agar $x=c>0$ uchun $f(x)$ ko‘phad va uning barcha $f'(x), f'(x), \dots, f^{(n)}(x)$ hosilalari nomanfiy bo‘lsa, u holda $R=c$ ni (2.5) tenglamaning musbat ildizlar uchun yuqori chegara deb hisoblash mumkin.

2.4-misol: Yuqoridagi 2.2-misolni Nyuton teoremasidan foydalanib yeching.

Yechish: Teorema 5 dan foydalanamiz. Bu yerda

$$f(x)=2x^4-2{,}5x^2+8x-8,$$
$$f'(x)=8x^3-5x+8,$$

$$f''(x) = 24x^2 - 5,$$
$$f'''^{(x)} = 48x,$$
$$f^{IV}(x) = 48.$$

Ko‘rinib turibdiki, $x > 1{,}5$ uchun $f(x) > 0, f'(x) > 0,\ f''(x) > 0,\ \ f'''(x) > 0,\ f^{IV}(x) > 0.$

Demak, $c = 1{,}5$ musbat ildizlarining yuqori chegarasi ekan. Endi (2.6) tenglamadagi x ni $-x$ bilan almashtirib $f_1(-x) = 2x^4 - 2{,}5x^2 - -8x - 8 = 0$ tenglamani hosil qilamiz. Yuqoridagiga o‘xshash hisoblashlardan keyin ko‘rinadiki $f_1(x) = 0$ tenglama musbat ildizlarining yuqori chegarasi $c = 2{,}5$ ekan. Demak, (2.1.2) tenglamaning ildizlari $(-2{,}5\ , 1{,}5)$ oraliqda yotar ekan.

2.1-, 2.2-, 2.3-misollarning natijalarini solishtirsak, Nyuton metodi ildizlar chegaralari uchun yaxshiroq natija berishi ko‘rinadi.

2.3-§. TENGLAMA ILDIZLARINING SONINI ANIQLASH

2.7-teorema (Dekart teoremasi)**.** (2.5) tenglama koeffitsienglaridan tuzilgan sistemada ishora almashishlar soni qancha bo‘lsa (sanashda nolga teng koeffitsienglarga e’tibor berilmaydi) , tenglamaning shuncha musbat ildizlari mavjud yoki musbat ildizlar soni ishora almashishlar sonidan juft songa kam bo‘ladi.

2.5-misol: $f(x) = 2x^4 - 2{,}5x^2 + 8x - 8 = 0$ tenglamaning ildizlar sonini toping.

Yechish: $f(x)$ ning koeffitsientlar 2, 0, -2,5, 8, -8 sonlardan tuzilgan sistemada ishora almashishlar soni 3 ta. Demak, mazkur tenglamada musbat ildizlarning soni 3 ta yoki 1 ta. Tenglamadagi x ni $-x$ bilan almashtirgandagi

$f_1(x) = 2x^4 - 2{,}5x^2 - 8x - 8 = 0$ tenglamada koeffitsientlar 2, 0, -2,5, -8, -8 sonlardan tuzilgan sistemada ishora almashishlar soni 1 ta. Demak berilgan tenglama 1 ta manfiy ildizga ega.

Faraz qilaylik, (2.5) tenglama karrali ildizga ega bo‘lmasin. $f_1(x)$ orqali $f'(x)$ hosilani, $f_2(x)$ orqali $f(x)$ ni $f_1(x)$ ga bo‘lganda hosil bo‘ladigan qoldiqning teskari ishora bilan olinganini, $f_3(x)$ orqali $f_1(x)$

ni $f_2(x)$ ga bo‘lganda hosil bo‘ladigan qoldiqning teskari ishora bilan olinganini, va h.k. belgilaymiz va bu jarayonni qoldiqda o‘zgarmas son hosil bo‘lguncha davom ettiramiz. Natijada *Shturm qatori* deb ataluvchi

$$f(x),\ f_1(x),\ f_2(x), \dots,\ f_n(x)$$

funksiyalar ketma-ketligiga ega bo‘lamiz.

2.8-teorema (Shturm teoremasi)**.** $f(x)$ ko‘phadning ildizlaridan farqli a va b $(a < b)$ sonlarni olib, x ni a dan b gacha o‘zgartirganda $f(x)$ uchun tuzilgan Shturm qatorida nechta ishora almashinishlar yo‘qolsa, $f(x) = 0$ ning (a, b) oraliqda xuddi shuncha haqiqiy ildizlari mavjud bo‘ladi.

2.6-misol: Shturm metodi yordamida

$$f(x) = 2x^4 - 2{,}5x^2 + 8x - 8 = 0$$

tenglamaning ildizlari ajratilsin.

Yechish: Shturm qatorini tuzamiz: $f(x) = 2x^4 - 2{,}5x^2 + 8x - 8$; $f_1(x) = f'(x) = 8x^3 - 5x + 8$. Endi $f(x)$ ni $f_1(x)$ ga bo‘lamiz.

$$\begin{array}{l|l} \;\;2x^4 - 2{,}5x^2 + 8x - 8 & \underline{8x^3 - 5x + 8} \\ \underline{-2x^4 + 1{,}25x^2 - 2x} & -0{,}25x + 8 \\ -1{,}25x^2 + 6x - 8 & \end{array}$$

Qoldiqni 1,25 soniga bo‘lami va $-x^2 + 4{,}8x - 6{,}4$ ni hosil qilamiz. Demak, $f_2(x) = x^2 - 4{,}8x + 6{,}4$. Endi $f_1(x)$ ni $f_2(x)$ ga bo‘lamiz.

$$\begin{array}{l|l} \;\;8x^3 - 5x + 8 & \underline{x^2 - 4{,}8x + 6{,}4} \\ \underline{-8x^3 + 38{,}4x^2 - 51{,}2x} & -8x - 38{,}4 \\ \;\;38{,}4x^2 - 56{,}2x + 8 & \\ \;\;\underline{-38{,}4x^2 + 184{,}32x - 245{,}76} & \\ \;\;\;\;\underline{128{,}12x - 237{,}76} & \end{array}$$

Qoldiqni 128,12 soniga bo‘lami va $x - 1{,}9$ ni hosil qilamiz. Demak, $f_3(x) = -x + 1{,}9$. Endi $f_2(x)$ ni $f_3(x)$ ga bo‘lamiz.

$$\begin{array}{l|l} x^2 - 4{,}8x + 6{,}4 & \underline{-x + 1{,}9} \\ \;\;\underline{-x^2 + 1{,}9x} & x - 2{,}9 \\ -2{,}9x + 6{,}4 & \\ \underline{2{,}9\,x - 5{,}51} & \\ \;\;\;0{,}89 & \end{array}$$

Hosil bo‘lgan qoldiqni 0,89 ga bo‘lib, teskari ishorasi bilan olsak $f_4(x) = -1$ kelib chiqadi.

Shunday qilib, Shturm qatorining elementlari quyidagi funksiyalardan iborat:

$f(x) = 2x^4 - 2{,}5x^2 + 8x - 8$; $f_1(x) = 8x^3 - 5x + 8$; $f_2(x) = x^2 - 4{,}8x + 6{,}4$; $f_3(x) = -x + 1{,}9$; $f_4(x) = -1$. Endi hosil qilingan Shturm qatoridagi ishora almashinishlar jadvalini tuzamiz.

X ning qiymatlari	$-\infty$	-3	-1	0	1	2	$+\infty$
sign $f(x)$	+	+	-	-	-	+	+
sign $f_1(x)$	-	-	+	+	+	+	+
sign $f_2(x)$	+	+	+	+	+	+	+
sign $f_3(x)$	+	+	+	+	+	-	-
sign $f_4(x)$	-	-	-	-	-	-	-
ishora almashinishlar soni	3	3	2	2	2	1	1

2-ustun va oxirgi ustunlarga qarasangiz ishora almashinishlar soni ikkitaga kamaygan demak, $(-\infty, +\infty)$ oraliqda berilgan

$$2x^4 - 2{,}5x^2 + 8x - 8 = 0$$

tenglama ikkita ildizga ega. E’tiborni 3-ustun va 4-ustunlar hamda 6-ustun va 7-ustunlarga qaratsangiz ishora almashinishlar soni bittaga kamaygan. Demak, (-3, -1) oraliqda bitta manfiy ildiz va (1; 2) oraliqda bitta musbat ildiz bor ekan.

2.4-§. TENGLAMALARNING HAQIQIY ILDIZLARINI TOPISH

- **Kesmani teng ikkiga bo‘lish usuli** Algebraik yoki transsendent tenglamalarning haqiqiy ildizlarini taqribiy topishning bir nechta usullari mavjud. Quyida shu usullar to‘g‘risida gaplashamiz.

Bulardan biri *dixotomiya* usuli deyiladi, ya’ni kesmani teng ikkiga bo‘lish usuli. (Dixotomiya termini grekchadan olingan bo‘lib,

$\delta\iota\chi o\tau o\mu\iota\alpha$: $\delta\iota\chi\eta$ ikkiga + $\tau o\mu\eta$ "$bo‘lish$" , ikkiga bo‘lish degan manoni bildiradi.)

Dixotomiya, ya’ni kesmani teng ikkiga bo‘lish usulidan foydalanish uchun oldin $f(x) = 0$ tenglamaning ildizlari biror (grafik yoki analitik) usul bilan ajratib olingan bo‘lish kerak.

Berilgan tenglamadagi $f(x)$ funksiya $[a, b]$ intervalda uzluksiz bo‘lsin, $[a, b]$ intervalda (2.1) tenglamaning yagona x^* ildizi joylashgan bo‘lsin. Bu ildizni topish uchun

$$\xi_1 = \frac{a+b}{2}$$

nuqta bilan $[a, b]$ kesmani teng ikkiga bo‘lamiz. Agar $f(\xi_1) = 0$ bo‘lsa, ξ_1 tenglamaning ildizi bo‘ladi. Aks holda hosil bo‘lgan $[a, \xi_1]$ yoki $[\xi_1, b]$ kesmalardan biri tanlab olinadi. Bu kesmalar qiyidagicha tanlanadi: kesmalarning uchlarida $f(x)$ funksiyaning ishorasi tekshiriladi, qaysi kesmaning uchlarida $f(x)$ funksiyaning ishorasi turli xil bo‘lsa, o‘sha kesma tanlab olinadi. Yuqorida aytilganlarga ko‘ra, tanlab olingan kesmada (2.1) tenglamaning ildizi yotadi. Shunday qilib, tanlangan $[a_1, b_1]$ interval (bu erda $a_1 = a, b_1 = \xi_1$ yoki $a_1 = \xi_1$, $b_1 = b$) dastlabki $[a, b]$ intervalning yarmiga teng va x^* ildizini o‘z ichiga oladi. Yuqoridagi ishlarni birma-bir bajaramiz, ya’ni $[a_1, b_1]$ intervalni ikkiga bo‘lib,

$$\xi_2 = \frac{a_1+b_1}{2}$$

nuqtani topamiz va $[a_1, \xi_2]$ $yoki$ $[\xi_2, b_1]$ kesmalarni hosil qilamiz va hokazo. Natijada, kichrayib boruvchi $[a_n, b_n]$ intervallar ketma-ketligi hosil bo‘ladi va uning uzunligi $a_n - b_n = \frac{1}{2^n}(b-a)$ga teng bo‘ladi. $[a_n, b_n]$ intervallarda (2.5) $tenglamaning\, x^*$ ildizi joylashgan bo‘ladi. x^* ildizni hisoblash aniqligi $[a, b]$ intervalni n marta bo‘lishdan hosil bo‘lgan $[a_n, b_n]$ intervalning o‘lchami bilan aniqlanadi. x^* ildizni hisoblash xatoligi $[a_n, b_n]$ intervalning o‘lchamidan oshib ketmaydi.

Demak, agar x^* ildizni hisoblash aniqligi ε bo‘lsa, u holda

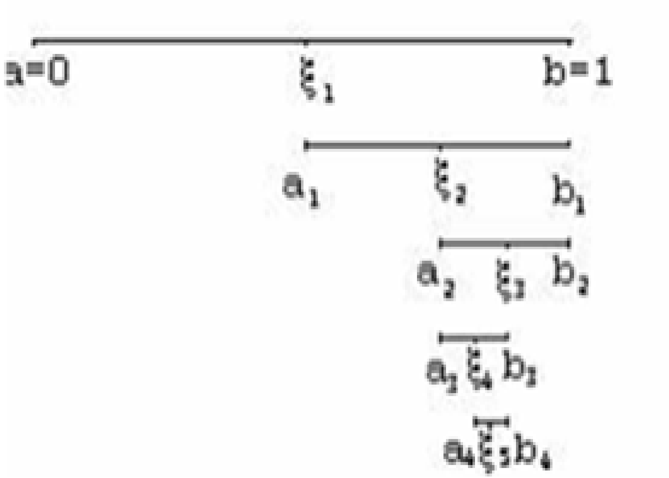
$$\frac{1}{2^n}(b-a) \le \varepsilon$$

shart albatta bajarilishi kerak. Bundan ko‘rinadiki, agar ε berilgan bo‘lsa, $[a,b]$ kesmani ikkiga bo‘lishlar soni n ni hisoblash mumkin:

$$n \geq \frac{lg\frac{b-a}{\varepsilon}}{lg2} \approx 3.32lg\frac{b-a}{\varepsilon}.$$

2.7-misol. $f(x) = 2x^4 - 2{,}5x^2 + 8x - 8 = 0$ tenglamaning $[1, 2]$ kesmada yotuvchi ildizlarini kesmani teng ikkiga bo‘lish metodi bilan toping. To‘rt qadamgacha aniqlansin.

Yechish: Bizda $a = 1, b = 2$. $f(1) = -0{,}5, f(2) = 30$. $f(1) \cdot f(2) < 0$ demak, $[1, 2]$ kesmada ildiz bor.



$$\xi_1 = \frac{1+2}{2} = 1.5$$

$f(1{,}5) = 2 \cdot (1{,}5)^4 - 2{,}5 \cdot (1{,}5)^2 + 8 \cdot 1{,}5 - 8$ =8,5. $f(1{,}5) \cdot f(1) < 0$, $f(1{,}5) \cdot f(2) > 0$. Demak, ildiz $[1, 1{,}5]$ kesmada. $[1, 1{,}5]$ kesmani teng ikkiga bo‘lamiz.

$$\xi_2 = \frac{1+1{,}5}{2} = 1{,}25.$$

$f(1{,}25) = 2 \cdot (1{,}25)^4 - 2{,}5 \cdot (1{,}25)^2 + 8 \cdot 1{,}25 - 8 = 2{,}98$. $f(1{,}25) \cdot f(1) < 0$,

$f(1{,}5) \cdot f(1{,}25) > 0$. Demak, ildiz $[1, 1{,}25]$ kesmada. $[1, 1{,}25]$ kesmani teng ikkiga bo‘lamiz.

$$\xi_3 = \frac{1+1{,}25}{2} = 1{,}125$$

$f(1{,}125) = 2 \cdot (1{,}125)^4 - 2{,}5 \cdot (1{,}125)^2 + 8 \cdot 1{,}125 - 8 = 1{,}04$. $f(1{,}125) \cdot f(1) < 0$,

$f(1{,}125) \cdot f(1{,}25) > 0$. Demak, ildiz $[1, 1{,}125]$ kesmada. $[1, 1{,}125]$ kesmani teng ikkiga bo‘lamiz.

$$\xi_4 = \frac{1 + 1{,}125}{2} = 1.0625$$

$f(1{,}0625) = 2 \cdot (1{,}0625)^4 - 2{,}5 \cdot (1{,}0625)^2 + 8 \cdot 10625 - 8 = 0{,}23.$ $f(1{,}0625) \cdot f(1) < 0,$

$f(1{,}125) \cdot f(1{,}0625) > 0$. Demak, ildiz $[1, 1{,}0625]$ kesmada. $[1, 1{,}0625]$ kesmani teng ikkiga bo‘lamiz.

$$\xi_5 = \frac{1 + 1{,}0625}{2} = 1{,}03125$$

$f(1{,}03125) = 2 \cdot (1{,}03125)^4 - 2{,}5 \cdot (1{,}03125)^2 + 8 \cdot 103125 - 8 = \quad = -0{,}15.$

$f(1{,}03125) \cdot f(1) > 0,$

$f(1{,}03125) \cdot f(1{,}0625) < 0$. Demak, ildiz $[1{,}03125, 1{,}0625]$ kesmada. $[1{,}03125, 1{,}0625]$ kesmani teng ikkiga bo‘lamiz.

$$\xi_6 = \frac{1{,}03125 + 1{,}0625}{2} = 1{,}046875$$

$f(1{,}046875) = 0{,}04$. $f(1{,}03125) \cdot f(1{,}046875) < 0$. Demak, ildiz $[1{,}03125, 1{,}046875]$ kesmada. Ya’ni $a_6 = 1{,}03125$, $b_6 = \xi_6 = 1{,}046875$. Izlanayotgan ildizni x^* deylik.

Dastlabki $[1, 2]$ kesmani oltinchi bo‘lishdan keyin ma’lum bo‘ldiki, izlanayotgan x^* ildiz

$1{,}03125 < x^* < 1{,}046875$ shartni bajaradi va uni hisoblashdagi ε maksimal xatolik

$$\varepsilon < 1{,}046875 - 1{,}03125 = 0{,}0156$$

Xatolikni yana ham kamaytirish uchun

$$x^* = \frac{1{,}046875 + 1{,}03125}{2} = 1{,}0390625$$

deb olsak bo‘ladi.

- **Oddiy iteratsiya metod.** (2.1) tenglamani bu metod bilan yechish uchun uni $x = \varphi(x)$ ko‘rinishda yozib olish kerak bo‘ladi (*Masalan:* $\sin x - 3x + 2 = 0$ tenglama berilgan bo‘lsa, uni $x = \frac{2+\sin x}{3}$ ko‘rinishda yozish mumkin). Izlanayotgan x^* ildizning dastlabki taqribiy qiymati x_0 biror usul bilan aniqlangan bo‘lsin. Endi

$$x_{n+1} = \varphi(x_n) \qquad (n = 0,1,2, \dots) \qquad (2.8)$$

ifodadan foydalanib, $x_1, x_2, \dots$ sonli ketma ketlikni tuzamiz.

Agar (2.8) ketma ketlikning

$$\lim_{n\to\infty} x_n = \xi$$

limiti mavjud bo‘lib, $\varphi(x)$ funksiya uzluksiz bo‘lsa, u holda (2.8) dan

$$\xi = \lim_{n\to\infty} x_n = \lim_{n\to\infty} \varphi(x_n) = \varphi\left(\lim_{n\to\infty} x_n\right) = \varphi\left(\lim_{n\to\infty} x_{n+1}\right).$$

Shunday qilib, $\lim_{n\to\infty} x_n = \varphi\left(\lim_{n\to\infty} x_n\right)$ dan ko‘rinadiki $\lim_{n\to\infty} x_n$ limit (2.8) tenglamaning ildizi bo‘ladi. Va demak, dastlabki berilgan (2. 1) tenglamaning ham ildizi bo‘ladi.

Agar $[a, b]$ kesmada

$$|\varphi'(x)| \le m < 1 \tag{2.9}$$

shart bajarilsa, u holda n –chi yaqinlashish xatoligi

$$\varepsilon = |x_n - x^*| \le \frac{m^*}{1-m}|x_0 - \varphi(x_0)| \tag{2.10}$$

tengsizlik bilan baholanishi mumkin va $[a, b]$ kesmadan olingan ixtiyoriy x_0 uchun ketma ket yaqinlashish x^* ga intiladi. Bu iteratsion protses

$$|x_{n+1} - x_n| \le \varepsilon$$

shart bajarilganda to‘xtaydi.

Oddiy iteratsiya usuli birinchi tartibli tezlik bilan yaqinlashuvchi uslublar sinfiga kiradi.

2.8-misol. Ushbu $x^4 - 5x^2 + 8x - 8 = 0$ tenglamaning ildizlari iteratsiya usuli bilan, $\varepsilon = 0{,}5 \cdot 10^{-3}$ aniqlikda topilsin.

Echish: Tenglamaning ildizlari (-3, -2,9) va (1,5; 2) oraliqlarda yotadi (1-misolga o‘xshash topilgan). Tenglamani turlicha $x = \varphi(x)$ kanonik ko‘rinishda yozish mumkin:

$$x = -x^4 + 5x^2 - 7x + 8,$$
$$x = (-x^4 + 5x^2 + 8)/8,$$
$$x = \sqrt{(\,x^4 + 8x - 8)/5}$$

va hokazo. Ularning ichidan biz qarayotgan oraliqlarda (2.9) shartni bajariladiganini olishimiz kerak. Jumladan, (-3, -2,9) oraliqda $|((-\,x^4 + 5x^2 + 8)/8)'| > 1$, ya’ni iteratsion jarayon uzoqlashadi, (1,5; 2) da esa oraliqning chap qismidagina $|\varphi'(x)| < 1$ shart bajariladi. Usulning qo‘llanish mumkin bo‘lgan chegaralarini aniqlashtirish maqsadida $\varepsilon = 0{,}75$ bo‘lsin deb olamiz va

$$|\varphi'(x)| = \left|\frac{-2x^3 + 5x}{4}\right| \le 0{,}75$$

tengsizlikni tuzamiz. Uning yechimi $[-1,8229;\ -1]$, $[-0,8229;\ 0,8]$ va $[1;\ 1,8229]$ lardan iborat. Bu oraliqlar va ildizlar yotgan oraliqlarning umumiy qismi $[1,5;\ 1,8229]$ bo‘ladi va shu oraliqa nisbatan iteratsiya usulini qo‘llaymiz. Boshlang‘ich yaqinlashish $x_0 = 1,7$ bo‘lsin. ε aniqlikka erishish uchun zarur bo‘ladigan iteratsiya qadamlari soni n ni (2.10) munosabatdan foydalanib aniqlaymiz:

$$\frac{\varphi(1,7) - 1,7}{1 - 0,75} \cdot 0,75^n \leq 0,5 \cdot 10^{-3} \text{ yoki } 0,75^n \leq \frac{0,5 \cdot 10^3}{0,0622} \cdot 0,25 \approx 0,002,$$

bundan $n \geq 22$. Hisoblashlarni (2.8) munosobat bo‘yicha kompyuterda bajarib natijada $x_{22} = 1,7444102163572999$ ni olamiz yoki ko‘rsatilgan aniqlikkacha yaxlitlansa $x \approx 1,744$.

2.9-misol. $\sin x - 2x + 0,5 = 0$ tenglamaning $\left[0,\ {}^{\pi}/_{2}\right]$ kesmada joylashgan ildizini $\varepsilon \leq 0,0001$ aniqlikda toping.

Yechish: Tenglamani unga teng kuchli bo‘lgan tenglik bilan almashtiramiz. Uholda

$$x = 0.25 + 0.5 \sin x = \varphi(x),$$
$$\varphi'(x) = 0.5 \cos x$$

va ixtiyoriy x lar uchun, shu jumladan $\left[0,\ {}^{\pi}/_{2}\right]$ kesma nuqtalari uchun ham

$$|\varphi'(x)| \leq 0.5 < 1$$

tengsizlik bajariladi. Ko‘rinib turibdiki yaqinlashish sharti bajarilayapdi.

(2.8) formula

$$x_{n+1} = 0.25 + 0.5 \sin x_n \qquad (n = 0, 1, \quad 2, \dots).$$

Ko‘rilayotgan intervalning $x_0 = 0.5$ nuqtasidan boshlab ketma-ket quyidagilarni topamiz:

$x_1 = 0.25 + 0.5 \sin 0.5 = 0.4897, x_2 = 0.25 + 0.5 \sin 0.4897 = 0.4852,$
$x_3 = 0.25 + 0.5 \sin 0.4852 = 0.4832, x_4 = 0.25 + 0.5 \sin 0.4832 = 0.4823,$
$x_5 = 0.25 + 0.5 \sin 0.4823 = 0.4819,\ x_6 = 0.25 + 0.5 \sin 0.4819 = 0.48175,$
$x_7 = 0.25 + 0.5 \sin 0.48175 = 0.48165, x_8 = 0.25 + 0.5 \sin 0.48165 = 0.4816,$

$$x_8 \approx x_9 \approx x_{10} \approx j.$$

Keyingi yaqinlashishlar

$$|x_{n+1} - x_n| \leq \varepsilon = 0.0001$$

shartni bajaradi va $x^* \approx 0.4816$. Uni izlanayotgan ildizning qiymati sifatida olish mumkin.

- **Vestgeyn usuli.** Oddiy iteratsiya usulida $\varphi(x)$ ni iteratsiya jarayonini yaqinlashtiruvchi qilib tanlash yengil ish emas, Shu jihatdan Vestgeyn usuli qulayroq: u $\varphi'(x)$ ning ixtiyoriy qiymatida qo'llanilishi mumkin, $|\varphi'(x)| < 1$ da esa Vestgeyn jarayoni oddiy iteratsiya jarayoniga nisbatan tezroq yaqinlashadi. Bu usulni qo'llashda oldin oddiy iteratsiya bo'yicha $x_1 = \varphi(x_0)$ topiladi, so'ng $z_0 = x_0\,, z_1 = x_1$ deb olinadi. Keyingi yaqinlashishlar $x_{n+1} = \varphi(z_n)$ formula bo'yicha ketma-ket topiladi, bunda $z_n = qz_{n-1} + (1-q)x_n$ yoki

$$z_n = x_n - q(x_n - z_{n-1}), \qquad n = 1,2,\ldots \tag{2.11}$$

va har qadamda q ning qiymatini hisoblab turiladi:

$$q \approx \frac{(x_{n+1} - x_n)}{\big((x_{n+1} - x_n) + (z_{n+1} - z_n)\big)} \tag{2.12}$$

2.10-misol. Vestgey usuli qo'llanilib $2x^4 - 2.5x^2 + 8x - 8 = 0$ tenglamaning ildizlari $\varepsilon = 0.005$ aniqlikda topilsin.

Yechish: $x = (-2x^4 + 2.5x^2 + 8)/8$ va $x_0 = 1.7$ (chunki 3-misoldan ma'lumki (1; 2) oraliqda bitta musbat ildiz bor) bo'lsin. x_1 ni oddiy iteratsiya bo'yicha topamiz. Qolgan hisoblashlar (2.2.5), (2.2.6) munosabatlar bo'yicha bajariladi (jadvalga qarang):

n	x_n	z_n	q_n	ε
0	1,7		0,5	
1	-0,1849	-0,09245	0,953245505	1,8849
2	1,002652675	-0,041249028	0,958667349	1,187552675
3	1,000530989	0,001810502	-0,051827009	0,002121686
4	1,000001024	1,051734254	-0,00050502	0,000529965
5	1,039781137	1,0397751	1,429861315	0,039780112
6	1,045642091	1,037253108	1,755322311	0,005860954

Demak, taqribiy ildiz $x^* = 1{,}045642091$ deb olsak bo'ladi.

- **Nyuton usuli (urinmalar usuli).** Faraz qilaylik $f(x)$ funksiya $[a,b]$ kesmaning chetki nuqtalarida har xil ishoraga ega bo'lib, $f'(x)$, $f''(x)$ hosilalar mavjud va uzluksiz va $[a,b]$ da ishorasini saqlasin. $f(x) = 0$ tenglama $[a,b]$ da yagona ildizga ega bo‘lsin. $x_0 \in (a, b)$ bo‘lsin. Lagranj formulasidan foydalanib $f(x)$ funksiyani biror chiziqli funksiya bilan almashtiramiz:

$$f(x) = f(x_0) + f'(\xi)(x - x_0),$$

bu yerda $x < \xi < x_0$ yoki $x_0 < \xi < x$.

ξ noma’lum bo‘lgani uchun, ξ ning o‘rniga x_0 ni olamiz:

$$f(x) \approx f(x_0) + f'(x_0)(x - x_0).$$

Oxirgi munasobatni hisobga olib, ko‘rilayotgan $f(x) = 0$ tenglama o‘rniga $f(x_0) + f'(x_0)(x - x_0) = 0$ tenglama bilan almashtiramiz va uning

$$x_1 = -\frac{1}{f'(x_0)} f(x_0) \qquad (2.13)$$

ildizini $f(x) = 0$ tenglama ildizini taqribiy hisoblashning birinchi yaqinlashishi sifatida olamiz.

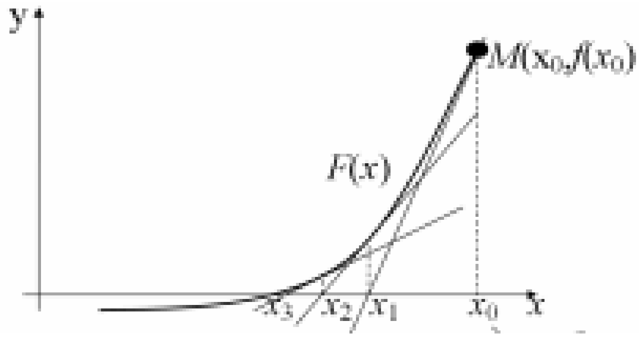
(2.13) ni umumlashtirib

$$x_n = x_{n-1} - \frac{1}{f'(x_{n-1})} f(x_{n-1}) \qquad (2.14)$$

ni olamiz.

Ko‘pincha Nyuton usulidan foydalanishda, dastlabki yaqinlashish x_0 sifatida (a, b) oraliqning $f(x)f''(x) > 0$ shart bajariladigan chekkasi olinadi va keyingi yaqinlashishlar (2.14) formula bilan aniqlanadi.

Nyuton usulining geometrik tahlili: Dastlabki yaqinlashish sifatida $x_0 \in (a, b)$ nuqtani olamiz. M$(x_0, f(x_0)$) nuqtadan $f(x)$ funksiya grafigiga urinma



o‘tkazamiz. Urinmaning Ox o‘qi bilan kesishish nuqtasini keyingi yaqinlashish sifatida olamiz, ya’ni x_1 deb olamiz. Keyin $M(x_1, f(x_1))$ nuqtadan urinma o‘tkazib, x_2 ni topamiz va hokoza.

Nyuton metodining yaqinlashishi haqida teoremalar.

2.9-teorema. Agar $f(x)$ va dastlabki yaqinlash x_0 quyidagi shartlarni qanoatlantirsin:

1. $f'(x_0) \neq 0$ va $\frac{1}{|f'(x_0)|} \leq B$;
2. $\left|\frac{f(x_0)}{f'(x_0)}\right| \leq \eta$ tengsizlik o‘rinli;
3. $f(x)$ funksiya $|x - x_0| \leq \delta$ oraliqda ikkinchi tartibli uzluksiz $f''(x_0)$ hosilaga ega va bu oraliqning barcha nuqtalarida $f''(x_0) \leq K$ bo‘lsa;
4. B, K, η sonlar uchun $h = BK\eta \leq \frac{1}{2}$ shart bajarilsa;
5. hamda

$$\frac{1 - \sqrt{1 - 2\,h}}{n}\,\eta \leq \delta$$

tengsizlik o‘rinli bo‘lsa, u holda:

1. $f(x) = 0$ tenglama $|x - x_0| \leq \delta$ oraliqda ξ yechimga ega bo‘ladi;
2. $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$ $\quad (n = 0, 1, 2, \ \dots)$ ketma-ketlik yaqinlashishlarni qurish mumkin va ular ξ yechimga yaqinlashadi:

$$\lim_{n \to \infty} x_n = \xi \; ;$$

3. Yaqinlashish tezligi uchun

$$|\xi - x_n| \leq t^* - t_n$$

baho o‘rinli bo‘lib, bu yerda t_n esa

$$P(t) = \frac{K}{2} t^2 - \frac{t}{B} + \frac{\eta}{B} = 0$$

kvadrat tenglamaning kichik ildizi t^* uchun $t_0 = 0$ dan boshlab qurilgan Nyuton ketma-ketligining $n -$ elementidir:

$$t_n = t_n - \frac{P(t_n)}{P'(t_n)}\,.$$

2.10-teorema. Agar 2.9-teoremaning shartlari bajarilsa, $\xi - x_n$ ayirma uchun

$$|\xi - x_n| \leq \frac{1}{2^{n-1}} (2h)^{2^{n-1}} \eta$$

baho o‘rinli bo‘ladi.

- **Nyuton usulining modifikatsiyasi**

1) *O‘zgarmas qadamli ayirmalar usuli.* $f(x)=0$ tenglamani Nyuton usuli bilan taqribiy yechish jarayonida $f'(x)$ ni hisoblashga to‘g‘ri keladi. Lekin murakkab funksiyalar uchun $f'(x)$ ni hisoblash juda katta mehnat talab qiladi, shuning uchun $f'(x)$ ning o‘rniga

$$f'(x)=\lim_{h\to 0}\frac{f(x_n+h)-f(x_n)}{h}$$

tenglikning o‘ng tomonidagi ifodani qo‘ysak, (2.14) tenglik

$$x_n=x_{n-1}-\frac{f(x_{n-1})\cdot h}{f(x_{n-1}+h)-f(x_{n-1})} \qquad (2.15)$$

ko‘rinishga o‘tadi.

2) *O‘zgaruvchi qadamli ayirmalar usuli***.** Har bir iteratsiyada h qadamni o‘zgartirib turilsa, $h_1, h_2, \ldots$ larga ega bo‘lamiz. U holda (2.15) quyidagi ko‘rinishga o‘tadi:

$$x_n=x_{n-1}-\frac{f(x_{n-1})\cdot h_k}{f(x_{n-1}+h_k)-f(x_{n-1})}, \quad n\neq k. \qquad (2.16)$$

Bunday usulning yaxshi tomoni hosilaning qatnashmaganida bo‘lsa, kamchiligi esa yaqinlashish tezligining pastligiga.

- **Vatarlar usuli.** Vatarlar metodi yaqinlashuvchi iteratsion jarayonga asoslangan usullardan biridir.

Nyuton metodidagi hisoblashlarni soddalashtirish uchun undagi $f'(x)$ ni

$$f'(x)\approx\frac{f(x_n)-f(x_{n-1})}{x_n-x_{n-1}},$$

taqribiy ifodasi bilan almashtirsak, u holda navbatdagi yaqinlashishni topish qoidasi quyidagicha bo‘ladi:

$$x_{n+1}=x_n-\frac{x_n-x_{n-1}}{f(x_n)-f(x_{n-1})}f(x_n). \qquad (2.17)$$

x_n ketma-ketlik x^* izlanadigan ildizga intiladi.

Ko‘pincha vatarlar usulidan foydalanishda, dastlabki yaqinlashish x_0 sifatida (a,b) oraliqning $f(x)f''(x)<0$ shart bajariladigan chekkasi olinadi va keyingi yaqinlashishlar (2.15) formula bilan aniqlanadi.

Vatarlar usulining geometrik tahlili: faraz qilaylik $f(x)$ funksiya $[a, b]$ da monoton bo‘lib, x^* son $f(x) = 0$ tenglamaning $[a, b]$ da ajratilgan yagona ildizi bo‘lsin.

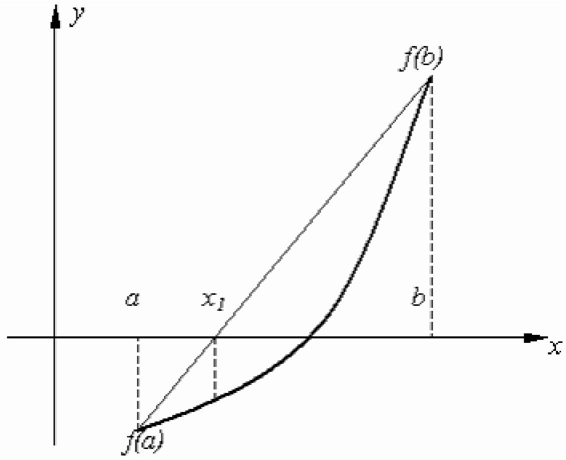


Рис. 2.8

$y = f(x)$ egri chiziqning $\big(a, f(a)\big)$ va $\big(b, f(b)\big)$ nuqtalardan o‘tuvchi vatarini o‘tkazamiz.

$f(x) = 0$ tenglama ildizining birinchi yaqinlashishi sifatida vatar bilan Ox o‘qining x_1 kesishish nuqtasini olamiz. Vatarning tenglamasini yozaylik:

$$\frac{y - f(a)}{f(b) - f(a)} = \frac{x - a}{b - a}.$$

bundan $\big(a, f(a)\big) = (x_{n-1};\ f(x_{n-1}))$, $(b, f(b)) = (x_n;\ f(x_n))$ desak,

$$\frac{y - f(x_n)}{f(x_n) - f(x_{n-1})} = \frac{x - x_n}{x_n - x_{n-1}}$$

hosil bo‘ladi.

2.11-misol. Nyuton usullari qo‘llanilib, $f(x) = 2x^4 - 2{,}5x^2 + 8x - 8 = 0$ tenglama yechilsin.

Yechish: $f(x)$ –uzluksiz differensiallanuvchi funksiya va $f(a)f(b) < 0$ bo‘lsin, ya’ni $f(x) = 0$ tenglamaning ildizi $\xi \in (a, b)$ bo‘lsin. x_0 boshlang‘ich qiymat sifatida (a, b) oraliqning $f(x)f''(x) > 0$ bajariladigan chekkasi olinadi. Keyingi yaqinlashishlar (2.14) munosobat bilan aniqlanadi.

Tenglamaning ildizlari $(-5,-0,5)$ va $(0{,}5,5)$ oraliqda yotar ekanligini ko‘rdik (1- misol). Birinchi oraliqni olamiz, ya’ni $(-5,-0,5)$ olamiz.

$f(x) = 2x^4 - 2{,}5 + 8x - 8$ funksiyaning hosilalarini topaylik:

$f'(x) = 8x^3 - 5x + 8$,

$f''(x) = 24x^2 - 5.$

$f(-5)f''(5) > 0$ bo‘lgani uchun dastlabki yaqinlashish sifatida $x_0 = -5$ ni olamiz, $x_1 = -0.5$ nuqta esa qo‘zg‘almas nuqta bo‘ladi. (2.14) formulasi bo‘yicha:

$$x_n = x_{n-1} - \frac{1}{f'(x_{n-1})} f(x_{n-1}).$$

n	x_n	$f(x_n)$	$f'(x_n)$	$\varepsilon = \lvert x_n - x_{n-1} \rvert$
0	-5	1139,5	-967	
1	-3,821613237	351,511419	-419,4009004	1,178386763
2	-2,983485759	104,3414154	-189,5350949	0,838127478
3	-2,432973382	27,81544754	-95,04828553	0,550512376
4	-2,14032794	5,396056902	-59,73716187	0,292645442
5	-2,049997956	0,415658924	-50,67080405	0,090329984
6	-2,041794831	0,003216219	-49,88776048	0,008203125
7	-2,041730362	1,97531E-07	-49,88163262	6,44691E-05
8	-2,041730358	0	-49,88163225	3,95999E-09

Jadvaldan ko‘rinib turibdiki aniq ildiz -2,041730362 $< x < -2{,}041730357$. Demak, taqribiy ildiz $x^* \approx -2{,}041730358$ bo‘ladi.

2.12-misol. Vatar usuli qo‘llanilib, $f(x) = 2^4 - 2{,}5x^2 + 8x - 8 = 0$ tenglama yechilsin.

Yechish:$f(x)$ –uzluksiz differensiallanuvchi funksiya va $f(a)f(b) < 0$ bo‘lsin, ya’ni $f(x) = 0$ tenglamaning ildizi $\xi \in (a,b)$ bo‘lsin. x_0 boshlang‘ich qiymat sifatida (a,b) oraliqning $f(x)f''(x) < 0$ bajariladigan chekkasi olinadi. Keyingi yaqinlashishlar (2.17) munosobat bilan aniqlanadi. Tenglamaning ildizlari $(-5,-0,5)$ va $(0{,}5,5)$ oraliqda yotar ekanligini ko‘rdik (1- misol). Birinchi oraliqni olamiz, ya'ni $(-5, -0,5)$ olamiz. $x_0 = -5$ nuqtani olamiz, chunki $f(-5)f''(5) > 0$. $x_0 = -5$ nuqtaning yaqin atrofida 9 -teoremaning shartlari bajariladi, shuning uchun $x_1 = -4.9$ deb olsak bo'ladi.

n	$x(n)$	$z(n)$
0	-5	1139,5
1	-4,9	1045,7352
2	-3,784725185	336,2740809
3	-3,256101412	164,2588349
4	-2,751314034	65,66675706
5	-2,415102942	26,13866791
6	-2,192777242	8,676061889
7	-2,082317697	2,10394794
8	-2,046955995	0,261963362
9	-2,041926924	0,009806871
10	-2,041731334	4,86551E-05
11	-2,041730358	9,11078E-09
12	-2,041730358	0
13	-2,041730358	0

Demak, taqribiy ildiz $x^* \approx -2,041730358$ bo'ladi.

2.1-turdagi topshiriqlar

Berilgan tenglamalar aytilgan usullar bilan yechilsin.

№	Tenglamalar	Taqribiy hisoblash usuli va uning aniqligi ε .	Parametlar
1	$\frac{a}{b+x^2} - cx = 0$	Kesmani teng ikkiga bo'lish $\varepsilon = 10^{-3}$	$a = 1,05; b = 0,1; c = 2.03$
2	$ax + bsinx + c = 0$	Iteratsiya $\varepsilon = 10^{-4}$	$a = 4,02; b = 0,2; c = -0,33$
3	$tg(ax+b) + cx^2 = 0$	Nyuton $\varepsilon = 10^{-3}$	$a = 3,01; b = 4; c = -1$
4	$ax + bxsinx = 0$	Vatarlar $\varepsilon = 10^{-5}$	$a = 2,01; b = -1;$
5	$\sqrt{A - x^2} + bx^2 = 0$	Kesmani teng ikkiga bo'lish $\varepsilon = 4 \cdot 10^{-5}$	$a = 1,23; b = -3,14;$
6	$a + bx^2 + ce^x = 0$	Iteratsiya $\varepsilon = 5 \cdot 10^{-3}$	$a = 1,03; b = 1,01; c = -2,33$

7	$sinax + bx^3 + cx = 0$	Nyuton $\varepsilon = 6 \cdot 10^{-4}$	$a = 2,23; b = -3,14; c = 1,02$
8	$ax^3 + b + c\sqrt{x+2} = 0$	Vatarlar $\varepsilon = 7 \cdot 10^{-5}$	$a = 1,11; b = -10,11; c = -2,02$
9	$\left(\frac{x+a}{b}\right) \cdot c^x + d = 0$	Kesmani teng ikkiga bo‘lish $\varepsilon = 9 \cdot 10^{-4}$	$a = 0,1; b = 2,23;$ $c = 2; d = -1,03$
10	$ax + b + cx = 0$	Iteratsiya $\varepsilon = 10^{-4}$	$a = 3,51; b = 1,47; c = 2,004;$
11	$(a+x)^2 + bx + c = 0$	Nyutona $\varepsilon = 10^{-5}$	$a = -2,13; b = 1,47; c = -3,14;$
12	$\mathrm{acos}(x+b) + cx^3 = 0$	Vatarlar $\varepsilon = 2 \cdot 10^{-4}$	$a = 2,13; b = 3,62; c = -4,12;$
13	$(x+a)^5 + bx = 0$	Kesmani teng ikkiga bo‘lish $\varepsilon = 3 \cdot 10^{-4}$	$a = 0,29; b = 2;$
14	$ax^2 + b + c\sqrt{x+d} = 0$	Iteratsiya $\varepsilon = 10^{-3}$	$a = 1,02; b = 4,37; c = -2,1; d = -0,5;$
15	$xa(bx+c)^2 - 14 = 0$	Nyuton $\varepsilon = 4 \cdot 10^{-3}$	$a = 3,23; b = 1,2; c = 3,22;$
16	$\ln(x+a) + (x+b)^5 = 0$	Vatarlar $\varepsilon = 3 \cdot 10^{-5}$	$a = 2,11; b = -4,03$
17	$(x+a)^2 - e^{bx} = 0$	Kesmani teng ikkiga bo‘lish $\varepsilon = 10^{-4}$	$a = -0,4; b = 0,53;$
18	$(a - bsinx)^2 + clnx = 0$	Iteratsiya $\varepsilon = 0,5 \cdot 10^{-3}$	$a = 1,23; b = 1,45;$ $c = 1,004;$
19	$(x+a)^3 + +bsinc^x = 0$	Nyuton $\varepsilon = 2 \cdot 10^{-4}$	$a = -3,21; b = -1,45; c = 2,12;$
20	$ax^2cosbx - cx = 0$	Vatarlar $\varepsilon = 7 \cdot 10^{-5}$	$a = 2,93; b = 3,01;$ $c = 2,1;$
21	$\frac{a}{\sqrt{bx+c}} + dcosbx = 0$	Kesmani teng ikkiga bo‘lish $\varepsilon = 2 \cdot 10^{-5}$	$a = 2,07; b = 1,19;$ $c = 1,13;\ d = -1,001$
22	$\frac{ax}{b+cx^2} + dlg(ax) = 0$	Iteratsiya $\varepsilon = 10^{-5}$	$a = 1,01; b = 0,98;$ $c = 2,03;$ $d = -2,04$
23	$a\sqrt{\lvert sinx \rvert} + blgx = 0$	Nyuton $\varepsilon = 4 \cdot 10^{-5}$	$a = 2,06; b = -1,06;$
24	$\frac{a}{x} + be^{ex} = 0$	Vatarlar $\varepsilon = 5 \cdot 10^{-5}$	$a = 2,37;$ $b = -0,99; c = 0,56;$

Bu 1-topshiriqni bajarish uchun namunalar yuqorida ko'rsatilgan 12 ta misollar bo'ladi.

2.2-turdagi topshiriqlar.

Topshiriqlarni bajarish uchun namuna.

1-misol. Ildizlarini analitik usulda ajrating

$$5^x - 6x - 3 = 0.$$

Yechish: $f(x) = 5^x - 6x - 3$ deb belgilab olamiz. Hosilasini hisoblaymiz $f'(x) = 5^x \ln 5 - 6$.

$f'(x) = 0$ tenglamani yechamiz

$$5^x \ln 5 - 6 = 0;\ \ 5^x = \frac{6}{\ln 5};\ \ xlg5 = lg6 - \lg(ln5);$$

$$x = \frac{lg6 - \lg(\ln 5)}{lg5} = \frac{0{,}7782 - 0{,}2065}{0{,}6990} = \frac{0{,}5717}{0{,}6990} \approx 0{,}82.$$

x quyidagi xolatlarda bo'ladi deb, $f(x)$ funksiyaning ishoralari jadvalini tuzamiz:

a) Funksiyaning kritik qiymati (hosilasining ildizi) yoki unga yaqin;

b) Chegaraviy qiymat (noma'lumning qabul qilish mumkin bo'lgan qiymatlar to'plamidan):

x	$-\infty$	1	$+\infty$
$signf(x)$	+	-	+

Jadvaldan ko'rinadiki funksiyaning ishorasi ikki marta o'zgaradi, demak tenglama ikkita ildizga ega. Ildizlarni yakkalash uchun, ildiz mavjud bo'lgan oraliqni uzunligi 1 dan oshmaydigan qilib, bo'laklarga ajratish kerak. Buning uchun $f(x)$ funksiyaning ishoralari jadvalini tuzamiz.

x	-1	0	1	$+\infty$
$signf(x)$	+	-	-	+

Bundan ko'rinadiki, x_1 va x_2 ildizlar quyidagi oraliqlarda bo'ladi:

$x_1 \in [-1,\ 0];\ \ \ x_2 \in [1,\ 2]$.

2-misol. Ildizlarini analitik usulda ajrating va ulardan birini tajriba usuli bilan 0.01 aniqlikkacha toping

$$x^4 - x^3 - 2x^2 + 3x - 3 = 0.$$

Yechish: $f(x) = x^4 - x^3 - 2x^2 + 3x - 3$ deb olsak,

$f'(x) = 4x^3 - 3x^2 - 4x + 3$

bo‘ladi. Hosilaning ildizini topamiz:

$$4x^3 - 3x^2 - 4x + 3 = 0; \quad 4x(x^2 - 1) - 3(x^2 - 1) = 0;$$

$$(x^2 - 1)(4x - 3) = 0; \quad x_1 = -1;\ x_2 = 1;\ x_3 = \frac{3}{4}.$$

$f(x)$ funksiyaning ishoralari jadvalini tuzamiz:

x	$-\infty$	-1	$3/4$	1	$+\infty$
$signf(x)$	+	-	-	-	+

Jadvaldan ko‘rinadiki, tenglama ikkita haqiqiy ildizga ega bo‘ladi:

$$x_1 \in [-\infty, -1]; \quad x_2 \in [1, +\infty].$$

Tenglamaning ildizlari joylashgan oraliqlarni yana ham kichik bo‘laklarga ajratamiz va quyidagiga ega bo‘lamiz:

x	-2	-1	1	2
$signf(x)$	+	-	-	+

Demak, $x_1 \in [-2, -1]; \quad x_2 \in [1,\ 2]$

Ildizlardan birini, masalan $x_1 \in [-2, -1]$ ni, tajribalar usuli bilan yuzdan bir aniqlikda topamiz. Barcha hisoblashlarni quyidagi jadvaldan foydalanib bajarsak qulay bo‘ladi:

n	a_n^+	b_n^-	$x_n = \frac{a_n + b_n}{2}$	x_n^4	$-x_n^3$	$-2x_n^2$	$3x_n$	$f(x_n)$
0	-2	-1	-1,5	5,0625	3,375	-4,5	-4,5	-35625
1	-2	-1,5	-1,75	9,3789	5,3594	-6,125	-5,25	0,3633
2	-	-1,5	-1,63	7,0591	4,3307	-	-4,89	-
3	1,75	-	-1,69	8,1573	4,8268	5,3138	-5,07	1,8140
4	-	1,63	-1,72	8,7521	5,0884	-	-5,16	-
5	1,75	-	-1,73	8,9575	5,1777	5,7122	-5,19	0,7981
6	-	1,69	-1,74	9,1664	5,2680	-	-5,22	-
7	1,75					5,9168		1,2363

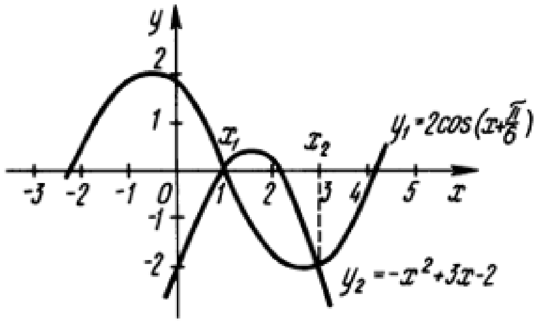
	-1,75 -1,75 -1,74	-1,72 -1,73 -1,73				-5,9858 -6,0552		-0,0406 0,1592

Javob: $x_1 \approx -1.73$.

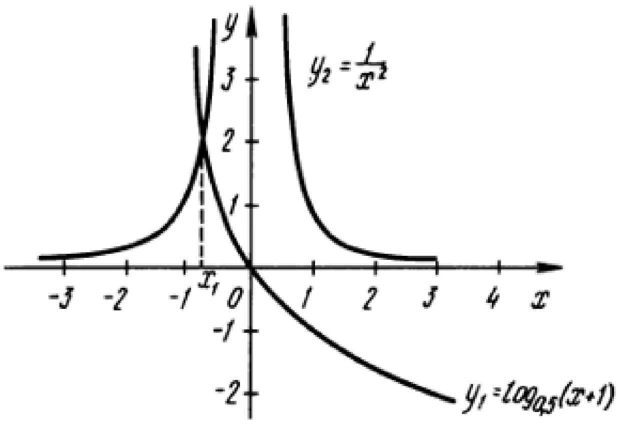
3-misol. Ildizlarini grafik usulda ajrating

$$2\cos(x+\frac{\pi}{6}) + x^2 = 3x - 2.$$

Yechish: $2\cos(x+\frac{\pi}{6}) + x^2 = 3x - 2$ tenglamani $2\cos\left(x+\frac{\pi}{6}\right) = -x^2+3x-2$ ko‘rinishda yozib olamiz. Tenglikning ikkala qismini $y_1=2\cos\left(x+\frac{\pi}{6}\right)$, $y_2 = -x^2+3x-2$ kabi belgilab olib, y_1 va y_2 funksiyalarning grafigini yasaymiz (1-rasm).



1-rasm



2-rasm

Grafikdan ko‘rinadiki, tenglama ikkita ildizga ega: $x_1 \approx 1{,}1$; $x_2 \approx 2{,}9$.

4-misol. Ildizlarini grafik usulda ajrating va ulardan birini tajriba usuli bilan 0.01 aniqlikkacha toping

$$x^2\log_{0,5}(x+1) = 1.$$

Yechish:

$x^2 \log_{0,5}(x+1) = 1$ tenglamani $\log_{0,5}(x+1) = \frac{1}{x^2}$ ko‘rinishda yozib olamiz. Tenglikning ikkala qismini y_1=$\log_{0,5}(x+1)$, $y_2 = \frac{1}{x^2}$ kabi belgilab olib, y_1 va y_2 funksiyalarning grafigini yasaymiz (2-rasm). Grafikdan ko‘rinadiki, tenglama bitta ildizga ega: $x_1 \approx$ -0,8.

Bu ildizni tajriba usuli bilan aniqlashtirish uchun, chekkalarida $f(x) == x^1 \log_{0,5}(x+1) - 1$ funksiya har xil ishoraga ega bo‘ladigan oraliqni tanlab olamiz. Jadval tuzamiz:

x	0.5	-0.8
$signf(x)$	-	+

Hisoblashlar qulay bo‘lsin uchun o‘nli logarifmga o‘tib olamiz.

$$f(x) = x^2 \frac{\lg(x+1)}{\lg 0,5} - 1 = x^2 \frac{\lg(x+1)}{-0,301} - 1.$$

Qolgan hisoblashlarni jadvalda bajaramiz:

n	a_n^+	b_n^-	$x_n = \frac{a_n + b_n}{2}$	x_n^2	$\lg(x_n+1)$	$f(x_n)$
0	$-0,8$	-0,5	-0,65	0,4225	-0,4559	-0,360
1	$-0,8$	-0,65	-0,73	0,5329	-0,5686	-0,0067
2	$-0,73$	-0,65	-0,69	0,4761	-0,5086	-0,196
3	$-0,73$	-0,69	-0,71	0,5041	-0,5376	-0,099
4	$-0,73$	-0,71	-0,72	0,5184	-0,5528	-0,048
5	$-0,73$	-0,72				

Javob: $x_1 \approx -73.$

Mustaqil yechish uchun:

Quyidagi misollarga mos amallarni bajaring (Yuqoridagi na’munaga qarang)**:**

1) Ildizlarini analitik usulda ajrating.

2) Ildizlarini analitik usulda ajrating va ulardan birini tajriba usuli bilan 0.01 aniqlikkacha toping.

3) Ildizlarini grafik usulda ajrating.

4) Ildizlarini grafik usulda ajrating va ulardan birini tajriba usuli bilan 0.01 aniqlikkacha toping.

Misollar:

№1. 1) $2^x + 5x - 3 = 0$;
2) $3x^4 + 4x^3 - 12x^2 - 5 = 0$;
3) $0{,}5^x + 1 = (x-2)^2$;
4) $(x-3)\cos x = 1,\ -2\pi \le x \le 2\pi$.

№ 3. 1) $5^x + 3x = 0$;
2) $x^4 - x - 1 = 0$;
3) $x^2 - 2 + 0{,}5^x = 0$;
4) $(x-1)^2 \cdot lg(x+11)=1$.

№ 5. 1) $3^{x-1} - 2 - x = 0$;
2) $3x^4 + 8x^3 + 6x^2 - 10 = 0$;
3) $(x-4)^2 \cdot \log_{0,5}(x-3) = -1$;
4) $5\sin x = x$.

№ 7. 1) $e^{-2x} - 2x + 1 = 0$;
2) $x^4 + 4x^3 - 8x^2 - 17 = 0$;
3) $0{,}5^x - 1 = (x+2)^2$;
4) $x^2\cos 2x = -1$.

№ 9. 1) $arctg(x-1) + 2x = 0$;
2) $3x^4 + 4x^3 - 12x^2 + 1 = 0$;
3) $(x-2)^2 2^x = 1$;
4) $x^2 - 20\sin x=0$.

№ 11. 1) $3^x + 2x - 2 = 0$;
2) $2x^4 - 8x^3 + 8x^2 - 1 = 0$;
3) $[(x-2)^2\text{-}1]2^x = 1$;
4) $(x-2)\cos x =1,\ -2\pi\le x \le 2\pi$.

№ 13. 1) $3^x + 2x - 5 = 0$;
2) $x^4 - 4x^3 - 8x^2 + 1 = 0$;
3) $x^2 - 3+0{,}5^x = 0$;
4) $(x-1)^2 \cdot lg(x+11)=1$.

№ 15. 1) $3^{x-1} - 4 - x = 0$;
2) $2x^3 - 9x^2 - 60x + 1 = 0$;
3) $(x-3)^2 \cdot \log_{0,5}(x-2) = -1$;
4) $5\sin x = x - 1$.

№ 17. 1) $e^x + x + 1 = 0$;
2) $2x^4 - x^2 - 10 = 0$;
3) $0{,}5^x - 3 = (x+2)^2$
4) $x^2\cos x =-1,\ -2\pi\le x \le 2\pi$.

№ 2. 1) $arctgx - \frac{1}{3x^3} = 0$;
2) $2x^3 - 9x^2 - 60x + 1 = 0$;
3) $[\log_2(-x)] \cdot (x+2) = -1$;
4) $\sin\left(x + \frac{\pi}{3}\right) - 0{,}5x = 0$

№ 4. 1) $2e^x = 5x + 2$;
2) $2x^4 - x^2 - 10 = 0$;
3) $xx \cdot \log_3(x+1) = 1$;
4) $\cos(x+0{,}5) = x^3$.

№ 6. 1) $2arctgx - \frac{1}{2x^3} = 0$;
2) $x^4 - 18x^2 + 6 = 0$;
3) $x^2 \cdot 2^x = 1$;
4) $\tan x = x + 1\ \ \pi/2 \le x \le \pi/2$.

№ 8. 1) $5^x - 6x - 3 = 0$;
2) $x^4 - x^3 - 2x^2 + 3x - 3 = 0$;
3) $2^x - 0{,}5^x - 3 = 0$;
4) $xlg(x+1) = 1$.

№ 10. 1) $2arctgx - x + 3 = 0$;
2) $3x^4 - 8x^3 - 18x^2 + 2 = 0$;
3) $2\sin\left(x + \frac{\pi}{3}\right) = 0{,}5x^2 - 1$;
4) $2lgx - \frac{x}{2} + 1 = 0$.

№ 12. 1) $2arctgx - 3x + 2 = 0$;
2) $2x^4 - 8x^3 + 8x^2 - 1 = 0$;
3) $[\log_2(x+2)](x-1) = 1$;
4) $\sin(x-0{,}5) - x + 0{,}8 = 0$.

№ 14. 1) $3e^x + 3x + 1 = 0$;
2) $3x^4 + 4x^3 - 12x^2 - 5 = 0$;
3) $x\log_3(x+1) = 2$;
4) $\cos(x+0{,}3) = x^2$.

№ 16. 1) $arctgx - \frac{1}{3x^3} = 0$;
2) $x^4 - x - 1 = 0$;
3) $(x-1)^2 2^x = 1$;
4) $tan^3 x = x - 1, -\pi/2 \le x \le \frac{\pi}{2}$.

№ 18. 1) $3^x - 2x + 5 = 0$;

№ 19. 1) $arctg(x-1)+3x-2=0$;
2) $x^4-18x^2+6=0$;
3) $(x-2)^2 2^x=1$;
4) $x^2-20\sin x=0$.

№ 21. 1) $2^x-3x-2=0$;
2) $x^4-x^3-2x^2+3x-3=0$;
3) $(0,5)^x+1=(x-2)^2$;
4) $(x-3)\cos x=1$, $-2\pi\le x\le 2\pi$.

№ 23. 1) $3^x+2x-3=0$;
2) $3x^4-8x^3-18x^2+2=0$;
3) $x^2-4+0,5^x=0$;
4) $(x-2)^2\cdot lg(x+11)=1$.

№ 5. 1) $3^x+2+x=0$;
2) $2x^3-9x^2-60x+1=0$;
3) $(x-4)^2\cdot\log_{0,5}(x-3)=-1$;
4) $5\sin x=x-0,5$.

№ 27. 1) $e^{-2x}-2x+1=0$;
2) $2x^4-x^2-10=0$;
3) $0,5^x-3=-(x+1)^2$
4) $x^2\cos 2x=-1$, $-2\pi\le x\le 2\pi$.

№ 29. 1) $arctg(x-1)+2x=0$;
2) $x^4-18x^2+6=0$;
3) $(x-2)^2 2^x=1$;
4) $x^2-10\sin x=0$.

2) $3x^4+8x^3+6x^2-10=0$;
3) $2x^2-0,5^x-2=0$;
4) $xlg(x+1)=1$.

№ 20. 1) $2arctgx-x+3=0$;
2) $x^4+4x^3-8x^2-17=0$;
3) $2\sin\left(x+\frac{\pi}{3}\right)=x^2-0,5$;
4) $2lgx-\frac{x}{2}+1=0$.

№ 22. 1) $arctgx+2x-1=0$;
2) $3x^4+4x^3-12x^2+1=0$;
3) $(x+2)\log_2(x)=1$;
4) $\sin(x+1)=0,5x$.

№ 24. 1) $2e^x-2x-3=0$;
2) $3x^4+4x^3-12x^2-5=0$;
3) $x\log_3(x+1)=1$;
4) $\cos(x+0,5)=x^3$.

№ 26. 1) $arctg(x-1)+2x-3=0$;
2) $x^4-x-1=0$;
3) $(x-1)^2 2^x=1$;
4) $tan^3x=x-1, -\pi/2\le x\le\frac{\pi}{2}$.

№ 28. 1) $3^x-2x-5=0$;
2) $3x^4+8x^3+6x^2-10=0$;
3) $2x^2-0,5^x-3=0$;
4) $xlg(x+1)=1$.

№ 30. 1) $3^x+5x-2=0$;
2) $3x^4+4x^3-12x^2+1=0$;
3) $(0,5)^x+1=(x-2)^2$;
4) $(x-3)\cos x=1$, $-2\pi\le x\le 2\pi$.

2.3-turdagi topshiriqlar.

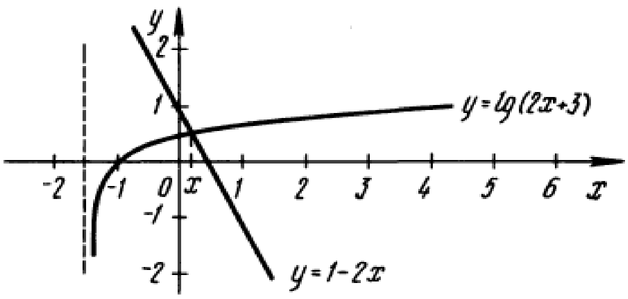
Topshiriqlarni bajarish uchun na'muna.

1) Tenglama ildizlarini grafik usulda ajrating va ulardan birini iteratsiya usuli bilan 0,001 aniqlikda toping.

2) Tenglama ildizlarini analitik usulda ajrating va ulardan birini iteratsiya usuli bilan 0,001 aniqlikda toping.

Misolning berilishi: 1) $2x+lg(2x+3)=1$; 2) $x^3-2x^2+7x+3=0$.

Yechish: 1) $2x + lg(2x + 3) = 1$ tenglamaning ildizlarining taqribiy qiymatini grafik usulda topamiz; buning uchun berilgan tenglamani



3-rasm

$lg(2x + 3) = 1 - 2x$ ko‘rinishda yozamiz (3-rasm). Grafikdan ko‘rinadiki, tenglama [0; 0,5] kesmada yotuvchi bitta ildizga ega. Uni iteratsiya usuli bilan berilgan aniqlikda topish uchun tenglamani $x = \varphi(x)$ ko‘rinishga keltiramiz. $|k| \geq Q/2$ deb olib $\varphi(x)$ funksiyani $\varphi(x) = x - \frac{f(x)}{k}$ munosobatdan qidiramiz, bunda $Q = ax|f'(x)|$;

[0; 0,5] kesmada k bilan $f'(x)$ bir xil ishorali bo‘ladi. Quyidagilarni topamiz:

$$f\ (x) = 2x + lg(2x + 3) - 1; \quad f\ '(x) = 2 + \frac{0{,}8686}{2x + 3};$$

$$Q = \max_{[0;0,5]} f\ '(x) = 2 + \frac{0{,}8686}{2 * 0 + 3} \approx 2{,}2895;\ f\ '(x) > 0\ pri\ 0 \leq x \leq 0{,}5.$$

$k = 2$ desak, u holda

$$\varphi(x) = x - \frac{f(x)}{2} = x - x - \frac{\lg(2x + 3)}{2} + \frac{1}{2} = \frac{1}{2} - \frac{1}{2}\lg(2x + 3).$$

Dastlabki yaqinlashish sifatida $x_0 = 0$ ni olamiz, boshqa barcha yaqinlashishlarni

$$x_{n+1} = \frac{1}{2} - \frac{1}{2}\lg(2x_n + 3)$$

tenglikdan aniqlaymiz.

Hisoblashlarni jadvalda bajarish qulayroq:

n	x_n	$2x_n + 3$	$\lg(2x_n + 3)$	$\frac{1}{2}\lg\ (2x_n + 3)$
0	0	3	0,4771	0,2386
1	0,2614	3,5228	0,5469	0,2734
2	0,2266	3,4532	0,5382	0,2691

3 4 5	0,2309 0,2303 0,2304	3,4618 3,4606	0,5394 0,5392	0,2697 0,2696

Javob: $x \approx 0{,}230$.

2) $x^3 - 2x^2 + 7x + 3 = 0$ tenglamaning ildizini analitik usulda ajratamiz.

Quyidagilarni topamiz

f (x)=$x^3 - 2x^2 + 7x + 3$; $f'(x) = 3x^2 - 4x + 7$; $D = 4 - 21 * 4 < 0$.

Jadval tuzamiz:

-	-∞	*-1*	*0*	+∞
sign f(x)	-	-	+	+

Tenglama [-1, 0] kesmada bitta haqiqiy ildizga ega bo‘ladi. Berilgan tenglamani

1≤ $x \leq 0$ da $|\varphi'(x)| < 1$ bo‘ladigan qilib $x = \varphi(x)$ ko‘rinishda keltirib olamiz. Q=$\max\limits_{[-1,0]}|f'(x)| = f'(-1) = 3 + 4 + 7 = 14$ bo‘lgani uchun, k=10 deb olsak bo‘ladi. U holda

$$\varphi(x) = x - \frac{\varphi(x)}{k} = x - 0{,}1x^3 + 0{,}2x^2 - 0{,}7x - 0{,}3 =$$
$$= -0{,}1x^3 + 0{,}2x^2 + 0{,}3x - 0{,}3.$$

Dastlabki yaqinlashish sifatida $x_0 = 0$ ni olamiz, boshqa barcha yaqinlashishlarni

$$x_{n+1} = \varphi(x_n)$$

tenglikdan aniqlaymiz.

Hisoblashlarni jadvalda bajarish qulayroq:

n	x_n	x_n^2	x_n^3	$\varphi(x_n)$
0 1 2 3 4 5	0 -0,3 -0,3693 -0,3785 -0,3795 -0,3796	0 0,09 0,1364 0,1433 0,1440	0 -0,027 -0,0504 -0,0542 -0,0546	-0,3 -0,3693 -0,3785 -0,3795 -0,3796

Javob: $x \approx -0{,}380$.

Mustaqil yechish uchun:

Quyidagi misollarga mos amallarni bajaring (Yuqoridagi na'munaga qarang)**:**

1) Tenglama ildizlarini grafik usulda ajrating va ulardan birini iteratsiya usuli bilan 0,001 aniqlikda toping.

2) Tenglama ildizlarini analitik usulda ajrating va ulardan birini iteratsiya usuli bilan 0,001 aniqlikda toping.

Misolning berilishi:

№ 1. 1) $lnx + (x+1)^3 = 0$; 2) $x^3 + 2x^2 + 2 = 0$.
№ 2. 1) $x \cdot 2^x = 1$; 2) $x^3 - 3x^2 + 9x - 10 = 0$.
№ 3. 1) $\sqrt{x+1} = \frac{1}{x}$; 2) $x^3 - 2x + 2 = 0$.
№ 4. 1) $x - \cos x = 0$; 2) $x^3 + 3x - 1 = 0$.
№ 5. 1) $3x + \cos x + 1 = 0$; 2) $x^3 + x - 3 = 0$.
№ 6. 1)$x + ln\, x = 0{,}5$; 2) $x^3 + 0{,}4x^2 + 0{,}6x - 1{,}6 = 0$.
№ 7. 1)$2 - x = lnx$; 2) $x^3 - 0{,}2x^2 + 0{,}4x - 1{,}4 = 0$.
№ 8. 1) $(x-1)^2 = \frac{1}{2}e^{x;}$ 2) $x^3 - 0{,}1x^2 + 0{,}4x + 2 = 0$.
№ 9. 1) $(2-x)e^x = 0{,}5$; 2) $x^3 + 3x^2 + 12x + 3 = 0$.
№ 10. 1) $2{,}2x - 2^x = 0$; 2) $x^3 - 0{,}2x^2 + 0{,}5x - 1 = 0$.
№ 11. 1) $x^2 + 4\sin x = 0$; 2) $x^3 - 0{,}1x^2 + 0{,}4x + 1{,}2 = 0$.
№ 12. 1) $2x - lgx = 7$; 2) $x^3 - 3x^2 + 6x - 5 = 0$.
№ 13. 1) $5x - 8lnx = 8$; 2) $x^3 - 0{,}2x^2 + 0{,}5x - 1{,}4 = 0$.
№ 14. 1) $3x - e^x = 0$; 2) $x^3 + 2x + 4 = 0$.
№ 15. 1) $x(x+1)^2 = 1$; 2) $x^3 - 3x^2 + 12x - 12 = 0$.
№ 16. 1)$x = (x+1)^3$; 2) $x^3 + 0{,}2x^2 + 0{,}5x + 0{,}8 = 0$.
№ 17. 1) $x^2 = \sin x$; 2) $x^3 + 4x - 6 = 0$.
№ 18. 1) $x^3 = \sin x$; 2) $x^3 + 0{,}1x^2 + 0{,}4x - 1{,}2 = 0$.
№ 19. 1)$x = \sqrt{lg(x+2)}$; 2) $x^3 + 3x^2 + 6x - 1 = 0$.
№ 20. 1) $x^2 = ln(x+1)$; 2) $x^3 - 0{,}1x^2 + 0{,}4x - 1{,}5 = 0$.
№ 21. 1) $2x + lgx = -0{,}5$; 2) $x^3 - 3x^2 + 6x - 2 = 0$.
№ 22. 1) $2x + \cos x = 0{,}5$; 2) $x^3 - 0{,}2x^2 + 0{,}3x - 1{,}2 = 0$.
№ 23. 1) $\sin 0{,}5x + 1 = x^2; x > 0$; 2) $x^3 - 3x^2 + 12x - 9 = 0$.
№ 24. 1) $0{,}5x + lg(x-1) = 0{,}5$; 2) $x^3 + 0{,}2x^2 + 0{,}5x - 2 = 0$.
№ 25. 1) $\sin(0{,}5 + x) = 2x - 0{,}5$; 2) $x^3 + 3x + 1 = 0$.
№ 26. 1) $lg(2+x) + 2x = 3$; 2) $x^3 + 0{,}2x^2 + 0{,}5x - 1{,}2 = 0$.
№ 27. 1) $lg(1+2x) = 2 - x$; 2) $x^3 - 3x^2 + 9x + 2 = 0$.
№ 28. 1) $2\sin(x - 0{,}6) = 1{,}5 - x$; 2) $x^3 - 0{,}1x^2 + 0{,}4x - 1{,}5 = 0$.
№ 29. 1) $x + lg(1+x)$=1,5; 2) $x^3 - 3x^2 + 6x + 3 = 0$.
№ 30. 1) $x + \cos x = 1$; 2) $x^3 - 0{,}1x^2 + 0{,}3x - 0{,}6 = 0$.

2.4-turdagi topshiriqlar.

Topshiriqlarni bajarish uchun na'muna.

Misol. Vatarlar va urinmalarning aralash (kombinirovanniy) usuli bilan uchinchi darajali tenglamani 0,001 aniqlikda hisoblang.

$$x^3 - 2x^2 - 4x + 7 = 0.$$

Yechish. Ildizlarni analitik usulda ajratamiz. Ma'lumki

$$f(x) = x^3 - 2x^2 - 4x + 7, f'(x) = 3x^2 - 4x - 4;$$

$$x_{1,2} = \frac{2 \pm \sqrt{4+12}}{3} = \frac{2 \pm 4}{3}\ x_1 = -\frac{2}{3};\quad x_2 = 2.$$

$f(x)$ funksiyaning ishoralar almashinish jadvalini tuzamiz:

x	$-\infty$	$-2/3$	2	$+\infty$
sign $f(x)$	–	+	–	+

Demak, tenglama uchta xaqiqiy ildizga ega:

$$x_1 \in \left]-\infty, -\frac{2}{3}\right];\ \ x_2 \in \left[-\frac{2}{3}, 2\right];\ \ x_3 \in [2, +\infty[.$$

Ildizni o'z ichiga olgan oraliqni uzunligi 1 ga teng bo'lgan oraliqlarga bo'lamiz:

x	-2	-1	0	1	2	3
sign $f(x)$	–	+	+	+	–	+

Demak,

$x_1 \in [-2, -1];\ \ x_2 \in [1, 2];\ \ x_3 \in [2, 3].$ (*)

Vatarlar va urinmalarning aralash (kombinirovanniy) usulini qo'llab ildizni aniqlaymiz:

1) x_1 ildizni topamiz. (*) dan $x_1 \in [-2, -1]$.

$$f(-2) < 0;\ \ f(-1) > 0;\ \ f''(x) = 6x - 4.$$

$-2 \le x \le -1$ da $f''(x) < 0$ bo'ladi.

Hisoblashlar uchun quyidagi formulalardan foydalanamiz

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}; \ \ \bar{x}_{n+1} = x_n - \frac{f(x_n)}{f(\bar{x}_n) - f(x_n)}(\bar{x}_n - x_n), \qquad (**)$$

bunda, x_n va $\bar{x}_n$ – lar ildizning kami bilan va ko‘pi bilan olingan qiymati. $x_0 = -2;\ \bar{x}_0 = -1$ deb

$$h_{1n} = \frac{f(x_n)}{f'(x_n)};\ h_{2n} = \frac{f(x_n)}{f(\bar{x}_n) - f(x_n)} \cdot (\bar{x}_n - x_n)$$

belgilash kiritib olamiz va hisoblashlarni jadvalda bajaramiz

n	x_n / $\bar{x}_n$	$\bar{x}_n - x_n$	x_n^2 / $\bar{x}_n^2$	x_n^3 / $\bar{x}_n^3$	$f(x_n)$ / $f(\bar{x}_n)$	$f'(x_n)$	$f(\bar{x}_n) - f(x_n)$	h_{1n} / h_{2n}
0	-2	1	4	-8	-1	16	9	-0,06
	-1		1	-1	8			-011
1	-1,94	0,05	3,7636	-7,3014	-0,0686	15,0508	0,7331	-0,0045
	-1,89		3,5721	-6,7513	0,6645			-0,0047
2	-1,9355	0,0002	3,7462	-7,2507	-0,0011	–	–	-
	-1,9353		3,7454	-7,2484	0,0020			-

Javob: $x_1 \approx -1{,}935.$

2) x_2 ildizni topamiz. (*) dan $x_2 \in [1, 2]$.

$f(1) > 0;\ f(2) < 0$, demak $1 \le x \le 2\ da\ f''^{(x)} > 0$ bo‘ladi. Keyingi hisoblashlar uchun $x_0 = 1, \bar{x}_0$=*2* deb olib, (**) formuladan foydalanamiz. Hisoblashni jadvalda bajaramiz:

n	x_n / $\bar{x}_n$	$\bar{x}_n - x_n$	x_n^2 / $\bar{x}_n^2$	x_n^3 / $\bar{x}_n^3$	$f(x_n)$ / $f(\bar{x}_n)$	$f'(x_n)$	$f(\bar{x}_n) - f(x_n)$	h_{1n} / h_{2n}
0	1	1	1	1	2	-5	-3	-0,4
	2		4	8	-1			-07
1	1,4	0,3	1,96	2,744	0,224	-3,72	-0,891	-0,06

	1,7		2,89	4,913	-0,667			-0,075
2	1,46	0,015	2,1316	3,1121	0,0089	-3.4452	-0,0511	-0,0025
	1,475		2,1756	3,2090	-0,0422			-0,0026
3	1,4625	0,0001	2,1389	3,1282	0,0004			
	1,4626		2,1392	3,1288	0			

Javob:$x_2 \approx 1{,}463.$

3) x_3 ildizni topamiz. (*) dan $x_3 \in [2, 3]$. $f(2) < 2;$ $f(3) > 0;$ $f''^{(x)} > 0$. Hisoblashlar uchun

$$x_{n+1} = x_n - \frac{f(x_n)}{f(\bar{x}_n) - f(x_n)} \cdot (\bar{x}_n - x_n);\ \bar{x}_{n+1} = \bar{x}_n - \frac{f(\bar{x}_n)}{f'^{(\bar{x}_n)}},$$

formulani qo‘llaymiz, bunda $x_0 = 2;\ \bar{x}_0 = 3.$

$$h_{1n} = \frac{f(x_n)}{f(\bar{x}_n) - f(x_n)} \cdot (\bar{x}_n - x_n);\ \ h_{2n} = \frac{f(\bar{x}_n)}{f'^{(\bar{x}_n)}}$$

Deb belgalash kiritib, barcha hisoblashlarni jadvalda bajaramiz.

. n	x_n / $\bar{x}_n$	$\bar{x}_n - x_n$	x_n^2 / $\bar{x}_n^2$	x_n^3 / $\bar{x}_n^3$	$f(x_n)$ / $f(\bar{x}_n)$	$f(\bar{x}_n) - f(x_n)$	$f'(x_n)$	h_{1n} / h_{2n}
0	*2*	1	4	8	-1	5	11	$-0{,}20$
	3		9	27	4			0,36
1	2,2	0,44	4,84	10,648	$-0{,}832$	1,7325	6,3488	$-0{,}126$
	2,64		6,9696	18,3997	$-0{,}9005$			0,142
2	2,326	0,172	5,4103	12,8430	$-0{,}2816$		4,728	$-0{,}122$
	2,498		6,2400	15,5875	0,1155	0,3971		0,024
3	2,448	0,026	5,9927	14,6701	$-0{,}1073$	0,1125	4,4661	$-0{,}0248$
	2,474		6,1207	15,1426	0,0052			0,0012
4	2,4728	0						
	2,4728							

Javob: $x_3 \approx 2{,}473.$

Mustaqil yechish uchun

Quyidagi amallarni va misollarni mos ravishda bajaring:

Vatarlar va urinmalarning aralash (kombinirovanniy) usuli bilan uchinchi darajali tenglamani 0,001 aniqlikda hisoblang.

№ 1. $2x^3 - 3x^2 - 12x - 5 = 0.$ **№ 2.** $x^3 - 3x^2 - 24x - 3 = 0.$

№ 3. $x^3 - 3x^2 + 3 = 0.$ **№ 4.** $x^3 - 12x + 6$ =0.

№ 5. $x^3 + 3x^2$ −24x−10= 0. **№ 6.** $2x^3 - 3x^2 - 12x + 10 = 0.$

№ 7. $2x^3 + 9x^2 - 21 = 0.$ **№ 8.** $x^3 - 3x^2 + 2,5 = 0.$

№ 9. $x^3 + 3x^2 - 2 = 0.$ **№ 10.** $x^3 + 3x^2 - 3,5 = 0.$

№ 11. $x^3 + 3x^2 - 24x + 10 = 0.$ **№ 12.** $x^3 - 3x^2 - 24x - 8 = 0.$

№ 13. $2x^3 + 9x^2 - 10 = 0.$ **№ 14.** $x^3 - 12x + 10 = 0.$

№ 15. $x^3 + 3x^2 - 3 = 0.$ **№ 16.** $2x^3 - 3x^2 - 12x + 1 = 0.$

№ 17. $x^3 - 3x^2 - 24x - 5 = 0.$ **№ 18.** $x^3 - 4x^2 + 2 = 0.$

№ 19. $x^3 - 12x - 5 = 0.$ **№ 20.** $x^3 + 3x^2 - 24x + 1 = 0.$

№ **21.** $2x^3 - 3x^2 - 12x + 12 = 0$ **№ 22.** $2x^3 + 9x^2 - 6 = 0.$

№ 23. $x^3 - 3x^2 + 1,5 = 0.$ **№ 24.** $x^3 - 3x^2 - 24x + 10 = 0.$

№ 25. $x^3 + 3x^2 - 24x - 3 = 0.$ **№ 26.** $x^3 - 12x - 10 = 0.$

№ 27. $2x^3 + 9x^2 - 4 = 0.$ № **28.** $2x^3 - 3x^2 - 12x + 8 = 0.$

№ 29. $x^3 + 3x^2 - 1 = 0.$ **№ 30.** $x^3 - 3x^2 + 3,5 = 0.$

2.5-turdagi topshiriqlar.

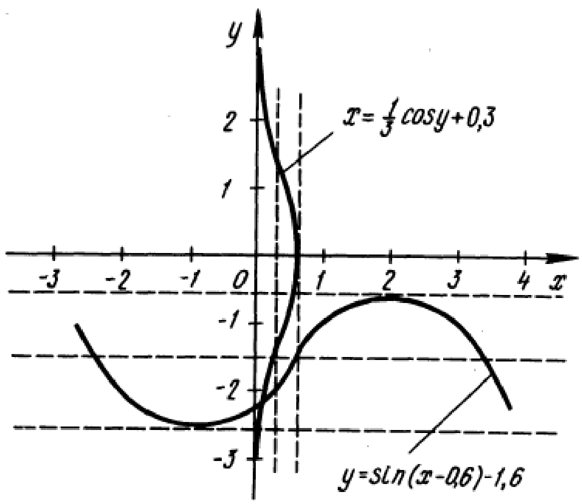
Topshiriqlarni bajarish uchun na'muna.

1-misol. Iteratsiya usulini qo‘llab, chiziqli bo‘magan tenglamani 0,001 aniqlikgacha yeching.

$$\begin{cases} \sin(x - 1,6) - y = 1,6; \\ 3x - \cos y = 0,9. \end{cases}$$

Yechish. 1) Berilgan $\begin{cases} \sin(x - 1,6) - y = 1,6; \\ 3x - \cos y = 0,9. \end{cases}$ sistemani quyidagi ko‘rinishda yozamiz

$$\begin{cases} y = \sin(x - 0,6) - 1,6; \\ x = \dfrac{1}{3}\cos y + 0,3. \end{cases}$$



4-rasm

Ildizlarni grafik usulda ajratamiz

(4-rasm). Grafikdan ko‘rinadiki sistema D: $0 < x < 0{,}3$; 2,2$< y < -1{,}8$ sohada bitta yechimga ega.

Sistemani iteratsiya usuli bilan yechish mumkinligini bilish uchun, uni quyidagi ko‘rinishda yozib olamiz:

$$\begin{cases} x = \varphi_1(x,y) = \frac{1}{3}\cos y + 0{,}3; \\ y = \varphi_2(x,y) = \sin(x - 0{,}6) - 1{,}6. \end{cases}$$

$$\frac{\partial\varphi_1}{\partial x} = 0, \frac{\partial\varphi_2}{\partial x} = \cos(x - 0{,}6)\,, \frac{\partial\varphi_1}{\partial y} = -\frac{1}{3}\sin y, \qquad \frac{\partial\varphi_2}{\partial y} = 0$$

bo‘lgani uchun, D sohada quyidagilarga egamiz

$$\left|\frac{\partial\varphi_1}{\partial x}\right| + \left|\frac{\partial\varphi_2}{\partial x}\right| = |\cos(x - 0{,}6)| \le \cos 0{,}3 = 0{,}2955 < 1;$$

$$\left|\frac{\partial\varphi_1}{\partial y}\right| + \left|\frac{\partial\varphi_2}{\partial y}\right| = \left|-\frac{1}{3}\sin y\right| \le \left|\frac{1}{3}\sin(-1{,}8)\right| < 1.$$

Demak, yaqinlashish sharti bajariladi.

Boshlang‘ich yaqinlashishni $x_0 = 0{,}15, y_0 = -2$ deb olib, hisoblashlarni

$$\begin{cases} x_{n+1} = \frac{1}{3}\cos y_n + 0{,}3; \\ y_{n+1} = \sin(x_n - 0{,}6) - 1{,}6. \end{cases}$$

formula bilan bajaramiz.

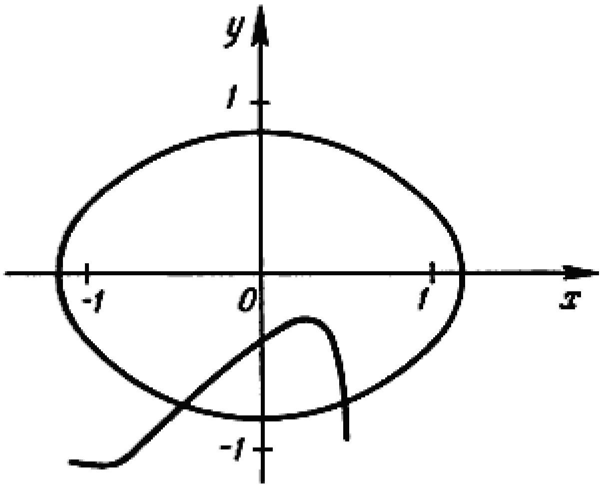
n	x_n	y_n	$x_n - 0{,}6$	$\sin(x_n - 0{,}6)$	$\cos y_n$	$\frac{1}{3}\cos y_n$
0	0,15	-2	-0,45	-0,4350	-0,4161	-0,1384
1	0,1616	-2,035	-0,4384	-0,4245	-0,4477	-0,1492
2	0,1508	-2,0245	-0,4492	-0,4342	-0,4382	-0,1461
3	0,1539	-2,0342	-0,4461	-0,4313	-0,4470	-0,1490
4	0,1510	-2,0313	-0,4490	-0,4341	-0,4444	-0,1481
5	0,1519	-2,0341	-0,4481	-0,4333	-0,4469	-0,1490
6	0,1510	-2,0333	-0,449	-0,4341	-0,4462	-0,1487
7	0,1513	-2,0341	-0,4487	-0,4340	-0,4469	-0,1490
8	0,1510	-2,0340				

Javob: x$\approx 0{,}151$; $y \approx -2{,}034.$

2-misol. Nyuton usulini qo‘llab, chiziqli bo‘magan tenglamani 0,001 anaiqlikgacha yeching.

$$\begin{cases} \sin(2x - y) - 1{,}2x = 0{,}4; \\ 0{,}8x^2 + 1{,}5y^2 = 1. \end{cases}$$

Yechish. Ildizlarni grafik usulda ajratamiz (5-rasm). Funksiyalar grafiklarini chizish uchun y_1 va y_2 funksiyalar qiymatlari jadvalini tuzamiz (jadval 1).



5-rasm

1-jadval

x	-1,1	-1	-0,8	-0,6	-0,2	-0,4	0	0,2	0,4	0,5
x^2	1,21	1	0,64	0,36	0,04	0,16	0	0,04	0,16	0,25
$0{,}8x^2$	0,97	0,8	0,51	0,29	0,032	0,13	0	0,032	0.13	0,2
$1\text{-}0{,}8x^2$	0,03	0,2	0,49	0,71	0,97	0,87	1	0,97	0,87	0,8
$\frac{1-0{,}8x^2}{1.5}$	0,02	0,13	0,33	0,47	0,65	0,58	0,67	0,65	0,58	0,53
y_2	±0,14	±0,36	±0,57	±0,69	±0,81	±0,76	±0,82	±0,81	±0,76	±0,73
$1{,}2\,x$	-1,32	-1,2	-0,96	-0,72	-0,24	-0,48	0	0,24	0,48	0,6
$0{,}4+1{,}2x$	-0,92	-0,8	-0,56	-0,32	0,16	-0,08	0,4	0,64	0,88	1
2x-y	-1,17	-0,93	-0,59	-0,33	0,16	-0,08	0,41	0,69	2,06 1,08	1,57
y_1	-1,03	-1,07	-1,01	-0,87	-0,56	-0,72	-0,41	-0,29	-1,26 -1,28	-0,57

Quyidagi shartlardan kelib chiqib x uchun qiymat olamiz: birinchi tenglamadan $-1 \leq 1{,}2x + 0{,}4 \leq 1$, ya’ni $-1{,}16 \leq x \leq 0{,}5$; ikkinchi tenglamadan $-\sqrt{1{,}25} \le x \le \sqrt{1{,}25},$, ya'ni $-1{,}12 \le x \le 1{,}12.$ Shunday qilib,

$-1{,}12 \le x \le 0{,}5.$

Sistema ikkita yechimga ega. Ulardan birini, D: $0{,}4 < x < 0{,}5;\ -0{,}76 < y < -0{,}73$ sohada yotadiganini aniqlaymiz. Dastlabki yoqinlashish sifatida

$x_0 = 0{,}4;\ y_0 = -0{,}75$ ni olib,

$$\begin{cases} F(x,y) = \sin(2x - y) - 1{,}2x - 0{,}4 \\ G(x,y) = 0{,}8x^2 + 1{,}5y^2 - 1; \end{cases}$$

deb belgilaymiz. U holda

$$\begin{cases} F_x^` = 2\cos(2x - y) - 1{,}2; \\ G_x^` = 1{,}6; \end{cases} \quad \begin{cases} F_y^` = -\cos(2x - y); \\ G_y^` = 3y. \end{cases}$$

Ildizlarni Nyuton usulida aniqlaymiz:

$$\begin{cases} x_{n+1} = x_n + h_n, \\ y_{n+1} = y_n + k_n, \end{cases}$$

bunda $h_n = \frac{\Delta_{h_n}}{\Delta_n};\ k_n = \frac{\Delta_{k_n}}{\Delta_n}.$

$$\Delta_n = \begin{vmatrix} F_x^`(x_n,y_n) & F_y^`(x_n,y_n) \\ G_x^`(x_n,y_n) & G_y^`(x_n,y_n) \end{vmatrix};\quad \Delta_{h_n} = \begin{vmatrix} F_y^`(x_n,y_n) & F(x_n,y_n) \\ G_y^`(x_n,y_n) & G(x_n,y_n) \end{vmatrix}.$$

$$\Delta_{k_n} = \begin{vmatrix} F(x_n,y_n) & F_y^`(x_n,y_n) \\ G(x_n,y_n) & G_y^`(x_n,y_n) \end{vmatrix}.$$

Barcha hisoblashlarni 2-jadvalda keltiramiz:

2-jadval

n	x_n	$0{,}8x_n^2$	$2x_n - y_n$	Sin(2 $x_n - y_n$)	F(x_n, y_n)	$F'_x(x_n, y_n)$	$F'_y(x_n, y_n)$	Δ_n	Δ_{h_n}	h_n
	y_n	$1{,}5y_n^2$		Cos(2 $x_n - y_n$)	G(x_n, y_n)	$G'_x(x_n, y_n)$	$G'_y(x_n, y_n)$		Δ_{k_n}	k_n
0	0,4	0,128	0,55	0,9988	0,1198	-1,1584	-0,0208	2,6917	0,2701	0,10
	0,75	0,8438		0,0208	-0,0282	0,64	-2,25		0,0440	0,017
1	0,50	0,2	0,733	0,9869	-0,0131	-1,523	0,1615	3,2199	-0,0193	-0,0060
	-0,733	0,8059		-0,1615	0,059	0,8	-2,199		0,0794	0,0247
2	0,4940	0,1952	1,6963	0,9921	-0,0007	-1,4502	0,1251	2,9827	-0,008	-0,0027

									0	
	-0,7083	0,7525		-0,1251	-0,0523	0,7904	-2,1249		-0,0764	-0,0256
3	0,4913	0,1931	1,7165	0,9894	-0,0002	-1,4904	0,1452	3,1673	-0,0003	-0,0001
	-0,7339	0,8079		-0,1452	-0,0010	0,7861	-2,2017		0,0013	0,0004
4	0,4912									
	-0,7335									

Javob: $x \approx 0{,}491; y \approx -0{,}734$.

Mustaqil yechish uchun:

Quyidagi misollarga mos amallarni bajaring (Yuqoridagi na'munaga qarang)**:**

1) Iteratsiya usulini qo'llab, berilgan chiziqli bo'lmagan tenglamalar sistemasini 0,001 aniqlikgacha yeching.

2) Nyuton usulini qo'llab, berilgan chiziqli bo'lmagan tenglamalar sistemasini 0,001 aniqlikgacha yeching.

№ 1. 1) $\begin{cases} \sin(x+1) - y = 1{,}2; \\ 2x + \cos y = 2. \end{cases}$ 2) $\begin{cases} \tan(xy + 0{,}4) = x^2; \\ 0{,}6x^2 + 2y^2 = 1, x > 0, y > 0. \end{cases}$

№ 2. 1) $\begin{cases} \cos(x-1) + y = 0{,}5; \\ x - \cos y = 3. \end{cases}$ 2) $\begin{cases} \sin(x+y) - 1{,}6x = 0; \\ x^2 + y^2 = 1, x > 0, y > 0. \end{cases}$

№ 3. 1) $\begin{cases} \sin x + 2y = 2; \\ \cos(y-1) + x = 0{,}7. \end{cases}$ 2) $\begin{cases} \tan(xy + 0{,}1) = x^2; \\ x^2 + 2y^2 = 1. \end{cases}$

№ 4. 1) $\begin{cases} \cos x + y = 1{,}5; \\ 2x - \sin(y - 0{,}5) = 1. \end{cases}$ 2) $\begin{cases} \sin(x+y) - 1{,}2x = 0{,}2; \\ x^2 + y^2 = 1. \end{cases}$

№ 5. 1) $\begin{cases} \sin(x + 0{,}5) - y = 1; \\ \cos(y-2) + x = 0. \end{cases}$ 2) $\begin{cases} \tan(xy + 0{,}3) = x^2; \\ 0{,}9x^2 + 2y^2 = 1. \end{cases}$

№ 6. 1) $\begin{cases} \cos(x+0{,}5)+y=0{,}8; \\ \sin y-2x=1{,}6. \end{cases}$ 2) $\begin{cases} \sin(x+y)-1{,}3x=0; \\ x^2+y^2=1. \end{cases}$

№ 7. 1) $\begin{cases} \sin(x-1)=1{,}3-y; \\ x-\sin(y+1)=0{,}8. \end{cases}$ 2) $\begin{cases} \tan xy=x^2; \\ 0{,}8x^2+2y^2=1. \end{cases}$

№ 8. 1) $\begin{cases} 2y-\cos(x+1)=0; \\ x+\sin y=-0{,}4. \end{cases}$ 2) $\begin{cases} \sin(x+y)-1{,}5x=0{,}1; \\ x^2+y^2=1. \end{cases}$

№ 9. 1) $\begin{cases} \cos(x+0{,}5)-y=2; \\ \sin y-2x=1. \end{cases}$ 2) $\begin{cases} \tan xy=x^2; \\ 0{,}7x^2+2y^2=1. \end{cases}$

№ 10. 1) $\begin{cases} \sin(x+2)-y=1{,}5; \\ x+\cos(y-2)=0{,}5. \end{cases}$ 2) $\begin{cases} \sin(x+y)-1{,}2x=0{,}1; \\ x^2+y^2=1. \end{cases}$

№ 11. 1) $\begin{cases} \sin(y+1)-x=1{,}2; \\ 2y+\cos x=2. \end{cases}$ 2) $\begin{cases} \tan(xy+0{,}2)=x^2; \\ 0{,}6x^2+2y^2=1. \end{cases}$

№ 12. 1) $\begin{cases} \cos(y-1)+x=0{,}5; \\ y-\cos x=3. \end{cases}$ 2) $\begin{cases} \sin(x+y)=1{,}5x-0{,}1; \\ x^2+y^2=1. \end{cases}$

№ 13. 1) $\begin{cases} \sin y+2x=2; \\ \cos(x-1)+y=0{,}7. \end{cases}$ 2) $\begin{cases} \tan(xy+0{,}4)=x^2; \\ 0{,}8x^2+2y^2=1. \end{cases}$

№ 14 1) $\begin{cases} \cos y+x=1{,}5; \\ 2y-\sin(x-0{,}5)=1. \end{cases}$ 2) $\begin{cases} \sin(x+y)=1{,}2x-0{,}1; \\ x^2+y^2=1. \end{cases}$

№ 15. 1) $\begin{cases} \sin(y+0{,}5)-x=1; \\ \cos(x-2)+y=0. \end{cases}$ 2) $\begin{cases} \tan(xy+0{,}1)=x^2; \\ 0{,}9x^2+2y^2=1. \end{cases}$

№ 16. 1) $\begin{cases} \cos(y+0{,}5)+x=0{,}8; \\ \sin x-2y=1{,}6. \end{cases}$ 2) $\begin{cases} \sin(x+y)-1{,}4x=0; \\ x^2+y^2=1. \end{cases}$

№ 17. 1) $\begin{cases} \sin(y-1)+x=1{,}3; \\ y-\sin(x+1)=0{,}8. \end{cases}$ 2) $\begin{cases} \tan(xy+0{,}1)=x^2; \\ 0{,}5x^2+2y^2=1. \end{cases}$

№ 18. 1) $\begin{cases} 2x-\cos(y+1)=0; \\ y+\sin x=-0{,}4. \end{cases}$ 2) $\begin{cases} \sin(x+y)=1{,}1x-0{,}1; \\ x^2+y^2=1. \end{cases}$

№ 19. 1) $\begin{cases} \cos(y+0{,}5)-x=2; \\ \sin x-2y=1. \end{cases}$ 2) $\begin{cases} \tan(x-y)-xy=0; \\ x^2+2y^2=1. \end{cases}$

№ 20. 1) $\begin{cases} \sin(y+2)-x=1{,}5; \\ y+\cos(x-2)=0{,}5. \end{cases}$ 2) $\begin{cases} \sin(x-y)-xy=-1; \\ x^2-y^2=\frac{3}{4}. \end{cases}$

№ 21. 1) $\begin{cases} \sin(x+1) - y = 1; \\ 2x + \cos y = 2. \end{cases}$ 2) $\begin{cases} \tan(xy + 0{,}2) = x^2; \\ x^2 + 2y^2 = 1. \end{cases}$

№ 22. 1) $\begin{cases} \cos(x-1) + y = 0.8; \\ x - \cos y = 2. \end{cases}$ 2) $\begin{cases} \sin(x+y) - 1{,}5x = 0; \\ x^2 + y^2 = 1. \end{cases}$

№ 23. 1) $\begin{cases} \sin x + 2y = 1{,}6; \\ \cos(y-1) + x = 1. \end{cases}$ 2) $\begin{cases} \tan xy = x^2; \\ 0{,}5x^2 + 2y^2 = 1. \end{cases}$

№ 24. 1) $\begin{cases} \cos x + y = 1{,}2; \\ 2x - \sin(y - 0{,}5) = 2. \end{cases}$ 2) $\begin{cases} \sin(x+y) = 1{,}2x - 0{,}2; \\ x^2 + y^2 = 1. \end{cases}$

№ 25. 1) $\begin{cases} \sin(x + 0{,}5) - y = 1{,}2; \\ \cos(y-2) + x = 0. \end{cases}$ 2) $\begin{cases} \tan(xy + 0{,}1) = x^2; \\ 0{,}7x^2 + 2y^2 = 1. \end{cases}$

№ 26. 1) $\begin{cases} \cos(x + 0{,}5) + y = 1; \\ \sin y - 2x = 2. \end{cases}$ 2) $\begin{cases} \sin(x+y) - 1{,}5x = 0{,}2; \\ x^2 + y^2 = 1. \end{cases}$

№ 27. 1) $\begin{cases} \sin(x-1) + y = 1{,}5; \\ x - \sin(y+1) = 1. \end{cases}$ 2) $\begin{cases} \tan xy = x^2; \\ 0{,}6x^2 + 2y^2 = 1. \end{cases}$

№ 28. 1) $\begin{cases} \sin(y+1) - x = 1; \\ 2y + \cos x = 2. \end{cases}$ 2) $\begin{cases} \sin(x+y) - 1{,}2x = 0; \\ x^2 + y^2 = 1. \end{cases}$

№ 29. 1) $\begin{cases} \cos(y-1) + x = 0{,}8; \\ y - \cos x = 2. \end{cases}$ 2) $\begin{cases} \tan(xy + 0{,}3) = x^2; \\ 0{,}5x^2 + 2y^2 = 1. \end{cases}$

№ 30. 1) $\begin{cases} \cos(x-1) + y = 1; \\ \sin y + 2x = 1{,}6. \end{cases}$ 2) $\begin{cases} \sin(x+y) - 1{,}1x = 0{,}1; \\ x^2 + y^2 = 1. \end{cases}$

III BOB. MATRITSALAR ALGEBRASI

3.1-§. MATRITSA TUSHUNCHASI.

Chiziqli tenglamalar sistemasini yechish masalasi bu sistemaning koeffitsientlaridan tuzilgan ushbu

$$A = \begin{pmatrix} a_{11}\ a_{12}\ \cdots\ a_{1n} \\ a_{21}\ a_{22}\ \cdots\ a_{2n} \\ \cdots\cdots\cdots\cdots\cdots \\ a_{n1}\ a_{12}\ \cdots\ a_{sn} \end{pmatrix}$$

to‘g‘ri burchakli to‘rtburchak jadvalning xossalariga bog‘liq. Bu jadval s ta satrli n ta ustunli matritsa $(s \times n - \text{matritsa})$ deyiladi va (a_{ij}) yoki $\|a_{ij}\|$ ko‘rinishda belgilanadi.

A -ixtiyoriy $s \times n -$ matritsa bo‘lsin. A matritsada qandaydir *k* ta va *k* ta ustunlarning kesishigan joyidagi elementlaridan tashkil topgan *k* -tartibli matritsaning determinanti *k -tartibli minor* deyiladi.

A -kvadrat matritsa bo‘lsin $(n = s)$. Bu holda $M = M_{i_1 i_2 \cdots i_k}^{j_1 j_2 \cdots j_k}$ minorning elementlaridan o‘tmaydigan satrlar va ustunlarning kesishishidan hosil bo‘lgan M' minor M ga to‘ldiruvchi minor deb aytiladi. Ushbu

$$A_{i_1 i_2 \cdots i_k}^{j_1 j_2 \cdots j_k} = (-1)^{i_1 + i_2 + \cdots i_k + j_1 + j_2 + \cdots + j_k} M'{}_{i_1 i_2 \cdots i_k}^{j_1 j_2 \cdots j_k}$$

son esa M minorning *algebraik to‘ldiruvchisi* deyiladi

Satrlari $E_1, E_2, \cdots, E_n$ ortlardan iborat n -tartibli ushbu

$$E = \begin{pmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & 1 \end{pmatrix}$$

matritsa n -tartibli birlik matritsa deyiladi.

A –haqiqiy $a_{ij},\ i, j = \overline{1, n}$ elementli (n×n)-o‘lchovli matritsa. Quyidagicha belgilashlar kiritamiz: $A^T - transponirlangan\ matritsa, A^{-1}\ - teskari\ matritsa, detA - matritsa\ diterminanti. E - (\text{n} \times \text{n}) -$ o‘lchovli $birlik$ matritsa.

Ta’rif 3.1. Agar $a_{ij} = 0,\ i \neq j$ bo‘lsa, u holda A matritsa diogonal matritsa deyiladi.

Ta’rif 3.2. Agar $a_{ij} = 0,\ i < j$ bo‘lsa, u holda A matritsa quyi uchburchak matritsa deyiladi.

Ta’rif 3.3. Agar $a_{ij} = 0,\ i > j$ bo‘lsa, u holda A matritsa yuqori uchburchak matritsa deyiladi.

Ta’rif 3.4. Agar $detA \neq 0$ bo‘lsa, u holda A matritsa aynimagan matritsa deyiladi.

Ta’rif 3.5. Agar $A^T A = E$ bo‘lsa, u holda A matritsa ortogonal matritsa deyiladi.

Ta’rif 3.6. Agar $A^T = A$ bo‘lsa, u holda A matritsa simmetrik matritsa deyiladi.

Ta’rif 3.7. Agar barcha $i\ va\ j$ lar uchun $a^*_{i,j} = \bar{a}_{i,j}$ bo‘lsa (bu yerda $\bar{a}_{i,j}$ qo‘shma kompleks son), elementlari $a^*_{i,j}$ dan iborat bo‘lgan A^* matritsa berilgan A matritsaga nisbatan qo‘shma matritsa deyiladi.

Ta’rif 3.8. Agar A kvadrat matritsa o‘zining qo‘shmasi A^* bilan ustma-ust tushsa, ya’ni $A^* = A$ bo‘lsa, u Ermit matritsasi yoki o‘z-o‘ziga qo‘shma matritsa deyiladi.

Ta’rif 3.9. Agar $AA^* = E$ tenglik bajarilsa, u holda A unitar matritsa deyiladi.

Ta’rif 3.10. Agar $AB = BA = E$ tenglik o‘rinli bo‘lsa, A matritsa teskarilanuvchi deyiladi va B matritsa A ga teskari matritsa deyiladi.

$B = A^{-1}$ bo‘ladi.

Ta’rif 3.11. Agar biror noldan farqli x vektor uchun

$$A\bar{x} = \lambda\bar{x}$$

tenglik bajarilsa, u holda λ son A kvadrat matritsaning xos soni yoki xarakteristik soni deyiladi. Bu tenglikni qanoatlantiradigan har qandan noldan farqli $\bar{x}$ vektor A matritsaning λ xos soniga mos keladigan xos vektori deyiladi.

Ta’rif 3.12. A matritsaning satrlari bilan ustunlarining o‘rnini almashtirishdan hosil bo‘lgan A^T -matritsa A matritsaning $transponirlangan\ matritsasi$ deyiladi

3.2- §. TESKARI MATRITSANI HISOBLASH .

Teskari matritsani topish algoritmi:

1. A matritsaning determinantini hisoblash.
2. A_n Algebraik to‘ldiruvchilarini topish.
3. $(A_j)^T$ larni topish.
4. Teskari matritsani hisoblash: $A^{-1} = \frac{1}{\Delta} \cdot (A_j)^T$.

Misol 3.1. Ushbu

$$A = \begin{pmatrix} 2 & 4 & 5 \\ 3 & -1 & 2 \\ -4 & 1 & 1 \end{pmatrix}$$

matritsaga teskari matritsani toping.

Yechish:

a) Matritsa determinantini topamiz

$$\Delta = \begin{vmatrix} 2 & 4 & 5 \\ 3 & -1 & 2 \\ -4 & 1 & 1 \end{vmatrix} = \begin{vmatrix} 18 & 0 & 1 \\ -1 & 0 & 3 \\ -4 & 1 & 1 \end{vmatrix} = 1 \cdot (-1)^{3+2} \begin{vmatrix} 18 & 1 \\ -1 & 3 \end{vmatrix} =$$

$= -(18 \cdot 3 + 1) = -55 \neq 0$, ya’ni teskari matritsa mavjud.

b) Algebraik to‘ldiruvchilarni topib, A ga qo‘shma A^* matritsani tuzamiz.

$A_{11} = (-1)^2 \begin{vmatrix} -1 & 2 \\ 1 & 1 \end{vmatrix} = -3;$ $A_{12} = (-1)^3 \begin{vmatrix} 3 & 2 \\ -4 & 1 \end{vmatrix} = -11;$ $A_{13} =$

$= (-1)^4 \begin{vmatrix} 3 & -1 \\ -4 & 1 \end{vmatrix}$ =-1; $A_{21} = (-1)^3 \begin{vmatrix} 4 & 5 \\ 1 & 1 \end{vmatrix} = 1;$ $A_{22} = (-1)^4 \begin{vmatrix} 2 & 5 \\ -4 & 1 \end{vmatrix} = 22;$

$$A_{31} = (-1)^4 \begin{vmatrix} 4 & 5 \\ -1 & 2 \end{vmatrix} = 13; \quad A_{32} = (-1)^5 \begin{vmatrix} 2 & 5 \\ 3 & 2 \end{vmatrix} = 11;$$

$A_{33} = (-1)^6 \begin{vmatrix} 2 & 4 \\ 3 & -1 \end{vmatrix} = -14$. U holda $A^* = \begin{pmatrix} -3 & -11 & -1 \\ 1 & 22 & -18 \\ 13 & 11 & -14 \end{pmatrix}$.

c) Topilgan A^* qo‘shma matritsani transponirlaymiz

$$(A^*)^T = \begin{pmatrix} -3 & 1 & 13 \\ -11 & 22 & 11 \\ -1 & -18 & -14 \end{pmatrix}$$

d) Teskari matritsani hosil qilamiz.

$$A^{-1} = -\frac{1}{55} \begin{pmatrix} -3 & 1 & 13 \\ -11 & 22 & 11 \\ -1 & -18 & -14 \end{pmatrix}.$$

3.1-turdagi topshiriqlar.

Topshiriqlarni bajarish uchun na’muna.

Misol. Matritsaga teskari matritsani uni bo‘laklarga bo‘lish usuli bilan toping.

$$S=\begin{pmatrix} 1 & 1 & 3 & 4 \\ -1 & 0 & 3 & -2 \\ 2 & 1 & 2 & -3 \\ 1 & 2 & -1 & 1 \end{pmatrix}$$

Yechish..

$$S=\left(\begin{array}{cc|cc} 1 & 1 & 3 & 4 \\ -1 & 0 & 3 & -2 \\ \hline 2 & 1 & 2 & -3 \\ 1 & 2 & -1 & 1 \end{array}\right)$$

$S=\left(\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right)$ bo‘lsin, u holda $S^{-1}=\left(\begin{array}{c|c} K & L \\ \hline M & N \end{array}\right)$ bo‘ladi , bunda

$N=(D-CA^{-1}B)^{-1}, L=-A^{-1}BN,$

$M=-NCA^{-1}, K=A^{-1}-A^{-1}BM.$

A^{-1} matritsa D^{-1} ga nisbatan osonroq topiladi.

Yuqoridagilarni ketma-ket topamiz:

1. $A^{-1};\ A=\begin{pmatrix} 1 & 1 \\ -1 & 0 \end{pmatrix};\ \ \Delta=1;\ A^{-1}=\begin{pmatrix} 0 & -1 \\ 1 & 1 \end{pmatrix};$

2. $A^{-1}B=\begin{pmatrix} 0 & -1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 3 & 4 \\ 3 & -2 \end{pmatrix}=\begin{pmatrix} -3 & 2 \\ 6 & 2 \end{pmatrix};$

3. $CA^{-1}=\begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 0 & -1 \\ 1 & 1 \end{pmatrix}=\begin{pmatrix} 1 & -1 \\ 2 & 1 \end{pmatrix};$

4. $CA^{-1}B=\begin{pmatrix} 1 & -1 \\ 2 & 1 \end{pmatrix}\begin{pmatrix} 3 & 4 \\ 3 & -2 \end{pmatrix}=\begin{pmatrix} 0 & 6 \\ 9 & 6 \end{pmatrix};$

5. $D-CA^{-1}B=\begin{pmatrix} 2 & -3 \\ -1 & 1 \end{pmatrix}-\begin{pmatrix} 0 & 6 \\ 9 & 6 \end{pmatrix}=\begin{pmatrix} 2 & -9 \\ -10 & -5 \end{pmatrix};$

6. $N=\begin{pmatrix} 2 & -9 \\ -10 & -5 \end{pmatrix}^{-1}$; $\Delta=-100;$ A_{11}= -5; A_{12}=10; A_{21}=9; A_{22}=2

$$N=\begin{pmatrix} 1/2 & -9/100 \\ -1/10 & -1/50 \end{pmatrix};$$

7. $L=-A^{-1}BN=\begin{pmatrix} -3 & 2 \\ 6 & 2 \end{pmatrix}\begin{pmatrix} 1/20 & -9/100 \\ -1/10 & -1/50 \end{pmatrix}=\begin{pmatrix} 7/20 & -23/100 \\ -1/10 & 29/50 \end{pmatrix};$

8. $M=-NCA^{-1}=-\begin{pmatrix} 1/20 & -9/100 \\ -1/10 & -1/50 \end{pmatrix}\begin{pmatrix} 1 & -1 \\ 2 & 1 \end{pmatrix}=\begin{pmatrix} 13/100 & 14/100 \\ 7/50 & -4/50 \end{pmatrix};$

9. $K=A^{-1}-A^{-1}BM=\begin{pmatrix} 0 & -1 \\ 1 & 1 \end{pmatrix}-\begin{pmatrix} -3 & 2 \\ 6 & 2 \end{pmatrix}\begin{pmatrix} 13/100 & 14/100 \\ 7/50 & -4/50 \end{pmatrix}=$

$=\begin{pmatrix} 0 & -1 \\ 1 & 1 \end{pmatrix}-\begin{pmatrix} 11/100 & -58/100 \\ 106/100 & 68/100 \end{pmatrix}=\begin{pmatrix} 11/100 & -42/100 \\ -6/100 & 32/100 \end{pmatrix};$

10. $S^{-1}=\begin{pmatrix} 11/100 & -42/100 & 7/20 & -23/100 \\ -6/100 & 32/100 & -1/10 & 29/50 \\ 13/100 & 14/100 & 1/20 & -9/100 \\ 7/50 & -4/50 & -1/10 & -1/50 \end{pmatrix}.$

Tekshirish:

$$\begin{pmatrix} 1 & 1 & 3 & 4 \\ -1 & 0 & 3 & -2 \\ 2 & 1 & 2 & -3 \\ 1 & 2 & -1 & 1 \end{pmatrix}\cdot\begin{pmatrix} 11/100 & -42/100 & 7/20 & -23/100 \\ -6/100 & 32/100 & -1/10 & 29/50 \\ 13/100 & 14/100 & 1/20 & -9/100 \\ 7/50 & -4/50 & -1/10 & -1/50 \end{pmatrix}=$$

$$=\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

Mustaqil yechish uchun:

Quyidagi amallarni va misollarni mos ravishda bajaring:

Topshiriq. Matritsaga teskari matritsani uni bo‘laklarga bo‘lish usuli bilan toping.

№ 1. $A=\begin{pmatrix} 1 & 4 & 1 & 3 \\ 0 & -1 & 3 & -1 \\ 3 & 1 & 0 & 2 \\ 1 & -2 & 1 & 1 \end{pmatrix}.$

№ 2. $A=\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 4 & 2 & 3 \\ 1 & 10 & 3 & 6 \\ 6 & 10 & 1 & 4 \end{pmatrix}.$

№ 3. $A=\begin{pmatrix} 1 & 2 & 3 & -2 \\ 2 & -1 & -2 & -3 \\ 3 & 2 & -1 & 2 \\ 2 & -3 & 2 & 1 \end{pmatrix}.$

№ 4. $A=\begin{pmatrix} -2 & 2 & 1 & 0 \\ 1 & -3 & 3 & 7 \\ 2 & -1 & 2 & -3 \\ -5 & 4 & -1 & 2 \end{pmatrix}.$

№ 5. $A=\begin{pmatrix} 1 & -1 & -1 & 1 \\ -1 & 2 & 2 & 0 \\ 0 & -1 & 1 & 4 \\ 1 & 1 & -1 & -1.5 \end{pmatrix}$

№ 6. $A=\begin{pmatrix} 3 & -2 & 2 & 0 \\ 2 & 1 & 1 & -2 \\ 3 & -1 & 2 & 1 \\ 1 & 2 & -1 & -1 \end{pmatrix}.$

№ 7. $A=\begin{pmatrix} 5 & -4 & 0 & 2 \\ -1 & 1 & 1 & -1 \\ 2 & 3 & 1 & -6 \\ 1 & 0 & 2 & -1 \end{pmatrix}$.

№ 8. $A=\begin{pmatrix} 4 & -1 & 0 & 1 \\ 3 & 2 & -1 & 2 \\ 0 & 2 & 2 & 1 \\ -1 & 1 & -3 & -1 \end{pmatrix}$.

№ 9. $A=\begin{pmatrix} 1 & 2 & 0 & 1 \\ -1 & -3 & 3 & -1 \\ 0 & 4 & -10 & 2 \\ 1 & -1 & 2 & -1 \end{pmatrix}$.

№ 10. $A=\begin{pmatrix} 1 & 4 & -3 & 0 \\ 0 & 4 & 1 & 2 \\ -1 & 2 & 4 & 1 \\ 1 & 0 & -1 & 5 \end{pmatrix}$.

№ 11. $A=\begin{pmatrix} 2 & -1 & 1 & 2 \\ 1 & 2 & -1 & 1 \\ 3 & 0 & -1 & -3 \\ 1 & -1 & 1 & 3 \end{pmatrix}$.

№ 12. $A=\begin{pmatrix} 2 & 1 & 2 & 0 \\ -1 & -3 & 3 & -1 \\ 1 & 3 & -8 & 1 \\ 1 & -1 & 2 & -1 \end{pmatrix}$.

№ 13. $A=\begin{pmatrix} 2 & 3 & 0 & 1 \\ -1 & 1 & 3 & 0 \\ 0 & 2 & -1 & 1 \\ 3 & -1 & 1 & -2 \end{pmatrix}$.

№ 14. $A=\begin{pmatrix} -1 & 3 & 3 & 2 \\ -2 & 2 & 2 & 1 \\ 0 & 1 & 2 & 0 \\ -1 & 3 & 3 & 3 \end{pmatrix}$.

№ 15. $A=\begin{pmatrix} 1 & 0 & 3 & 1 \\ 0 & 1 & 5 & 1 \\ -3 & 4 & 10 & 1 \\ 0 & -6 & 0 & -1 \end{pmatrix}$.

№ 16. $A=\begin{pmatrix} 3 & 1 & 3 & 3 \\ 2 & 2 & 1 & 3 \\ 1 & 0 & 2 & 0 \\ 1 & 1 & 1 & 3 \end{pmatrix}$.

№ 17. $A=\begin{pmatrix} 2 & 1 & 0 & 1 \\ 2 & 3 & 2 & 0 \\ 0 & 1 & 2 & 0 \\ 1 & 0 & -2 & 1 \end{pmatrix}$.

№ 18. $A=\begin{pmatrix} -2 & -2 & -1 & 3 \\ 2 & 1 & 0 & -1 \\ 3 & 2 & 1 & -3 \\ 4 & 3 & 2 & -4 \end{pmatrix}$.

№ 19. $A=\begin{pmatrix} 1 & 1 & 2 & 3 \\ 3 & -1 & -1 & -2 \\ 2 & 3 & -1 & -1 \\ 1 & 2 & 3 & -1 \end{pmatrix}$.

№ 20. $A=\begin{pmatrix} 1 & 2 & 3 & -2 \\ 2 & -1 & -2 & -3 \\ 3 & 2 & -1 & 2 \\ 2 & -3 & 2 & 1 \end{pmatrix}$.

№ 21. $A=\begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 2 & 3 \\ 3 & 2 & 1 & 2 \\ 4 & 3 & 2 & 1 \end{pmatrix}$.

№ 22. $A=\begin{pmatrix} 1 & 1 & -3 & 0 \\ 2 & 5 & 1 & 2 \\ 0 & 6 & 4 & 1 \\ 6 & -1 & -1 & 5 \end{pmatrix}$.

№ 23. $A=\begin{pmatrix} 1 & 2 & -1 & 1 \\ 2 & -1 & 1 & 2 \\ 2 & 1 & -2 & -6 \\ 1 & -1 & 1 & 3 \end{pmatrix}$.

№ 24. $A=\begin{pmatrix} 2 & 5 & -3 & 2 \\ 0 & -4 & 5 & -2 \\ 1 & 3 & -8 & 1 \\ 1 & -1 & 2 & -1 \end{pmatrix}$.

№ 25. A=$\begin{pmatrix} 2 & 1 & -3 & 0 \\ 1 & 5 & 1 & 2 \\ 3 & 6 & 4 & 1 \\ 0 & -1 & -1 & 5 \end{pmatrix}$. **№ 26**. A=$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 2 & -3 \\ 3 & 2 & 1 & -2 \\ 4 & 3 & 2 & 1 \end{pmatrix}$.

№ 27. A=$\begin{pmatrix} 2 & -1 & 3 & 2 \\ 3 & 3 & 3 & 2 \\ 3 & -1 & -1 & 2 \\ 3 & -1 & 3 & -1 \end{pmatrix}$. **№ 28**. A=$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \\ 3 & 4 & 1 & 2 \\ 4 & 1 & 2 & 3 \end{pmatrix}$.

№ 29. A=$\begin{pmatrix} 2 & 1 & 1 & -1 \\ 2 & -1 & 0 & -3 \\ 3 & 0 & -1 & 1 \\ 2 & 2 & -2 & 5 \end{pmatrix}$. **№ 30**. A=$\begin{pmatrix} 2 & 1 & -3 & 4 \\ 1 & 0 & -2 & 3 \\ 3 & 2 & 0 & -5 \\ 4 & 3 & -5 & 0 \end{pmatrix}$.

3.2- turdagi topshiriq.

Topshiriqlarni bajarish uchun namuna.

Misol. Matritsani ikkita uchburchak matritsalar ko‘paytmasiga ajratish usuli bilan uning teskari matritsasini toping.

$$A = \begin{pmatrix} 1 & 1 & 3 & 4 \\ -1 & 0 & 3 & -2 \\ 2 & 1 & 2 & -3 \\ 1 & 2 & -1 & 1 \end{pmatrix}$$

Yechish. Bu misolni yechish quyidagi ketma-ketlikda bajariladi:

1. A matritsani $A = T_1T_2$ ko‘paytma ko‘rinishda yozish, bunda T_1 va T_2 – uchburchak matritsa.

2. T_1^{-1} va T_2^{-1} matritsalarni topish.

3. Izlanayotgan A^{-1} matritsani topilgan T_1^{-1} va T_2^{-1} matritsalar ko‘paytmasi ko‘rinishida yozish: A^{-1}=$T_2^{-1}T_1^{-1}$.

1, $T_1\, va\, T_2$ matritsalarni topish uchun quyidagi sxemadan foydalanamiz

Matritsa elementlari				$\sum$
a_{11}	a_{12}	a_{13}	a_{14}	C_1
a_{21}	a_{22}	a_{23}	a_{24}	C_2
a_{31}	a_{32}	a_{33}	a_{34}	C_3

a_{41}	a_{42}	a_{43}	a_{44}	C_4
t_{11} 1	r_{12}	r_{13}	r_{14}	C'_1
t_{21}	t_{22} 1	r_{23}	r_{24}	C'_2
t_{31}	t_{32}	t_{33} 1	r_{34}	C'_3
t_{41}	t_{42}	t_{43}	t_{44} 1	C'_4

$\sum$ ustun nazorat ustuni bo‘ladi; C_1 , C_2 , C_3 , C_4 – sonlar satr elementlari yig‘indisi.

Sxema elementlari quyidagi tartibda:

1. $t_{11}= a_{11}$; $t_{21} = a_{21}$; $t_{31}= a_{31}$; $t_{41}= a_{41}$.

2. $r_{12} = \frac{a_{12}}{t_{11}}$; $r_{13}= \frac{a_{13}}{t_{11}}$; $r_{14}= \frac{a_{14}}{t_{11}}$; $C'_1= \frac{c_1}{t_{11}}$

Tekshirish ustuni: c'_1=1+r_{12}+r_{13}+r_{14} .

3. t_{22}=a_{22}-$t_{21}r_{22}$; t_{32} =a_{32}-$t_{31}r_{12}$; t_{42} = a_{42}-$t_{41}r_{12}$

4. $r_{23}=\frac{a_{23}-t_{21}r_{13}}{t_{22}}$; $r_{24} =\frac{a_{24}-t_{21}r_{14}}{t_{22}}$; $c'_2=\frac{c_2-t_{21}c'_1}{t_{22}}$

Tekshirish ustuni: c'_2=1+r_{23}+ r_{24}.

5. t_{33}=a_{33}-$t_{31}r_{13}$-$t_{32}r_{23}$; t_{43}=a_{43}-$t_{41}r_{13}$-$t_{42}r_{23}$

6. $r_{34}=\frac{a_{34}-t_{31}r_{14}-t_{32}r_{24}}{t_{23}}$; $c'_3=\frac{c_3-t_{31}c'_1-t_{32}c'_2}{t_{33}}$.

Tekshirish ustuni: c'_3=1+r_{34}.

7. t_{44}=a_{44}-$t_{41}r_{14}$-$t_{42}r_{24}$-$t_{43}r_{34}$;

Tekshirish ustuni: $c'_4=\frac{c_4-t_{11}c'-t_{42}\,c'_2-t_{43}c'_3}{t_{44}}$ c'_4=1

Topilgan elementlardan matritsalar tuziladi

$$T_1 = \begin{pmatrix} t_{11} & 0 & 0 & 0 \\ t_{21} & t_{22} & 0 & 0 \\ t_{31} & t_{32} & t_{33} & 0 \\ t_{41} & t_{42} & t_{43} & t_{44} \end{pmatrix}, T_2=\begin{pmatrix} 1 & r_{12} & r_{13} & r_{14} \\ 0 & 1 & r_{23} & r_{24} \\ 0 & 0 & 1 & r_{34} \\ 0 & 0 & 0 & 0 \end{pmatrix}.$$

Ayni holda quyidagicha bo‘ladi:

Matritsa elementlari				∑
1	1	3	4	9
-1	0	3	-2	0
2	1	2	-3	2
1	2	-1	1	3
1 1	1	3	4	9
-1	1 1	6	2	9
2	-1	2 1	-4,5	-3,5
1	1	-10	-50 | 1	

$t_{22} = 0 - (-1) \cdot 1 = 1$; $t_{32} = 1 - 2 \cdot 1 = -1$; $t_{42} = 2 - 1 \cdot 1 = 1$,

$t_{23} = (3 - (-1) \cdot 3)/1 = 6$; $t_{24} = (-2 - (-1) \cdot 4)/1 = 2$;

$$c_2' = (0 - (-1) * 9)/1 = 9$$

(Tekshirish ustuni: 1+6+2=9=c'_2) ;

$t_{33} = 2 - 2 \cdot 3 - (-1) \cdot 6 = 2$; $t_{43} = -1 - 1 \cdot 3 - 1 \cdot 9 - 6 = -10$;

(Tekshirish ustuni: 1+(-4.5)=-3.5=c'_3)

$$r_{34} = \frac{-3 - 2 * 4 - (-1) * 32}{2} = 4.5; \quad c'_3 = \frac{2 - 2 * 9 - (-1) * 9}{2} = -3.5.$$

$$t_{44} = 1 - 1 \cdot 4 - 1 \cdot 2 - (-10) \cdot (-4.5) = -50 ;$$

$$c'_4 = \frac{3 - 1 * 9 - 1 * 9 - (-10) * (-3.5)}{-50} = 1$$

(tekshirish ustuni: s'_4 =1 bajariladi) .

Shunday qilib,

$$T_1 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 2 & -1 & 2 & 0 \\ 1 & -1 & -10 & -50 \end{pmatrix}, \quad T_2 = \begin{pmatrix} 1 & 1 & 3 & 4 \\ 0 & 1 & 6 & 2 \\ 0 & 0 & 1 & -4.5 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

2. T_1^{-1} matritsani aniqlash uchun $T_1 T_1^{-1} = E$ tenglikdan foydalanamiz va quyidagi tenglikga ega bo‘lamiz

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 2 & -1 & 2 & 0 \\ 1 & 1 & -10 & -50 \end{pmatrix} \begin{pmatrix} x_{11} & 0 & 0 & 0 \\ x_{21} & x_{22} & 0 & 0 \\ x_{31} & x_{32} & x_{33} & 0 \\ x_{41} & x_{42} & x_{43} & x_{44} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

Bu tenglikka mos ravishda quyidagi tenglamalar sistemasini topamiz

$$\begin{cases} x_{11} = 1, \\ -x_{11} + x_{21} = 0, \\ 2x_{11} - x_{21} + 2x_{31=} = 0, \\ x_{11} + x_{21} - 10x_{31} - 50x_{41} = 0\ , \\ x_{22} = 1\ . \end{cases} \qquad \begin{array}{c} x_{22} + 2x_{32} = 0, \\ x_{22} - 10x_{32} + 50x_{42} = 0, \\ 2x_{33} = 1, \\ -10x_{33} - 50x_{43} = 0, \\ -50x_{44} = 1. \end{array}$$

Bundan

$x_{11} = 1;\ x_{21} = 1\ ;\ x_{31} = -\frac{1}{2}\ ; x_{41} = \frac{7}{50}\ ;\ \ x_{22} = 1;\quad x_{32} = \frac{1}{2}\ ;$
$x_{42} = -\frac{2}{25};\ x_{33} = \frac{1}{2}\ ;\ x_{43} = -\frac{1}{10}\ ;\ x_{44} = -\frac{1}{50}.$

Bundan ko‘rinadiki,

$$T_1 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ -\frac{1}{2} & \frac{1}{0} & \frac{1}{2} & 0 \\ \frac{7}{50} & -\frac{2}{25} & -\frac{1}{10} & -\frac{1}{50} \end{pmatrix}$$

T_2^{-1} matritsani aniqlash uchun $T_2T_2^{-1}$=E tenglikni o‘rnatamiz:

$$\begin{pmatrix} 1 & 1 & 3 & 4 \\ 0 & 1 & 6 & 2 \\ 0 & 0 & 1 & -4.5 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & x_{12} & x_{13} & x_{14} \\ 0 & 1 & x_{23} & x_{24} \\ 0 & 0 & 1 & x_{34} \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Shunday qilib

$$\begin{cases} x_{12} + 1 = 0, \\ x_{13} + x_{23} + 3 = 0, \\ x_{23} + 6 = 0, \end{cases} \qquad \begin{cases} x_{14} + x_{24} + 3x_{34} + 4 = 0, \\ x_{24} + 6x_{34} + 2 = 0, \\ x_{34} - 4.5 = 0, \end{cases}$$

Bundan $x_{12} = -1;\quad x_{23} = -6\ ;\quad x_{13} = 3\ ;\quad x_{34} = 4.5\ ;\ \ x_{24} = -29\ ;\ x_{14} = 11.5$.

Demak

$$T_2^{-1} = \begin{pmatrix} 1 & -1 & 3 & 11.5 \\ 0 & 1 & -6 & -29 \\ 0 & 0 & 1 & 4.5 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

3. $A^{-1} = T_2^{-1}T_1^{-1}$ tenglikdan foydalanib, quyidagini topamiz

$$A^{-1} = \begin{pmatrix} 1 & -1 & 3 & 11.5 \\ 0 & 1 & -6 & -29 \\ 0 & 0 & 0 & 4.5 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{7}{50} & -\frac{2}{25} & -\frac{1}{10} & -\frac{1}{50} \end{pmatrix} =$$

$$= \begin{pmatrix} \frac{11}{100} & -\frac{21}{50} & \frac{7}{20} & -\frac{23}{100} \\ -\frac{6}{100} & \frac{8}{25} & -\frac{1}{10} & \frac{29}{50} \\ \frac{13}{100} & \frac{7}{50} & \frac{1}{20} & -\frac{9}{100} \\ \frac{7}{50} & -\frac{2}{25} & -\frac{1}{10} & -1/50 \end{pmatrix}$$

Hisoblash tugadi.

Mustaqil yechish uchun:

Quyidagi amallarni va misollarni mos ravishda bajaring:

Topshiriq. Matritsani ikkita uchburchak matritsalar koʻpaytmasiga ajratish usuli bilan uning teskari matritsasini toping.

№ 1. $A=\begin{pmatrix} 1 & 4 & 1 & 3 \\ 0 & -1 & 3 & -1 \\ 3 & 1 & 0 & 2 \\ 1 & -2 & 1 & 1 \end{pmatrix}$.

№ 2. $A=\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 4 & 2 & 3 \\ 1 & 10 & 3 & 6 \\ 6 & 10 & 1 & 4 \end{pmatrix}$.

№ 3. $A=\begin{pmatrix} 1 & 2 & 3 & -2 \\ 2 & -1 & -2 & -3 \\ 3 & 2 & -1 & 2 \\ 2 & -3 & 2 & 1 \end{pmatrix}$.

№ 4. $A=\begin{pmatrix} -2 & 2 & 1 & 0 \\ 1 & -3 & 3 & 7 \\ 2 & -1 & 2 & -3 \\ -5 & 4 & -1 & 2 \end{pmatrix}$.

№ 5. $A=\begin{pmatrix} 1 & -1 & -1 & 1 \\ -1 & 2 & 2 & 0 \\ 0 & -1 & 1 & 4 \\ 1 & 1 & -1 & -1.5 \end{pmatrix}$.

№ 6. $A=\begin{pmatrix} 3 & -2 & 2 & 0 \\ 2 & 1 & 1 & -2 \\ 3 & -1 & 2 & 1 \\ 1 & 2 & -1 & -1 \end{pmatrix}$.

№ 7. $A=\begin{pmatrix} 5 & -4 & 0 & 2 \\ -1 & 1 & 1 & -1 \\ 2 & 3 & 1 & -6 \\ 1 & 0 & 2 & -1 \end{pmatrix}$.

№ 8. $A=\begin{pmatrix} 4 & -1 & 0 & 1 \\ 3 & 2 & -1 & 2 \\ 0 & 2 & 2 & 1 \\ -1 & 1 & -3 & -1 \end{pmatrix}$.

№ 9. $A=\begin{pmatrix} 1 & 2 & 0 & 1 \\ -1 & -3 & 3 & -1 \\ 0 & 4 & -10 & 2 \\ 1 & -1 & 2 & -1 \end{pmatrix}$.

№ 10. $A=\begin{pmatrix} 1 & 4 & -3 & 0 \\ 0 & 4 & 1 & 2 \\ -1 & 2 & 4 & 1 \\ 1 & 0 & -1 & 5 \end{pmatrix}$.

№ 11. $A=\begin{pmatrix} 2 & -1 & 1 & 2 \\ 1 & 2 & -1 & 1 \\ 3 & 0 & -1 & -3 \\ 1 & -1 & 1 & 3 \end{pmatrix}$.

№ 12. $A=\begin{pmatrix} 2 & 1 & 2 & 0 \\ -1 & -3 & 3 & -1 \\ 1 & 3 & -8 & 1 \\ 1 & -1 & 2 & -1 \end{pmatrix}$.

№ 13. $A=\begin{pmatrix} 2 & 3 & 0 & 1 \\ -1 & 1 & 3 & 0 \\ 0 & 2 & -1 & 1 \\ 3 & -1 & 1 & -2 \end{pmatrix}$.

№ 14. $A=\begin{pmatrix} -1 & 3 & 3 & 2 \\ -2 & 2 & 2 & 1 \\ 0 & 1 & 2 & 0 \\ -1 & 3 & 3 & 3 \end{pmatrix}$.

№ 15. $A=\begin{pmatrix} 1 & 0 & 3 & 1 \\ 0 & 1 & 5 & 1 \\ -3 & 4 & 10 & 1 \\ 0 & -6 & 0 & -1 \end{pmatrix}$.

№ 16. $A=\begin{pmatrix} 3 & 1 & 3 & 3 \\ 2 & 2 & 1 & 3 \\ 1 & 0 & 2 & 0 \\ 1 & 1 & 1 & 3 \end{pmatrix}$.

№ 17. $A=\begin{pmatrix} 2 & 1 & 0 & 1 \\ 2 & 3 & 2 & 0 \\ 0 & 1 & 2 & 0 \\ 1 & 0 & -2 & 1 \end{pmatrix}$.

№ 18. $A=\begin{pmatrix} -2 & -2 & -1 & 3 \\ 2 & 1 & 0 & -1 \\ 3 & 2 & 1 & -3 \\ 4 & 3 & 2 & -4 \end{pmatrix}$.

№ 19. $A=\begin{pmatrix} 1 & 1 & 2 & 3 \\ 3 & -1 & -1 & -2 \\ 2 & 3 & -1 & -1 \\ 1 & 2 & 3 & -1 \end{pmatrix}$.

№ 20. $A=\begin{pmatrix} 1 & 2 & 3 & -2 \\ 2 & -1 & -2 & -3 \\ 3 & 2 & -1 & 2 \\ 2 & -3 & 2 & 1 \end{pmatrix}$.

№ 21. $A=\begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 2 & 3 \\ 3 & 2 & 1 & 2 \\ 4 & 3 & 2 & 1 \end{pmatrix}$.

№ 22. $A=\begin{pmatrix} 1 & 1 & -3 & 0 \\ 2 & 5 & 1 & 2 \\ 0 & 6 & 4 & 1 \\ 6 & -1 & -1 & 5 \end{pmatrix}$.

№ 23. $A=\begin{pmatrix} 1 & 2 & -1 & 1 \\ 2 & -1 & 1 & 2 \\ 2 & 1 & -2 & -6 \\ 1 & -1 & 1 & 3 \end{pmatrix}$.

№ 24. $A=\begin{pmatrix} 2 & 5 & -3 & 2 \\ 0 & -4 & 5 & -2 \\ 1 & 3 & -8 & 1 \\ 1 & -1 & 2 & -1 \end{pmatrix}$.

№ 25. $A=\begin{pmatrix} 2 & 1 & -3 & 0 \\ 1 & 5 & 1 & 2 \\ 3 & 6 & 4 & 1 \\ 0 & -1 & -1 & 5 \end{pmatrix}$.

№ 26. $A=\begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 2 & -3 \\ 3 & 2 & 1 & -2 \\ 4 & 3 & 2 & 1 \end{pmatrix}$.

№ 27. A= $\begin{pmatrix} 2 & -1 & 3 & 2 \\ 3 & 3 & 3 & 2 \\ 3 & -1 & -1 & 2 \\ 3 & -1 & 3 & -1 \end{pmatrix}$.   **№ 28**. A= $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \\ 3 & 4 & 1 & 2 \\ 4 & 1 & 2 & 3 \end{pmatrix}$.

№ 29. A= $\begin{pmatrix} 2 & 1 & 1 & -1 \\ 2 & -1 & 0 & -3 \\ 3 & 0 & -1 & 1 \\ 2 & 2 & -2 & 5 \end{pmatrix}$.   **№ 30**. A= $\begin{pmatrix} 2 & 1 & -3 & 4 \\ 1 & 0 & -2 & 3 \\ 3 & 2 & 0 & -5 \\ 4 & 3 & -5 & 0 \end{pmatrix}$.

3.3-§. MATRITSALARNING XOS SON VA XOS VEKTORLARINI HISOBLASH

Noldan farqli $\vec{x}$ vektor uchun

$$A\vec{x} = \lambda\vec{x} \qquad (3.1)$$

tenglik bajarilsin. Undagi λ soni A kvadrat *matritsaning xos soni yoki xarakteristik soni*, $\vec{x}$ vektor A kvadrat matritsaning λ mos *xos vektori* (umuman, $a\vec{x}$ ham A matritsaning xos vektori bo‘ladi, bunda a - ixtiyoriy son). A matritsaning barcha xos sonlari to‘plami A matritsaning *spektori*, xos sonlar modullarining maksimumi A matritsaning $\rho(A)$ spektral radiusi,

$$D(\lambda) = \det(A - \lambda E) = \begin{vmatrix} a_{11} - \lambda & a_2 & \cdots & a_{1n} \\ a_{21} & a_{22} - \lambda & \cdots & a_{1n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{n1} & a_{n2} & \cdots & a_{11} - \lambda \end{vmatrix} = 0 \qquad (3.2)$$

A matritsaning *asriy yoki xarakteristik tenglamasi*, (3.2) tenglamaning chap qismidan iborat

$$\varphi(\lambda) = \det(A - \lambda E) = (-1)^n(\lambda^n - p_1\lambda^{n-1} - p_2\lambda^{n-2} - \cdots - p_n) \qquad (3.3)$$

n -darajali ko‘phad A matritsaning *xarakteristik ko‘phadi,*

$$P(\lambda) = \lambda^n - p_1\lambda^{n-1} - p_2\lambda^{n-2} - \cdots - p_n \qquad (3.4)$$

esa A matritsaning xos ko‘phadidir. Xos sonlar va xos vektorlarni topish uchun: 1) $P(\lambda)$ tuziladi, 2) $P(\lambda) = 0$ tenglamadan barcha λ_i $(i = \overline{1,n})$ xos sonlar topiladi, 3) ushbu

$$(A - \lambda_i E)\vec{x} = \vec{0} \tag{3.5}$$

bir jinsli tenglamalar sistemasidan $\vec{x}$ xos vektorlar aniqlanadi.

(3.4) ko‘phadning p_i koeffitsientlari A matritsaning $(-1)^{i-1}$ ishora bilan olingan i −tartibli minorlar yig‘indisiga teng:

$$p_1 = \sum_{j=1}^{n} a_{jj}, \qquad p_2 = \sum_{j\le k} \begin{vmatrix} a_{jj} & a_{jk} \\ a_{kj} & a_{kk} \end{vmatrix},$$

$$p_3 = \sum\nolimits_{j<k<i} \begin{vmatrix} a_{jj} & a_{jk} & a_{ji} \\ a_{kj} & a_{kk} & a_{ki} \\ a_{ij} & a_{ik} & a_{ii} \end{vmatrix}, \cdots, p_n = (-1)^{n-1}\text{detA}.$$

Iteratsiya usullarda xarakteristik ko‘pxad tuzib o‘tirilmaydi, to‘g‘ridan-to‘g‘ri xos sonlar va xos vektorlar bir vaqtning o‘zida topiladi. Aniq usullar xos sonlarning hammasini topishga (muammoni to‘liq hal qilishga), iteratsion usullar esa bitta yoki bir nechta xos son va xos vektorni topishga (muammoni qisman hal qilishga) imkon beradi. Hisoblashlarda

$$\lambda_1 + \lambda_2 + \cdots + \lambda_n = a_{11} + a_{22} + \cdots + a_{nn} = trA,$$
$$\lambda_1 \cdot \lambda_2 \cdot \cdots \cdot \lambda_n = \det A \tag{3.6}$$

tengliklardan keng foydalaniladilar.

A.N. Krilov usuli. 1) Noldan farqli ixtiyoriy $\vec{c}^{(0)} = (c_{01}, c_{02}, \cdots, c_{0n})'$ vektor tanlanadi, qolgan $\vec{c}^{(i)}$ $(i = \overline{1,n})$ vektorlar $\vec{c}^{(i)} = A^i \vec{c}^{(0)}$ yoki $\vec{c}^{(i)} = \overrightarrow{As}^{(i-1)}$ munosabat bo‘yicha aniqlanadi.

2) Ushbu

$$\begin{cases} p_1 c_{n-1,1} + p_2 c_{n-2,1} + \cdots + p_n c_{0,1} = c_{n,1}, \\ p_1 c_{n-1,2} + p_2 c_{n-2,2} + \cdots + p_n c_{0,2} = c_{n,2}, \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ p_1 c_{n-1,n} + p_2 c_{n-2,n} + \cdots + p_n c_{0,n} = c_{n,n} \end{cases} \tag{3.7}$$

sistema tuziladi va undan p_i lar aniqlanadi;

3) (3) xarakteristik ko‘phad tuziladi va undan λ_i xos sonlar aniqlanadi,

4) xos vektorlar aniqlanadi:

$$\vec{x}^{(i)} = \beta_{i1}\vec{c}^{(0)} + \beta_{i2}\vec{c}^{(2)} + \cdots + \beta_{im}\vec{c}^{(m-1)}, \quad m \le n, \tag{3.8}$$

bunda

$$\begin{cases} \beta_{i,m} = 1, \\ \beta_{i,m-1} = \lambda_i - p_1, \\ \beta_{i,m-2} = \lambda_i^2 - p_1\lambda_i - p_2 \\ \dots\dots\dots\dots\dots\dots\dots\dots \\ \beta_{i1} = \lambda_i^{m-1} - p_1\lambda_i^{m-2} - \dots - p_{m-1}. \end{cases} \qquad (3.9)$$

Misol. Ushbu $A = \begin{bmatrix} 5 & 30 & -48 \\ 3 & 14 & -24 \\ 3 & 15 & -25 \end{bmatrix}$ matritsaning xos sonlari va ularga mos xos vektorlari topilsin.

Yechish. 1) $\boldsymbol{s}^{(\mathbf{0})} = (1,0,0)'$ bo‘lsin. (6) munosabatga asosan:

$$\vec{s}^{(1)} = \begin{bmatrix} 5 & 30 & -48 \\ 3 & 14 & -24 \\ 3 & 15 & -25 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 5 \\ 3 \\ 3 \end{bmatrix}.$$

$$\vec{s}^{(2)} = \overrightarrow{As}^{(1)} = \begin{bmatrix} -29 \\ -15 \\ -15 \end{bmatrix}, \ \vec{s}^{(3)} = \overrightarrow{As}^{(2)} = \begin{bmatrix} 125 \\ 63 \\ 63 \end{bmatrix}.$$

2) (7) sistema:

$$\begin{cases} -29p_1 + 5p_2 + p_3 = 125, \\ -15p_1 + 3p_2 = 63, \\ -15p_1 + 3p_2 = 63. \end{cases}$$

Ikkinchi va uchinchi tenglamalarrning bir xil bo‘layotgani oldingi $\vec{s}^{(0)}, \vec{s}^{(1)}, \vec{s}^{(2)}$ vektorlar chiziqli bog‘langaniligini bildiradi. Sistemani shu vektorlarining chiziqli kombinatsiyasini tuzamiz:

$$\begin{cases} 5p_1 + p_2 = -29, \\ 3p_1 = -15, \end{cases}$$

Bundan $p_1 = -5,\ p_2 = -4;\ \ \varphi(\lambda) = \lambda^2 + 5\lambda + 4,\ \lambda_1 = -4,$ $\lambda_2 = -1;$

(6) munosabatga ko‘ra $\lambda_3 = 5 + 14 - 25 + 4 + 1 = -1. \lambda_1\ va\ \lambda_2$ ga mos bo‘lgan $\vec{x}^{(1)}\ va\ \vec{x}^{(2)}$ xos vektorlarni topamiz ($\vec{x}^{(3)}$ ni topish uchun $\vec{s}^{(0)}$ boshqacha tanlanishi kerak). $m = 2$ bo‘lganda β_{12}=1,

$\beta_{11} = \lambda_1 - p_1 = -4 + 5 = 1,\ \beta_{22} = 1, \beta_{21} = \lambda_2 - p_1 = -1 + 5 = 4,$

shunga ko‘ra $\vec{x}^{(1)} = \beta_{11}\vec{s}^{(0)} + \beta_{12}\vec{s}^{(1)} = 1 \cdot \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + 4 \cdot \begin{pmatrix} 5 \\ 3 \\ 3 \end{pmatrix} = \begin{pmatrix} 21 \\ 12 \\ 12 \end{pmatrix},$

$$\vec{x}^{(2)} = \beta_{21}\vec{s}^{(0)} + \beta_{22}\vec{s}^{(1)} = 4 \cdot \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + 1 \cdot \begin{pmatrix} 5 \\ 3 \\ 3 \end{pmatrix} = \begin{pmatrix} 9 \\ 3 \\ 3 \end{pmatrix}.$$

Levere usuli. Quyidagi

$$s_1 = \lambda_1 + \lambda_2 + \cdots + \lambda_n = SpA,$$
$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$
$$s_k = \lambda_1^k + \lambda_2^k + \cdots + \lambda_n^k = SpA^k, \qquad (3.10)$$
$$A^k = A^{k-1}A, k = \overline{0,n}, k \le n,$$

Munosabatlardan foydalanib,

$$\det(\lambda E - A) = \lambda^n + p_1\lambda^{n-1} + p_2\lambda^{n-2} + \cdots + p_n)$$

ko‘phadning p_k koeffitsientlari ketma-ket topiladi:

$$p_k = -\frac{1}{k}(s_k + p_1 s_{k-1} + p_2 s_{k-2} + \cdots + p_{k-1}s_1), \quad k = \overline{1,n}.$$

Misol 3.4. $A = \begin{bmatrix} 5 & 30 & -48 \\ 3 & 14 & -24 \\ 3 & 15 & -25 \end{bmatrix}$ matritsaning xarakteristik polinomi topilsin va λ_i xos sonlari topilsin.

Yechish. $s_1 = SpA = 5 + 14 - 25 = -6,$

$A^2 = A \cdot A = \begin{bmatrix} -29 & -150 & 240 \\ -15 & -74 & 120 \\ -15 & -75 & 121 \end{bmatrix},\ s_2 = SpA^2 = -29 - 74 + 121 = 18,$

$A^3 = A^2 \cdot A = \begin{bmatrix} 125 & 630 & -1008 \\ 63 & 314 & -504 \\ 63 & 315 & -505 \end{bmatrix}, s_3 = SpA^3 = 125 + 314 - 505 = = 66,$

$$p_1 = -\frac{1}{1}s_1 = 6, p_2 = -\frac{1}{2}(s_2 + p_1 s_1) = -\frac{1}{2}\big(18 + 6 \cdot (-6)\big) = 9,$$

$$p_3 = -\frac{1}{3}(s_3 + p_1 s_2 + p_2 s_1) = -\frac{1}{3}\big(-66 + 6 \cdot 18 + 9 \cdot (-6)\big) = 4,$$

$\det(\lambda E - A) = \lambda^{n3} + 6\lambda^2 + 9\lambda + 4 = 0$, bundan

$$\lambda_1 = -1, \qquad \lambda_2 = -4, \qquad \lambda_3 = -1.$$

3.3-turdagi topshiriq.

Topshiriqlarni bajarish uchun namuna.

Misol. *Matritsaning* xos son va xos vektorlari *bevosita o'rniga qo'yish usuli bilan 0,001 aniqlikda topilsin.*

$$A = \begin{pmatrix} 2 & -1 & 3 \\ -2 & 4 & 5 \\ 3 & 2 & -1 \end{pmatrix}.$$

Yechish: *Matritsaning xarakteristik tenglamasini tuzamiz. Bu tenglamaning ildizlari matritsaning xos sonlari bo'ladi.*

$$D(\lambda) = \begin{vmatrix} 2-\lambda & -1 & 3 \\ -2 & 4-\lambda & 5 \\ 3 & 2 & -1-\lambda \end{vmatrix} = 0.$$

Bundan

$$(2-\lambda)(4-\lambda)(-1-\lambda) - 15 - 12 - 9(4-\lambda) - 10(2-\lambda) - 2(-1-\lambda) = 0,$$

Qavslarni ochib, o'xshash hadlarni ixchamlaymiz. Hosil bo'lgan tenglamaning ikkala qismini -1 ga ko'paytirib

$$\lambda^3 - 5\lambda^2 - 19\lambda + 89 = 0$$

tenglamani hosil qilamiz.

Oldin bu tenglamaning ildizlarini ajratib olib, keyin uni Nyuton usuli bilan yechamiz va quyidagini hosil qilamiz

$$f(\lambda) = \lambda^3 - 5\lambda^2 - 19\lambda + 89; \quad f'(x) = 3\lambda^2 - 10\lambda - 19;$$

$$\lambda_{1,2} = \frac{5 \pm \sqrt{25+57}}{3} = \frac{5 \pm 9{,}1}{3}; \quad \lambda_1 = -1{,}2; \ \lambda_2 = 4{,}7.$$

$f(\lambda)$ funksiyaning ishoradari jadvalini tuzamiz:

λ	$-\infty$	$-1{,}2$	$4{,}7$	$+\infty$
sign$f(\lambda)$	$-$	$+$	$-$	$+$

Ishoralar jadvalidan ko'rinadiki, tenglama uchta haqiqiy ildizga ega:

$\lambda_1 \in]-\infty; -1{,}2]$; $\lambda_2 \in [-1{,}2; 4{,}7]$; $\lambda_3 \in [4{,}7; +\infty[$. Aniqlash uchun ulardan birini tanlaymiz, masalan λ_2.

Bu ildiz jovlashgan $[-1{,}2; 4{,}7]$ oraliqni kichraytiramiz.

Buning uchun $f(\lambda)$ funksiyaning qiymatini shu oraliqning ba’zi nuqtalarida tekshirib ko‘ramiz:

$f(2)$=$39 > 0$; $f(3)$=$14 > 0$; $f(4)$=$-3 < 0$. Shunday qilib λ_2 ildiz [3,4] oraliqning ichida joylashadi.

Ildizni

$$\lambda_{n+1} = \lambda_n + \frac{f(\lambda_n)}{f'(\lambda_n)}.$$

formuladan foydalanib aniqlaymiz.

Boshlang‘ich λ_0 yaqinlashishni tanlash uchun $[3, 4]$ oraliqda $f''(\lambda)$ ikkinchi tartibli hosilaning ishorasini aniqlaymiz; quyidagilarga egamiz

$3 \le \lambda \le 4\ da \quad f''(\lambda) = 6\lambda - 10; f''(\lambda) > 0\ ;\ \ demak, \lambda_0 = 3.$

Funksiya va uning hosilasining qiymatini hisoblash uchun Gorner sxemasidan foydalanamiz. Ildizni to‘rtta ishinchli o‘nli raqam bilan olamiz.

Barcha hisoblashlarni 3 ta jadvalga joylashtiramiz. Jadval III Jadval I da $f(\lambda)$ funksiyaning qiymatlari, Jadval II da $f'(\lambda)$ hosilaning qiymatlari, Jadval III da esa λ ning qiymatlarini aniqlaymiz.

Jadval I

N	λ_n	1	-5	-10	89
0	3		3	-6	-75
		1	-2	-25	14
1	3.63		3.63	4.9731	-87.0224
		1	-1.37	-23.9731	1.9776
2	3.75		3.75	-4.6875	-88.8281
		1	-1.25	-23.6875	0.1719
3	3.762		3.762	-4.6574	-88.9991
		1	-1.238	-23.6574	0.0009
4	3.7621		3.7621	-4.65710	-89.0004
		1	-1.2379	-23.65710	-0.0004

Jadval II

n	λ_n	3	-10	- 19
0	3		9	-3
		3	-1	-22
1	3.63		10.89	3.2307
		3	0.89	-15.7693
2	3.75		11.25	4.6875
		3	1.25	-14.3125
3	3.762		11.286	4.8379
		3	1.286	-14.1621

Jadval III

n	λ_n	$f(\lambda_n)$	$f'(\lambda_n)$	$f(\lambda_n)f'(\lambda_n)$
0	3	14	-22	-0,63
1	3,63	1,9776	-15,7693	-0,12
2	3,75	0,1719	-14,3125	-0,012
3	3,762	0,0009	-14,1621	-0,00006
4	3,7621			

Demak, $\lambda_2 \approx 3,7621$.

Ikki qolgan xos sonlarni topish uchun $f(\lambda)$ ko‘phadni $\lambda - 3,7621$ ga bo‘lishdan hosil bo‘lgan kvadrat tenglamani yechamiz:

$\lambda^2 - 1,2379\lambda - 23,6571 = 0$; $\lambda_{1,3}$

$$= 0,61895 \pm \sqrt{0,3831 + 23,6571} =$$

$$= 0,61895 \pm \sqrt{24,0402} = 4,90302; \;\; \lambda_1 = 4,2841; \;\; \lambda_3 = 5,5220.$$

1. Topilgan xos sonlarga mos xos vektorlarni topish uchun $(A - \lambda E)X = 0$ tenglikdan olingan chiziqli tenglamalar sistemasi foydalanamiz. $\lambda_1 = -4,2841$ da quyidagi sistemani olamiz

$$\begin{cases} 6,2841x_1 - x_2 + 3x_3 = 0, \\ -2x_1 + 8,28415x_2 + 5x_3 = 0, \\ 3x_1 + 2x_2 + 3,2841x_3 = 0. \end{cases}$$

Bu chiziqli bir jinsli tenglamalar sistemasi aniqlanmagan sistema bo‘ladi, chunki uning asosiy determinanti nolga teng.

Sistemaning ildizini topish uchun, uning ixtiyoriy ikkita tenglamasi olib birgalikda qaraladi, masalan ikkinchi va uchinchi tenglamalar:

$$\frac{x_1}{\begin{vmatrix} 8,2841 & 5 \\ 2 & -3,2841 \end{vmatrix}} = \frac{x_2}{\begin{vmatrix} 5 & -2 \\ 3,2841 & 3 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} -2 & 8,2841 \\ 3 & 2 \end{vmatrix}} = C,$$

yoki

$$\frac{x_1}{17,2058} = \frac{x_2}{21,5622} = \frac{x_3}{-28,8523} = S.$$

Vektorning normasi $\|X_1\|$ birga teng bo‘lishi uchun, uning hamma koordinatalarini ularning modul bo‘yicha eng kattasiga bo‘lamiz va $X_1 == S(-0,597;\ -0,746; 1)$ ga ega bo‘lamiz.

Qolgan ikki xos vektorni ham xuddi shunday topamiz

$\lambda_2 = 3,7621$ da

$$\begin{cases} -1,7621x_1 - x_2 + 3x_3 = 0, \\ -2x_1 + 0,2379x_2 + 5x_3 = 0, \\ 3x_1 + 2x_2 - 4,7621x_3 = 0; \end{cases}$$

bo‘ladi.

$$\frac{x_1}{\begin{vmatrix} 0,2379 & 4 \\ 2 & -4,7621 \end{vmatrix}} = \frac{x_2}{\begin{vmatrix} 5 & 2 \\ -4,7621 & 3 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} -2 & 0,2379 \\ 3 & 2 \end{vmatrix}} = \mathrm{C};$$

$$\frac{x_1}{-11,13229} = \frac{x_2}{5,4758} = \frac{x_3}{-4,7137} = C;\ X_2 = C(1;\ -0,492; 0,423).$$

$\lambda_3 = 5,220$ da

$$\begin{cases} -3,522x_1 - x_2 + 3x_3 = 0, \\ -2x_1 - 1,522x_2 + 5x_3 = 0, \\ 3x_1 + 2x_2 - 6,522x_3 = 0; \end{cases}$$

bo‘ladi.

$$\frac{x_1}{\begin{vmatrix} -1,522 & 5 \\ 2 & -6,522 \end{vmatrix}} = \frac{x_2}{\begin{vmatrix} 5 & -2 \\ -6,522 & 3 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} -2 & -1,522 \\ 3 & 2 \end{vmatrix}} = C;$$

$$\frac{x_1}{-0,0735} = \frac{x_2}{1,956} = \frac{x_3}{8,566} = C;\ X_3 = C(-0,00858; 0,228; 1).$$

Shunday qilib, javob:

λ_1	x_{i1}	x_{i2}	x_{i3}
-4,284	-0,597	-0,746	1
3,762	1	-0,492	0,423
5,522	-0,00858	0,228	1

Mustaqil yechish uchun:

Topshiriqlar. Matritsaning *xos son va xos vektorlari* bevosita o‘rniga qo‘yish usuli bilan 0,001 aniqlikda topilsin.

№ **1**. $A = \begin{pmatrix} 1 & -1 & 2 \\ 1 & 2 & 1 \\ 2 & 0 & 3 \end{pmatrix}$. №**2**. $A = \begin{pmatrix} -1 & 1 & -1 \\ 4 & 4 & 2 \\ 1 & 2 & 3 \end{pmatrix}$.

№**3**. $A = \begin{pmatrix} 3 & 1 & 1 \\ 1 & 2 & 3 \\ -2 & 5 & -2 \end{pmatrix}$. №**4**. $A = \begin{pmatrix} 3 & 2 & -1 \\ 2 & 4 & 1 \\ -1 & 0 & -2 \end{pmatrix}$.

№**5**. $A = \begin{pmatrix} 5 & 2 & 5 \\ -2 & -1 & -8 \\ 2 & -3 & -2 \end{pmatrix}$. №**6**. $A = \begin{pmatrix} 6 & 3 & 8 \\ 3 & 2 & 3 \\ -1 & 0 & -4 \end{pmatrix}$.

№**7**. $A = \begin{pmatrix} 1 & 4 & 1 \\ -1 & 1 & 3 \\ 1 & 2 & -1 \end{pmatrix}$. №**8**. $A = \begin{pmatrix} 1 & 2 & 3 \\ 7 & -1 & 5 \\ 2 & 3 & -3 \end{pmatrix}$.

№**9**. $A = \begin{pmatrix} 1 & -1 & -1 \\ 0 & 6 & 2 \\ -6 & -3 & -2 \end{pmatrix}$. №**10**. $A = \begin{pmatrix} 1 & 1 & -4 \\ 3 & -3 & 2 \\ -5 & 1 & 4 \end{pmatrix}$.

№**11**. $A = \begin{pmatrix} 2 & 5 & -6 \\ 4 & 3 & -1 \\ -1 & 2 & -2 \end{pmatrix}$. №**12**. $A = \begin{pmatrix} 3 & 1 & 1 \\ 2 & -1 & 3 \\ 4 & 1 & 1 \end{pmatrix}$.

№**13**. $A = \begin{pmatrix} 2 & 1 & -1 \\ 1 & 1 & 2 \\ -1 & 2 & 0 \end{pmatrix}$. №**14**. $A = \begin{pmatrix} -2 & 4 & 3 \\ 1 & 5 & 1 \\ 2 & -4 & -1 \end{pmatrix}$.

№**15**. $A = \begin{pmatrix} -2 & 1 & -1 \\ 0 & 4 & 2 \\ -1 & 2 & 3 \end{pmatrix}$. №**16**. $A = \begin{pmatrix} 2 & 3 & 2 \\ 4 & -6 & -4 \\ -1 & 4 & 7 \end{pmatrix}$.

№**17**. $A = \begin{pmatrix} -5 & -4 & 4 \\ 2 & 3 & 1 \\ 5 & 6 & 0 \end{pmatrix}$. №**18**. $A = \begin{pmatrix} -1 & 3 & -1 \\ 4 & -2 & 1 \\ 3 & -2 & 4 \end{pmatrix}$.

$$№\mathbf{19}.\ A=\begin{pmatrix} 3 & 1 & 0 \\ 4 & 2 & 3 \\ -4 & 2 & -1 \end{pmatrix}. \qquad №\mathbf{20}.\ A=\begin{pmatrix} 6 & -3 & 5 \\ 1 & 3 & 5 \\ -2 & 4 & -3 \end{pmatrix}.$$

$$№\mathbf{21}.\ A=\begin{pmatrix} 1 & -2 & 2 \\ -2 & 1 & 1 \\ 3 & -1 & 2 \end{pmatrix}. \qquad №\mathbf{22}.\ A=\begin{pmatrix} 2 & 1 & -4 \\ -2 & 4 & 1 \\ -3 & 0 & 3 \end{pmatrix}.$$

$$№\mathbf{23}.\ A=\begin{pmatrix} 1 & 2 & -1 \\ -2 & 4 & -6 \\ -1 & 0 & -2 \end{pmatrix}. \qquad №\mathbf{24}.\ A=\begin{pmatrix} -1 & 3 & 1 \\ 7 & 2 & 4 \\ 5 & 1 & 2 \end{pmatrix}.$$

$$№\mathbf{25}.\ A=\begin{pmatrix} -3 & 1 & 0 \\ 4 & -1 & 3 \\ 6 & 2 & 3 \end{pmatrix}. \qquad №\mathbf{26}.\ A=\begin{pmatrix} -4 & 1 & -1 \\ 2 & -2 & 3 \\ 3 & 1 & 3 \end{pmatrix}.$$

$$№\mathbf{27}.\ A=\begin{pmatrix} 3 & 1 & -5 \\ -3 & 1 & 1 \\ -4 & 3 & -4 \end{pmatrix}. \qquad №\mathbf{28}.\ A=\begin{pmatrix} 2 & 1 & -1 \\ -2 & 3 & 6 \\ 1 & 1 & -2 \end{pmatrix}.$$

$$№\mathbf{29}.\ A=\begin{pmatrix} 0 & 3 & 4 \\ 7 & 1 & 4 \\ 2 & 1 & -2 \end{pmatrix}. \qquad №\mathbf{30}.\ A=\begin{pmatrix} 1 & 2 & 1 \\ 9 & 1 & 2 \\ 2 & -1 & 0 \end{pmatrix}.$$

3.4-turdagi topshiriq.

Topshiriqlarni bajarish uchun namuna.

Misol. Matritsaning xos son va xos vektorlarini Krilov usuli bilan toping. Xos sonlarni to‘rtta ishonchli raqam bilan, xos vektorlarni esa uchta o‘nli raqam bilan toping

$$\begin{pmatrix} 2{,}2 & 1 & 0{,}5 & 2 \\ 1 & 1{,}3 & 2 & 1 \\ 0{,}5 & 2 & 0{,}5 & 1{,}6 \\ 2 & 1 & 1{,}6 & 2 \end{pmatrix}.$$

Yechish:

$$\lambda^4 - p_1\lambda^3 - p_2\lambda^2 - p_3\lambda - p_4 = 0$$

Xarakteristik tenglamaning koeffitsientini aniqlash uchun quyidagi vektorlar ketma ketligini quramiz:

B_0—ixtiyoriy vektor; $B_1 = AB_0$; $B_2 = AB_1$; $B_3 = AB_2$; $B_4 = AB_3$

Agar B_0, B_1, B_2, B_3 vektorlar chiziqli erkli bo‘lsa, u holda p_1, p_2, p_3, p_4 koeffitsientlar

$$B_4 = p_1 B_3 + p_2 B_2 + p_3 B_1 + p_4 B_0$$

tenglikka mos keluvchi chiziqli tenglamalar sistemasi yechimidan aniqlanadi. Chiziqli tenglamadar sistemasini Xaletskiy sxemasidan foydalanib yechamiz. Barcha hisoblashlarni jadvalga joylashtiramiz.

1 -jadval

A				B_0	B_1	B_2	B_3	B_4	Σ
2,2	1	0,5	2	1	2,2	10,1	52,373	291	356,6636
1	1	2	1	0	1	6,5	41,84	239,61	288,945
0,5	2	0,5	1,6	0	0,5	6,55	37,64	220,78	265,4725
2	1	1,6	2	0	2	10,2	57,56	321,93	391,69
				1 1	2,2	10	52,373	291	356,6636
				0	1 1	6,5	41,84	239,61	288,945
				0	2	3 1	5,066667	30,6	36,666667
				0	2	-2,8	-11,93333 1	6,00000	7,00000
							1	6	7
						1		0,2	1,2
					1			-12,74	-11,735
				1				2,7616	3,7616

Shunday qilib matritsaning xarakteristik tenglamasi

$$\lambda^4 - 6\lambda^3 - 0{,}2\lambda^2 + 12{,}735\lambda - 2{,}7616 = 0. \qquad (*)$$

ko‘rinishda bo‘ladi.

1) Matritsaning xos sonlarini topish uchun xarakteristik tenglamani biror usul bilan yechish kerak.

(*) tenglamani Lobachevskiy usuli bilan yechamiz va hisoblashlarni 2-jadvalda keltiramiz.

2-jadval

m	2^m	a_0	a_1	a_2	a_3	a_4
0	2	1	-6	-0,2	12,735	-2,7616
1	2	1	36	0,04	-162,1802	7,6264

			-0,4	+152,82 -5,5232	-1,1046	
		1	$3{,}64\cdot 10^{1}$	$1{,}4734\cdot 10^{2}$	$1{,}6108\cdot 10^{2}$	7,6264
2	4	1	$13{,}2496\cdot 10^{2}$ $-2{,}9468\cdot 10^{2}$	$2{,}1707\cdot 10^{4}$ $-1{,}1727\cdot 10^{4}$ $-0{,}0015\cdot 10^{4}$	$2{,}5947\cdot 10^{4}$ $-2{,}2247\cdot 10^{4}$	$58{,}162\cdot 10^{0}$
		1	$1{,}0303\cdot 10^{3}$	$9{,}967\cdot 10^{3}$	$2{,}3700\cdot 10^{4}$	$5{,}8162\cdot 10^{1}$
3	8	1	$1{,}0615\cdot 10^{6}$ $-0{,}0199\cdot 10^{6}$	$9{,}9341\cdot 10^{7}$ $-4{,}8836\cdot 10^{7}$	$5{,}6169\cdot 10^{8}$ $-0{,}0116\cdot 10^{8}$	$33{,}828\cdot 10^{2}$
		1	$1{,}0416\cdot 10^{6}$	$5{,}0505\cdot 10^{7}$	$5{,}6053\cdot 10^{8}$	$3{,}3828\cdot 10^{3}$
4	16	1	$1{,}0849\cdot 10^{12}$ $-0{,}0001\cdot 10^{12}$	$2{,}5508\cdot 10^{15}$ $-1{,}1677\cdot 10^{15}$	$3{,}1419\cdot 10^{17}$ $-0{,}0000\cdot 10^{17}$	$11{,}443\cdot 10^{6}$
		1	$1{,}0848\cdot 10^{12}$	$1{,}3831\cdot 10^{15}$	$3{,}1419\cdot 10^{17}$	$1{,}1443\cdot 10^{7}$
5	32	1	$1{,}1768\cdot 10^{24}$	$1{,}9130\cdot 10^{34}$ $-0{,}6817\cdot 10^{30}$	$9{,}8715\cdot 10^{34}$ 0	$1{,}3094\cdot 10^{14}$
		1	$1{,}1768\cdot 10^{24}$	$1{,}2313\cdot 10^{30}$	$9{,}8715\cdot 10^{34}$	$1{,}3094\cdot 10^{14}$
6	64	1	$1{,}3849\cdot 10^{48}$ 0	$1{,}5161\cdot 10^{60}$ $-0{,}2334\cdot 10^{60}$	$9{,}7447\cdot 10^{69}$ 0	$1{,}7145\cdot 10^{28}$
		1	$1{,}3849\cdot 10^{48}$	$1{,}2827\cdot 10^{60}$	$9{,}7447\cdot 10^{69}$	$1{,}7145\cdot 10^{28}$
7	123	1	$1{,}9179\cdot 10^{96}$ 0	$1{,}6453\cdot 10^{120}$ $-0{,}0270\cdot 10^{120}$	$9{,}4959\cdot 10^{139}$ 0	$2{,}9395\cdot 10^{56}$
		1	$1{,}9179\cdot 10^{96}$	$1{,}6183\cdot 10^{120}$	$9{,}4959\cdot 10^{139}$	$2{,}9395\cdot 10^{56}$
8	256	1	$3{,}6783\cdot 10^{192}$	$2{,}6189\cdot 10^{240}$ $-0{,}0004\cdot 10^{240}$	$9{,}0172\cdot 10^{279}$	$8{,}6407\cdot 10^{112}$
		1	$3{,}6783\cdot 10^{192}$	$2{,}6185\cdot 10^{240}$	$9{,}0172\cdot 10^{279}$	$8{,}6407\cdot 10^{112}$

Quyidagiga egamiz:

$lg\lfloor\lambda\rfloor = \frac{1}{256} lg3.6735 \cdot 10^{192} = \frac{1}{256} \cdot 192.5657 = 0.7522;\ \lfloor\lambda_1\rfloor = 5.652;$

$$lg\lfloor\lambda_2\rfloor = \frac{1}{256} \cdot lg\frac{2.6158 \cdot 10^{240}}{3.6783 \cdot 10^{192}} = \frac{1}{256} \cdot (48 + 0.4180 - 0.5657) =$$

$$= \frac{1}{256} \cdot 47.8523 = 0.1869; \ \lfloor\lambda_2\rfloor = 1.538;$$

$$lg\lfloor\lambda_3\rfloor = \frac{1}{256} lg \frac{9.0172 \cdot 10^{279}}{2.6185 \cdot 10^{240}} = \frac{1}{256} \cdot (39 + 0.9550 - 0.4180) =$$
$$= \frac{1}{256} \cdot 39.537 = 0.1544; \ \lfloor\lambda_3\rfloor = 1.427.$$
$$lg\lfloor\lambda_4\rfloor = \frac{1}{256} \cdot lg \frac{8.6407 \cdot 10^{112}}{9.0172 \cdot 10^{279}} = \frac{1}{256} \cdot (-167 + 0.9366 - 0.9550) =$$
$$= \frac{1}{256} \cdot (-167.0184) = -0.6524 = 1.3476; \ \lfloor\lambda_4\rfloor = 0.2226.$$

Topilgan ildizlarni tenglamaga bevosita qo‘yib, ildizlarning ishoralarini topamiz. Bundan

$$\lambda_1 = 5{,}652; \ \lambda_2 = 1.538; \ \lambda_3 = -1.427; \ \lambda_4 = 0.2226$$

ni topamiz. Hisoblash ishlari uchun Gorner sxemasi qo‘llab, ildizlarning topilgan qiymatining oxirgi raqamini topamiz 3-jadval.

3 - jadval

	1	-6	-0,2	12,735	-2,7616
5,652		5,652	-1,9669	-12,24752	2,7584
	1	-0,348	-2,1669	0,48768	-0,0052
5,653		5,653	-1,96159	-12,21947	2,9143
1,538	1	-0,347	-2,16159	0,51553	0,1527
		1,538	-6,86256	-10,86222	2,8808
	1	-4,462	-7,06256	1,87278	0,118
1,540		1,540	-6,8684	-10,8853	2,8485
	1	-4,46	-7,0684	1,8497	0,0869
1,544		1,544	-6,88006	-10,9316	2,7844
	1	-4,456	-7,08006	1,8034	0,0228
	1	-6	-0,2	12,735	-2,7616
2		1,545	-6,88298	10,9432	2,7683
	1	-4,455	-708298	1,7918	0,0067

2		1,546	-6,88588	-10,95477	2,7522
	1	-4,454	-7,08588	1,78023	-0,0094
-1		-1,427	10,59833	-14,83842	3,0016
	1	-7,427	10,39833	-2,10342	0,24
-1		-1,425	10,58062	-14,79239	2,9318
	1	-7,425	10,38062	-2,05739	0,1702
-1		-1,421	10,54524	-14,70059	2,7931
	1	-7,421	10,34524	-1,96555	0,0315
-1		-1,42	10,5364	-14,6777	2,7586
	1	-7,42	10,3364	-1,9427	-0,003
0		0,2226	-1,28605	-0,33079	2,7612
	1	-5,7774	-1,48605	12,40421	-0,0004
0		0,2227	-1,286605	-0,33107	2,7626
	1	-5,7773	-1,486605	12,40393	0,0008

Matritsaning xos sonlari

$$\lambda_1 = 5{,}652;\ \lambda_2 = 1.538;\ \lambda_3 = -1.427;\ \lambda_4 = 0.2226$$

bo‘ladi.

2. λ_i xos sonlarga mos kelo‘vchi X_i xos

$$X_i = \beta_{i3}B_0 + \beta_{i2}B_1 + \beta_{i2}B_1 + \beta_{i1}B_2 + \beta_{i0}B_3,$$

formula bilan aniqlanadi. Oldindan topilgan B_0, B_1, B_2, B_3 vektor oldidagi koeffitsientlar

$$\frac{D(\lambda)}{\lambda - \lambda_i} = \beta_{i0}\lambda^3 + \beta_{i1`}\lambda^2 + \beta_{i2`}\lambda + \beta_{i3}$$

tenglikdan topiladi.

Xos vektorlarning oxirgi qiymati birga teng normaga $\|X_i\|_j$ ega bo‘lish kerak.

Barcha hisoblashlarni 4-jadvalda keltiramiz:

4 -jadval

λ_i	$\beta_{i3}B_0$	$\beta_{i2}B_1$	$\beta_{i1}B_2$	$\beta_{i0}B_3$	X_i	$\bar{X}_i$
5,652	0,4877	-4,7672	-3,5113	52,373	44,5822	0,879
	0	-2,1669	-2,2620	41,84	37,4111	0,753
	0	-1,0334	-2,2794	37,64	34,2772	0,690
	0	-4,3338	-3,5496	57,56	49,6766	1,0
1,545	1.7918	-15,5826	-44,9510	52,373	-6,3688	1
	0	-7,08298	-28,9575	41,84	5,7995	-0,911
	0	-3,5415	-29,1802	37,64	4,9183	-0,772
	0	-14,1660	-45,4410	57,56	-2,0470	0,321
-1,420	-1,9427	22,7400	-74,8678	52,373	-1,6975	0,293
	0	10,3364	-48,2300	41,84	3,9464	-0,681
	9	5,1682	-48,6010	37,64	-5,7928	1
	0	20,6728	-75,6840	57,56	2,5488	-0,440
0,2226	12,4042	-5,2692	-58,2940	52,373	3,2140	-0,740
	0	-1,4860	-37,5531	41,84	2,8009	-0,645
	0	-0,7430	-37,8420	37,64	-0,9450	-0,218
	0	-2,9720	-58,9295	57,56	-4,3415	1

IV BOB. CHIZIQLI ALGEBRAIK TENGLAMALAR SISTEMASINI YECHISH

4.1-§. GAUSS USULI

Chiziqli tenglamalar sistemasini yechish uchun Gausning kompakt sxemasini ko‘rib chiqamiz. Bizga

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n = a_{1,n+1}, \\ a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_{2,n} = a_{2,n+1}, \\ \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \\ a_{n1}x_1 + a_{n2}x_2 + \ldots + a_{nn}x_{2,n} = a_{n,n+1}. \end{cases} \tag{4.1}$$

chiziqli algebraik tenglamalar sistemasi berilgan bo‘lsin.

Birinchi tenglamadagi x_1 noma’lum oldidagi a_{11} koeffitsientni noldan farqli deb faraz qilamiz va uni yetakchi element deb ataymiz (aks holda shu sistemadagi x_1 noma’lum oldidagi koeffitsient noldan farqli bo‘lgan boshqa bir tenglamani tanlab olamiz. Bu holda yetakchi element tanlab olingan tenglamadagi x_1 noma’lum oldidagi koeffitsient bo‘ladi). (4.1) sistemadagi birinchi tenglamaning barcha koeffitsientlarini yetakchi elementga bo‘lib, quyidagilar tenglamani hosil qilamiz:

$$x_1 + b_{12}^{(1)}x_2 + \ldots + b_{1n}^{(1)}x_n = b_{1,n+1}^{(1)}. \tag{4.2}$$

Bu yerda

$$b_{1j}^{(1)} = \frac{a_{ij}}{a_{11}} \ (j \geq 2).$$

(4.2) tenglamani ketma-ket a_{21}, a_{31}, . . . larga ko‘paytirib, (4.1) sistemaning ikkinchi, uchinchi, to‘rtinchi va hokazo tenglamalaridan ayirsak ikkinchi, uchinchi, to‘rtinchi va hokazo tenglamalaridagix_1 noma’lum yo‘qoladi. Va quyidagi sistemani hosil qilamiz:

$$\begin{cases} a_{22}^{(1)}x_2 + a_{23}^{(1)}x_3 + \ldots + a_{2n}^{(1)}x_n = a_{2,n+1}^{(1)}, \\ a_{32}^{(1)}x_2 + a_{33}^{(1)}x_3 + \ldots + a_{3n}^{(1)}x_{3,n} = a_{3,n+1}^{(1)}, \\ \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \\ a_{n2}^{(1)}x_1 + a_{n3}^{(1)}x_2 + \ldots + a_{nn}^{(1)}x_{n,n} = a_{n,n+1}^{(1)}, \end{cases} \tag{4.3}$$

bu yerda

$$a_{ij}^{(1)} = a_{ij} - a_{i1} b_{1j}^{(1)} \quad (i, j \geq 2).$$

Yuqorida (4.1) sistema uchun qilingan ishlarni (4.3) sistema uchun ham aynan bajaramiz (faqat bu safar yetakchi element $a_{22}^{(1)}$ bo‘ladi). (4.3) sistemadagi birinchi tenglamaning barcha koeffitsientlarini yetakchi element $a_{22}^{(1)}$ ga bo‘lib, quyidagi tenglamani hosil qilamiz:

$$x_2 + b_{23}^{(2)} x_3 + \ldots + b_{2n}^{(2)} x_n = b_{2,n+1}^{(2)}, \qquad (4.4)$$

bu yerda

$$b_{2j}^{(2)} = \frac{a_{2j}^{(1)}}{a_{22}^{(1)}} \; (j \geq 3).$$

Endi (4.3) sistemadagi boshqa tenglamalaridan x_2 ni yuqorida ko‘rsatilgan tartibda yo‘qotib,

$$\begin{cases} a_{33}^{(2)} x_3 + a_{34}^{(2)} x_4 + \ldots + a_{3n}^{(2)} x_n = a_{3,n+1}^{(2)}, \\ a_{43}^{(2)} x_2 + a_{43}^{(2)} x_3 + \ldots + a_{4n}^{(2)} x_{3,n} = a_{4,n+1}^{(2)}, \\ \ldots\ldots\ldots\ldots\ldots \\ a_{n3}^{(2)} x_1 + a_{n3}^{(2)} x_2 + \ldots + a_{n3}^{(2)} x_{2,n} = a_{n,n+1}^{(2)}, \end{cases}$$

sistemaga kelamiz, bu yerda

$$a_{ij}^{(2)} = a_{ij}^{(1)} - a_{i2}^{(1)} b_{2j}^{(2)} \quad (i, j \geq 3).$$

Va hakoza davom etadi. n –qadamda

$$\begin{cases} x_1 + b_{12}^{(1)} x_2 + \ldots + b_{1n}^{(1)} x_n = b_{1,n+1}^{(1)} \\ \qquad x_2 + \ldots + b_{2n}^{(2)} x_n = b_{2,n+1}^{(2)}, \\ \ldots\ldots\ldots\ldots\ldots \\ \qquad\qquad x_n = b_{n,n+1}^{(n)} \end{cases} \qquad (4.5)$$

sistemaga ega bo‘lamiz. Bunda ixtiyoriy m –qadam uchun

$$b_{mj}^{(m)} = \frac{a_{mj}^{(m)}}{a_{mm}^{(m)}}, \quad a_{ij}^{(m)} = a_{ij}^{(m-1)} - a_{im}^{(m-1)} b_{mj}^{(m)} \quad (i, j \geq m+1)$$

bo‘ladi. (4.5) sistemadan $x_n,\ x_{n-1}, \ldots, x_1$ larni topish mumkin.

$$\begin{cases} x_n = b_{n,n+1}^{(n)} \\ x_{n-1} = b_{n-1,n+1}^{(n-1)} - b_{n-1,n}^{(n-1)} x_n, \\ \quad . \; . \; . \; . \; . \; . \; . \; . \; . \\ x_1 = b_{1,n+1}^{(1)} - b_{12}^{(1)} x_2 - \ldots - b_{1n}^{(1)} x_n \,. \end{cases} \qquad (4.6)$$

(4.5) uchburchak sistemaning koeffitsientlarini topish Gaus usulining to‘g‘ri yurishi, (4.6) sistemaning ildizini topish jarayoni teskari yurish deyiladi.

Hisoblashlarda xatoga yo‘l qo‘ymaslik maqsadida

$$a_{i,n+2} = \sum_{j=1}^{n+1} a_{ij} \qquad (i = \overline{1,n}) \qquad (4.7)$$

yig‘indini hisoblaymiz va uni kontrol yig‘indi deb ataymiz.

Agar satrlar ustida bajarilgan amallarni kontrol yig‘indi ustida ham bajarsak va hisoblashlar xatosiz bajarilgan bo‘lsa, u holda kontrol yig‘indida almashtirishlar bajarganidan keyingi elementlarning yig‘indisiga teng bo‘ladi.

n ta noma’lumli tenglamalar sistemasini Gaus usuli bilan yechish jarayonida $\frac{1}{3}(n^3 + 3n^2 - n)$ ta ko‘paytirish va bo‘lish va $\frac{1}{6}(2n^3 + +3n^2 - 5n)$ qo‘shish amali bajariladi.

1-misol. Chiziqli tenglamalar sistemasini Gaus usuli bilan yeching.

$$\begin{cases} 8.1x_1 + 1.2x_2 - 9.1x_3 + 1.7x_4 = 10 \,, \\ 1.1x_1 - 1.7x_2 + 7.2x_3 - 3.4x_4 = 1.7 \,, \\ 1.7x_1 - 1.8x_2 + 10x_3 + 2.3x_4 = 2.1 \,, \\ 1.3x_1 + 1.7x_2 - 9.9x_3 + 3.5x_4 = 27.1 \,, \end{cases}$$

Yechish: Bu sistemani yechishdan oldin Gauss sxemasini tuzib olaylik.

Gauss sxemasi

Qism	i	a_{i1}	a_{i2}	a_{i3}	a_{i4}	Ozod had a_{i5}	Kontrol	
							Σ	Σ'
	1	a_{11}	a_{12}	a_{13}	a_{14}	a_{15}	$\sum a_{1j} (= a_{16})$	

I	2	$a_{2\,1}$	$a_{2\,2}$	a_{23}	a_{24}	a_{25}	$\sum a_{2j}(=a_{26})$	
	3	$a_{3\,1}$	$a_{3,2}$	a_{33}	a_{34}	a_{35}	$\sum a_{3j}(=a_{36})$	
	4	$a_{4\,1}$	$a_{4\,2}$	a_{43}	a_{44}	a_{45}	$\sum a_{4\,j}(=a_{46})$	
	(d)	$b_{1\,1}$	$b_{1,2}$	b_{13}	b_{14}	b_{15}	$b_{16}=a_{16}/a_{11}$	
II	2		$a_{22}^{(1)}$	$a_{23}^{(1)}$	$a_{24}^{(1)}$	$a_{25}^{(1)}$	$a_{26}^{(1)}$	$\sum a_{2j}^{(1)}$
	3		$a_{32}^{(1)}$	$a_{33}^{(1)}$	$a_{34}^{(1)}$	$a_{35}^{(1)}$	$a_{36}^{(1)}$	$\sum a_{3j}^{(1)}$ $j=\overline{2,5}$
	4		$a_{42}^{(1)}$	$a_{43}^{(1)}$	$a_{44}^{(1)}$	$a_{45}^{(1)}$	$a_{46}^{(1)}$	$\sum a_{4j}^{(1)}$
	$(d^{(1)})$		1	$b_{23}^{(1)}$	$b_{24}^{(1)}$	$b_{25}^{(1)}$	$b_{26}^{(1)}=a_{26}^{(1)}/a_{22}^{(1)}$	$1+\sum b_{2j}^{(1)}$ $j=\overline{3,5}$
III	3			$a_{33}^{(2)}$	$a_{34}^{(2)}$	$a_{35}^{(2)}$	$a_{36}^{(2)}$	$\sum a_{3j}^{(2)}$ $j=\overline{3,5}$
	4			$a_{43}^{(2)}$	$a_{44}^{(2)}$	$a_{45}^{(2)}$	$a_{46}^{(2)}$	$\sum a_{4j}^{(2)}$
				1	$b_{34}^{(2)}$	$b_{25}^{(1)}$	$b_{36}^{(2)=}=a_{36}^{(2)}/a_{33}^{(2)}$	$1+\sum b_{3j}^{(1)}$ $j=\overline{4,5}$
IV	4				$a_{44}^{(3)}$	$a_{45}^{(3)}$	$a_{46}^{(3)}$	$\sum a_{4j}^{(3)}$ $j=\overline{4,5}$
					1	x_1		
				1		x_2		
			1			x_3		
		1				x_4		

Bizning misolda:

	1	2	3	4	ozod had	Kontrol	
						$S'=\sum a_{i,j}$	S''
a_{1j}	8,1	1,2	-9,1	1,7	10	11,9	
a_{2j}	1,1	-1,7	7,2	-3,4	1,7	4,9	
a_{3j}	1,7	-1,8	10	2,3	2,1	14,3	

a_{4j}	1,3	1,7	-9,9	3,5	27,1	23,7	
$b_{1j}^{(1)}$	1	0,148148	-1,12346	0,209877	1,234568	1,469136	1,469135802
$a_{1j}^{(1)}$		-1,86296	8,435802	-3,63086	0,341975	3,283951	3,283950617
$a_{2j}^{(1)}$		-2,05185	11,90988	1,94321	0,001235	11,80247	11,80246914
$a_{3j}^{(1)}$		1,507407	-8,43951	3,22716	25,49506	21,79012	21,79012346
$b_{2j}^{(2)}$		1	-4,52816	1,948973	-0,18357	-1,76276	-1,762756793
$a_{1j}^{(2)}$			2,618763	5,94221	-0,37541	8,185558	8,185556611
$a_{2j}^{(2)}$			-1,61372	0,289265	25,77177	24,44731	24,44731539
$b_{3j}^{(3)}$			1	1,029093	-0,06502	1,964078	
$a_{1j}^{(3)}$				1,949933	25,66685	27,61679	
x_4				1	13,16294		
x_3			1		-13,6109		
x_2		1			-87,4702		
x_1	1				-3,86074		

Shunday qilib, quyidagi $x_1 = -3{,}8607,\ x_2 = -87{,}4702\ , x_3 = -13{,}6109, x_4 = 13{,}16294$ taqribiy yechimga ega bo‘lamiz.

4.2-§. BOSH ELEMENTLAR USULI

Gauss metodida yetakchi elementlar doim noldan farqli bo‘lavermaydi. Yoki ular nolga yaqin sonlar bo‘lishi mumkin: bunday sonlarga bo‘lganda katta absolyut xatoga ega bo‘lgan sonlar hosil bo‘ladi. Buning natijasida taqribiy yechim aniq yechimdan sezilarli darajada chetlashib ketadi.

Hisoblash xatosining bunday halokatli ta’siridan qutulish uchun Gauss usuli bosh elementni topish yo‘li bilan qo‘llaniladi. Bu usulning Gauss usuliningg kompakt sxemasidan farqi quyidagidan iborat. Faraz qilaylik, noma’lumlarni yo‘qotish jarayonida quyidagi sistemaga ega bo‘lgan bo‘laylik:

$$\begin{cases} x_1 + b_{12}^{(1)} x_2 + b_{13}^{(1)} x_2 \; \ldots + b_{1n}^{(1)} x_n = b_{1,n+1}^{(1)}, \\ . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \\ x_m + b_{m,m+1}^{(m)} x_{m+1} + \; \ldots + b_{m,n}^{(m)} x_n = b_{m,n+1}^{(m)} \quad , \\ a_{m+1,m+1}^{(m)} x_{m+1} + \ldots + a_{m+1,n}^{(m)} x_n = a_{m+1,n+1}^{(m)}, \\ \ldots . \; \ldots .. \; \ldots \ldots .. \; .. \\ a_{n,m+1}^{(m)} x_{m+1} + \ldots + a_{n,n}^{(m)} x_n = a_{n,n+1}^{(m)}. \end{cases}$$

Endi $\left|a_{m+1,k}^{(m)}\right| = \max_j \left|a_{m+1,j}^{(m)}\right|$ tenglikni qanoatlantiradigan k nomerni topib, o‘zgaruvchilarni qayta belgilaymiz: $x_{m+1} = x_k$ va $x_k = x_{m+1}$ $so‘ngra$ $(m+2)$ -tenglamadan boshlab, barchasidan x_{m+1} no’malumni yo‘qotamiz. Bunday qayta belgilashlar yo‘qotish tartibini o‘zgartirishga olib keladi va ko‘p hollarda hisoblash xatosini kamaytirishga xizmat qiladi.

4.3-§. OPTIMAL YO‘QOTISH USULI

Bu usulning dastlabki qadamlari Gauss usuliga o‘xshaydi. Yetakchi element $a_{11} \neq 0$ deb faraz qilib, (2.1) sistemaning birinchi tenglamasini

$$x_1 + b_{12}^{(1)} x_2 + \; \ldots + b_{1n}^{(1)} x_n = b_{1,n+1}^{(1)} \qquad (4.2)$$

ko‘rinishga keltiramiz. So‘ngra (2.1) sistemaning faqat ikkinchi tenglamasidan x_1 ni yo‘qotamiz:

$$a_{22}^{(1)} x_2 + a_{23}^{(1)} x_3 + \; \ldots + a_{2n}^{(1)} x_n = a_{2,n+1}^{(1)}$$

Enti $a_{22}^{(1)} \neq 0$ deb faraz qilib, bu tenglamani (3.4) ko‘rinishga keltiramiz:

$$x_2 + b_{23}^{(2)} x_3 + \; \ldots + b_{2n}^{(2)} x_n = b_{2,n+1}^{(2)}.$$

Bu tenglama yordamida (2.2) tenglamadan x_2 ni yo‘qotamiz. Natijada

$$x_1 + s_{13}^{(2)} x_3 + \; \ldots + s_{1n}^{(2)} x_n = s_{1,n+1}^{(2)}$$
$$x_2 + s_{23}^{(2)} x_3 + \; \ldots + s_{2n}^{(2)} x_n = s_{2,n+1}^{(2)}$$

hosil bo‘ladi. Bu yerda

$$s_{1j}^{(2)} = b_{1j}^{(1)} - b_{12}^{(1)} b_{2j}^{(2)}, \;\; s_{2j}^{(2)} = b_{2j}^{(2)} \qquad (j \geq 3).$$

Faraz qilaylik, avvalgi k ta tenglamalar ustida almashtirishlar bajarish natijasida (2.1) sistema quyidagi teng kuchli sistemaga keltirilgan bo‘lsin:

$$\begin{cases} x_1 + c_{1k+1}^{(k)} x_{k+1} + \cdots + c_{1n}^{(k)} x_n = c_{1,n+1}^{(k)}, \\ \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \\ x_k + c_{k,k+1}^{(k)} x_{k+1} + \cdots + c_{k,n}^{(k)} x_n = c_{k,n+1}^{(k)}, \\ a_{k+1,1} x_1 + \cdots + a_{k+1,k+1} x_{k+1} + a_{k+1,n} x_n = a_{k+1,n+1}, \\ \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \\ a_{n,m+1}^{(m)} x_{m+1} + \ldots + a_{n,n}^{(m)} x_n = a_{n,n+1}^{(m)}. \end{cases} \qquad (2.12)$$

Bu sistemaning avvalgi k ta tenglamasini mos ravishda $a_{k+1,1}, a_{k+1,2}, \cdots, a_{k+1,k}$ larga ko‘paytirib, natijalarni $(k+1)$ - tenglamadan ayiramiz va hosil bo‘lgan tenglamani x_{k+1} noma’lum oldidagi koeffitsientga bo‘lamiz. Natijada $(k+1)$ - tenglama quyidagi ko‘rinishga ega bo‘ladi:

$$x_{k+1} + c_{k+1,k+2}^{(k)} x_{k+2} + \cdots + c_{k+1,n}^{(k)} x_n = c_{k+1,n+1}^{(k)}.$$

Endi bu tenglama yordamida (2.12) sistemaning avvalgi k ta tenglamasidan x_{k+1} ni yo‘qotsak, u holda yana (2.12) ko‘rinishdagi sistemaga, faqat k ning $k+1$ ga almashgan holga, ega bo‘lamiz.

Shu bilan birga, agar

$$a_{k+1,k+1} - \sum_{r=1}^{k} c_{r,k+1}^{(k)} a_{k+1,r} \neq 0$$

bo‘lsa, quyidagi formulalarga ega bo‘lamiz:

$$s_{k+1,p}^{(k+1)} = \frac{a_{k+1,p} - \sum_{r=1}^{k} a_{k+1,r} s_{r,p}^{(k)}}{a_{k+1,k+1} - \sum_{r=1}^{k} a_{k+1,r} s_{r,k+1}^{(k)}},$$

$$s_{ip}^{(k+1)} = s_{ip}^{(k)} - s_{i,k+1}^{(k)} s_{k+1,p}^{(k+1)}.$$

$$(i = 1, 2, \cdots, k; \; p = k+2, k+3, \cdots, n+1).$$

Almashtirishlarning n- qadami ham bajarilgandan so‘ng (2.1) sistemaning yechimi uchun quyidagi formulalar hosil bo‘ladi:

$$x_i = c_{i,n+1}^{(n)} \quad (i = 1,2,\cdots,n).$$

Bu yerda ham hisoblash jarayonini kontrol qilish Gauss usulidagiga o‘xshashdir. Optimal yo‘qatish usulida ham barcha yetakchi elementlar noldan farqli bo‘lishi zarurdir. Agar bu fakt oldindan ma’lum bo‘lmasa, u holda hisoblash sxemasini o‘zgartirib bosh elementlarni satr bo‘yicha tanlash yo‘li bilan noma’lumlarni yo‘qotish maqsadga muvoffiqdir. Buning uchun, agar $(k+1)$ - tenglamada $x_1, x_2, \cdots, x_k$ noma’lumlarni yo‘qotgandan keyin,

$$a_{k+1,k+1} - \sum_{s=1}^{k} a_{k+1,s} c_{s,p}^{(k+1)} \qquad (p > k+1)$$

modul bo‘yicha eng katta element bo‘lsa, u holda o‘zgaruvchilarni boshqatdan belgilab:

$x_{k+1} = x_p \; va \; x_p = x_{k+1}$, so‘ngra optimal yo‘qotish qoidasiga ko‘ra noma’lumlarni yo‘qotishni davom ettirish kerak.

Optimal yo‘qotish usulining ustunligi shundan iboratki n – tartibli sistemani yechish uchun zarur bo‘lgan arifmetik amallarning soni Gauss usulidagidek bo‘lsa ham, bu metod EHM lar xotirasidan effektiv ravishda foydalanishga imkon beradi, ya’ni sistemaning tartibini ikki marta orttirish mumkin.

(2.12) sistemadan ko‘rinib turibdiki, optimal yo‘qotishning k-qadami bajarilgach, berilgan sistemaning oxirgi $(n-k)$ ta tenglamasi o‘zgarishsiz qoladi. Buni hisobga olgan holda xotiraga matritsaning barcha elementlarini to‘la kiritmasdan, har bir qadamdan oldin bittadan satr kiritamiz. U holda $(k+1)$ qadamni amalga oshirish uchun xotiraning

$$= k(n-k+1) + n + 1$$

ta yacheykasi yetarli bo‘ladi, bular

$$\begin{pmatrix} s_{1,k+1}^{(k)} & \ldots s_{1,n+1}^{(k)} \\ \ldots\ldots & \ldots\ldots \\ s_{k,k+1}^{(k)} & \ldots s_{k,n+1}^{(k)} \end{pmatrix}$$

matritsani va (2.12) sistemadagi $(k+1)$ –tenglama koeffitsientlarini joylashtirish uchun xizmat qiladi. Endi $f(k)$ ning maksimumini topib, n –tartibli sistemani yechish uchun $\frac{(n+1)(n+5)}{4}$ ta yacheykaga ega bo‘lgan maydon yetarli ekanligiga ishonch hosil qilamiz.

Misol tariqasida

$$\begin{cases} 1{,}7x_1 + 9{,}9x_2 - 20x_3 - 1{,}7x_4 = 1{,}7\,, \\ 20x_1 + 0{,}5x_2 - 30{,}1x_3 - 1{,}1x_4 = 2{,}1\,, \\ 10x_1 - 20x_2 + 30{,}2x_3 + 0{,}5x_4 = 1{,}8\,, \\ 3{,}3x_1 - 0{,}7x_2 + 3{,}3x_3 + 20x_4 = -1{,}7. \end{cases}$$

Sistemani optimal yo‘qotish usuli bilan yechaylik. Birinchi tenglamadan

$$x_1 + 5{,}8x_2 - 11{,}76x_3 - x_4 = 1 \qquad (2.13)$$

ni hosil qilamiz va buni 20 ga ko‘paytirib, sistemaning ikkinchi tenglamasidan ayiramiz:

$$-115{,}5x_2 + 205{,}1x_3 + 18{,}9x_4 = -17{,}9 \qquad (2.13^*)$$

(2.13*) ni -115,5 ga bo‘lamiz va

$$x_2 - 1{,}78x_3 - 0{,}16x_4 = 0{,}155 \qquad (2.14)$$

ni hosil qilamiz. Endi (2.13) dan x_2 ni yo‘qotsak,

$$x_1 - 1{,}436x_3 - 0{,}075x_4 = 0{,}101 \qquad (2.15)$$

(2.15) ni 10 ga va (2.14) ni -20 ga ko‘paytirib, sistemaning uchinchi tenglamasidan ayiramiz va hosil bo‘lgan tenglamani x_3 oldidagi koeffitsientga bo‘lsak,

$$x_3 + 0{,}0126x_4 = 0{,}0555 \qquad (2.16)$$

kelib chiqadi. Bu tenglamalar yordamida (2.14) va (2.15) lardan x_3 yo‘qotsak,

$$\begin{cases} x_1 - 0{,}0569x_4 = 0{,}181 \\ x_2 - 0{,}1376x_4 = 0{,}2538 \end{cases} \qquad (2.17)$$

Endi (2.16)- (2.17) tenglamalar yordamida sistemaning to‘rtinchi tenglamasidan $x_1, x_{2,}, x_3$ ni yo‘qotamiz:

$$20{,}04982x_4 = -2{,}30279.$$

Bundan va (2.13)- (2.17) dan noma’lumlarni ketma-ket topamiz:

x_4 =-0,11485; $x_3 = 0{,}05695$; $x_2 = 0{,}23800$; $x_1 = 0{,}17447$.

4.4-§. KVADRAT ILDIZLAR USULI.

Chiziqli tenglamalar sistemasini yechishning kvadrat ildizlar usulini ko‘rib chiqamiz. (3.1) sistemaning noma’lumlar oldidagi koeffitsient-lardan tuzilgan A matritsa Ermit matritsasi bo‘lsin. Kvadrat ildizlar usulininig g‘oyasi A matritsani uchburchak va diogonal matritsalar ko‘paytmasi shaklda tasvirlashdan iboratdir:

$$A = T^* DT$$

bu yerda

$$T = \begin{bmatrix} t_{11} & t_{12} & \dots t_{1n} \\ 0 & t_{22} & \dots t_{2n} \\ . & . & . \; . \; . \; . \; . \; . \\ 0 & 0 & . \; . \; . \, t_{nn} \end{bmatrix}$$

yuqori uchburchak matritsa bo‘lib, D esa d_{ii} elementlari +1 yoki -1 dan iborat bo‘lgan

$$D = \begin{bmatrix} d_{11} & 0 \dots & 0 \\ 0 & d_{22} \dots & 0 \\ . & . \; . \; . \; . \; . & . \\ 0 & 0 \; . \; . \; . & d_{nn} \end{bmatrix}$$

dioganal matritsadir. Bu matritsalarning d_{ii} va t_{ii} elementlari quyidagi formulalar yordamida hisoblanadi:

$$\begin{cases} d_{11} = signa_{11}, \qquad t_{11} = \sqrt{|a_{11}|}\; t_{1j} = \dfrac{a_{1j}}{d_{11}t_{11}} \\ d_{ii} = sign\left(a_{ii} - \sum\limits_{s=1}^{i-1} |t_{si}|^2 \, d_{ss} \right), \\ t_{ii} = \sqrt{\left| a_{ii} - \sum\limits_{s=1}^{i-1} |t_{si}|^2 d_{ss} \right|} \quad (i > 1), \\ t_{ii} = \dfrac{a_{ij} - \sum_{s=1}^{i-1} \overline{t_{s\imath}}\, d_{ss} t_{sj}}{d_{ii}t_{ii}} \qquad (j = \overline{\imath + 1, n}\,). \end{cases} \tag{3.8}$$

Bu yerda $\overline{t_{s\imath}}$ lar t_{si} lar o‘zaro qo‘shma kompleks sonlardir.

A –haqiqiy matritsa va uning bosh minori musbat bo‘lgan holda (3.8) formulalar quyidagi ko‘rinishga keladi.

$$\begin{cases} t_{11} = \sqrt{a_{11}}, \quad t_{1j} = \dfrac{a_{1j}}{t_{11}}, \\ t_{ii} = \sqrt{a_{ii} - \sum\limits_{s=1}^{i-1} t_{si}^2} \quad (i > 1), \\ t_{ij} = \dfrac{a_{ij} - \sum_{s=1}^{i-1} t_{si} t_{sj}}{t_{ii}} \quad (j = \overline{\imath + 1, n}\,) \end{cases} \qquad (3.9)$$

$A\vec{x} = \vec{b}$ sistemani yechish uchun uni $A = T^*DT$ yoyilmadan foydalanib, quyidagi ikkita uchburchak matritsali sistemalar shaklida yozib olamiz:

$$T^*D\vec{y} = \vec{b}\,, \qquad T\vec{x} = \vec{y},$$

bu yerda $\vec{x} = (x_1, x_2, \dots, x_n), \vec{y} = (y_1, y_2, \dots, y_n), \vec{b} = (b_1, b_2, \dots, b_n)$.

Bu sistemalarni yoyib yozsak,

$$\begin{cases} t_{11} d_{11} y_1 = b_1\,, \\ \bar{t}_{12} d_{11} y_1 + t_{22} d_{22} y_2 = b_2\,, \\ . \; . \; . \; . \; . \; . \; . \; . \; . \; . \\ \bar{t}_{1n} d_{11} y_1 + \bar{t}_{2n} d_{22} y_2 + \dots + t_{nn} d_{nn} y_n = b_n \end{cases}$$

va

$$\begin{cases} t_{11} x_1 + t_{12} x_2 + \dots + t_{1n} x_n = y_1\,, \\ t_{22} x_2 + \dots + t_{2n} x_n = y_2\,, \\ . \; . \; . \; . \; . \; . \; . \; . \; . \; . \\ t_{nn} x_n = y_n \end{cases}$$

ga ega bo‘lamiz. Bundan

$$y_1 = \frac{b_1}{t_{11} d_{11}}\,, \quad y_i = \frac{b_i - \sum_{s=1}^{i-1} \bar{t}_{si} y_s d_{ss}}{t_{ii} d_{ii}} \quad (i > 1) \qquad (3.10)$$

va

$$x_n = \frac{y_n}{t_{nn}}\,, \qquad x_i = \frac{y_i - \sum_{s=i+1}^{n} t_{is} x_s}{t_{ss}} \qquad (i < n) \qquad (3.11)$$

Qo‘lda hisoblashlarda xatolikka yo‘l qo‘ymaslik uchun nazorat qilish maqsadida nazorat(kontrol) o‘tkaziladi.

Birinchi kontrol: r elementlari

$$r_1 = \frac{\sum_1}{t_{11}d_{11}}\ ,\quad bu\ erda\ \textstyle\sum_i = \displaystyle\sum_{j=1}^{n} a_{ij}\ .$$

$$r_k = \frac{\sum_k - \sum_{s=1}^{k-1} \bar{t}_{ks} r_s d_{ss}}{\bar{t}_{kk} d_{kk}}$$

formulalar bo‘yicha hisoblanadi. Bu qiymatlar noma’lum x_i lar oldidagi koeffitsientlar va ozod hadlar yig‘indisiga teng bo‘lishi kerak.

Oxirgi kontrol: (3.11) formulalarda y_i lar r_i larga almashtirilib, $\bar{x}_i$ lar topiladi, $\bar{x}_i = x_i + 1$ bo‘lishligi kerak.

	$\boldsymbol{a_{i1}}$	$\boldsymbol{a_{i2}}$	$\boldsymbol{a_{i3}}$	$\boldsymbol{a_{i4}}$	$\boldsymbol{b_i}$	Kontrol	
						$\sum$	(r)
I	a_{11}	a_{12}	a_{13}	a_{14}	b_1	$\sum \boldsymbol{a_{1j}} + b_1$	$\sum_1$
	a_{21}	a_{22}	a_{23}	a_{24}	b_2	$\sum \boldsymbol{a_{2j}} + b_2$	$\sum_2$
	a_{31}	a_{32}	a_{33}	a_{34}	b_3	$\sum \boldsymbol{a_{3j}} + b_3$	$\sum_3$
	a_{41}	a_{42}	a_{43}	a_{44}	b_4	$\sum \boldsymbol{a_{4j}} + b_4$	$\sum_4$
	$\boldsymbol{t_{i1}}$	$\boldsymbol{t_{i2}}$	$\boldsymbol{t_{i3}}$	$\boldsymbol{t_{i4}}$	$\boldsymbol{y_i}$		
II	t_{11}	t_{12}	t_{13}	t_{14}	y_1	$\sum \boldsymbol{t_{1j}} + y_1$	r_1
		t_{22}	t_{23}	t_{24}	y_2	$\sum \boldsymbol{t_{2j}} + y_2$	r_2
			t_{33}	t_{34}	y_3	$\sum \boldsymbol{t_{3j}} + y_3$	r_3
				t_{44}	y_4	$\sum \boldsymbol{t_{4j}} + y_4$	r_4
III	x_1	x_2	x_3	x_4			
	$\bar{x}_1$	$\bar{x}_2$	$\bar{x}_3$	$\bar{x}_4$			

2-misol. Kvadrat ildizlar usulidan foydalanib, quyidagi simmetrik matritsaga ega bo‘lgan tenglamalar sistemasini yeching.

$$\begin{cases} 2x_1 - 3x_2 + 4x_3 + x_4 = 11\ , \\ -3x_1 + 5x_2 - x_3 + 2x_4 = -6\ , \\ 4x_1 - x_2 + x_3 + 3x_4 = 1\ , \\ x_1 + 2x_2 + 3x_3 + 2x_4 = 1\ , \end{cases}$$

Yechish: Jadval tuzib olamiz. Jadvalning **I** qismiga noma’lum oldidagi a_{ij} koeffitsientlar va b_i ozod hadlarni joylashtiramiz. $\sum$ ustunni hisoblab chiqamiz.

II qismiga t_{ij} hamda y_i larning (3.9), (3.10) formulalar bilan topiladigan qiymatlarini yozamiz. $\sum \boldsymbol{t_{ij}} + y_i$ va r_{i1} lar hisoblanib yoziladi.

III qismiga x_i larning (3.11) formulalar bilan topiladigan qiymatlarini yozamiz, hamda $\bar{x}_i$ (hisoblash usuli yuqorida ko‘rsatilgan) ning qiymatlari yoziladi.

	$\boldsymbol{a_{i1}}$	$\boldsymbol{a_{i2}}$	$\boldsymbol{a_{i3}}$	$\boldsymbol{a_{i4}}$	$\boldsymbol{b_i}$	**Kontrol**	
						$\sum$	**(r)**
I	2	-3	4	1	11	15	
	-3	5	-1	2	-6	-3	
	4	-1	1	3	1	8	
	1	2	3	2	1	9	
	$\boldsymbol{t_{i1}}$	$\boldsymbol{t_{i2}}$	$\boldsymbol{t_{i3}}$	$\boldsymbol{t_{i4}}$	$\boldsymbol{y_i}$		
II	1.41421	-2.12133	2.82843	0.70711	7.77819	10.60661	10.60661
		0.70708	7.07138	4.94995	30.40691	43.13538	43.13538
			7.55013	4.50363i	28.61122i	40.66504i	40.66504i
				1.64904 i	4.94718	6.59622i	6.59622i
III	2.99958	1.99975	2.00002	3.00004	x_i		
	3.99970	2.99980	3.00004	4.00004	$\bar{x}_i$		

Jadvaldagi yechimni verguldan keyin uch xonasigacha yaxlitlab olsak, quyidagiga ega bo‘lamiz:

$$x_1 = 3{,}000; \quad x_2 = 2{,}000; \quad x_3 = 2{,}000; \quad x_4 = 3{,}000.$$

Bu esa aniq yechimni beradi.

4.5-§. MATRITSA DITERMINANTINI HISOBLASHNING GAUSS USULI

Quyidagi

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{pmatrix}$$

matritsaning determinantini topish talab qilinsin. Bu determinantni topish uchun bir jinsli, chiziqli

$$A\bar{x} = \bar{0} \qquad (4.1)$$

tenglamalar sistemasini yechishga Gauss usulini qo‘llaymiz.

Sistemani yechish jarayonida A matritsa $B = \begin{pmatrix} 1 & b_{12}^{(1)} & b_{13}^{(1)} & \cdots & b_{1n}^{(1)} \\ 0 & 1 & b_{21}^{(2)} & \cdots & b_{2n}^{(2)} \\ & & \cdots\cdots\cdots & & \\ 0 & 0 & 0 & \cdots & 1 \end{pmatrix}$

uchburchak matritsaga almashtiriladi. Bu yerda

$$b_{1j}^{(1)} = \frac{a_{ij}}{a_{11}} \ (j \geq 2).$$

$$b_{mj}^{(m)} = \frac{a_{mj}^{(m-1)}}{a_{mm}^{(m-1)}}, \quad a_{ij}^{(m)} = a_{ij}^{(m-1)} - a_{im}^{(m-1)} b_{mj}^{(m)} .$$

(2.18) sistema

$$B\bar{x} = \bar{0}$$

sistemaga o‘tadi.

B matritsaning elementlari A matritsa va keyingi yordamchi A_1, $A_2, \cdots, A_{n-1}$ matritsalardan quyidagi ikkita elementar almashtirishlar natijasida hosil bo‘lgan:

a) Noldan farqli deb faraz qilingan $a_{11}, a_{22}^{(1)}, \cdots, a_{nn}^{(n-1)}$ *yetakchi elementlarga bo‘lish*;

b) A matritsa va yordamchi $A_1, A_2, \cdots, A_{n-1}$ larning satrlaridan mos ravishdaga yetakchi satrlarga proporsional bo‘lgan satrlarni ayirish.

Birinchi almashtirish natijasida matritsaning determinanti ham mos ravishdagi yetakchi elementga bo‘linadi, ikkinchi almashtirish esa determinantni o‘zgarishsiz qoldiradi. Shuning uchun ham

$$1 = \det B = \frac{\det A}{a_{11}, a_{22}^{(1)}, \cdots, a_{nn}^{(n-1)}},$$

bundan

$$\det A = a_{11}, a_{22}^{(1)}, \cdots, a_{nn}^{(n-1)}. \qquad (4.2)$$

Demak, A determinant (4.1) sistema uchun Gaussning kompakt sxemasidagi yetakchi elementlarning ko‘paytmasiga (4.2) teng ekan.

4.1 -turdagi topshiriq.

Topshiriqlarni bajarish uchun namuna.

Misol. 1) Sistemani Kramer formulasi bilan yeching.

2) Sistemani teskari matritsa yordamida yeching.

3) Matritsalar ustida amallar bajaring.

4) Tenglamani yeching.

$$1)\begin{cases} 2x_1+x_2-2x_3+x_4=1; \\ 3x_1+4x_2+x_3-3x_4=-7; \\ 4x_1-2x_2+3x_3-4x_4=3; \\ 2x_1+2x_2-3x_3-x_4=-11. \end{cases} \qquad 2)\begin{cases} 2x_1-4x_2+3x_3=1; \\ x_1+3x_2+2x_3=4; \\ 3x_1-5x_2+4x_3=1. \end{cases}$$

3) $(3A+B)(2A-B)$,

bunda $A=\begin{pmatrix} 1 & 2 & -1 \\ 0 & 2 & -3 \\ 1 & -1 & 2 \end{pmatrix}$, $\quad B=\begin{pmatrix} 2 & 1 & 3 \\ 1 & 0 & 1 \\ -1 & 3 & 1 \end{pmatrix}$.

$$4)\ \begin{pmatrix} 1 & 3 & -1 \\ 0 & 1 & -2 \\ 1 & -1 & 0 \end{pmatrix}\cdot X=\begin{pmatrix} 13 & -4 & 6 \\ 2 & -4 & 2 \\ -2 & 5 & 5 \end{pmatrix}$$

Yechish: **1.** 1) *ni yechilish jarayoni:*

$$\Delta=\begin{vmatrix} 2 & 1 & -2 & 1 \\ 3 & 4 & 1 & -3 \\ 4 & -2 & 3 & -4 \\ 2 & 2 & -3 & -1 \end{vmatrix}=\begin{vmatrix} 0 & 1 & 0 & 0 \\ -5 & 4 & 4 & -7 \\ 8 & -2 & 7 & -2 \\ -2 & 2 & -1 & 2 \end{vmatrix}=$$

$$=-\begin{vmatrix} -5 & 4 & -7 \\ 8 & 7 & -2 \\ -2 & -1 & -3 \end{vmatrix}=-(105+16+56-98+10+96)=-185;$$

$$\Delta_{x_1}=\begin{vmatrix} 1 & 1 & -2 & 1 \\ -7 & 4 & 1 & -3 \\ 3 & -2 & 3 & -4 \\ -11 & 2 & -3 & -1 \end{vmatrix}=\begin{vmatrix} 0 & 1 & 0 & 0 \\ -11 & 4 & 9 & -7 \\ 5 & -1 & -1 & -2 \\ -13 & 2 & 1 & -3 \end{vmatrix}=$$

$$= -\begin{vmatrix} -11 & 9 & -7 \\ 5 & -1 & -2 \\ -13 & 1 & -3 \end{vmatrix} = -(-33 + 234 - 35 + 91 - 22 + 135) = -370;$$

$$\Delta_{x_2} = \begin{vmatrix} 2 & 1 & -2 & 1 \\ 3 & -7 & 1 & -3 \\ 4 & 3 & 3 & -4 \\ 2 & -11 & 3 & -1 \end{vmatrix} = \begin{vmatrix} 0 & 1 & 0 & 0 \\ 4 & -7 & -13 & 4 \\ 7 & 3 & 9 & -7 \\ -1 & 11 & -25 & 10 \end{vmatrix} =$$

$$= -\begin{vmatrix} 4 & -13 & 4 \\ 7 & 9 & -7 \\ -1 & -25 & 10 \end{vmatrix} = -(360 - 91 - 700 + 36 - 700 + 910) = 185;$$

$$\Delta_{x_3} = \begin{vmatrix} 2 & 1 & 1 & 1 \\ 3 & 4 & -7 & -3 \\ 4 & -2 & 3 & -4 \\ 2 & 2 & -11 & -1 \end{vmatrix} = \begin{vmatrix} 0 & 1 & 0 & 0 \\ -5 & 4 & -11 & -7 \\ 8 & -2 & 5 & -2 \\ -2 & 2 & -13 & -3 \end{vmatrix} =$$

$$= \begin{vmatrix} -5 & -11 & -7 \\ 8 & 5 & -2 \\ -2 & -13 & -3 \end{vmatrix} = -(75 - 44 + 728 - 70 + 130 - 264) = -555;$$

$$\Delta_{x_4} = \begin{vmatrix} 2 & 1 & -2 & 1 \\ 3 & 4 & 1 & -7 \\ 4 & -2 & 3 & 3 \\ 2 & 2 & -3 & -11 \end{vmatrix} = \begin{vmatrix} 0 & 1 & 0 & 0 \\ -5 & 4 & 4 & -11 \\ 8 & -2 & 7 & 5 \\ -2 & 2 & -1 & -13 \end{vmatrix} =$$

$$= -\begin{vmatrix} -5 & 4 & -11 \\ 8 & 7 & 5 \\ -2 & -1 & -13 \end{vmatrix} = -(455 - 40 + 88 - 154 + 416 - 25) = -470;$$

$$x_1 = \frac{\Delta x_1}{\Delta} = \frac{-370}{-185} = 2; \quad x_2 = \frac{\Delta x_2}{\Delta} = \frac{185}{-185} = -1$$

$$x_3 = \frac{\Delta x_3}{\Delta} = \frac{-555}{-185} = 3; \quad x_4 = \frac{\Delta_{x_4}}{\Delta} = \frac{-740}{-185} = 4$$

Javob:

$$x_1 = 2; \ x_2 = -1; \ x_3 = 3; \ x_4 = 4.$$

2. 2) *ni yechilish jarayoni:*

$$\begin{pmatrix} 2 & -4 & 3 \\ 1 & 3 & 2 \\ 3 & -5 & 4 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 4 \\ 1 \end{pmatrix}$$

$$\Delta = \begin{vmatrix} 2 & -4 & 3 \\ 1 & 3 & 2 \\ 3 & -5 & 4 \end{vmatrix} = 24 - 24 - 15 - 27 + 16 + 20 = -6$$

$A_{11} = \begin{vmatrix} 3 & 2 \\ -5 & 4 \end{vmatrix} = 1$ $\quad A_{12} = -\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix}$=*-1* $\quad A_{13} = \begin{vmatrix} 1 & 3 \\ 3 & -5 \end{vmatrix}$=*-14*

$A_{21} = \begin{vmatrix} -4 & 3 \\ -5 & 4 \end{vmatrix}$=*1* $\quad A_{22} = \begin{vmatrix} 2 & 3 \\ 3 & 4 \end{vmatrix}$=*-1* $\quad A_{23} = -\begin{vmatrix} 2 & -4 \\ 3 & -5 \end{vmatrix}$=*-2*

$A_{31} = \begin{vmatrix} -4 & 3 \\ 3 & 2 \end{vmatrix}$=*-17* $\quad A_{32} = \begin{vmatrix} 2 & 3 \\ 1 & 2 \end{vmatrix}$=*-1* $\quad A_{33} = \begin{vmatrix} 2 & -4 \\ 1 & 3 \end{vmatrix}$=*-10*

$$A^{-1}=-\frac{1}{6}\begin{pmatrix}22 & 1 & -17\\ 2 & -1 & -1\\ -14 & -2 & 10\end{pmatrix}$$

$$\begin{pmatrix}x_1\\ x_2\\ x_3\end{pmatrix}=-\frac{1}{6}\begin{pmatrix}22 & 11 & -17\\ 2 & -1 & -1\\ -14 & -1 & 10\end{pmatrix}\cdot\begin{pmatrix}1\\ 4\\ 1\end{pmatrix}=\begin{pmatrix}-1{,}5\\ 0{,}5\\ 2\end{pmatrix}$$

Javob: x_1=-1,5; x_2 = 0,5; x_3=2

3. *3) ni yechilish jarayoni:*

$$3A+V=\begin{pmatrix}3 & 6 & -3\\ 0 & 6 & -9\\ 3 & -3 & 6\end{pmatrix}+\begin{pmatrix}2 & 1 & 3\\ 1 & 0 & 1\\ -1 & 3 & 1\end{pmatrix}=\begin{pmatrix}5 & 7 & 0\\ 1 & 6 & -8\\ 2 & 0 & 7\end{pmatrix};$$

$$(3A+V)(2A-V)=$$

$$\begin{pmatrix}5 & 7 & 0\\ 1 & 6 & -8\\ 2 & 0 & 7\end{pmatrix}\cdot\begin{pmatrix}0 & 3 & -5\\ -1 & 1 & -7\\ 3 & -1 & 3\end{pmatrix}=\begin{pmatrix}-7 & 43 & -74\\ -30 & 67 & -71\\ 21 & -29 & 11\end{pmatrix}.$$

4. *4) ni yechilish jarayoni:*

AX=V ga egamiz, *bundan X=A^{1}V. Quyidagilarni topamiz*

$$\Delta=\begin{vmatrix}1 & 3 & -1\\ 0 & 1 & -2\\ 1 & -1 & 0\end{vmatrix}=-6+1-2=7;$$

$$A_{11}=\begin{vmatrix}1 & -2\\ -1 & 0\end{vmatrix}=-2\quad A_{12}=\begin{vmatrix}0 & -2\\ 1 & 0\end{vmatrix}=-2;\quad A_{13}=\begin{vmatrix}0 & -2\\ 1 & 0\end{vmatrix}=-1$$

$$A_{21}=-\begin{vmatrix}3 & -1\\ -1 & 0\end{vmatrix}=1\quad A_{22}==\begin{vmatrix}1 & -1\\ 1 & 0\end{vmatrix}=1\quad A_{23}=-\begin{vmatrix}1 & 3\\ 1 & -1\end{vmatrix}=4$$

$$A_{31}=\begin{vmatrix}3 & 1\\ 1 & -2\end{vmatrix}=-5\quad A_{32}=-\begin{vmatrix}1 & -1\\ 0 & -2\end{vmatrix}=2\quad A_{33}=-\begin{vmatrix}1 & 3\\ 0 & 1\end{vmatrix}=1$$

$$A^{-1}=-\frac{1}{7}\begin{pmatrix}-2 & 1 & -5\\ -2 & 1 & 2\\ -1 & 4 & 1\end{pmatrix};$$

$$X=-\frac{1}{7}\begin{pmatrix}-2 & 1 & -5\\ -2 & 1 & 2\\ -1 & 4 & 1\end{pmatrix}.\begin{pmatrix}13 & -4 & 6\\ 2 & -4 & 2\\ -2 & 5 & 5\end{pmatrix}=$$

$$=\frac{1}{7}\begin{pmatrix}-14 & -21 & -35\\ -28 & 14 & 0\\ -7 & -7 & 7\end{pmatrix}=\begin{pmatrix}2 & 3 & 5\\ 4 & -2 & 0\\ 1 & 1 & -1\end{pmatrix}$$

Javob:

$$X=\begin{pmatrix}2 & 3 & 5\\ 4 & -2 & 0\\ 1 & 1 & -1\end{pmatrix}.$$

Mustaqil yechish uchun:

Quyidagi amallarni va misollarni mos ravishda bajaring:

Topshiriqlar.

1) Sistemani Kramer formulasi bilan yeching.
2) Sistemani teskari matritsa yordamida yeching.
3) Matritsalar ustida amallar bajaring.
4) Tenglamani yeching.

№ 1) $\begin{cases} x_1 + x_2 + 2x_3 - 2x_4 = 1;\\ 3x_1 - x_2 - x_3 - 2x_4 = -4;\\ 2x_1 + 3x_2 - x_3 - x_4 = -6;\\ x_1 + 2x_2 + 3x_3 - x_4 = -4.\end{cases}$ 2) $\begin{cases} 5x + 8y - z = -7;\\ x + 2y + 3z = 1;\\ 2x - 3y + 2z = 9.\end{cases}$

3) $2(A+B)(2B-A)$, bunda A=$\begin{pmatrix}2 & 3 & -1\\ 4 & 5 & 2\\ -1 & 0 & 7\end{pmatrix}$, B=$\begin{pmatrix}-1 & 0 & 5\\ 0 & 1 & 3\\ 2 & -2 & 4\end{pmatrix}$.

4) $\begin{pmatrix}2 & 3 & 1\\ -1 & 2 & 4\\ 5 & 3 & 0\end{pmatrix}\cdot X = \begin{pmatrix}-1 & 0 & 5\\ 0 & 1 & 3\\ 2 & -2 & 4\end{pmatrix}$.

№ 2 . 1) $\begin{cases} x_1 + 2x_2 + 3x_3 - 2x_4 = 6;\\ x_1 - x_2 - 2x_3 - 3x_4 = 8;\\ 3x_1 + 2x_2 - x_3 + 2x_4 = 4;\\ 2x_1 - 3x_2 + 2x_3 + x_4 = -8.\end{cases}$ 2) $\begin{cases} x + 2y + z = 4;\\ 3x - 5y + 3z = 1;\\ 2x + 7y - z = 8.\end{cases}$

3) $3A - (A + 2B)B$ .0

bunda A=$\begin{pmatrix}4 & 5 & -2\\ 3 & -1 & 0\\ 4 & 2 & 7\end{pmatrix}$, B=$\begin{pmatrix}2 & 1 & -1\\ 0 & 1 & 3\\ 5 & 7 & 3\end{pmatrix}$

4) X$\cdot\begin{pmatrix}-1 & -2 & 3\\ 2 & 3 & 5\\ 1 & 4 & -1\end{pmatrix} = \begin{pmatrix}4 & 11 & 3\\ 1 & 6 & 1\\ 2 & 2 & 16\end{pmatrix}$.

№ 3. 1) $\begin{cases} x_1 + 2x_2 + 3x_3 + 4x_4 = 5; \\ 2x_1 + x_2 + 2x_3 + 3x_4 = 1; \\ 3x_1 + 2x_2 + x_3 + 2x_4 = 1; \\ 4x_1 + 3x_2 + 2x_3 + x_4 = -5. \end{cases}$ 2) $\begin{cases} 3x + 2y + z = 5; \\ 2x + 3y + z = 1; \\ 2x + y + 3z = 11. \end{cases}$

3) $2(A-B)(A^2+B)$,

bunda A=$\begin{pmatrix} 5 & 1 & 7 \\ -10 & -2 & 1 \\ 0 & 1 & 2 \end{pmatrix}$, $B = \begin{pmatrix} 2 & 4 & 1 \\ 3 & 1 & 0 \\ 7 & 2 & 1 \end{pmatrix}$.

4) $\begin{pmatrix} 4 & -2 & 0 \\ 1 & 1 & 2 \\ 3 & -2 & 0 \end{pmatrix} \cdot X = \begin{pmatrix} 0 & -2 & 6 \\ 2 & 4 & 3 \\ 0 & -3 & 4 \end{pmatrix}$.

№ 4. 1) $\begin{cases} x_2 - 3x_3 + 4x_4 = -5; \\ x_1 - 2x_3 + 3x_4 = -4; \\ 3x_1 + 2x_2 - 5x_4 = 12; \\ 4x_1 + 3x_2 - 5x_3 = 5\ . \end{cases}$ 2) $\begin{cases} x_1 + 2x_2 + 4x_3 = 31; \\ 5x_1 + x_2 + 2x_3 = 29; \\ 3x_1 - x_2 + x_3 = 10. \end{cases}$

3) $(A^2-B^2)(A+B)$,

bunda A=$\begin{pmatrix} 7 & 2 & 0 \\ -7 & -2 & 1 \\ 1 & 1 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 0 & 2 & 3 \\ 1 & 0 & -2 \\ 3 & 1 & 1 \end{pmatrix}$.

4) X$\cdot \begin{pmatrix} 2 & 1 & 3 \\ 1 & -2 & 0 \\ 4 & -3 & 0 \end{pmatrix} = \begin{pmatrix} 22 & -14 & 3 \\ 6 & -7 & 0 \\ 11 & 3 & 15 \end{pmatrix}$.

№ 5. 1) $\begin{cases} x_1 + 3x_2 + 5x_3 + 7x_4 = 12; \\ 3x_1 + 5x_2 + 7x_3 + x_4 = 0\ ; \\ 5x_1 + 7x_2 + x_3 + 3x_4 = 4; \\ 7x_1 + x_2 + 3x_3 + 5x_4 = 16. \end{cases}$ 2) $\begin{cases} 4x - 3y + 2z = 9; \\ 2x + 5y - 3z = 4; \\ 5x + 6y - 2z = 18. \end{cases}$

3) (A-B^2)$(2A+B)$,

bunda A=$\begin{pmatrix} 5 & 2 & 0 \\ 10 & 4 & 1 \\ 7 & 3 & 2 \end{pmatrix}$, $B = \begin{pmatrix} 3 & 6 & -1 \\ -1 & -2 & 0 \\ 2 & 1 & 3 \end{pmatrix}$.

4) $\begin{pmatrix} 2 & 3 & 1 \\ 4 & -1 & 0 \\ 0 & 1 & 2 \end{pmatrix} \cdot X = \begin{pmatrix} 9 & 8 & 7 \\ 2 & 7 & 3 \\ 4 & 3 & 5 \end{pmatrix}$.

№ 6. 1) $\begin{cases} x_1 + 5x_2 + 3x_3 - 4x_4 = 20; \\ 3x_1 + x_2 - 2x_3 = 9; \\ 5x_1 - 7x_2 + 10x_4 = -9; \\ 3x_2 - 5x_3 = 1. \end{cases}$ 2) $\begin{cases} 2x_1 - x_2 - x_3 = 4 \\ 3x_1 + 4x_2 - 2x_3 = 11 \\ 3x_1 - 2x_2 + 4x_3 = 11. \end{cases}$

3) (A-B)A+2B,

bunda A=$\begin{pmatrix} 5 & -1 & 3 \\ 0 & 2 & -1 \\ -2 & -1 & 0 \end{pmatrix}$, $B = \begin{pmatrix} 3 & 7 & -2 \\ 1 & 1 & -2 \\ 0 & 1 & 3 \end{pmatrix}$.

4) X$\cdot \begin{pmatrix} 5 & 2 & 2 \\ -1 & 2 & 0 \\ 1 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 8 & 1 & 5 \\ -2 & 2 & -1 \\ 17 & 1 & 7 \end{pmatrix}$.

№ 7. 1) $\begin{cases} 2x_1 + x_2 - 5x_3 + x_4 = 8; \\ x_1 - 3x_2 - 6x_4 = 9; \\ 2x_2 - x_3 + 2x_4 = -5 \\ x_1 + 4x_2 - 7x_3 + 6x_4 = 0. \end{cases}$ 2) $\begin{cases} x_1 + x_2 + 2x_3 = -1; \\ 2x_1 - x_2 + 2x_3 = -4; \\ 4x_1 + x_2 + 4x_3 = -2. \end{cases}$

3) 2(A-0,5B)+AB,

bunda A=$\begin{pmatrix} 5 & 3 & -1 \\ 2 & 0 & 4 \\ 3 & 5 & -1 \end{pmatrix}$, B=$\begin{pmatrix} 1 & 4 & 16 \\ -3 & -2 & 0 \\ 5 & 7 & 2 \end{pmatrix}$.

4) $\begin{pmatrix} 4 & 2 & 1 \\ 3 & -2 & 0 \\ 0 & -1 & 2 \end{pmatrix}$$\cdot$X=$\begin{pmatrix} 2 & 0 & 2 \\ 5 & -7 & -2 \\ 1 & 0 & -1 \end{pmatrix}$.

№ 8. 1) $\begin{cases} 2x_1 - x_2 + 3x_3 + 2x_4 = 4; \\ 3x_1 + 3x_2 + 3x_3 + 2x_4 = 6; \\ 3x_1 - x_2 - x_3 + 2x_4 = 6; \\ 3x_1 - x_2 + 3x_3 - x_4 = 6. \end{cases}$ 2) $\begin{cases} 3x_1 - x_2 = 5; \\ -2x_1 + x_2 + x_3 = 0; \\ 2x_1 - x_2 + 4x_3 = 15. \end{cases}$

3) (A-B)A+3B,

bunda A=$\begin{pmatrix} 3 & 2 & -5 \\ 4 & 2 & 0 \\ 1 & 1 & 2 \end{pmatrix}$, B=$\begin{pmatrix} -1 & 2 & 4 \\ 0 & 3 & 2 \\ -1 & -3 & 4 \end{pmatrix}$.

4) X$\cdot\begin{pmatrix} 1 & 4 & 2 \\ 2 & 1 & -2 \\ 0 & 1 & -1 \end{pmatrix} = \begin{pmatrix} 4 & 6 & -2 \\ 4 & 10 & 1 \\ 2 & 4 & -5 \end{pmatrix}$.

№ 9. 1) $\begin{cases} x_1 + 2x_2 - x_3 + x_4 = 8; \\ 2x_1 + x_2 + x_3 + x_4 = 5; \\ x_1 - x_2 + 2x_3 + x_4 = -1; \\ x_1 + x_2 - x_3 + 3x_4 = 10. \end{cases}$ 2) $\begin{cases} 3x_1 - x_2 + x_3 = 4; \\ 2x_1 - 5x_2 - 3x_3 = -17; \\ x_1 + x_2 - x_3 = 0. \end{cases}$

3) 2A-$(A^2 + B)B$,

bunda A=$\begin{pmatrix} 3 & 4 & 2 \\ 4 & 1 & -2 \\ 0 & 1 & -1 \end{pmatrix}$, B=$\begin{pmatrix} 4 & 6 & -2 \\ 4 & 10 & 1 \\ 2 & 4 & -5 \end{pmatrix}$.

4) $\begin{pmatrix} 3 & 2 & -5 \\ 4 & 2 & 0 \\ 1 & 1 & 2 \end{pmatrix} \cdot X = \begin{pmatrix} -1 & 2 & 4 \\ 0 & 3 & 2 \\ -1 & -3 & 4 \end{pmatrix}$.

№ 10. 1) $\begin{cases} 4x_1 + x_2 - x_4 = -9; \\ x_1 - 3x_2 + 4x_3 = -7; \\ 3x_2 - 2x_3 + 4x_4 = 12; \\ x_1 + 2x_2 - x_3 - 3x_4 = 0. \end{cases}$ 2) $\begin{cases} x_1 + x_2 + x_3 = 2; \\ 2x_1 - x_2 - 6x_3 = -1; \\ 3x_1 - 2x_2 = 8. \end{cases}$

3) $3(A^2 - B^2) - 2AB$,

bunda A=$\begin{pmatrix} 4 & 2 & 1 \\ 3 & -2 & 0 \\ 0 & -1 & 2 \end{pmatrix}$, B=$\begin{pmatrix} 2 & 0 & 2 \\ 5 & -7 & -2 \\ 1 & 0 & -1 \end{pmatrix}$.

4) X$\cdot\begin{pmatrix} 5 & 3 & -1 \\ -2 & 0 & 4 \\ 3 & 5 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 4 & 16 \\ -3 & -2 & 0 \\ 5 & 7 & 2 \end{pmatrix}$.

№ 11. 1) $\begin{cases} 2x_1 - x_2 + x_3 - x_4 = 1; \\ 2x_1 - x_2 - 3x_4 = 2; \\ 3x_1 - x_3 + x_4 = -3; \\ 2x_1 + 2x_2 - 2x_3 + 5x_4 = -6. \end{cases}$ 2) $\begin{cases} 2x_1 + x_2 - x_3 = 1; \\ x_1 + x_2 + x_3 = 6; \\ 3x_1 - x_2 + x_3 = 4. \end{cases}$

3) (2A-B)(3A+B) -2AB,

bunda $A=\begin{pmatrix} 1 & 0 & 3 \\ -2 & 0 & 1 \\ -1 & 3 & 1 \end{pmatrix}$, $B=\begin{pmatrix} 7 & 5 & 2 \\ 0 & 1 & 2 \\ -3 & -1 & -1 \end{pmatrix}$.

4) $X\cdot\begin{pmatrix} 5 & -1 & 3 \\ 0 & 2 & -1 \\ -2 & -1 & 0 \end{pmatrix} = \begin{pmatrix} 3 & 7 & -2 \\ 1 & 1 & -2 \\ 0 & 1 & 3 \end{pmatrix}$.

№ 12. 1) $\begin{cases} x_1 + x_2 - x_3 - x_4 = 0; \\ x_2 + 2x_3 - x_4 = 2; \\ x_1 - x_2 - x_4 = -1; \\ -x_1 + 3x_2 - 2x_3 = 0. \end{cases}$ 2) $\begin{cases} 2x_1 - x_2 - 3x_3 = 3; \\ 3x_1 + 4x_2 - 5x_3 = 8; \\ 2x_2 + 7x_3 = 17. \end{cases}$

3) $A(A^2 - B) - 2(B+A)B$,

bunda $A=\begin{pmatrix} 2 & 3 & 1 \\ -1 & 2 & 4 \\ 5 & 3 & 0 \end{pmatrix}$, $B=\begin{pmatrix} 2 & 7 & 13 \\ -1 & 0 & 5 \\ 5 & 13 & 21 \end{pmatrix}$.

4) $\begin{pmatrix} 4 & 5 & -2 \\ 3 & -1 & 0 \\ 4 & 2 & 7 \end{pmatrix}\cdot X = \begin{pmatrix} 2 & 1 & -1 \\ 0 & 1 & 3 \\ 5 & 7 & 3 \end{pmatrix}$.

№ 13. 1) $\begin{cases} 5x_1 + x_2 - x_4 = -9; \\ 3x_1 - 3x_2 + x_3 + 4x_4 = -7; \\ 3x_1 - 2x_3 + x_4 = -16; \\ x_1 - 4x_2 + x_4 = 0. \end{cases}$ 2) $\begin{cases} x_1 + 5x_2 + x_3 = -7; \\ 2x_1 - x_2 - x_3 = 0; \\ x_1 - 2x_2 - x_3 = 2. \end{cases}$

3) (A+B)A-B(2A+3B),

bunda $A=\begin{pmatrix} 2 & -2 & 3 \\ 2 & 3 & 5 \\ 1 & 4 & -1 \end{pmatrix}$, $B=\begin{pmatrix} 4 & 11 & 3 \\ 1 & 6 & 1 \\ 2 & 2 & 16 \end{pmatrix}$.

4) $X\cdot\begin{pmatrix} 2 & -8 & 5 \\ -1 & 1 & 1 \\ -2 & -2 & -3 \end{pmatrix} = \begin{pmatrix} 10 & -2 & 6 \\ 0 & 4 & -2 \\ -4 & -2 & 0 \end{pmatrix}$.

№ 14. 1) $\begin{cases} 2x_1 + x_3 + 4x_4 = 9; \\ x_1 + 2x_2 - x_3 + x_4 = 8; \\ 2x_1 + x_2 + x_3 + x_4 = 5; \\ x_1 - x_2 + 2x_3 + x_4 = -1. \end{cases}$ 2) $\begin{cases} x - 2y + 3z = 6; \\ 2x + 3y - 4z = 16; \\ 3x - 2y - 5z = 12. \end{cases}$

3) A(2A+B)-B(A-B),

bunda A= $\begin{pmatrix} 2 & 3 & 1 \\ 4 & -1 & 0 \\ 0 & 1 & 2 \end{pmatrix}$, B= $\begin{pmatrix} 9 & 8 & 7 \\ 2 & 7 & 3 \\ 4 & 3 & 5 \end{pmatrix}$.

4) $\begin{pmatrix} 5 & 3 & -1 \\ 2 & 0 & 4 \\ 3 & 5 & -1 \end{pmatrix} \cdot X = \begin{pmatrix} 1 & 4 & 16 \\ -3 & -2 & 0 \\ 5 & 7 & 2 \end{pmatrix}$.

№ 15. 1) $\begin{cases} 2x_1 - 6x_2 + 2x_3 + 2x_4 = 12; \\ x_1 + 3x_2 + 5x_3 + 7x_4 = 12; \\ 3x_1 + 5x_2 + 7x_3 + x_4 = 0; \\ 5x_1 + 7x_2 + x_3 + 3x_4 = 4; \end{cases}$ 2) $\begin{cases} 3x + 4y + 2z = 8; \\ 2x - y - 3z = -1; \\ x + 5y + z = 0. \end{cases}$

3) 3(A+B)(AB-2A),

bunda A= $\begin{pmatrix} 2 & 1 & 3 \\ 1 & -2 & 0 \\ 4 & -3 & 0 \end{pmatrix}$, B= $\begin{pmatrix} 22 & -14 & 3 \\ 6 & -7 & 0 \\ 11 & 3 & 15 \end{pmatrix}$.

4) X$\cdot \begin{pmatrix} 2 & 3 & -1 \\ 4 & 5 & 2 \\ -1 & 0 & 7 \end{pmatrix} = \begin{pmatrix} -1 & 0 & 5 \\ 2 & 1 & 3 \\ 0 & -2 & 4 \end{pmatrix}$.

№ 16. 1) $\begin{cases} x_1 + 5x_2 = 2; \\ 2x_1 - x_2 + 3x_3 + 2x_4 = 4; \\ 3x_1 - x_2 + 3x_3 + 2x_4 = 6; \\ 3x_1 - x_2 + 3x_3 - x_4 = 6. \end{cases}$ 2) $\begin{cases} 2x_1 - x_2 + 3x_3 = 7; \\ x_1 + 3x_2 - 2x_3 = 0; \\ 2x_2 - x_3 = 2. \end{cases}$

3) 2AB-(A+B)(A-B),

bunda A= $\begin{pmatrix} 4 & -2 & 0 \\ 1 & 1 & 2 \\ 3 & -2 & 0 \end{pmatrix}$, B= $\begin{pmatrix} 0 & -2 & 6 \\ 2 & 4 & 3 \\ 0 & -3 & 4 \end{pmatrix}$.

4) $\begin{pmatrix} 12 & 15 & -6 \\ 9 & -3 & 0 \\ 12 & 0 & 21 \end{pmatrix} \cdot X = \begin{pmatrix} 8 & 7 & -4 \\ 3 & 1 & 6 \\ 16 & 16 & 13 \end{pmatrix}$.

№ 17. 1) $\begin{cases} x_1 - 4x_2 - x_4 = 2; \\ x_1 + x_2 + 2x_3 + 3x_4 = 1; \\ 2x_1 + 3x_2 - x_3 - x_4 = -6; \\ x_1 + 2x_2 + 3x_3 - x_4 = -4; \end{cases}$ 2) $\begin{cases} 2x_1 + x_2 + 4x_3 = 20; \\ 2x_1 - x_2 - 3x_3 = 3; \\ 3x_1 + 4x_2 - 5x_3 = -8. \end{cases}$

3) 2A+3B(AB-2A),

bunda A=$\begin{pmatrix} 1 & -1 & 0 \\ 2 & 0 & -1 \\ 1 & 1 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 5 & 3 & 1 \\ -1 & 2 & 0 \\ -3 & 0 & 0 \end{pmatrix}$.

4) X$\cdot\begin{pmatrix} 1 & 3 & 4 \\ 6 & 6 & 5 \\ -1 & -2 & 11 \end{pmatrix} = \begin{pmatrix} -4 & -3 & 11 \\ 0 & -3 & 4 \\ 1 & -4 & 1 \end{pmatrix}$.

№ 18. 1) $\begin{cases} 5x_1 - x_2 + x_3 + 3x_4 = -4; \\ x_1 + 2x_2 + 3x_3 - 2x_4 = 6; \\ 2x_1 - x_2 - 2x_3 - 3x_4 = 8; \\ 3x_1 + 2x_2 - x_3 + 2x_4 = 4. \end{cases}$ 2) $\begin{cases} x_1 - x_2 = 4; \\ 2x_1 + 3x_2 + x_3 = 1; \\ 2x_1 + x_2 + 3x_3 = 11. \end{cases}$

3) (A-B)(A+B)-2AB,

bunda A=$\begin{pmatrix} 3 & 4 & 5 \\ -1 & 0 & 2 \\ -2 & -1 & 0 \end{pmatrix}$, B=$\begin{pmatrix} 0 & 1 & -2 \\ -1 & 1 & 2 \\ 3 & -1 & 0 \end{pmatrix}$.

4) $\begin{pmatrix} 8 & -5 & -1 \\ -4 & 7 & -1 \\ -4 & 1 & 5 \end{pmatrix}\cdot$X=$\begin{pmatrix} 5 & 4 & -1 \\ 10 & 12 & -1 \\ 0 & 1 & 1 \end{pmatrix}$.

№ 19. 1) $\begin{cases} 4x_1 - 2x_2 + x_3 - 4x_4 = 3; \\ 2x_1 - x_2 + x_3 - x_4 = 1; \\ 3x_1 - x_2 + x_4 = -3; \\ 2x_1 + 2x_2 - 2x_3 + 5x_4 = -6. \end{cases}$ 2) $\begin{cases} x_1 + 5x_2 - x_3 = 7; \\ 2x_1 - x_2 - x_3 = 4; \\ 3x_1 - 2x_2 + 4x_3 = 11. \end{cases}$

3) $2A - AB(B - A) + B$,

bunda $A = \begin{pmatrix} 3 & 2 & -1 \\ 0 & -1 & 2 \\ 5 & 7 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 0 & 3 & -1 \\ 2 & -1 & 2 \\ -3 & 1 & 4 \end{pmatrix}$.

4) $X\cdot\begin{pmatrix}3 & 2 & -5\\ 4 & 2 & 0\\ 1 & 1 & 2\end{pmatrix}=\begin{pmatrix}-1 & 2 & 4\\ 0 & 3 & 2\\ -1 & -3 & 4\end{pmatrix}$.

№20 1) $\begin{cases}2x_1 - x_3 - 2x_4 = -1;\\ x_2 + 2x_3 - x_4 = 2;\\ x_1 - x_2 - x_1 = -1;\\ -x_1 + 3x_2 - 2x_3 = 0.\end{cases}$ 2) $\begin{cases}11x + 3y - z = 2;\\ 2x + 5y - 5z = 0;\\ x + y + z = 2.\end{cases}$

3) $A^2 - (A+B)(A-3B)$,

bunda $A=\begin{pmatrix}4 & 5 & 6\\ -1 & 0 & 3\\ -1 & 2 & -1\end{pmatrix}$, $B=\begin{pmatrix}0 & -1 & 2\\ 1 & 0 & -2\\ 3 & 1 & 2\end{pmatrix}$.

4) $\begin{pmatrix}-2 & 1 & 2\\ 3 & 0 & 4\\ 2 & 1 & -1\end{pmatrix}\cdot X=\begin{pmatrix}1 & 1 & 2\\ 2 & 1 & 1\\ -1 & 0 & 1\end{pmatrix}$.

№21 1) $\begin{cases}-x_1 + x_2 + x_3 + x_4 = 4;\\ 2x_1 + x_2 + 2x_3 + 3x_4 = 1;\\ 3x_1 + 2x_2 + x_3 + 2x_4 = 1;\\ 4x_1 + 3x_2 + 2x_3 + x_4 = -5.\end{cases}$ 2) $\begin{cases}7x + 5y + 2z = 18;\\ x - y - z = 3;\\ x + y + 2z = -2.\end{cases}$

3) $B(A+2B) - 3AB$,

bunda $A=\begin{pmatrix}7 & -3 & 0\\ 1 & -1 & 0\\ 2 & 0 & 3\end{pmatrix}$, $B=\begin{pmatrix}-4 & 2 & 1\\ 1 & 0 & 1\\ 3 & 2 & 1\end{pmatrix}$.

4) $X\cdot\begin{pmatrix}-1 & 2 & 0\\ -3 & 2 & 1\\ 1 & 2 & 3\end{pmatrix}=\begin{pmatrix}5 & -1 & 3\\ 4 & 2 & 1\\ -1 & 0 & 2\end{pmatrix}$.

№22 1) $\begin{cases}5x_1 + 3x_2 - 7x_3 + 3x_4 = 1;\\ x_2 - 3x_3 + 4x_4 = -5;\\ x_1 - 2x_3 - 3x_4 = -4;\\ 4x_1 + 3x_2 - 5x_3 = 5.\end{cases}$ 2) $\begin{cases}2x + 3y + z = 1;\\ x + z = 0;\\ x - y - z = 2.\end{cases}$

3) $3(A+B) - (A-B)A$,

bunda $A = \begin{pmatrix} 1 & 2 & 3 \\ 0 & -2 & 3 \\ 1 & 1 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 4 & 2 & 1 \\ -1 & 2 & 0 \\ 2 & 3 & -1 \end{pmatrix}$.

4) $\begin{pmatrix} 1 & 4 & -1 \\ 4 & -3 & 1 \\ 0 & 2 & 1 \end{pmatrix} \cdot X = \begin{pmatrix} 7 & 0 & -5 \\ 4 & 11 & 2 \\ 1 & 3 & 1 \end{pmatrix}$.

№23 1) $\begin{cases} x_1 + x_2 - x_3 - x_4 = 0; \\ x_1 + 2x_3 - 2x_4 = 1; \\ x_1 - x_2 - x_4 = -1; \\ -x_1 + 3x_2 - 2x_3 = 0. \end{cases}$ 2) $\begin{cases} x - 2y - 2z = 3; \\ x + y - 2z = 0; \\ x - y - z = 1. \end{cases}$

3) $A(A - B) + 2B(A + B)$,

bunda $A = \begin{pmatrix} 1 & -2 & -2 \\ 1 & 1 & -2 \\ 1 & -1 & -1 \end{pmatrix}$, $B = \begin{pmatrix} 0 & 3 & 5 \\ 4 & 1 & 0 \\ 1 & 1 & 2 \end{pmatrix}$.

4) $X \cdot \begin{pmatrix} 2 & 3 & 1 \\ 1 & 0 & 1 \\ 1 & -1 & -1 \end{pmatrix} = \begin{pmatrix} 7 & 5 & 2 \\ 1 & -1 & -1 \\ 1 & 1 & 2 \end{pmatrix}$.

№24 1) $\begin{cases} 2x_1 + x_2 - x_3 + 3x_4 = -6; \\ 3x_1 - x_2 + x_3 + 5x_4 = 3; \\ x_1 + 2x_2 - x_3 + 2x_4 = 28; \\ 2x_1 + 3x_2 + x_3 - x_4 = 0. \end{cases}$ 2) $\begin{cases} 3x_1 + x_2 - 5x_3 = -7; \\ 2x_1 - 3x_2 + 4x_3 = -1; \\ 5x_1 - x_2 + 3x_3 = 0. \end{cases}$

3) $(2A + B)B - 0{,}5A$,

bunda $A = \begin{pmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 2 & -1 & 1 \end{pmatrix}$, $B = \begin{pmatrix} -1 & 0 & -2 \\ 2 & 1 & 1 \\ -2 & 0 & 1 \end{pmatrix}$.

4) $X \cdot \begin{pmatrix} 2 & 3 & 1 \\ 3 & 0 & -2 \\ 1 & 2 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 5 & 0 \\ 3 & -1 & 2 \\ 0 & 2 & 1 \end{pmatrix}$.

№25 1) $\begin{cases} 2x_1 - x_2 + 2x_3 + 2x_4 = -3; \\ 3x_1 + 2x_2 + x_3 - x_4 = 3; \\ x_1 - 3x_2 - x_3 - 3x_4 = 0; \\ 4x_1 + 2x_2 + 2x_3 + 5x_4 = -15. \end{cases}$ 2) $\begin{cases} x_1 - 2x_2 + x_3 = 15; \\ 2x_1 + x_2 + 3x_3 = 9; \\ 2x_1 + 3x_2 + 2x_3 = -2. \end{cases}$

3) $AB - 2(A - B)A$,

bunda $A = \begin{pmatrix} 2 & 1 & -1 \\ 1 & 0 & 1 \\ 3 & 1 & -2 \end{pmatrix}$, $\quad B = \begin{pmatrix} 2 & -1 & 0 \\ 0 & 2 & 1 \\ 1 & 3 & -1 \end{pmatrix}$.

4) $\begin{pmatrix} -2 & 1 & 2 \\ 3 & 0 & 4 \\ 2 & 1 & -1 \end{pmatrix} \cdot X = \begin{pmatrix} 1 & 1 & 2 \\ 2 & 1 & 1 \\ -1 & 0 & 1 \end{pmatrix}$.

№26 1) $\begin{cases} x_1 - 2x_2 + 3x_3 - 4x_4 = -2; \\ 2x_1 + 3x_2 + 4x_3 - 5x_4 = 8; \\ 3x_1 - x_2 - x_3 + 7x_4 = -2; \\ 2x_1 - x_2 + 6x_3 - 3x_4 = 7. \end{cases}$ 2) $\begin{cases} 2x_1 - x_2 - 2x_3 = 1; \\ 3x_1 + 2x_2 + x_3 = 1; \\ 2x_1 + 3x_2 + 3x_3 = 0. \end{cases}$

3) $(A + 2B)(3A - B)$,

bunda $A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & -2 & 1 \\ 0 & 1 & -1 \end{pmatrix}$, $\quad B = \begin{pmatrix} 2 & 3 & -1 \\ -2 & 0 & -1 \\ 1 & 0 & 1 \end{pmatrix}$.

4) $X \cdot \begin{pmatrix} 2 & 3 & 1 \\ -2 & 0 & 2 \\ 0 & 2 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 3 & 2 \\ 4 & 2 & 1 \\ -1 & 0 & 1 \end{pmatrix}$.

№27 1) $\begin{cases} 3x_1 + 2x_2 + 5x_3 - x_4 = 3; \\ 2x_1 - 3x_2 - 3x_3 + 4x_4 = 1; \\ 4x_1 + x_2 + 3x_3 + 2x_4 = 3; \\ 5x_1 - 2x_2 + x_3 + 3x_4 = 5. \end{cases}$ 2) $\begin{cases} 2x_1 + 3x_2 + 4x_3 = 5; \\ 3x_1 + 4x_2 - x_3 = 3; \\ 4x_1 + 5x_2 - 2x_3 = 3. \end{cases}$

3) $2AB + A(B - A)$,

bunda $A = \begin{pmatrix} 1 & 2 & -1 \\ 2 & 3 & 0 \\ 0 & 2 & -1 \end{pmatrix}$, $\quad B = \begin{pmatrix} 1 & 2 & -1 \\ 2 & -1 & 0 \\ 1 & 2 & 1 \end{pmatrix}$.

4) $X \cdot \begin{pmatrix} 3 & 2 & -1 \\ 0 & 1 & -2 \\ 1 & 1 & 3 \end{pmatrix} = \begin{pmatrix} -2 & 0 & 1 \\ 2 & 0 & -1 \\ 1 & 2 & 1 \end{pmatrix}$.

№28 1) $\begin{cases} 2x_1 + x_2 + 5x_3 - x_4 = 1; \\ 3x_1 + 3x_2 - 2x_3 - 5x_4 = 2; \\ x_1 - x_2 + 2x_3 + 3x_4 = 10; \\ 3x_1 + 2x_2 + 7x_3 - 2x_4 = 1. \end{cases}$ 2) $\begin{cases} 2x_1 - x_2 - 3x_3 = -9; \\ x_1 + 2x_2 + x_3 = 3; \\ 3x_1 + x_2 - x_3 = -1. \end{cases}$

3) $(3A + 0{,}5)(2B - A)$,

bunda $A = \begin{pmatrix} 1 & 2 & 3 \\ -1 & 0 & 2 \\ 1 & 2 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 0 & 2 \\ 2 & 3 & 1 \\ 3 & 1 & 0 \end{pmatrix}$.

4) $\begin{pmatrix} 1 & 2 & 1 \\ -1 & 0 & 2 \\ 3 & 0 & 1 \end{pmatrix} \cdot X = \begin{pmatrix} 2 & 3 & 5 \\ 0 & 2 & 1 \\ 1 & 0 & 2 \end{pmatrix}$.

№29 1) $\begin{cases} 3x_1 + x_2 + 2x_3 - x_4 = 8; \\ 2x_1 - 3x_2 - 3x_3 + x_4 = -3; \\ 4x_1 + 2x_2 + 5x_3 + 3x_4 = 6; \\ x_1 + 2x_2 - 4x_3 - 3x_4 = -3. \end{cases}$ 2) $\begin{cases} 3x_1 + x_2 - 2x_3 = 4; \\ 2x_1 - 3x_2 + x_3 = 9; \\ 5x_1 + x_2 + 3x_3 = -4. \end{cases}$

3) $2A(A + B) - 3AB$,

bunda $A = \begin{pmatrix} 2 & 3 & 4 \\ 1 & -2 & 0 \\ 0 & 1 & 2 \end{pmatrix}$, $B = \begin{pmatrix} 2 & 0 & -2 \\ 1 & 1 & 0 \\ 1 & -1 & 1 \end{pmatrix}$.

4) $X \cdot \begin{pmatrix} 2 & 1 & 0 \\ 1 & 2 & 2 \\ 0 & 1 & -1 \end{pmatrix} = \begin{pmatrix} -1 & 2 & 3 \\ 0 & 2 & -1 \\ 1 & 2 & 3 \end{pmatrix}$.

№30 1) $\begin{cases} 2x_1 + 3x_2 + 5x_3 + x_4 = 6; \\ 3x_1 + x_2 - x_3 + 5x_4 = 0; \\ 2x_1 - x_2 + 3x_4 = -5; \\ 2x_1 + 2x_2 - x_3 + 7x_4 = -3. \end{cases}$ 2) $\begin{cases} 2x_1 - x_2 + 3x_3 = -4; \\ x_1 + 3x_2 - x_3 = 2; \\ 5x_1 + 2x_2 + x_3 = 5. \end{cases}$

3) $3AB + (A - B)(A + 2B)$,

bunda $A = \begin{pmatrix} 2 & 5 & -1 \\ 0 & 2 & 1 \\ 1 & 0 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 1 & -2 & 0 \\ 1 & 0 & 2 \\ 0 & 0 & 3 \end{pmatrix}$.

4) $\begin{pmatrix} 1 & 2 & 1 \\ 0 & -1 & 2 \\ 2 & -1 & 0 \end{pmatrix} \cdot X = \begin{pmatrix} 1 & 1 & 3 \\ 3 & 2 & 1 \\ 0 & 2 & -1 \end{pmatrix}$.

4.2 -turdagi topshiriq.

Topshiriqlarni bajarish uchun namuna.

Misol. Gauss sxemasidan foydalanib, tenglamalar sistemasi 0,001 aniqlikda toping.

$$\begin{cases} 0{,}68x_1 + 0{,}05x_2 - 0{,}11x_3 + 0.08x_4 = 0{,}15, \\ 0{,}21x_1 - 0{,}13x_2 + 0{,}27x_3 - 0{,}8x_4 = 0{,}44, \\ -0.11x_1 - 0{,}84x_2 + 0{,}28x_3 + 0{,}06x_4 = -0{,}83, \\ -0{,}08x_1 + 0{,}15x_2 - 0{,}5x_3 - 0{,}12x_4 = 1{,}16, \end{cases}$$

Yechish: Hisoblashlarni yagona bo‘lish sxemasi bo‘yicha bajaramiz:

Noma’lumlar oldidagi koeffitsient				Ozod had	Nazorat summasi $\sum$	Satr summasi $\sum$
x_1	x_2	x_3	x_4			
0.68 0.21 -0.11 -0.08	0.05 -0.13 -0.84 0.15	-0.11 0.27 0.28 -0.5	0.08 -0.8 0.06 -0.12	2.15 0.44 -0.83 1.16	2.85 -0.01 -1.44 0.61	2.85 -0.01 -1.44 0.61
1	0.0735	-0.1618	0.1176	3.1618	4.1912	4.1912
	-0.1454 -0.8319 0.1559	0.30398 0.2622 -0.5129	-0.8247 0.0729 -0.1106	-0.22398 -0.4822 1.4129	-0.89015 -0.97897 0.9453	-0.8901 -0.97896 0.9453
	1	-2.0906	5.6719	1.5404	6.1221	6.1217
		-1.47697 -0.18697	4.79139 -0.9948	0.7992 1.1723	4.1140 -0.00913	4.1136 -0.0095
		1	-3.2441	-0.5411	-2.7854	-2.7851
			-1.6013	1.0711	-0.5299	-0.5302
			1	-0.6689	0.3309	0.3311
2.8264	-0.3337	-2.7110	-0.6689			
3.8263	0.6664	-1.7119	0.3309			

Javob: $x_1 = 2.826$; $x_2 = -0.334$; $x_3 = -2.711$; $x_4 = -0.669$.

Mustaqil yechish uchun:

Quyidagi amallarni va misollarni mos ravishda bajaring:

Topshiriqlar: Gauss sxemasidan foydalanib, tenglamalar sistemasi 0,001 aniqlikda toping.

№1.

$$\begin{cases} 4{,}4x_1 - 2{,}5x_2 + 19{,}2x_3 - 10{,}8x_4 = 4{,}3\ ; \\ 5{,}5x_1 - 9{,}3x_2 - 14{,}2x_3 + 13{,}2x_4 = 6{,}8\ ; \\ 7{,}1x_1 - 11{,}5x_2 + 5{,}3x_3 - 6{,}7x_4 = -1{,}8; \\ 14{,}2x_1 - 23{,}x_2 - 8{,}8x_3 + 5{,}3x_4 = 7{,}2. \end{cases}$$

№2

$$\begin{cases} 8{,}2x_1 - 3{,}2x_2 + 14{,}2x_3 - 14{,}8x_4 = -8{,}4; \\ 5{,}6x_1 - 12x_2 + 15x_3 - 6{,}4x_4 = 4{,}5; \\ 5{,}7x_1 + 3{,}6x_2 - 12{,}4x_3 - 2{,}3x_4 = -3{,}3; \\ 6{,}8x_1 + 13{,}2x_2 - 6{,}3x_3 + 8{,}7x_4 = 14{,}3. \end{cases}$$

№3

$$\begin{cases} 5{,}7x_1 - 7{,}8x_2 - 5{,}6x_3 - 8{,}3x_4 = -2{,}7; \\ 6{,}6x_1 + 13.1x_2 - 6.3x_3 + 4.3x_4 = -5{,}5; \\ 14{,}7\text{x}1x_1 - 2{,}8x_2 + 5{,}6x_3 - 12{,}1x_4 = 8{,}6; \\ 8{,}5x_1 + 12{,}7x_2 - 23{,}7x_3 + 5{,}7x_4 = 14{,}7. \end{cases}$$

№4

$$\begin{cases} 3{,}8x_1 + 14{,}2x_2 + 6{,}3x_3 - 15{,}5x_4 = 2{,}8; \\ 8{,}3x_1 - 6{,}6x_2 + 5{,}8x_3 + 12{,}2\text{x}4x_4 = -4{,}7; \\ 6{,}4x_1 - 8{,}5x_2 - 4{,}3x_3 + 8{,}8x_4 = 7{,}7; \\ 17{,}1x_1 - 8{,}3x_2 + 14{,}4x_3 - 7{,}2x_4 = 13{,}5; \end{cases}$$

№5

$$\begin{cases} 15{,}7x_1 + 6{,}6x_2 - 5{,}7x_3 - 11{,}5x_4 = -2{,}4; \\ 8{,}3x_1 - 6{,}6x_2 + 5{,}8x_3 + 12{,}2x_4 = -4{,}7; \\ 6{,}4x_1 - 8{,}5x_2 - 4{,}3x_3 + 8{,}8x_4 = 7{,}7; \\ 17{,}1x_1 - 8{,}3x_2 + 14{,}4x_3 - 7{,}2x_4 = 13{,}5. \end{cases}$$

№6

$$\begin{cases} 4{,}3x_1 - 12{,}1x_2 + 23{,}2x_3 - 14{,}1x_4 = 15{,}5; \\ 2{,}4x_1 - 4{,}4x_2 + 3{,}5x_3 + 5{,}5x_4 = 2{,}5; \\ 5{,}4x_1 + 8{,}3x_2 - 7{,}4x_3 - 12{,}7x_4 = 8{,}6; \\ 6{,}3x_1 - 7{,}6x_2 + 1{,}34x_3 + 3{,}7x_4 = 12{,}1; \end{cases}$$

№7

$$\begin{cases} 14{,}4x_1 - 5{,}3x_2 + 14{,}3x_3 - 12{,}7x_4 = -14{,}4; \\ 23{,}4x_1 - 14{,}2x_2 - 5{,}4x_3 + 2{,}1x_4 = 6{,}6; \\ 6{,}3x_1 - 13{,}2x_2 - 6{,}5x_3 + 14{,}3x_4 = 9{,}4; \\ 5{,}6x_1 + 8{,}8x_2 - 6{,}7x_3 - 23{,}8x_4 = 7{,}3. \end{cases}$$

№8

$$\begin{cases} 1{,}7x_1 + 10x_2 - 1{,}3x_3 + 2{,}1x_4 = 3{,}1; \\ 3{,}1x_1 - 1{,}4x_2 - 2{,}1x_3 + 5{,}4x_4 = 2{,}1; \\ 3{,}3x_1 - 7{,}7x_2 + 4{,}4x_3 - 5{,}1x_4 = 1{,}9; \\ 10x_1 - 20{,}1x_2 + 20{,}4x_3 + 1{,}7x_4 = 1{,}8. \end{cases}$$

№9

$$\begin{cases} 1{,}7x_1 - 1{,}8x_2 + 1{,}9x_3 - 57{,}4x_4 = 10; \\ 1{,}1x_1 - 4{,}3x_2 + 1{,}5x_3 - 1{,}7x_4 = 19; \\ 1{,}2x_1 + 1{,}4x_2 + 1{,}6x_3 + 1{,}8x_4 = 20; \\ 7{,}1x_1 - 1{,}3x_2 - 4{,}1x_3 + 5{,}2x_4 = 10. \end{cases}$$

№10

$$\begin{cases} 6{,}1x_1 + 6{,}2x_2 - 6{,}3x_3 + 6{,}4x_4 = 6{,}5; \\ 1{,}1x_1 - 1{,}5x_2 + 2{,}2x_3 - 3{,}8x_4 = 4{,}2; \\ 5{,}1x_1 - 5{,}0x_2 + 4{,}9x_3 - 4{,}8x_4 = 4{,}7; \\ 1{,}8x_1 + 1{,}9x_2 + 2{,}0x_3 - 2{,}1x_4 = 2{,}2. \end{cases}$$

№ 11.

$$\begin{cases} 2{,}2x_1 - 3{,}1x_2 + 4{,}2x_3 - 5{,}1x_4 = 6{,}01; \\ 1{,}3x_1 + 2{,}2x_2 - 1{,}4x_3 + 1{,}5x_4 = 10; \\ 6{,}2x_1 - 7{,}4x_2 + 8{,}5x_3 - 9{,}6x_4 = 1{,}1; \\ 1{,}2x_1 + 1{,}3x_2 + 1{,}4x_3 + 4{,}5x_4 = 1{,}6\,; \end{cases}$$

№12.

$$\begin{cases} 35{,}8x_1 + 2{,}1x_2 - 34{,}5x_3 - 11{,}8x_4 = 0{,}5; \\ 27{,}1x_1 - 7{,}5x_2 + 11{,}7x_3 - 23{,}5x_4 = 12{,}8; \\ 11{,}7x_1 + 1{,}8x_2 - 6{,}5x_3 + 7{,}1x_4 = 1{,}7; \\ 6{,}3x_1 + 10x_2 + 7{,}1x_3 + 3{,}4x_4 = 20{,}8. \end{cases}$$

№ 13.

$$\begin{cases} 35{,}1x_1 + 1{,}7x_2 + 37{,}5x_3 - 2{,}8x_4 = 7{,}5; \\ 45{,}2x_1 + 21{,}1x_2 - 1{,}1x_3 - 1{,}2x_4 = 11{,}1; \\ -21{,}1x_1 + 31{,}7x_2 + 1{,}2x_3 - 1{,}5x_4 = 2{,}1; \\ 31{,}1x_1 + 18{,}1x_2 - 31{,}7x_3 + 2{,}2x_4 = 0{,}5. \end{cases}$$

№ 14.

$$\begin{cases} 1{,}1x_1 + 11{,}2x_2 + 11{,}1x_3 - 13{,}1x_4 = 1{,}3; \\ -3{,}3x_1 + 1{,}1x_2 + 30{,}1x_3 - 20{,}1x_4 = 1{,}1; \\ 7{,}5x_1 + 1{,}3x_2 + 1{,}1x_3 + 10x_4 = 20; \\ 1{,}7x_1 + 7{,}5x_2 - 1{,}8x_3 + 2{,}1x_4 = 1{,}1. \end{cases}$$

№ 15.

$$\begin{cases} 7{,}5x_1 + 1{,}8x_2 - 2{,}1x_3 - 7{,}7x_4 = 1{,}1; \\ -10x_1 + 1{,}3x_2 - 20x_3 - 1{,}4x_4 = 1{,}5; \\ 2{,}8x_1 - 1{,}7x_2 + 3{,}9x_3 + 4{,}8x_4 = 1{,}2; \\ 10x_1 + 31{,}4x_2 - 2{,}1x_3 - 10x_4 = -1{,}1. \end{cases}$$

№ 16.

$$\begin{cases} 30{,}1x_1 - 1{,}4x_2 + 10x_3 - 1{,}5x_4 = 10; \\ -17{,}5x_1 + 11{,}1x_2 + 1{,}3x_3 - 7{,}5x_4 = 1{,}3; \\ 1{,}7x_1 - 21{,}1x_2 + 7{,}1x_3 - 17{,}1x_4 = 10; \\ 2{,}1x_1 + 2{,}1x_2 + 3{,}5x_3 + 3{,}3x_4 = 1{,}7; \end{cases}$$

№ 17.

$$\begin{cases} 7{,}3x_1 - 8{,}1x_2 + 12{,}7x_3 - 6{,}7x_4 = 8{,}8; \\ 11{,}5x_1 + 6{,}2x_2 - 8{,}3x_3 + 9{,}2x_4 = 21{,}5; \\ 8{,}2x_1 - 5{,}4x_2 + 4{,}3x_3 - 2{,}5x_4 = 6{,}2; \\ 2{,}4x_1 + 11{,}5x_2 - 3{,}3x_3 + 14{,}2x_4 = -6{,}2. \end{cases}$$

№ 18.

$$\begin{cases} 4{,}8x_1 + 12{,}5x_2 - 6{,}3x_3 - 9{,}7x_4 = 3{,}5; \\ 22x_1 - 31{,}7x_2 + 12{,}4x_3 - 8{,}7x_4\ 4{,}6; \\ 15x_1 + 21{,}1x_2 - 4{,}5x_3 + 14{,}4x_4 = 15; \\ 8{,}6x_1 - 14{,}4x_2 + 6{,}2x_3 + 2{,}8x_4 = -1{,}2. \end{cases}$$

№ 19.

$$\begin{cases} 6{,}4x_1 + 7{,}2x_2 - 8{,}3x_3 + 42x_4 = 2{,}23; \\ 5{,}8x_1 - 8{,}3x_2 + 14{,}3x_3 - 6{,}2x_4 = 17{,}1; \\ 8{,}6x_1 + 7{,}7x_2 - 18{,}3x_3 + 8{,}8x_4 = -5{,}4; \\ 13{,}2x_1 - 5{,}5x_2 - 6{,}5x_3 + 12{,}2x_4 = 6{,}5. \end{cases}$$

№ 20.

$$\begin{cases} 14{,}2x_1 + 3{,}2x_2 - 4{,}2x_3 + 8{,}5x_4 = 13{,}2; \\ 6{,}3x_1 - 4{,}3x_2 + 12{,}7x_3 - -5{,}8x_4 = -4{,}4; \\ 8{,}4x_1 - 22{,}3x_2 - 5{,}2x_3 + 4{,}7x_4 = 6{,}4; \\ 2{,}7x_1 + 13{,}7x_2 + 6{,}4x_3 - 12{,}7x_4 = 8{,}5. \end{cases}$$

№ 21.

$$\begin{cases} 7{,}3x_1 + 12{,}4x_2 - 3{,}8x_3 - 14{,}3x_4 = 5{,}8; \\ 10{,}7x_1 - 7{,}7x_2 + 12{,}5x_3 + 6{,}6x_4 = -6{,}6; \\ 15{,}6x_1 + 6{,}6x_2 + 14 + 4x_3 - 8{,}7x_4 = 12{,}4; \\ 7{,}5x_1 + 12{,}2x_2 - 8{,}3x_3 + 3{,}7x_4 = 9{,}2. \end{cases}$$

№ 22.

$$\begin{cases} 13{,}2x_1 — 8{,}3x_2 - 4{,}4x_3 + 6{,}2x_4 = 6{,}8; \\ 8{,}3x_1 + 4{,}2x_2 - 5{,}6x_3 + 7{,}7x_4 = 12{,}4; \\ 5{,}8x_1 - 3{,}7x_2 + 12{,}4x_3 - 6{,}2x_4 = 8{,}7; \\ 3{,}5x_1 + 6{,}6x_2 - 13{,}8x_3 - 9{,}3x_4 = -10{,}8 \end{cases} .$$

№ 23.

$$\begin{cases} 8{,}1x_1 + 1{,}2x_2 - 9{,}1x_3 + 1{,}7x_4 = 10; \\ 1{,}1x_1 - 1{,}7x_2 + 7{,}2x_3 - -3{,}4x_4 = 1{,}7; \\ 1{,}7x_1 - 1{,}8x_2 + 10x_3 + 2{,}3x_4 = 2{,}1; \\ 1{,}3x_1 + 1{,}7x_2 - 9{,}9x_3 + 3{,}5x_4 = 27{,}1. \end{cases}$$

№ 24.

$$\begin{cases} 3{,}3x_1 - 2{,}2x_2 - 10x_3 + 1{,}7x_4 = 1{,}1; \\ 1{,}8x_1 + 21{,}1x_2 + 1{,}3x_3 - 2{,}2x_4 = 2{,}2; \\ -10x_1 + 1{,}1x_2 + 20x_3 - 4{,}5x_4 = 10; \\ 70x_1 - 1{,}7x_2 - 2{,}2x_3 + 3{,}3x_4 = 2{,}1. \end{cases}$$

№ 25.

$$\begin{cases} 1{,}7x_1 + 9{,}9x_2 - 20x_3 - 1{,}7x_4 = 1{,}7; \\ 20x_1 + 0{,}5x_2 - 30{,}1x_3 - 1{,}1x_4 = 2{,}1; \\ 10x_1 - 20x_2 + 30{,}2x_3 + 0{,}5x_4 = 1{,}8; \\ 3{,}3x_1 - 0{,}7x_2 + 3{,}3x_3 + 20x_4 = -1{,}7. \end{cases}$$

№ 26.

$$\begin{cases} 1{,}7x_1 - 1{,}3x_2 - 1{,}1x_3 - 1{,}2x_4 = 2{,}2; \\ 10x_1 - 10x_2 - 1{,}3x_3 + 1{,}3x_4 = 1{,}1; \\ 3{,}5x_1 + 3{,}3x_2 + 1{,}2x_3 + 1{,}3x_4 = 1{,}2; \\ 1{,}3x_1 + 1{,}1x_2 - 1{,}3x_3 - 1{,}1x_4 = 10. \end{cases}$$

№ 27.

$$\begin{cases} 1{,}1x_1 + 11{,}3x_2 - 1{,}7x_3 + 1{,}8x_4 = 10; \\ 1{,}3x_1 - 11{,}7x_2 + 1{,}8x_3 + 1{,}4x_4 = 1{,}3; \\ 1{,}1x_1 - 10{,}5x_2 - 1{,}7x_3 - 1{,}5x_4 = 1{,}1; \\ 1{,}5x_1 - 0{,}5x_2 + 1{,}8x_3 - 1{,}1x_4 = 10 . \end{cases}$$

№ 28.

$$\begin{cases} 1{,}4x_1 + 2{,}1x_2 - 3{,}3x_3 + 1{,}1x_4 = 10; \\ 10x_1 - 1{,}7x_2 + 1{,}1x_3 - 1{,}5x_4 = 1{,}7; \\ 2{,}2x_1 + 34{,}4x_2 - 1{,}1x_3 - 1{,}2x_4 = 20; \\ 1{,}1x_1 + 1{,}3x_2 + 1{,}2x_3 + 1{,}4x_4 = 1{,}3. \end{cases}$$

4.3 -turdagi topshiriq.

Topshiriqlarni bajarish uchun na'muna.

Misol. Chiziqli tenglamalar sistemasini bosh elementlar usuli bilan 0,001 aniqlikda yeching.

$$\begin{cases} 2.74x_1 - 1.18x_2 + 3.17x_3 = 2.18 \\ 1.12x_1 + 0.83x_2 - 2.16x_3 = -1.15 \\ 0.18x_1 + 1.27x_2 + 0.76x_3 = 3.23 \end{cases}$$

Yechish: Hisoblashlarni quyidagi sxema bilan olib boramiz:

m_i	Noma'lumlar oldidagi koeffitsientlar			Ozod had	$\sum$	$\sum_1$
	x_1	x_2	x_3			
-1 0,6814 -0,2397	2,74 1,12 0,18	-1,18 0,83 1,27	3,17 -2,16 0,76	2,18 -1,15 3,23	6,91 -1,36 5,44	6,91 -1,36 5,44
-1 0,1596	2,9870 -0,4768	0,0259 1,5528	- -	0,3355 2,7075	3,3485 3,7837	3,3484 3,7835
-	-	1,5569	-	2,7601	4,3181	4,3170
	0,0970	1,7728	1,2638			
	1,0970	2,7735	2,2638			

$$x_2 = \frac{2.7602}{1.5569} = 1.7728;\ \ x_2 = \frac{4.3181}{1.5569} = 2.7735$$

$$x_1 = \frac{0.3355 - 0.0259 * 1.7728}{2.9870} = 0.0970;\ x_1 = \frac{3.3485 - 0.0259 * 2.7735}{2.9870} =$$

$$= 1.0970$$

$$x_3 = \frac{2.18 - 2.74 * 0.0970 + 1.18 * 1.7728}{3.17} = 1.2638;$$

$$x'_3 = \frac{6.91 - 2.74 * 1.0970 + 1.18 * 2.7735}{3.17} = 2.2638;$$

$x_1 \approx 0.097;\ x_2 \approx 1.773;\quad x_3 \approx 1.264/$

Mustaqil yechish uchun:

Quyidagi amallarni va misollarni mos ravishda bajaring:

Topshiriqlar. Chiziqli tenglamalar sistemasini bosh elementlar usuli bilan 0,001 aniqlikda yeching.

№ 7

$$\begin{cases} 1.24x_1 - 0.87x_2 - 3.17x_3 = 0.46; \\ 2.11x_1 - 0.45x_2 + 1.44x_3 = 1.50; \\ 0.48x_1 + 1.25x_2 - 0.63x_3 = 0.35. \end{cases}$$

№ 8

$$\begin{cases} 0.64x_1 - 0.83x_2 + 4.2x_3 = 2.23; \\ 0.58x_1 - 0.83x_2 + 1.43x_3 = 1.71; \\ 0.86x_1 + 0.77x_2 + 0.88x_3 = -0.54. \end{cases}$$

№ 9

$$\begin{cases} 1.24x_1 - 0.42x_2 + 0.85x_3 = 1.32; \\ 0.63x_1 - 1.43x_2 - 0.58x_3 = -0.44; \\ 0.84x_1 - 2.23x_2 - 0.52x_3 = 0.64. \end{cases}$$

№ 10

$$\begin{cases} 0.83x_1 + 2.18x_2 - 1.27x_3 = 0.28; \\ 2.18x_1 - 1.41x_2 + 1.03x_3 = -1.18. \\ -1.73x_1 + 1.03x_2 + 2.27x_3 = 0.72. \end{cases}$$

№ 11

$$\begin{cases} 0.62x_1 - 0.44x_2 - 0.86x_3 = 0.68; \\ 0.83x_1 + 0.42x_2 - 0.56x_3 = 1.24; \\ 0.58x_1 - 0.37x_2 - 0.62x_3 = 0.87. \end{cases}$$

№ 12

$$\begin{cases} 1.35x_1 - 0.72x_2 + 1.38x_3 = 0.88; \\ -0.72x_1 + 1.45x_2 - 2.18x_3 = 1.72; \\ 1.38x_1 - 2.18x_2 + 0.93x_3 = -0.72. \end{cases}$$

№ 13

$$\begin{cases} 0.46x_1 + 1.72x_2 + 2.53x_3 = 2.44; \\ 1.53x_1 - 2.32x_2 - 1.83x_3 = 2.83; \\ 0.75x_1 + 0.86x_2 + 3.72x_3 = 1.06. \end{cases}$$

№ 14

$$\begin{cases} 2.16x_1 - 3.18x_2 + 1.26x_3 = 1.83; \\ -3.18x_1 + 0.63x_2 - 2.73x_3 = 0.54; \\ 1.26x_1 - 0.63x_2 + 3.15x_3 = 1.72. \end{cases}$$

№ 15

$$\begin{cases} 4.24x_1 + 2.73x_{2_1} - 1.55x_3 = 1.87; \\ 2.34x_1 + 1.27x_2 + 3.15x_3 = 2.16; \\ 3.05x_1 - 1.05x_2 - 0.63x_3 = -1.25. \end{cases}$$

№ 16

$$\begin{cases} 1.36x_1 + 0.92x_2 - 1.87x_3 = 2.1; \\ 0.92x_1 - 2.24x_1 + 0.77x_3 = -2; \\ -1.87x_1 + 0.77x_2 - 1.16x_3 = 0.17. \end{cases}$$

№ 17

$$\begin{cases} 2.32x_1 + 1.17x_2 - 0.28x_3 = 1.4; \\ 1.17x_1 - 1.43x_2 + 0.88x_3 = -0; \\ -0.28x_1 - 0.88x_2 + 1.45x_3 = 1.09. \end{cases}$$

№ 18

$$\begin{cases} 0.75x_1 - 1.24x_2 + 1.56x_3 = 0.4\ ; \\ -1.24x_1 + 0.18x_2 - 1.72x_3 = -0.5; \\ 1.56x_1 - 1.72x_2 + 0.79x_3 = 1.03. \end{cases}$$

№ 19

$$\begin{cases} 1.18x_1 + 2.32x_2 - 0.67x_3 = 1.83; \\ 2.32x_1 + 1.87x_2 + 1.35x_3 = -0.73; \\ -0.67x_1 + 1.35x_2 - 0.88x_3 = 0.68. \end{cases}$$

№ 20

$$\begin{cases} 0.78x_1 + 1.13x_2 + 1.87x_3 = 0.83; \\ 1.13x_1 - 0.68x_2 + 2.16x_3 = -0.27; \\ 1.87x_1 - 2.16x_2 - 2.63x_3 = 1.37. \end{cases}$$

№ 21

$$\begin{cases} 1.17x_1 - 0.65x_2 + 1.54x_3 = -1.43; \\ -0.65x_1 + 1.16x_2 - 1.73x_3 = 0.68; \\ 1.54x_1 - 1.73x_2 + 2.15x_3 = 1.87. \end{cases}$$

№ 22

$$\begin{cases} 0.87x_1 + 1.35x_2 - 0.44x_3 = 1.51; \\ 1.35x_1 - 1.22x_2 + 2.32x_3 = 0.71; \\ -0.44x_1 + 1.22x_2 - 3.73x_3 = 0.53. \end{cases}$$

№ 23

$$\begin{cases} 1.17x_1 + 2.23x_2 - 0.77x_3 = 1.11; \\ 2.23x_1 - 0.81x_2 + 1.72x_3 = 1.88; \\ -0.77x_1 + 1.72x_2 - 0.65x_3 = 0.57. \end{cases}$$

№ 24

$$\begin{cases} 2.16x_1 + 1.45x_2 - 0.89x_3 = 0.61; \\ 1.45x_1 - 2.44x_2 + 1.18x_3 = 1.05; \\ -0.89x_1 + 1.18x_2 - 2.07x_3 = -0.83. \end{cases}$$

№ 25

$$\begin{cases} 0.64x_1 + 1.05x_2 - 2.93x_3 = 1.18; \\ 1.05x_1 - 1.41x_2 + 0.16x_3 = -0.27; \\ -2.93x_1 + 0.16x_2 - 1.56x_3 = 0.72. \end{cases}$$

№ 26

$$\begin{cases} 1.54x_1 - 0.75x_2 + 1.36x_3 = 2.45; \\ -0.75x_1 + 0.87x_2 - 0.79x_3 = 1.07; \\ 1.36x_1 - 0.87x_2 - 0.79x_3 = 0.54. \end{cases}$$

№ 27

$$\begin{cases} 2.44x_1 - 1.16x_2 + 0.83x_3 = 0.65; \\ -1.16x_1 - 3.45x_2 + 0.57x_3 = 1.88; \\ 0.83x_1 + 0.57x_2 - 1.71x_3 = 0.74. \end{cases}$$

№ 28

$$\begin{cases} 2.56x_1 + 0.67x_2 - 1.78x_3 = 1.14; \\ 0.67x_1 - 2.67x_2 + 1.35x_3 = 0.66; \\ -1.78x_1 + 1.35x_2 - 0.55x_3 = 1.72. \end{cases}$$

№ 29

$$\begin{cases} 0.53x_1 - 0.75x_2 + 1.83x_3 = 0.68; \\ -0.75x_1 + 0.68x_2 - 1.19x_3 = 0.95; \\ 1.83x_1 - 1.19x_2 + 2.15x_3 = 1.27. \end{cases}$$

№ 30

$$\begin{cases} 1.65x_1 - 1.76x_2 + 0.77x_3 = 2.15; \\ -1.76x_1 + 1.04x_2 - 0.77x_3 = 0.82; \\ 0.77x_1 - 2.61x_2 - 3.18x_3 = -0.73. \end{cases}$$

4.4 -turdagi topshiriq.

Topshiriqlarni bajarish uchun na'muna.

Misol. Chiziqli tenglamalar sistemasini kvadrat ildizlar usuli bilan 0,001 aniqlikda yeching.

$$\begin{cases} 4.25x_1 - 1.48x_2 + 0.73x_3 = 1.44 \\ -1.48x_1 + 1.73x_2 - 1.85x_3 = 2.73 \\ 0.73x_1 - 1.85x_2 + 1.93x_3 = -0.64 \end{cases}$$

Yechish: Hisoblashlarni quyidagi sxema bo'yicha bajaramiz

Noma'lumlar oldidagi koeffitsientlar			Ozod had	$\sum$	$\sum_1$
x_1	x_2	x_3			
4.25 -1.48 0.73	-1.48 1.73 -1.85	0.73 -1.85 1.93	1.44 2.73 -0.64	4.94 1.13 0.17	4.94 1.13 0.17
2.0616	-0.7179 1.1021	0.3541 -1.4480 0.5405i	0.6985 2.9323 -6.2141i	2.3962 2.5862 -5.6731i	2.3963 2.5864 -5.6736i
-2.0200	-12.4446	-11.4969			
-1.0199	-11.4436	-10.4960			

$$x_3 = -\frac{6.2141i}{0.5405i} = -11.4969; \;\; x'_3 = -\frac{5.6731i}{0.5405i} = -10.4960;$$

$$x_2 = \frac{2.9323 - 1.4480 * 10.4969}{1.1021} = -12.4446;$$

$$x_2 = \frac{2.5862 - 1.4480 * 10.4960}{1.1021} = -11.4436;$$

$$x_1 = \frac{0.6985 + 0.3541 * 11.4469 - 0.7179 * 12.4446}{2.0616} = -2.0200;$$

$$x'_1 = \frac{2.3962 + 0.3541 * 10.4960 - 0.7179 * 11.4436}{2.0616} = -1.0199;$$

$$x_1 \approx -2.020; \;\; x_2 \approx -12.445; \quad\; x_3 \approx -11.497;$$

Mustaqil yechish uchun:

Quyidagi amallarni va misollarni mos ravishda bajaring:

Topshiriqlar. Chiziqli tenglamalar sistemasini kvadrat ildizlar usuli bilan 0,001 aniqlikda yeching.

N1

$$\begin{cases} 3.14x_1 \_ 2.12x_2 + 1.17x_3 = 1.27; \\ -2.12x_1 + 1.32x_2 - 2.45x_3 = 2.13; \\ 1.17x_1 - 2.45x_2 + 1.18x_3 = 3.14. \end{cases}$$

N2

$$\begin{cases} 2.45x_1 + 1.75x_2 + 3.24x_3 = 1.23; \\ 1.75x_1 - 1.16x_2 + 2.18x_3 = 3.43; \\ -3.24x_1 + 2.18x_2 - 1.85x_3 = -0.16. \end{cases}$$

N3

$$\begin{cases} 1.65x_1 - 2.27x_2 + 0.18x_3 = 2.25; \\ -2.27x_1 + 1.73x_2 - 0.46x_3 = 0.93; \\ 0.18x_1 - 0.46x_2 + 2.16x_3 = 1.33. \end{cases}$$

N4

$$\begin{cases} 3.23x_1 + 1.62x_2 + 0.65x_3 = 1.28; \\ 1.62x_1 - 2.33x_2 - 1.53x_3 = 0.87; \\ 0.65x_1 - 1.43x_2 + 2.18x_3 = -2.87. \end{cases}$$

N5

$$\begin{cases} 0.93x_1 + 1.42x_2 - 2.55x_3 = 2.48; \\ 1.42x_1 - 2.87x_2 + 2.36x_3 = -0.75; \\ -2.55x_1 + 2.36x_2 - 1.44x_2 = 1.83. \end{cases}$$

N6

$$\begin{cases} 1.42x_1 - 2.15x_2 + 1.07x_3 = 2.48; \\ -2.15x_1 - 0.76x_2 - 2.18x_3 = 1.15; \\ 1.07x_1 - 2.18x_2 + 1.23x_3 = 0.88. \end{cases}$$

N7

$$\begin{cases} 2.23x_1 - 0.71x_2 + 0.63x_3 = 1.28; \\ -0.71x_1 + 1.45x_2 - 1.34x_3 = 0.64. \\ 0.63x_1 - 1.34x_2 + 0.77x_3 = -0.87. \end{cases}$$

N8

$$\begin{cases} 1.63x_1 + 1.27x_2 - 0.84x_3 = 1.51; \\ 1.27x_1 + 0.65x_2 - 1.27x_3 = -0.63; \\ -0.84x_1 + 1.27x_2 - 1.21x_3 = 2.15. \end{cases}$$

N9

$$\begin{cases} 0.78x_1 + 1.08x_2 - 1.35x_3 = 0.57; \\ 1.08x_1 - 1.28x_2 + 0.37x_3 = 1.27; \\ -1.35x_1 + 0.37x_2 + 2.86x_3 = 0.47. \end{cases}$$

N10

$$\begin{cases} 0.83x_1 + 2.18x_2 - 1.73x_3 = 0.28; \\ 2.18x_1 - 1.41x_2 + 1.03x_3 = -1.18; \\ -1.73x_1 + 1.03x_2 + 2.27x_3 = 0.72. \end{cases}$$

N11

$$\begin{cases} 2.74x_1 - 1.18x_2 + 1.23x_3 = 0.16; \\ -1.18x_1 + 1.71x_2 - 0.52x_3 = 1.81; \\ 1.23x_1 - 0.52x_2 + 0.62x_3 = -1.25. \end{cases}$$

N12

$$\begin{cases} 1.35x_1 - 0.72x_2 + 1.38x_3 = 0.88; \\ -0.72x_1 + 1.45x_2 - 2.18x_3 = 1.72 ; \\ 1.38x_1 - 2.18x_2 + 0.93x_3 = -0.72. \end{cases}$$

N13

$$\begin{cases} 1.48x_1 + 0.75x_2 - 1.23x_3 = 0.83; \\ 0.75x_1 - 0.96x_2 + 1.64x_3 = -1.12; \\ -1.23x_1 + 1.64x_2 - 0.55x_3 = 0.47. \end{cases}$$

N14

$$\begin{cases} 2.16x_1 - 3.18x_2 + 1.26x_3 = 1.83; \\ -3.18x_1 + 0.63x_2 - 2.73x_3 = 0.54; \\ 1.26x_1 - 2.73x_2 + 3.15x_3 = 1.72. \end{cases}$$

N15

$$\begin{cases} 0.63x_1 - 1.72x_2 + 3.37x_3 = -0.75; \\ -1.72x_1 - 2.27x_2 + 1.62x_3 = 1.27; \\ 3.27x_1 + 1.62x_2 - 0.43x_3 = 2.74. \end{cases}$$

N16

$$\begin{cases} 1.36x_1 + 0.92x_2 - 1.87x_3 = 2.15; \\ 0.92x_1 - 2.24x_2 + 0.77x_3 = -2.06; \\ -1.87x_1 + 0.77x_2 - 1.16x_3 = 0.17. \end{cases}$$

N17

$$\begin{cases} 2.32x_1 + 1.17x_2 - 0.28x_3 = 1.43; \\ 1.17x_1 - 1.43x_2 + 0.88x_3 = -0.47; \\ -0.28x_1 + 0.88x_2 - 1.45x_3 = 1.09. \end{cases}$$

N18

$$\begin{cases} 0.75x_1 - 1.24x_2 + 1.56x_3 = 0.49; \\ 1.24x_1 + 0.18x_2 - 1.72x_3 = -0.57; \\ 1.56x_1 - 1.27x_2 + 0.79x_3 = 1.03. \end{cases}$$

N19

$$\begin{cases} 1.18x_1 + 2.32x_2 - 0.67x_3 = 1.83; \\ 2.32x_1 + 1.87x_2 - 1.35x_3 = -0.73 ; \\ -0.67x_1 + 1.35x_2 - 0.88x_3 = 0.68. \end{cases}$$

N20

$$\begin{cases} 0.78x_1 + 1.13x_2 + 1.87x_3 = 0.83; \\ 1.13x_1 - 0.68x_2 + 2.16x_3 = -0.27; \\ 1.87x_1 + 2.16x_2 - 2.63x_3 = 1.37. \end{cases}$$

N21

$$\begin{cases} 1.17x_1 - 0.65x_2 + 1.54x_3 = -1.43; \\ -0.65x_1 + 1.16x_2 - 1.73x_3 = 0.68; \\ 1.54x_1 - 1.73x_2 + 2.15x_3 = 1.87. \end{cases}$$

N22

$$\begin{cases} 0.87x_1 + 1.35x_2 - 0.44x_3 = 1.51; \\ 1.35xx_1 - 1.22x_2 + 2.32x_3 = 0.71; \\ 0.44x_1 + 2.32x_2 - 3.73x_3 = 0.53. \end{cases}$$

N23

$$\begin{cases} 1.17x_1 + 2.23x_2 - 0.77x_3 = 1.11; \\ 2.23x_1 - 0.81x_2 + 1.72x_3 = 1.88; \\ -0.77x_1 + 1.72x_2 - 0.65x_3 = 0.72. \end{cases}$$

N24

$$\begin{cases} 2.16x_1 - 1.45x_2 + 0.89x_3 = 0.61; \\ 1.45x_1 - 2.44x_2 + 1.18x_3 = 1.05; \\ -0.89x_1 + 1.18x_2 - 2.07x_3 = -0.83. \end{cases}$$

N25

$$\begin{cases} 0.64x_1 + 1.05x_2 - 2.93x_3 = 1.18; \\ 1.05x_1 - 1.41x_2 + 0.16x_3 = -0.27; \\ -2.93x_1 + 0.16x_2 - 1.51x_3 = 0.72. \end{cases}$$

N26

$$\begin{cases} 1.54x_1 - 0.75x_2 + 1.36x_3 = 2.45; \\ -0.75x_1 + 0.87x_2 - 0.79x_3 = 1.07; \\ 1{,}36x_1 - 0{,}79x_2 + 0{,}64x_3 = 0{,}54. \end{cases}$$

N27

$$\begin{cases} 2.44x_1 - 1.16x_2 + 0.83x_3 = 0.65; \\ -1.16x_1 - 3.45x_2 + 0.57x_3 = 1.88; \\ 0.83x_1 + 0.57x_2 - 1.71x_3 = 0.74. \end{cases}$$

N28

$$\begin{cases} 2.56x_1 + 0.67x_2 - 1.78x_3 = 1.14; \\ 0.67x_1 - 2.67x_2 + 1.35x_3 = 0.66; \\ -1.78x_1 + 1.35x_2 - 0.55x_3 = 1.72. \end{cases}$$

N29

$$\begin{cases} 0.53x_1 - 0.75x_2 + 1.83x_3 = 0.68; \\ -0.75x_1 + 0.68x_2 - 1.19x_3 = 0.95; \\ 1.83x_1 - 1.19x_2 + 2.15x_3 = 1.27. \end{cases}$$

N30

$$\begin{cases} 1.65x_1 - 1.76x_2 + 0.77x_3 = 2.15; \\ -1.76x_1 + 1.04x_2 - 2.61x_3 = 0.82; \\ 0.77x_1 - 2.61x_2 - 3.18x_3 = -0.73. \end{cases}$$

4.5 -turdagi topshiriq

Topshiriqlarni bajarish uchun na'muna.

Misol. Kerakli qadamlar sonini oldindan baholash bilan tenglamalar sistemasini iteratsiya usuli yordamida 0,001 aniqlikda yeching

Yechish:

$$\begin{cases} x_1 = 0{,}32x_1 - 0{,}05x_2 + 0{,}11x_3 - 0{,}08x_4 + 2{,}15; \\ x_2 = 0{,}11x_1 + 0{,}16x_2 - 0{,}28x_3 - 0{,}06x_4 - 0{,}83; \\ x_3 = 0{,}08x_1 - 0{,}15x_2 + 0{,}12x_4 + 0.16; \\ x_4 = -0{,}21x_1 + 0{,}13x_2 - 0{,}27x_3 + 0{,}44\,. \end{cases}$$

0,001 aniqlikda yechim beradigan qadamlar sonini

$$\|X^* - x^k\| \le \frac{\|A\|^{k+1}}{1 - \|A\|} \cdot \|F\| \le 0{,}001.$$

formula bilan topamiz. Bunda

$$\|A\|_1 = max\{0,56; 0,61; 0,35; 0,61\} < 1;$$

Demak iteratsion protsess yaqinlashuchi; $\|F\|_1 = 2,15$. Quyidagilarga egamiz

$$\frac{0,61^{k+1}}{0,39} \cdot 2,15 < 0,001;\ 0,61^{k+1} < \frac{0,001 \cdot 0,39}{2,15};$$

$$(k+1) \cdot \log 0,61 < -3 + \log 0,39 - \log 2,15$$

$$\text{K} + 1 > \frac{-3 + 1,5911 - 0,3324}{1,7853} = 17,5;\ \ \text{k} \geq 17.$$

Hisoblashlarni jalvalda joylaymiz:

K	x_1	x_2	x_3	x_4
0	2,15	-0,83	1,16	0,44
1	2,9719	-1,0775	1,5093	-0,4326
2	2,3555	-1,0721	1,5075	-0,7317
3	3,5017	-1,0106	1,5015	-0,8111
4	3,5511	-0,9277	1,4944	-0,8321
5	3,5637	-0,9563	1,4834	-0,8298
6	3,5678	-0,9566	1,4890	-0,8332
7	3,5700	-0,9575	1,4889	-0,8656
8	3,5709	-0,9573	1,4890	-0,8362
9	3,5712	-0,9571	1,4889	-0,8364
10	3,5713	-0,9570	1,4890	-0,8364

10-qadamda yaqinlashish yetarli darajada bo‘ladi.

Javob: $x_1 \approx 3,571$; $x_2 \approx$-0,957; $x_3 \approx 1,489$; $x_4 \approx -0,836$.

Mustaqil yechish uchun:

Quyidagi amallarni va misollarni mos ravishda bajaring:

Topshiriqlar. Kerakli qadamlar sonini oldindan baholash bilan tenglamalar sistemasini iteratsiya usuli yordamida 0,001 aniqlikda yeching

$$\text{№ 1.} \begin{cases} x_1 = 0.23x_1 - 0.04x_2 + 0.21x_3 - 0.18x_4 + 1.24; \\ x_2 = 0.45x_1 - 0.23x_2 + 0.06x_3 - 0.88; \\ x_3 = 0.26x_1 + 0.34x_2 - 0.11x_3 + 0.62; \\ x_4 = 0.05x_1 - 0.26x_2 + 0.34x_3 - 0.12x_4 - 1.17; \end{cases}$$

$$
\text{№ 2.}\begin{cases}
x_1 = 0.21x_1 + 0.12x_2 - 0.34x_3 - 0.16x_4 - 0.64;\\
x_2 = 0.34x_1 - 0.08x_2 + 0.17x_3 - 0.18x_4 + 1.42;\\
x_3 = 0.16x_1 + 0.34x_2 + 0.15x_3 - 0.31x_4 - 0.42;\\
x_4 = 0.12x_1 - 0.26x_2 - 0.08x_3 + 0.25x_4 + 0.83;
\end{cases}
$$

$$
\text{№ 3.}\begin{cases}
x_1 = 0.32x_1 - 0.18x_2 + 0.02x_3 + 0.21x_4 + 1.18;\\
x_2 = 0.16x_1 + 0.12x_2 - 0.14x_3 + 0.27x_4 - 0.65;\\
x_3 = 0.37x_1 + 0.27x_2 - 0.02x_3 - 0.24x_4 + 2.23;\\
x_4 = 0.12x_1 + 0.21x_2 - 0.18x_3 + 0.25x_4 - 1.13;
\end{cases}
$$

$$
\text{№ 4.}\begin{cases}
x_1 = 0.42x_1 - 0.32x_2 + 0.03x_3 + 0.44;\\
x_2 = 0.11x_1 - 0.26x_2 - 0.36x_3 + 1.42;\\
x_3 = 0.12x_1 + 0.08x_2 - 0.14x_3 - 0.24x_4 - 0.83;\\
x_4 = 0.15x_1 - 0.35x_2 - 0.18x_3 - 1.42;
\end{cases}
$$

$$
\text{№ 5.}\begin{cases}
x_1 = 0.18x_1 - 0.34x_2 - 0.12x_3 + 0.15x_4 - 1.33;\\
x_2 = 0.11x_1 + 0.23x_2 - 0.15x_3 + 0.32x_4 + 0.84;\\
x_3 = 0.05x_1 - 0.12x_2 + 0.14x_3 - 0.18x_4 - 1.16;\\
x_4 = 0.12x_1 + 0.08x_2 + 0.06x_3 - 0.57;
\end{cases}
$$

$$
\text{№ 6.}\begin{cases}
x_1 = 0.13x_1 + 0.23x_2 - 0.44x_3 - 0.05x_4 + 2.13;\\
x_2 = 0.24x_1 - 0.31x_2 + 0.15x_3 - 0.18;\\
x_3 = 0.06x_1 + 0.15x_2 - 0.23x_3 + 1.44;\\
x_4 = 0.72x_1 - 0.08x_2 - 0.05x_3 + 2.42;
\end{cases}
$$

$$
\text{№ 7.}\begin{cases}
x_1 = 0.17x_1 + 0.31x_2 - 0.18x_3 + 0.22x_4 - 1.71;\\
x_2 = -0.21x_1 + 0.33x_2 + 0.22x_3 + 0.62;\\
x_3 = 0.32x_1 - 0.18x_2 + 0.05x_3 - 0.19x_4 - 0.89;\\
x_4 = 0.12x_1 + 0.28x_2 - 0.14x_3 + 0.94;
\end{cases}
$$

$$
\text{№ 8.}\begin{cases}
x_1 = 0.13x_1 + 0.27x_2 - 0.22x_3 - 0.18x_4 + 1.21;\\
x_2 = -0.21x_1 - 0.45x_3 + 0.18x_4 - 0.33;\\
x_3 = 0.12x_1 + 0.13x_2 - 0.33x_3 + 0.18x_4 - 0.48;\\
x_4 = 0.33x_1 - 0.05x_2 + 0.06x_3 - 0.28x_4 - 0.17;
\end{cases}
$$

$$
\text{№ 9.}\begin{cases}
x_1 = 0.19x_1 - 0.07x_2 + 0.38x_3 - 0.21x_4 - 0.81;\\
x_2 = -0.22x_1 + 0.08x_2 + 0.11x_3 + 0.33x_4 - 0.64;\\
x_3 = 0.51x_1 - 0.07x_2 + 0.09x_3 - 0.11x_4 + 1.71;\\
x_4 = 0.33x_1 - 0.41x_2 - 1.21
\end{cases}
$$

$$
\text{№ 10.}\begin{cases}
x_1 = 0.22x_2 - 0.11x_3 + 0.31x_4 + 2.7;\\
x_2 = 0.38x_1 - 0.12x_3 + 0.22x_4 - 01.5;\\
x_3 = 0.11x_1 + 0.23x_2 - 0.51x_4 + 1.2;\\
x_4 = 0.17x_1 - 0.21x_2 + 0.31x_3 - 0.17;
\end{cases}
$$

№ 11. $\begin{cases} x_1 = 0.07x_1 - 0.08x_2 + 0.11x_3 - 0.18x_4 + 1.18; \\ x_2 = 0.18x_1 + 0.52x_2 + 0.21x_4 + 1.17; \\ x_3 = 0.13x_1 + 0.31x_2 - 0.21x_4 - 1.02; \\ x_4 = 0.08x_1 - 0.33x_3 + 0.28x_4 - 0.28; \end{cases}$

№ 12. $\begin{cases} x_1 = 0.05x_1 - 0.06x_2 - 0.12x_3 + 0.14x_4 - 1.18; \\ x_2 = 0.04x_1 - 0.12x_2 + 0.08x_3 + 0.11x_4 + 0.65; \\ x_3 = 0.34x_1 + 0.08x_2 - 0.06x_3 + 0.14x_4 - 2.23; \\ x_4 = 0.11x_1 + 0.12x_2 - 0.03x_3 - 0.8; \end{cases}$

№ 13. $\begin{cases} x_1 = 0.08x_1 - 0.03x_2 - 0.04x_4 - 1.2; \\ x_2 = 0.31x_1 + 0.27x_3 - 0.27x_4 - 0.81; \\ x_3 = 0.33x_1 - 0.07x_3 + 0.07x_4 - 0.92; \\ x_4 = 0.11x_1 + 0.03x_3 + 0.03x_4 + 0.17; \end{cases}$

№ 14. $\begin{cases} x_1 = 0.12x_1 - 0.23x_2 + 0.25x_3 - 0.16x_4 + 1.24; \\ x_2 = 0.14x_1 + 0.34x_2 - 0.18x_3 + 0.24x_4 - 0.89; \\ x_3 = 0.33x_1 + 0.03x_2 + 0.16x_3 - 0.32x_4 + 1.15; \\ x_4 = 0.12x_1 - 0.05x_2 + 0.15x_4 - 0.57; \end{cases}$

№ 15. $\begin{cases} x_1 = 0.23x_1 - 0.14x_2 + 0.06x_3 - 0.12x_4 + 1.21; \\ x_2 = 0.12x_1 + 0.32x_3 - 0.18x_4 - 0.72; \\ x_3 = 0.08x_1 - 0.12x_2 + 0.23x_3 + 0.32x_4 - 0.58; \\ x_4 = 0.25x_1 + 0.22x_2 + 0.14x_3 + 1.56; \end{cases}$

№ 16. $\begin{cases} x_1 = 0.14x_1 + 0.23x_2 + 0.18x_3 + 0.17x_4 - 1.42; \\ x_2 = 0.12x_1 - 0.14x_2 + 0.08x_3 + 0.09x_4 - 0.83; \\ x_3 = 0.16x_1 + 0.24x_2 - 0.35x_4 + 1.21 \\ x_4 = 0.23x_1 - 0.08x_2 + 0.05x_3 + 0.25x_4 + 0.65; \end{cases}$

№ 17. $\begin{cases} x_1 = 0.24x_1 + 0.21x_2 + 0.06x_3 - 0.34x_4 + 1.18; \\ x_2 = 0.05x_1 + 0.32x_3 + 0.12x_4 - 0.57; \\ x_3 = 0.35x_1 - 0.27x_2 - 0.05x_4 + 0.68; \\ x_4 = 0.12x_1 - 0.43x_2 + 0.04x_3 - 0.21x_4 - 2.14; \end{cases}$

№ 18. $\begin{cases} x_1 = 0.17x_1 + 0.27x_2 - 0.13x_3 - 0.11x_4 - 1.42; \\ x_2 = 0.13x_1 - 0.12x_2 + 0.09x_3 - 0.06x_4 + 0.48; \\ x_3 = 0.11x_1 + 0.05x_2 - 0.02x_3 + 0.12x_4 - 2.34; \\ x_4 = 0.13x_1 + 0.18x_2 + 0.24x_3 + 0.43x_4 + 0.72; \end{cases}$

№ 19. $\begin{cases} x_1 = 0.15x_1 + 0.05x_2 - 0.08x_3 + 0.14x_4 - 0.48; \\ x_2 = 0.32x_1 - 0.13x_2 - 0.12x_3 + 0.11x_4 + 1.24; \\ x_3 = 0.17x_1 + 0.06x_2 - 0.08x_3 + 0.12x_4 + 1.15; \\ x_4 = 0.21x_1 - 0.16x_2 + 0.36x_3 - 0.88; \end{cases}$

№ 20. $\begin{cases} x_1 = 0.28x_2 - 0.17x_3 + 0.06x_4 - 0.21; \\ x_2 = 0.52x_1 + 0.12x_3 + 0.17x_4 - 1.17; \\ x_3 = 0.17x_1 - 0.18x_2 + 0.21x_3 - 0.81; \\ x_4 = 0.11x_1 + 0.22x_2 + 0.03x_3 + 0.05x_4 + 0.72; \end{cases}$

№ 21. $\begin{cases} x_1 = 0.52x_2 + 0.08x_3 + 0.13x_4 - 0.22; \\ x_2 = 0.07x_1 - 0.38x_2 - 0.05x_3 + 0.41x_4 + 1.8; \\ x_3 = 0.04x_1 + 0.42x_2 + 0.11x_3 - 0.07x_4 - 1.3; \\ x_4 = 0.17x_1 + 0.18x_2 - 0.13x_3 + 0.19x_4 + 0.33; \end{cases}$

№ 22. $\begin{cases} x_1 = 0.01x_1 + 0.02x_2 - 0.62x_3 + 0.08x_4 - 1.3; \\ x_2 = 0.03x_1 + 0.28x_2 + 0.33x_3 - 0.07x_4 + 1.1; \\ x_3 = 0.09x_1 + 0.13x_2 + 0.42x_3 + 0.28x_4 - 1.7; \\ x_4 = 0.19x_1 - 0.23x_2 + 0.08x_3 + 0.37x_4 + 1.5; \end{cases}$

№ 23. $\begin{cases} x_1 = 0.17x_2 - 0.33x_3 + 0.18x_4 - 1.2; \\ x_2 = 0.18x_2 + 0.43x_3 - 0.08x_4 + 0.33; \\ x_3 = 0.22x_1 + 0.18x_2 + 0.21x_3 + 0.07x_4 + 0.48; \\ x_4 = 0.08x_1 + 0.07x_2 + 0.21x_3 + 0.04x_4 - 1.2; \end{cases}$

№ 24. $\begin{cases} x_1 = 0.03x_1 - 0.05x_2 + 0.22x_3 - 0.33x_4 + 0.43; \\ x_2 = 0.22x_1 + 0.55x_2 - 0.08x_3 + 0.07x_4 - 1.8; \\ x_3 = 0.33x_1 + 0.13x_2 - 0.08x_3 - 0.05x_4 - 0.8; \\ x_4 = 0.08x_1 + 0.17x_2 + 0.29x_3 + 0.33x_4 + 1.7; \end{cases}$

№ 25. $\begin{cases} x_1 = 0.13x_1 + 0.22x_2 - 0.33x_3 + 0.07x_4 + 0.11; \\ x_2 = 0.45x_2 - 0.23x_3 + 0.07x_4 - 0.33; \\ x_3 = 0.11x_1 - 0.08x_3 + 0.18x_4 + 0.85; \\ x_4 = 0.08x_1 + 0.09x_2 + 0.33x_3 + 0.21x_4 - 1.7; \end{cases}$

№ 26. $\begin{cases} x_1 = 0.32x_1 - 0.16x_2 - 0.08x_3 + 0.15x_4 + 2.42; \\ x_2 = 0.16x_1 - 0.23x_2 + 0.11x_3 - 0.21x_4 + 1.43; \\ x_3 = 0.05x_1 - 0.08x_2 + 0.34x_4 - 0.16; \\ x_4 = 0.12x_1 + 0.14x_2 - 0.18x_3 + 0.06x_4 + 1.62; \end{cases}$

№ 27. $\begin{cases} x_1 = 0.08x_2 - 0.23x_3 + 0.32x_4 + 1.34; \\ x_2 = 0.16x_1 - 0.23x_2 + 0.18x_3 + 0.16x_4 - 2.33; \\ x_3 = 0.15x_1 + 0.12x_2 + 0.32x_3 - 0.18x_4 + 0.34; \\ x_4 = 0.25x_1 + 0.21x_2 - 0.16x_3 + 0.03x_4 + 0.63; \end{cases}$

№ 28. $\begin{cases} x_1 = 0.06x_1 + 0.18x_2 + 0.33x_3 + 0.16x_4 + 2.43; \\ x_2 = 0.32x_1 + 0.23x_3 - 0.05x_4 - 1.12; \\ x_3 = 0.16x_1 - 0.08x_2 - 0.12x_4 + 0.43; \\ x_4 = 0.09x_1 + 0.22x_2 - 0.13x_3 + 0.83; \end{cases}$

$$№ 29. \begin{cases} x_1 = 0.34x_2 + 0.23x_3 - 0.06x_4 + 1.42x_4 - 1.42; \\ x_2 = 0.11x_1 - 0.23x_2 - 0.18x_3 + 0.36x_4 - 0.66; \\ x_3 = 0.23x_1 - 0.12x_2 + 0.16x_3 - 0.35x_4 + 1.08; \\ x_4 = 0.12x_1 + 0.12x_2 - 0.47x_3 + 0.16x_4 + 1.72; \end{cases}$$

$$№ 30. \begin{cases} x_1 = 0.32x_1 - 0.23x_2 + 0.11x_3 - 0.06x_4 - 1.42; \\ x_2 = 0.18x_1 + 0.12x_2 - 0.33x_3 - 0.88; \\ x_3 = 0.12x_1 + 0.32x_2 - 0.05x_3 + 0.07x_4 - 2.34; \\ x_4 = 0.05x_1 - 0.11x_2 + 0.09x_3 - 0.12x_4 + 0.72; \end{cases}$$

4.6-turdagi topshiriq.

Topshiriqlarni bajarish uchun namuna.

Misol. Chiziqli tenglamalar sistemasini 0,001 aniqlikda Zeydel usuli bilan yechish uchun, uni iteratsiya uchun qulay ko‘rinishda yozing va yeching.

Yechish:

$$\begin{cases} 4{,}5x_1 - 1{,}8x_2 + 3{,}6x_3 = -1{,}7; & \text{(I)} \\ 3{,}1x_1 + 2{,}3x_2 - 1{,}2x_3 = 3{,}6; & \text{(II)} \\ 1{,}8x_1 + 2{,}5x_2 + 4{,}6x_3 = 2{,}2; & \text{(III)} \end{cases}$$

Sistemani bosh diogonal elementlari shu satrning boshqa elementlaridan katta bo‘lgan sistemaga keltiramiz:

$$\begin{cases} 7{,}6x_1 + 0{,}5x_2 + 2{,}4x_3 = \ \ 1{,}9; & \text{(I + II)} \\ 2{,}2x_1 + 9{,}1x_2 + 4{,}4x_3 = \ \ 9{,}7; & \text{(2III + II − I)} \\ -1{,}3x_1 + 0{,}2x_2 + 5{,}8x_3 = -1{,}4; & \text{(III − II)} \end{cases}$$

$$\begin{cases} 10x_1 = 2{,}4x_1 - 0{,}5x_2 - 2{,}4x_3 + 1{,}9; \\ 10x_2 = -2{,}2x_1 + 0{,}9x_2 - 4{,}4x_3 + 9{,}7; \\ 10x_3 = 1{,}3x_1 - 0{,}2x_2 - 4{,}2x_3 + 1{,}4; \end{cases}$$

$$\begin{cases} x_1 = 0{,}24x_1 - 0{,}05x_2 - 0{,}24x_3 + 0{,}19; \\ x_2 = -0{,}22x_1 + 0{,}09x_2 - 0{,}44x_3 + 0{,}97; \\ x_3 = 0{,}13x_1 - 0{,}02x_2 + 0{,}42x_3 - 0{,}14; \end{cases}$$

Tenglamalarning o‘ng tomondagi noma’lumlar oldidagi koeffitsientlardan tuzilgan matritsaning $\|A\|_1$ normasi $\{0{,}53; 0.77; 0{,}57\}$ =0,77 ga teng, demak Zeydel protsetsi yaqinlashuvchi bo‘ladi.

Hisoblashlarni jalvalga joylaymiz:

N	x_1	x_2	x_3	N	x_1	x_2	x_3
0	0,19	0,97	-0,14	5	0,2467	1,1138	-0,2237
1	0,2207	1,0703	-0,1915	6	0,2472	1,1143	-0,2241
2	0,2354	1,0988	-0,2118	7	0,2474	1,1145	-0,2243
3	0,2424	1,1088	-0,2196	8	0,2475	1,1145	-0,2243
4	0,2454	1,1124	-0,2226				

Javob; x_1 ≈0.248; x_2 ≈1.115; x_3 ≈-0.224.

Mustaqil yechish uchun:

Quyidagi amallarni va misollarni mos ravishda bajaring:

Topshiriqlar. Chiziqli tenglamalar sistemasini 0,001 aniqlikda Zeydel usuli bilan yechish uchun, uni iteratsiya uchun qulay ko‘rinishda yozing va yeching.

№ 1. $\begin{cases} 2.7x_1 + 3{,}3x_2 + 1{,}3x_3 = 2{,}1; \\ 3{,}5x_1 - 1{,}7x_2 + 2{,}8x_3 = 1{,}7; \\ 4{,}1x_1 + 5{,}8x_2 - 1{,}7x_3 = 0{,}8; \end{cases}$ № 2 . $\begin{cases} 1.7x_1 + 2{,}8x_2 + 1{,}9x_3 = 0{,}7; \\ 2{,}1x_1 + 3{,}4x_2 + 1{,}8x_3 = 1{,}1; \\ 4{,}2x_1 - 1{,}7x_2 + 1{,}3x_3 = 2{,}8; \end{cases}$

№ 3 . $\begin{cases} 3{,}1x_1 + 2{,}8x_2 + 1{,}9x_3 = 0{,}2; \\ 1{,}9x_1 + 3{,}1x_2 + 2{,}1x_3 = 2{,}1; \\ 7{,}5x_1 + 3{,}8x_2 + 4{,}8x_3 = 5{,}6; \end{cases}$ № 4. $\begin{cases} 9{,}1x_1 + 5{,}6x_2 + 7{,}8x_3 = 9{,}8; \\ 3{,}8x_1 + 5{,}1x_2 + 2{,}8x_3 = 6{,}7; \\ 4{,}1x_1 + 5{,}7x_2 + 1{,}2x_3 = 5{,}8; \end{cases}$

№ 5. $\begin{cases} 3{,}3x_1 + 2{,}1x_2 + 2{,}8x_3 = 0{,}8; \\ 4{,}1x_1 + 3{,}7x_2 + 4{,}8x_3 = 5{,}7; \\ 2{,}7x_1 + 1{,}8x_2 + 1{,}7x_3 = 3{,}2; \end{cases}$ № 6. $\begin{cases} 7{,}6x_1 + 5{,}8x_2 + 4{,}7x_3 = 10{,}1; \\ 3{,}8x_1 + 4{,}1x_2 + 2{,}7x_3 = 9{,}7; \\ 2{,}9x_1 + 2{,}1x_2 + 3{,}8x_3 = 7{,}8; \end{cases}$

№ 7. $\begin{cases} 3{,}2x_1 - 2{,}5x_2 + 3,7x_3 = 6{,}5; \\ 0{,}5x_1 + 0{,}34x_2 + 1{,}7x_3 = -0{,}24; \\ 1{,}6x_1 + 2{,}3x_2 - 1{,}5x_3 = 4{,}3; \end{cases}$ № 8. $\begin{cases} 5{,}4x_1 - 2{,}3x_2 + 3{,}4x_3 = 3{,}5; \\ 4{,}2x_1 + 1{,}7x_2 - 2{,}3x_3 = 2{,}7; \\ 3{,}4x_1 + 2{,}4x_2 + 7{,}4x_3 = 1{,}9; \end{cases}$

№ 9. $\begin{cases} 3{,}6x_1 + 1{,}8x_2 - 4{,}7x_3 = 3{,}8; \\ 2{,}7x_1 - 3{,}6x_2 + 1{,}9x_3 = 0{,}4; \\ 1{,}5x_1 + 4{,}5x_2 + 3{,}3x_3 = -1{,}6; \end{cases}$ № 10. $\begin{cases} 5{,}6x_1 + 2{,}7x_2 - 1{,}7x_3 = 1{,}9; \\ 3{,}4x_1 - 3{,}6x_2 - 6{,}7x_3 = -2{,}4; \\ 0{,}8x_1 + 1{,}3x_2 + 3{,}7x_3 = 1{,}2; \end{cases}$

№ 11. $\begin{cases} 2{,}7x_1 + 0{,}9x_2 - 0{,}5x_3 = 3{,}5; \\ 4{,}5x_1 - 2{,}8x_2 + 6{,}7x_3 = 2{,}6; \\ 5{,}1x_1 + 3{,}7x_2 - 1{,}4x_3 = -0{,}14; \end{cases}$ № 12. $\begin{cases} 4{,}5x_1 - 3{,}5x_2 + 7{,}4x_3 = 2{,}5; \\ 3{,}1x_1 - 0{,}6x_2 - 2{,}3x_3 = -1{,}5; \\ 0{,}8x_1 + 7{,}4x_2 - 0{,}5x_3 = 6{,}4; \end{cases}$

№ 13. $$\begin{cases} 3{,}8x_1 + 6{,}7x_2 - 1{,}2x_3 = 5{,}2; \\ 6{,}4x_1 + 1{,}3x_2 - 2{,}7x_3 = 3{,}8; \\ 2{,}4x_1 - 4{,}5x_2 + 3{,}5x_3 = -0{,}6; \end{cases}$$

№ 14. $$\begin{cases} 5{,}4x_1 - 6{,}2x_2 - 0{,}5x_3 = 0{,}52; \\ 3{,}4x_1 - 3{,}6x_2 - 6{,}7x_3 = -2{,}4; \\ 2{,}4x_1 - 1{,}1x_2 + 3{,}8x_3 = 1{,}8; \end{cases}$$

№ 15. $$\begin{cases} 7{,}8x_1 + 5{,}3x_2 + 4{,}8x_3 = 1{,}8; \\ 3{,}3x_1 + 1{,}1x_2 + 1{,}8x_3 = 2{,}3; \\ 4{,}5x_1 + 3{,}3x_2 + 2{,}8x_3 = 3{,}4; \end{cases}$$

№ 16. $$\begin{cases} 3{,}8x_1 + 4{,}1x_2 - 2{,}3x_3 = 4{,}8; \\ -2{,}14x_1 + 3{,}9x_2 - 5{,}8x_3 = 3{,}3; \\ 1{,}8x_1 + 1{,}1x_2 - 2{,}1x_3 = 5{,}8; \end{cases}$$

№ 17. $$\begin{cases} 1{,}7x_1 - 2{,}2x_2 + 3x_3 = 1{,}8; \\ 2{,}1x_1 + 1{,}9x_2 - 2{,}3x_3 = 2{,}8; \\ 4{,}2x_1 + 3{,}9x_2 - 3{,}1x_3 = 5{,}1; \end{cases}$$

№ 18. $$\begin{cases} 2{,}8x_1 + 3{,}8x_2 - 3{,}2x_3 = 4{,}5; \\ 2{,}5x_1 - 2{,}8x_2 + 3{,}3x_3 = 7{,}1; \\ 6{,}5x_1 - 7{,}1x_2 + 4{,}8x_3 = 6{,}3; \end{cases}$$

№ 21. $$\begin{cases} 3{,}7x_1 + 3{,}1x_2 + 4x_3 = 5; \\ 4{,}1x_1 + 4{,}5x_2 - 4{,}8x_3 = 4{,}9; \\ -2{,}1x_1 - 3{,}7x_2 + 1{,}8x_3 = 2{,}7; \end{cases}$$

№ 22. $$\begin{cases} 4{,}1x_1 + 5{,}2x_2 - 5{,}8x_3 = 7; \\ 3{,}8x_1 - 3{,}1x_2 + 4x_3 = 5{,}3; \\ 7{,}8x_1 + 5{,}3x_2 - 6{,}3x_3 = 5{,}8; \end{cases}$$

№ 21. $$\begin{cases} 3{,}7x_1 + 3{,}1x_2 + 4x_3 = 5; \\ 4{,}1x_1 + 4{,}5x_2 - 4{,}8x_3 = 4{,}9; \\ -2{,}1x_1 - 3{,}7x_2 + 1{,}8x_3 = 2{,}7; \end{cases}$$

№ 22. $$\begin{cases} 4{,}1x_1 + 5{,}2x_2 - 5{,}8x_3 = 7; \\ 3{,}8x_1 - 3{,}1x_2 + 4x_3 = 5{,}3; \\ 7{,}8x_1 + 5{,}3x_2 - 6{,}3x_3 = 5{,}8; \end{cases}$$

№ 23. $$\begin{cases} 3{,}7x_1 - 2{,}3x_2 + 4{,}5x_3 = 2{,}5; \\ 2{,}5x_1 + 4{,}7x_2 - 7{,}8x_3 = 3{,}5; \\ 1{,}6x_1 + 5{,}3x_2 + 1{,}3x_3 = -2{,}4; \end{cases}$$

№ 24. $$\begin{cases} 6{,}3x_1 + 5{,}2x_2 - 0{,}6x_3 = 1{,}5; \\ 3{,}4x_1 - 2{,}3x_2 + 3{,}4x_3 = 2{,}7; \\ 0{,}8x_1 + 1{,}4x_2 + 3{,}5x_3 = -2{,}3; \end{cases}$$

№ 25. $$\begin{cases} 1{,}5x_1 + 2{,}3x_2 - 3{,}7x_3 = 4{,}5; \\ 2{,}8x_1 + 3{,}4x_2 + 5{,}8x_3 = -3{,}2; \\ 1{,}2x_1 + 7{,}3x_2 - 2{,}3x_3 = 5{,}6; \end{cases}$$

№ 26. $$\begin{cases} 0{,}9x_1 + 2{,}7x_2 - 3{,}8x_3 = 2{,}4; \\ 2{,}5x_1 + 5{,}8x_2 - 0{,}5x_3 = 3{,}5; \\ 4{,}5x_1 - 2{,}1x_2 + 3{,}2x_3 = -1{,}2; \end{cases}$$

№ 27. $$\begin{cases} 2{,}4x_1 + 2{,}5x_2 - 2{,}9x_3 = 4{,}5; \\ 0{,}8x_1 + 3{,}5x_2 - 1{,}4x_3 = 3{,}2; \\ 1{,}5x_1 - 2{,}3x_2 + 8{,}6x_3 = -5{,}5; \end{cases}$$

№ 28. $$\begin{cases} 5{,}4x_1 - 2{,}4x_2 + 3{,}8x_3 = 5{,}5; \\ 2{,}5x_1 + 6{,}8x_2 - 1{,}1x_3 = 4{,}3; \\ 2{,}7x_1 - 0{,}6x_2 + 1{,}5x_3 = -3{,}5; \end{cases}$$

№ 29. $$\begin{cases} 2{,}4x_1 + 3{,}7x_2 - 8{,}3x_3 = 2{,}3; \\ 1{,}8x_1 + 4{,}3x_2 + 1{,}2x_3 = -1{,}2; \\ 3{,}4x_1 - 2{,}3x_2 + 5{,}2x_3 = 3{,}5; \end{cases}$$

№ 30. $$\begin{cases} 3{,}2x_1 - 11{,}5x_2 - 3{,}8x_3 = 2{,}8; \\ 0{,}8x_1 + 1{,}3x_2 - 6{,}4x_3 = -6{,}5; \\ 2{,}4x_1 + 7{,}2x_2 - 1{,}2x_3 = 4{,}5; \end{cases}$$

V BOB. FUNKSIYANI INTERPOLYASIYALASH

Ko‘pchilik hisoblash usullari masalaning qo‘yilishida qatnashadigan funksiyalarni unga muayyan ma’noda yaqin va tuzilishi soddaroq bo‘lgan funksiyalarga almashtirish g‘oyasiga asoslangan .

Funksiyalarni yaqinlashtirish masalasining eng sodda va juda keng qo‘llaniladigan qismi – funksiyalarni interpolyasiyalash masalasi qaraladi.

Dastlab interpolyasiyalash deganda funksiyaning qiymatlarini argumentning jadvalda berilmagan qiymatlari uchun topish tushunilar edi.

Hozirgi vaqtda interpolyasiyalash tushunchasi juda keng ma’noda tushuniladi. Interpolyasiyalash masalasining mohiyati quyidagidan iborat. Faraz qilaylik, $[a, b]$ oraliqda $y = f(x)$ funksiya berilgan yoki hech bo‘lmaganda uning $f(x_0)$, $f(x_1), \cdots, f(x_n)$ qiymatlari ma’lum bo‘lsin. Shu oraliqda aniqlangan va hisoblash uchun qulay bo‘lgan qandaydir funksiyalar $\{P(x)\}$ sinfini, masalan ko‘phadlar sinfini olamiz. Berilgan $y = f(x)$ funksiyani $[a, b]$ oraliqda interpolyasiyalash masalasi shu funksiyani berilgan sinfning shunday $P(x)$ funksiyasi bilan taqribiy ravishda

$$f(x) \approx P(x)$$

almashtirishdan iboratki, $P(x)$ berilgan $x_0, x_1, \cdots, x_n$ nuqtalarda $f(x)$ bilan bir xil qiymatlarni qabul qilsin:

$$P(x_i) = f(x_i) \quad (i = \overline{0, n}).$$

Bu yerda ko‘rsatilgan $x_0, x_1, \cdots, x_n$ nuqtalar *interpolyasiya tugunlari* yoki *tugunlar* deyiladi, $P(x)$ esa *interpolyasiyalovchi funksiya deyiladi*. Agar $\{P(x)\}$ sinfi sifatida darajali ko‘phadlar sinfi olinsa, u holda interpolyasiyalash algebraik deyiladi.

5.1-§. LAGRANJ INTERPOLYASION FORMULASI

Bizga $y = f(x)$ berilgan bo‘lsin. $f(x)$ funksiyani interpolyasiyalash uchun

$$P_n(x) = \sum_{i=0}^{n} y_i \frac{(x - x_0)(x - x_1) \dots (x - x_{i-1})(x - x_{i+1}) \dots (x - x_n)}{(x_i - x_0)(x_i - x_1) \dots (x_i - x_{i-1})(x_i - x_{i+1}) \dots (x_i - x_n)}$$

Lagranj interpolyasion ko‘phadidan foydalanamiz. Bunda $y_i = f(x_i)$. Lagranj koeffitsientlarini hisoblashda ayirmalarni quyidagi tartibda joylashtirish qulay:

$[x - x_0]$	$x_0 - x_1$	$x_0 - x_2$	... ...	$x_0 - x_n$
$x_1 - x_0$	$[x - x_1]$	$x_1 - x_2$	... ...	$x_1 - x_n$
$x_2 - x_0$	$x_2 - x_1$	$[x - x_2]$	... ...	$x_2 - x_n$
... ...	... ...	... ...	... ...	... ...
$x_n - x_0$	$x_n - x_1$	$x_n - x_2$	... ...	$[x - x_n]$

Agar satr(yo‘l) elementlarininig ko‘paytmasini D_i $(i = 0,1,2,\ldots,n)$ bilan, bosh diagonal elementlarininig ko‘paytmasini $\prod_{n+1}(x)$ bilan belgilasak, u holda

$$P_n(x) = \prod\nolimits_{n+1}(x)\sum_{i=0}^{n}\frac{y_i}{D_i}.$$

formula hosil bo‘ladi.

Teng uzoqlashgan tugunlar uchun Lagranj interpolyasiyalash formulasi

$$P_n(x) = \prod\nolimits_{n+1}(t)\sum_{i=0}^{n}\frac{y_i}{(t-1)i!\,(n-1)!\,(-1)^{n-i}},$$

bunda $t = \frac{x-x_0}{h}, h = x_{i+1} - x_i$ $(i = 1,2,3,\ldots n)$, ko‘rinishni olinadi.

Lagranj interpolyasiyalash formulasi baholash uchun

$$|R_n(x)| \le \frac{M_{n+1}|\prod_{n+1}(x)|}{(n+1)!},$$

bunda

$$M_{n+1} = \max_{[a,b]}\left|f^{(n+1)}(x)\right|$$

munosabatdan foydalanish mumkin.

5.2-§. NYUTON INTERPOLYASION FORMULASI:

a) Nyutonning birinchi interpolyasion formulasi:

$$P_n(x) = y_0 + q\Delta y_0 + \frac{q(q-1)}{2!}\Delta^2 y_0 + \cdots + \frac{q(q-1)\ldots(q-n+1)}{n!}\Delta^n y_0,$$

bunda $q = \frac{x - x_0}{h}, h = x_{i+1} - x_i \ (i = 0,1,2, \dots n),\ \Delta^i y_0 -\ \ i -$ tartibli chekli chekli ayirma, $\Delta^i y_0 = \Delta^{i-1} y_1 - \Delta^{i-1} y_0\ (i = 1,2, \dots, n)$.

Agar $n = 1$ bo‘lsa, u holda chiziqli interpolyasiya formulasi hosil bo‘ladi

$$P_1(x) = y_0 + q\Delta y_0.$$

Agar $n = 2$ bo‘lsa, u holda kvadrat interpolyasiya formulasi hosil bo‘ladi

$$P_2(x) = y_0 + q\Delta y_0 + \frac{q(q-1)}{2}\Delta^2 y_0.$$

b) Nyutonning ikkinchi interpolyasion formulasi:

$$P_n(x) = y_n + q\Delta y_{n-1} + \frac{q(q+1)}{2!}\Delta^2 y_{n-2} + \cdots + \frac{q(q+1)\dots(q+n-1)}{n!}\Delta^n y_0,$$

bunda $q = \frac{x - x_n}{h}$.

v) Argumentning teng uzoqlashmagan qiymatlari uchun Nyuton interpolyasion formulasi:

$$P_n(x) = f(x_0) + (x - x_0)f(x_0, x_1) + (x - x_0)(x - x_1)f(x_0, x_1, x_2) + \cdots + (x - x_0)(x - x_1)(x - x_2)\dots(x - x_{n-1})f(x_0, x_1, \dots, x_n),$$

bunda $f(x_0, x_1, \dots, x_i) = \frac{f(x_1, x_2, \dots, x_i) - f(x_0, x_1, \dots, x_n)}{x_i - x_0} - i -$tartibli bo‘lingan ayirma.

5.3-§. EYTKIN INTERPOLYASION SXEMA:

x_i	y_i	$P_{i-1,i}$	$P_{i-2,i-1,i}$	$P_{i-3,i-2,i-1,i}$	$x_i - x$
x_0	y_0	$-$	$-$	$-$	$x_0 - x$
x_1	y_1	$P_{0,1}(x)$	$-$	$-$	$x_1 - x$
x_2	y_2	$P_{1,2}(x)$	$P_{0,1,2}(x)$	$-$	$x_2 - x$
x_3	y_3	$P_{2,3}(x)$	$P_{1,2,3}(x)$	$P_{1,2,3,0}(x)$	$x_3 - x$
...	...	...	...	...	...

Bu yerda

$$P_{i,i+1\dots,i+k}(x) = \frac{1}{x_{i+k} - x_i} \begin{vmatrix} P_{i,i+1,\dots,i+k-1}(x) x_{i-k} \\ P_{i,i+1,i+2,\dots,i+k}(x) x_{i+k} - x \end{vmatrix},$$

bundan

$$P_{i,i+1}(x) = \frac{1}{x_{i+1} - x_i} \begin{vmatrix} y_i & x_i - x \\ y_{i+1} & x_{i+1} - x \end{vmatrix}.$$

Hisoblash ishlarining oxiri $P(x)$ ketma-ket qiymatlarini solishtirish yo‘li bilan aniqlanadi.

5.4-§. GAUSS INTERPOLYASION FORMULASI:

Gaussning birinchi formula **:**

$$P(x) = y_0 + q\Delta y_0 + \frac{q(q-1)}{2!}\Delta^2 y_{-1} + \frac{(q+1)q(q-1)}{3!}\Delta^3 y_{-1} +$$
$$+ \frac{(q+1)q(q-1)(q-2)}{4!}\Delta^4 y_{-2} + \frac{(q+2)(q+1)q(q-1)(q-2)}{5!}\Delta^5 y_{-2}$$
$$+ \dots + \frac{(q+n-1)\dots(q-n+1)}{(2n-1)!}\Delta^{2n-1} y_{-(n-1)}$$
$$+ \frac{(q+n-1)\dots(q-n)}{(2n)!}\Delta^{2n} y_{-n},$$

bunda $q = \frac{x-x_0}{h}$.

Gaussning ikkinchi formula :

$$\mathrm{P(x) = y_0 + q\Delta y_{-1}} + \frac{(q+1)q}{2!}\Delta^2 y_{-1} + \frac{(q+1)q(q-1)}{3!}\Delta^3 y_{-2} +$$
$$+ \frac{(q+1)q(q-1)}{4!}\Delta^4 y^{-2} + \dots + \frac{(q+n-1)\dots.(q-n+1)}{(2n-1)!}\Delta^{2n-1} y_{-n} +$$
$$+ \frac{(q+n)(q+n-1)\dots.(q-n+1)}{(2n)!}\Delta^{2n} y_{-n},$$

bunda q=(x-x_0)/h.

5.5-§. STIRLING INTERPOLYASION FORMULASI:

$$P(x) = y_0 + q\frac{\Delta y_{-1} + \Delta y_0}{2} + \frac{q^2}{2}\Delta^2 y_{-1} + \frac{q(q^2-1)}{3!} \cdot \frac{\Delta^3 y_{-2} + \Delta^3 y_{-1}}{2} +$$
$$+ \frac{q^2(q^2-1)}{4!}\Delta^4 y_{-2} + \frac{q(q^2-1)(q^2-2^2)}{5!} \cdot \frac{\Delta^5 y_{-3} + \Delta^5 y_{-2}}{2} +$$
$$+ \frac{q^2(q^2-1)(q^2-2^{2)}}{6!}\Delta^6 y_{-3} + \dots + \frac{q(q^2-1)(q^2-2^{2)}\dots[q^2-(n-1)^2]}{(2n-1)!} \times$$

$$\times \frac{\Delta^{2n-1}y_{-n} + \Delta^{2n-1}y_{-(n-1)}}{2}$$

$$+ \frac{q^2(q^2-1)(q^2-2^2)\ldots[q^2-(n-1)^2]}{(2n)!}\Delta^{2n}y_{-n},$$

bunda q=(x-$x_0)/h$. Odatda bu formula |q|≤0.25 da qo‘llaniladi.

5.6-§. BESSEL INTERPOLYASION FORMULASI:

$$P(x) = \frac{y_0 - y_{-1}}{2} + \left(q - \frac{1}{2}\right)\Delta y_0 + \frac{q(q-1)}{2!} * \frac{\Delta^2 y_{-1} + \Delta^2 y_0}{2}$$

$$+ \frac{(q-0{,}5)q(q-1)}{3!}\Delta^3 y_{-1} +$$

$$+ \frac{q(q-1)(q+1)(q-2)}{4!} \cdot \frac{\Delta^4 y_{-2} + \Delta^4 y_{-1}}{2}$$

$$+ \frac{(q-0{,}5)q(q-1)(q+1)(q-2)}{5!}\Delta^5 y_{-2} +$$

$$+ \frac{q(q-1)(q+1)(q-2)(q+2)(q+3)}{6!} \cdot \frac{\Delta^6 y_{-3} + \Delta^6 y_{-2}}{2} +$$

$$+ \cdots + \frac{q(q-1)(q+1)(q-2)(q+2)\ldots(q-n)(q+n-1)}{(2n)!}$$

$$* \frac{\Delta^{2n} y_{-n} + \Delta^{2n} y_{-n+1}}{2} +$$

$$+ \frac{(q-0{,}5)q(q-1)(q+1)(q-2)(q+2)\ldots(q-n)(q+n-1)}{(2n+1)!}\Delta^{2n+1} y_{-n},$$

bunda q=(x-x_0)/h. Odatda bu formula 0.25$\leq q \leq 0.75$ da qo‘llaniladi.

5.1 -turdagi topshiriq

Topshiriqlarni bajarish uchun namuna.

Misol. Agar funksiyaning qiymatlari: 1) teng uzoqlashmagan tugun nuqtalarda; 2) teng uzoqlashgan tugun nuqtalarda jadval ko‘rinishda berilgan bo‘lsa, uning $= x^*$ nuqtadagi taqribiy qiymatini Lagranj interpolyasion ko‘phadi yordamida toping.

1) holda $x^* = 0{,}263$.

x_i	y_i
0,05	0,050042
0,10	0,100335
0,17	0,171657
0,25	0,255342
0,30	0,309336
0,36	0,376403

2) holda $x^* = 0{,}1157$

x_i	y_i
0,101	1,26183
0,106	1,27644
0,111	1,29122
0,116	1,30617
0,121	1,32130
0,126	1,32660

Yechish: 1. 1) *ni yechilish jarayoni:*

$$f(x) \approx P_{n+1} \cdot \sum_{i=1}^{n} \left(\frac{y_i}{D_i}\right).$$

bunda $P_{n+1} = (x - x_0)(x - x_1) \dots (x - x_n)$,

$$D_i = (x_i - x_0)(x_i - x_1) \dots (x_i - x_{i-1})(x - x_i)(x_i - x_{i+1}) \dots (x_i - x_n).$$

Hisoblashlarni jadvalda keltiramiz.

I		Ayirmalar					D_i	$\frac{y_i}{D_i}$
0	0,213	-0,05	-0,12	-0,20	-0,25	-0,31	$-0{,}19809 \cdot 10^{-4}$	-2526,2
1	0,05	0,163	-0,07	-0,15	-0,20	-0,26	$0{,}44499 \cdot 10^{-5}$	25547,7
2	0,12	0,07	0,093	-0,08	-0,13	-0,19	$-0{,}154365 \cdot 10^{-5}$	-111202,0
3	0,20	0,15	0,08	0,013	-0,05	-0,11	$0{,}1716 \cdot 10^{-6}$	1488007,0
4	0,25	0,20	0,13	0,05	-0,037	-0,06	$0{,}7215 \cdot 10^{-6}$	428740,0
5	0,31	0,26	0,19	0,11	0,06	-0,097	$-0{,}980402 \cdot 10^{-6}$	-38392,7

Shunday qilib,

$$P_{5+1} = 0{,}1506492 \cdot 10^{-6}, \qquad \sum_{i=0}^{5} \left(\frac{y_i}{D_i}\right) = 1790173{,}8.$$

Bundan,

$$f(0{,}263) \approx P_{5+1} \cdot \sum_{i=0}^{5} \left(\frac{y_i}{D_i}\right) = 0{,}1506492 \cdot 10^{-6} \cdot 1790173{,}8 = 0{,}299678.$$

1) ning javobi: $f(0{,}263) = 0{,}299678.$

Endi 2)ni ishlaymiz.

2) Hisoblash uchun quyidagi formuladan foydalanamiz.

$$f(x) = y(x) \approx P_{n+1} \sum_{i=0}^{n} \frac{y_i}{(t-i)C_i},$$

bunda

$$P_{n+1}(t) = t(t-1) \ldots (t-n);\ t = (x-x_0)/h;\ h = x_{i+1} - x_i;$$

$$C_1 = (-1)^{n-i} \cdot i! \cdot (n-i)!\,.$$

Bu erda t=(0,1157-0,101)/0,005=2,94. Hisoblashlarni jadvalga joylashtiramiz.

i	x_i	l_1'	$t-i$	C_i	$(t-i)\cdot C_i$	$\frac{y_i}{(t-i)C_i}$
0	0,101	1,26183	2,94	-120	-352,8	-0,0035766
1	0,106	1,27644	1,94	24	46,56	0,0274149
2	0,111	1,29122	0,94	-12	-11,28	-0,1144691
3	0,116	1,30617	-0,06	12	-0,72	-1,8141250
4	0,121	1,32130	-1,06	-24	25,44	0,0519379
5	0,126	1,33660	-2,06	120	-247,2	-0,0054069

Shunday qilib,

$$P_{(5+1)}(t) = P_{(5+1)}(2{,}94) = -0{,}7024271; \quad \sum_{i=0}^{5} \frac{y_i}{(t-i)C_i} = -1{,}858225.$$

Demak, $f(0{,}1157) \approx 1{,}30527.$

Topshiriqlar. Agar funksiyaning qiymatlari: 1) teng uzoqlashmagan tugun nuqtalarda; 2) teng uzoqlashgan tugun nuqtalarda jadval ko‘rinishda berilgan bo‘lsa, uning $= x^*$ nuqtadagi taqribiy qiymatini Lagranj interpolyasion ko‘phadi yordamida toping.

1) topshiriqqa variantlar.

Bitta jadvalda ko‘rsatilgan barcha raqamli variantlar uchun x_i va y_i larning qiymatlari bir xil, faqat x^* o‘zgaradi.

1-jadval

Vari-ant №	x^*	x_i	y_i
1	0,702	0,43	1,63597
7	0,512	0,48	1,73234
13	0,645	0,55	1,87686
19	0,736	0,62	2,03345
25	0,608	0,70	2,22846
		0,75	2,35973

2-jadval

Vari-ant №	x^*	x_i	y_i
1	0,102	0,02	1,02316
7	0,114	0,08	1,09590
13	0,125	0,12	1,14725
19	0,203	0,17	1,21483
25	0,154	0,23	1,30120
		0,30	1,40976

3-jadval

Vari-ant №	x^*	x_i	y_i
3	0,526	0,35	2,73951
9	0,453	0,41	2,30080
15	0,482	0,47	1,96864
21	0,552	0,51	1,78776
27	0,436	0,56	1,59502
		0,64	1,34310

4-jadval

Vari-ant №	x^*	x_i	y_i
4	0,616	0,41	2,57418
10	0,478	0,46	2,32513
16	0,665	0,52	2,09336
22	0,537	0,60	1,86203
28	0,673	0,65	1,74926
		0,72	1,62098

5-jadval

Vari-ant №	x^*	x_i	y_i
5	0,896	0,68	0,808866
11	0,812	0,73	0,89492
17	0,774	0,80	1,02964
23	0,955	0,88	1,20966
29	0,715	0,93	1,34087
		0,99	1,52368

6-jadval

Vari-ant №	x^*	x_i	y_i
6	0,314	0,11	9,05421
12	0,235	0,15	6,61659
18	0,332	0,21	4,69170
24	0,275	0,29	3,35106
30	0,186	0,35	2,73951
		0,40	2,36522

2). topshiriqqa variantlar

1-jadval

Variant №	x^*	x_i	y_i
1,3832	1	1,375	5,04192
1,3926	7	1,380	5,17744
1,3862	13	1,385	5,32016
1,3934	19	1,390	5,47069
1,3866	25	1,395	5,62968
		1,400	5,79788

2-jadval

Variant №	x^*	x_i	y_i
2	0,1264	0,115	8,65729
8	0,1315	0,120	8,29329
14	0,1232	0,125	7,95829
20	0,1334	0,130	7,64893
26	0,1285	0,135	7,36235
		0,140	7,09613

3-jadval

Vari-ant №	x^*	x_i	y_i
3	0,1521	0,150	6,61659
9	0,1611	0,155	6,39989
15	0,1662	0,160	6,19658
21	0,1542	0,165	6,00551
27	0,1625	0,170	5,82558
		0,175	5,65583

4-jadval

Vari-ant №	x^*	x_i	y_i
4 10	0,1838	0,180	5,61543
16 22	0,1875	0,185	5,46693
28	0,1944	0,190	5,32634
	0,1976	0,195	5,19304
	0,2038	0,200	5,06649
		0,205	4,94619

5-jadval

Vari-ant №	x^*	x_i	y_i
5	0,2121	0,210	4,83170
11	0,2165	0,215	4,72261
17	0,2232	0,220	4,61855
23	0,2263	0,225	4,51919
29	0,2244	0,230	4,42422
		0,235	4,33337

6-jadval

Vari-ant №	x^*	x_i	y_i
6 12	1,4179	1,415	0,888551
18 24	1,4258	1,420	0,889599
30	1,4396	1,425	0,890637
	1,4236	1,430	0,891667
	1,4315	1,435	0,892687
		1,440	0,893698

5.2-turdagi topshiriq.

Topshiriqlarni bajarish uchun namuna.

Misol. Argumentning jadval bilan berilgan qiymatlarida funksiyaning taqribiy qiymatlarni Eytkin sxemasidan foydalanib hisoblang. 2-jadvaldan foydalanib $y(x)$ funksiyaning qiymatini x $=$ 0,8925 nuqtada aniqlang.

Yechish:

2-jadvaldan oltita qiymatini shunday tanlaymizki, argumentning 0,8925 qiymati oʻrtada qolsin.

f(0,8925) ni Eytkin sxemasi bilan verguldan keyin besh xonagacha ustma-ust tushadigan qiymat hosil boʻlgancha hisoblaymiz. Hisoblashlarni jadvalga keltiramiz.

x_i	y_i	$P_1(x_i, x_{i+1})$	$P_1(x_i, x_{i+1}, x_{i+2})$	$P_1(x_i, x_{i+1}, x_{i+2}, x_{i+3})$	$x_i - x$
0,8902	1,23510				-0,0023
0,8909	1,23687	1,240916			-0,0016

0,8919	1,23941	1,240934	1,240940		-0,0006
0,8940	1,24475	1,240936	1,240935	1,240937	0,0015
0,8944	1,24577	1,240925	1,240933	1,240934	0,0019
0,8955	1,24858	1,240916	1,240934	1,240933	0,0030

Olingan natijalarni taqqoslab quyidagini olamiz $f(0{,}8925) \approx 1{,}24093$.

Mustaqil yechish uchun:

Quyidagi amallarni va misollarni mos ravishda bajaring:

Topshiriqlar: Argumentning jadval bilan berilgan qiymatlarida funksiyaning taqribiy qiymatlarni Eytkin sxemasidan foydalanib hisoblang. Mos jadvaldan foydalanib $y(x)$ funksiyaning qiymatini x ning variantda koʻrsatilgan qiymatida aniqlang

1-jadval

Variant №	x	x_i	y_i
1	0,2054	0,2050	0,207921
7	0,2063	0,2052	0,208130
13	0,2072	0,2060	0,208964
19	0,2079	0,2065	0,209486
25	0,2088	0,2069	0,209904
		0,2075	0,210530
		0,2085	0,211575
		0,2090	0,212097
		0,2096	0,212724
		0,2100	0,213142

2-jadval

Variant №	x	x_i	y_i
2	*0,8942*	*0,8902*	*1,23510*
8	*0,8973*	*0,8909*	*1,23687*
14	*0,8958*	*0,8919*	*1,23941*
20	*0,8948*	*0,8940*	*1,24475*

26	*0,8934*	*0,8944*	*1,24577*
		0,8955	*1,24858*
		0,8965	*1,25114*
		0,8975	*1,25371*
		0,9010	*1,26275*
		0,9026	*1,26691*

3-jadval

Variant №	x	x_i	y_i
3	*0,6111*	*0,6100*	*1,83781*
9	*0,6124*	*0,6104*	*1,83686*
15	*0,6142*	*0,6118*	*1,83354*
21	*0,6163*	*0,6139*	*1,82860*
27	*0,6192*	*0,6145*	*1,82720*
		0,6158	*1,82416*
		0,6167	*1,82207*
		0,6185	*1,81791*
		0,6200	*1,81446*
		0,6225	*1,80876*

4-jadval

Variant №	x	x_i	y_i
4	*0,5415*	x_i	y_i
10	*0,5424*	*0,5400*	*1,66825*
16	*0,5436*	*0,5405*	*1,66636*
22	*0,5452*	*0,5410*	*1,66448*
28	*0,5461*	*0,5420*	*1,66071*
		0,5429	*1,65734*
		0,5440	*1,64322*
		0,5449	*1,64987*
		0,5455	*1,64764*
		0,5465	*1,64393*
		0,5473	*1,64097*

5-jadval

Variant №	x	x_i	y_i
5	*0,846*	*0,62*	*0,537944*

11	*0,864*	*0,67*	*0,511709*
17	*0,683*	*0,74*	*0,477114*
23	*0,785*	*0,80*	*0,449329*
29	*0,866*	*0,87*	*0,418952*
		0,96	*0,382893*
		0,99	*0,371577*

6-jadval

Variant №	x	x_i	y_i
6	*1,277*	*1,03*	*2,80107*
12	*1,118*	*1,08*	*2,94468*
18	*1,204*	*1,16*	*3,18993*
24	*1,255*	*1,23*	*3,42123*
30	*1,282*	*1,26*	*3,52542*
		1,33	*3,78104*
		1,39	*4,01485*

5.3 -turdagi topshiriq.

Topshiriqlarni bajarish uchun namuna.

Misol. Nyutonning birinchi yoki ikkinchi interpolyasion formalasidan foydalanib, argumentning berilgan qiymatlarida funksiyaning qiymatlarini hisoblang. Ayirmalar jadvalini tuzishda hisoblashlarni nazorat qiling

x_i	y_i	$y(x)$ funksiyaning qiymatini argumentning quyidagi qinmatlarida aniqlang
1,215	0,106044	
1,220	0,113276	
1,225	0,119671	1) x_1=1,2273; 3) x_3=1,210;
1,230	0,125324	2) x_2=1,253; 4) x_4=1,2638.
1,235	0,130328	
1,240	0,134776	
1,245	0,138759	
1,250	0,142367	
1,255	0,145688	
1,260	0,148809	

Yechish. Chekli ayirmalar jadvalini tuzamiz. Hisoblashlarni nazorat qilish uchun unga ikkita satr qo‘shamiz: $\sum$ satrga chekli ayirmalar jadvalinining ustun elementlarining summasini, R satrga esa – ustunlarning chetki elementlari ayirmasini yozamiz.

x_i	y_i	Δy_i	$\Delta^2 y_i$	$\Delta^3 y_i$
1,215	0,106044	0,007232	-0,000837	0,000095
1,220	0,113276	0,006395	-0,000742	0,000093
1,225	0,119671	0,005653	-0,000649	0,000093
1,230	0,125324	0,005004	-0,000556	0,000091
1,235	0,130328	0,004448	-0,000465	0,000090
1,240	0,134776	0,003983	-0,000375	0,000088
1,245	0,138759	0,003608	-0,000287	0,000087
1,250	0,142367	0,003321	-0,000200	---
1,255	0,145688	0,003121	---	---
1,260	0,148809	---	---	---
$\sum$	----	0,042765	-0,004111	0,000637
P	0,0042765	-0,004111	0,000637	----

Ayirmalar jadvalini tuzishda uchinchi tartibli ayirma bilan chegaralanamiz, chunki uchinchi tartibli ayirma deyarli o‘zgarmas bo‘lib qoldi.

Funksiyaning *x*=1,2273 va *x*=1,210 lardagi qiymatini topish uchun Nyutonning birinchi interpolyasion formulasidan foydalanamiz:

$$u(x) \approx y_0 + q\Delta y_0 + \frac{q(q-1)}{2!}\,\Delta^2 y_0 + \frac{q(q-1)(q-2)}{3!}\Delta^3 y_0$$

bunda, $q = (x - \mathrm{x}_0)/\,h$.

1) Agar *x*=1,2273 bo‘lsa, x_0=1,225 deb olamiz; u holda

$$q = \frac{1{,}2273 - 1{,}225}{0{,}005} = 0{,}46,$$

$$u\ (1{,}2273) \approx 0{,}119671 + 0{,}46 \cdot 0{,}005653 + (-0{,}000649) + \frac{0{,}46(-0{,}54)}{2} +$$

$$+\frac{0{,}46(-0{,}54)(-1{,}54)}{6} \cdot 0{,}000093$$

$$= 0{,}119671 + 0{,}0026004 + 0{,}0000806 +$$

$$+0{,}0000059 = 0{,}1223579 \approx 0{,}122358.$$

2) Agar *x*=1,210 bo‘lsa, x_0=1,215 deb olamiz; u holda

$$q = \frac{1{,}210 - 1{,}215}{0{,}005} = -1,$$

$$y(1{,}210) \approx 0{,}106044 + (-1) \cdot 0{,}007232 + \frac{(-1)(-2)}{2} \cdot (-0{,}000837) +$$

$$+ \frac{(-1)(-2)(-3)}{6} \cdot 0{,}000095 = 0{,}097880.$$

Funksiyaning *x*=1,253 va *x*=1,2638 lardagi qiymatini topish uchun Nyutonning ikkinchi interpolyasion formulasidan foydalanamiz:

$$u(\mathrm{x}) \approx y_n + q\Delta y_{n-1} + \frac{q(q+1)}{2!} \Delta^2 y_{n-2} + \frac{q(q+1)(\mathrm{q}+2)}{3!} \Delta^3 y_{n-3}$$

bunda $q = (x - x_n)/h$.

3) Agar x=1,253 bo‘lsa, x_n=1,255 deb olamiz, u holda

$$q = \frac{1{,}253 - 1{,}255}{0{,}005} = -0{,}4$$

$$u(1{,}253) \approx 0{,}145688 + (-0{,}4) \cdot 0{,}003321 + \frac{(-0{,}4) \cdot 0{,}6}{2} \cdot (-0{,}000287) +$$

$$+ \frac{(-0{,}4)(0{,}6)(1{,}6)}{6} \cdot (0{,}000088) = 0{,}145688 - 0{,}0013284 + 0{,}0000344 -$$

$$-0{,}0000056 = = 0{,}1443884 \approx 0{,}144388.$$

4) Agar $x = 1{,}2638$ bo‘lsa, x_0=1,260 deb olamiz; u holda

$$q = \frac{1{,}2638 - 1{,}260}{0{,}005} = 0{,}76,$$

$$u(1{,}2638) \approx 0{,}148809 + 0{,}76 \cdot 0{,}003121 + \frac{0{,}76 * 1{,}76}{2} \cdot (-0{,}000200) +$$

$$+ \frac{0{,}76 \cdot 1{,}76 \cdot 276}{6} \cdot (0{,}000087) = 0{,}148809 + 0{,}0023720 + 0{,}0001338 +$$

$$+ 0{,}0000535 = 0{,}1511007 \approx 0{,}151101.$$

Mustaqil yechish uchun:

Quyidagi amallarni va misollarni mos ravishda bajaring:

Topshiriqlar: Nyutonning birinchi yoki ikkinchi interpolyasion formulasidan foydalanib, argumentning berilgan qiymatlarida funksiyaning qiymatlarini hisoblang. Ayirmalar jadvalini tuzishda hisoblashlarni nazorat qiling.

1 -jadval

Variant №	Argument qiymati				*x*	*y*
	x_1	x_3	x_4	x_5	*1,415*	*0,898551*
1	1,4161	1,4625	1,4135	1,470	1,420	0,899599
11	1,4179	1,4633	1,4124	1,4655	1,425	0,890637
21	1,4263	1,4575	1,410	1,4662	1,430	0,891667
					1,435	0,892687
					1,440	0,893698
					1,445	0,894700
					1,450	0,895693
					1,455	0,896677
					1,460	0,897653
					1,465	0,898619

2 -jadval

Variant №	Argument qiymati				x	y
	x_1	x_3	x_4	x_5	0,101	1,26183
2	0,1026	0,1440	0,099	0,161	0,106	1,27644
12	0,1035	0,1492	0,096	0,153	0,111	1,29122
22	0,1074	0,1485	0,1006	0,156	0,116	1,30617
					0,121	1,32130
					0,126	1,33660
					0,131	1,35207
					0,136	1,36773
					0,141	1,38357
					0,146	1,39959
					0,151	1,41579

2 *-jadval*

Variant №	Argument qiymati				x	y
	x_1	x_3	x_4	x_5	0,15	0,860708
3	0,1511	0,7250	0,1430	0,80	0,20	0,818731
13	0,1535	0,7333	0,100	0,7540	0,25	0,778801
23	0,1525	0,6730	0,1455	0,85	0,30	0,740818
					0,35	0,704688

0,40	0,670320
0,45	0,637628
0,50	0,606531
0,55	0,576950
0,60	0,548812
0,65	0,522046
0,70	0,496585
0,75	0,4722367

4 -jadval

Variant №	Argument qiymati				x	y
	x_1	x_3	x_4	x_5	*0,180*	*5,61543*
4	*0,1817*	*0,2275*	*0,175*	*0,2375*	*0,185*	*5,46693*
14	*0,1827*	*0,2292*	*0,1776*	*0,240*	*0,190*	*5,32634*
24	*0,1873*	*0,2326*	*0,1783*	*0,245*	*0,195*	*5,19304*
					0,200	*5,06649*
					0,205	*4,94619*
					0,210	*4,83170*
					0,215	*4,72261*
					0,220	*4,61855*
					0,225	*4,51919*
					0,230	*4,42422*
					0,235	*4,33337*

5 -jadval

Variant №	Argument qiymati				x	y
	x_1	x_3	x_4	x_5	3,50	33,1154
5	3,522	4,176	3,475	4,25	3,55	34,8133
15	3,543	4,184	3,488	4,30	3,60	36,5982
25	3,575	4,142	3,45	4,204	3,65	38,4747
					3,70	40,4473
					3,75	42,5211
					3,80	44,7012
					3,85	46,9931
					3,90	49,4024
					3,95	51,9354
					4,00	54,5982
					4,05	57,3975
					4,10	60,3403

6 -jadval

Variant №	Argument qiymati				x	y
	x_1	x_3	x_4	x_5		
7	1,3617	1,3921	1,3359	1,400	1,340	4,25562
17	1,3463	1,3868	1,335	1,3990	1,345	4,35325
27	1,3432	1,3936	1,3365	1,3975	1,350	4,45522
					1,355	4,56184
					1,360	4,67344
					1,365	4,79038
					1,370	4,91306
					1,375	5,04192
					1,380	5,17744
					1,385	5,32016
					1,390	5,47069
					1,395	5,62968

7 -jadval

Variant №	Argument qiymati				x	y
	x_1	x_3	x_4	x_5	0,01	0,991824
8	0,027	0,525	0,008	0,61	0,06	0,951935
18	0,1243	0,492	0,0094	0,66	0,11	0,913650
28	0,083	0,5454	0,0075	0,573	0,16	0,876905
					0,21	0,841638
					0,26	0,807789
					0,31	0,775301
					0,36	0,744120
					0,41	0,714193
					0,46	0,685470
					0,51	0,657902
					0,56	0,631442

8 -jadval

Variant №	Argument qiymati				x	y
	x_1	x_3	x_4	x_5	0,15	4,4817
9	0,1539	0,2569	0,14	0,2665	0,16	4,9530
19	0,1732	0,2444	0,1415	0,27	0,17	5,4739
29	0,1648	0,2550	0,1387	0,28	0,18	6,0496
					0,19	6,6859
					0,20	7,3891

0,21	8,1662
0,22	9,0250
0,23	9,9742
0,24	11,0232
0,25	12,1825
0,26	13,4637

9 -jadval

Variant №	Argument qiymati				x	y
	x_1	x_3	x_4	x_5	0,45	20,1946
10	0,455	0,5575	0,44	0,5674	0,46	19,6133
20	0,4732	0,5568	0,445	0,57	0,47	18,9425
30	0,4675	0,5511	0,4423	0,058	0,48	18,1746
					0,49	17,3010
					0,50	16,3123
					0,51	15,1984
					0,52	13,9484
					0,53	12,5508
					0,54	10,9937
					0,55	9,2647
					0,56	7,3510

5.4 -turdagi topshiriq.

Topshiriqlarni bajarish uchun namuna.

Topshiriq sharti. 1) Chiziqli interpolyasiyani qo‘llab, argumentning berilgan qiymatlaridan foydalanib, funksiyaning qiymatlarini hisoblang. Formulani qo‘llash mumkinligini oldindan aniqlash uchun, Bradis jadvalidan oltita qiymatini tanlab ayirmalar jadvalini tuzing.

1) Kvadratik interpolyasiyadan foydalanib, argumentning berilgan qiymatlarida funksiyaning qiymatini toping. Formulani qo‘llash mumkinligini oldindan aniqlang.

Misol. 1) sin 0,6682 va cos 0,3033 larni aniqlang.

2) 2-jadvaldan foydalaninib, $y(x)$ funksiyaning qiymatini x_1 1,5306 va x_2 1,5282 larda aniqlang.

Yechish: 1) Sinuslar jadvalidan bir nechta qiymat olib birinchi va ikkinchi tartibli ayirmalar jadvalini tuzamiz:

x	$\sin x$	Δy_i	$\Delta^2 y_i$
0,63	0,5891	0,0081	-0,0001
0,64	0,5972	0,0080	-0,0001
0,65	0,6052	0,0079	0,0000
0,66	0,6131	0,0079	-0,0001
0,67	0,6210	0,0078	-
0,68	0,6288	-	-

Chiziqli interpolyasiyadan foydalanish mumkinligini quyidagilar ko‘rsatadi: Birinchidan birinchi tartibli ayirma deyarli bir xil, ikkinchidan

$\frac{1}{8}\max_i|\Delta^2 y_i| < 10^{-4}$ munosobat bajariladi; haqiqatdan ham $\frac{1}{8} \cdot 0{,}0001 < 0{,}0001$.

Hisoblash jarayonida

$$f(x) = f(x_0) + q\Delta f(x_0),$$

formuladan foydalanamiz, bunda q x x_0 / h, x_0 esa 0,6682 dan kichik va unga eng yaqin bo‘lgan jadvaldagi son.

x_0 0,66; q 0,6682 0,66 / 0,01 0,82;

sin 0, 6682 0,6131 0,82 0, 0079 0,6131 0, 0065 0, 6196 ga egamiz.

Kosinuslar jadvalidan bir nechta qiymat olib birinchi va ikkinchi tartibli ayirmalar jadvalini tuzamiz:

x	$\cos x$	Δy_i	$\Delta^2 y_i$
0,28	0,9611	-0,0029	0
0,29	0,9582	-0,0029	-0,001
0,30	0,9553	-0,0030	-0,001
0,31	0,9523	-0,0031	-
0,32	0,9492	-	-

Birinchi tartibli ayirma deyarli bir xil va $\frac{1}{8}\max_i|\Delta^2 y_i| < 10^{-4}$ munosobat bajariladi $\left(\frac{1}{8} \cdot 0{,}0001 < 0{,}0001 \text{ bo‘lgani uchun}\right)$, bu esa chiziqli interpolyasiyalash mumkinligini bildiradi.

x_0 0,30 deb olaylik, u holda q 0,3033 0,30 / 0,01 0,33 demak,

$$\cos 0{,}3033 \quad 0{,}9553 \quad 0{,}33 \quad 0{,}0030 \quad 0{,}9553 \quad 0{,}0010 \quad 0{,}9543.$$

Shunday qilib, $\sin 0{,}6682 \quad 0{,}6196$; $\cos 0{,}3033 \quad 0{,}9543$. Birinchi misolning yechilishi tugadi.

2) Endi topshiriqdagi ikkinchi misolni ishlaymiz.

2- jadvalidan bir nechta qiymat olib birinchi, ikkinchi va uchinchi tartibli ayirmalar jadvalini tuzamiz:

x_i	y_i	Δy_i	$\Delta^2 y_i$	$\Delta^3 y_i$
1,527	22,818	0,534	0,025	0,003
1,528	23,352	0,559	0,028	0,002
1,529	23,911	0,587	0,030	0,001
1,530	24,498	0,617	0,031	-
1,531	25,115	0,648	-	-
1,532	25,763	-	-	-

Bu jadvalda ikkinchi tartibli ayirma deyarli bir xil, undan tashqari $\frac{1}{15}\max\limits_i|\Delta^3 y_i| < 10^{-3}$ munosobat bajariladi $\left(\frac{1}{15}\cdot 0{,}003 < 0{,}001 \text{ bo‘lgani uchun}\right)$, bularning hammasi kvadratik interpolyasiyalashni qo‘llash mumkinligini bildiradi.

$$f(x) = y_0 + q\Delta y_0 + \frac{q(q-1)}{2}\Delta^2 y_0,$$

bunda, $q = (x - x_0)/h$.

Agar x = 1,5306 bo‘lsa, u holda x_0 1,530;

$$q \quad 1{,}5306 \quad 1{,}530 \quad / \, 0{,}001 \quad 0{,}6;$$

$$f \quad 1{,}5282 \quad 24{,}498 \quad 0{,}6 \quad 0{,}617 \quad \frac{0{,}6 \quad 0{,}4}{} \quad 0{,}031$$

$f \quad 1{,}5306 \quad 24{,}498 \quad 0{,}3702 \quad 0{,}0037 \quad 24{,}8645.$ 24,864 deb olsak bo‘ladi.

Agar x = 1,5282 bo‘lsa, u holda x_0 1,528; q 1,5282 1,528 / 0,001 0,2;

$$f(x) = 23{,}352 + 0{,}2\cdot 0{,}559 + \frac{0{,}2\cdot(-0{,}8)}{2}\cdot 0{,}028 =$$

$$23{,}352 \quad 0{,}1118 \quad 0{,}0022 \quad 23{,}4616.$$

f 1,5282 $23,462$ deb olsak bo‘ladi.

Javob: f 1,5306 24,864; f 1,5282 $23,462$.

Mustaqil yechish uchun:

Quyidagi amallarni va misollarni mos ravishda bajaring:

Topshiriqlar: **1-misolga variantlar**.

№ 1. *a)* $\sin 0{,}1436$; *b)* $\cos 1{,}1754$.

№ 2. *a)* $\sin 0{,}4974$; *b)* $\cos 0{,}9818$.

№ 3. *a)* $\sin 0{,}2453$; *b)* $\cos 1{,}0938$.

№ 4. *a)* $tg\ 0{,}3864$; *b)* $\cos 0{,}9222$.

№ 5. *a)* $\sin 0{,}4456$; *b)* $\cos 1{,}0045$.

№ 6. *a)* $\mathrm{tg}\, 0{,}3224$; *b)* $\cos 0{,}8465$.

№ 7. *a)* $\sin 0{,}6235$; *b)* $\cos 0{,}9464$.

№ 8. *a)* $\mathrm{tg}\, 0{,}2816$; *b)* $\cos 0{,}8065$.

№ 9. *a)* $\sin 0{,}7243$; *b)* $\cos 0{,}8675$.

№ 10. *a)* $\mathrm{tg}\, 0{,}2464$; *b)* $\cos 0{,}7312$.

№ 11. *a)* $\sin 0{,}8453$; *b)* $\cos 0{,}4324$.

№ 12. *a)* $\mathrm{tg}\, 0{,}2016$; *b)* $\cos 0{,}7075$.

№ 13. *a)* $\sin 0{,}9675$; *b)* $\cos 0{,}3436$.

№ 14. *a)* $\mathrm{tg}\, 0{,}1636$; *b)* $\cos 0{,}6865$.

№ 15. *a)* $\sin 1{,}0618$; *b)* $\cos 0{,}1458$.

№ 16. *a)* $\mathrm{tg}\, 0{,}1858$; *b)* $\cos 0, 5635$.

№ 17. *a)* $\sin 1{,}1238$; *b)* $\cos 0{,}1658$.

№ 18. *a)* $\sin 0{,}1362$; *b)* $\cos 0{,}5423$.

№ 19. *a)* $\mathrm{tg}\, 0{,}4052$; *b)* $\cos 0{,}7645$.

№ 20. *a)* $\sin 0{,}2134$; *b)* $\cos 1{,}1274$.

№ 21. *a)* $\mathrm{tg}\, 0{,}4527$; *b)* $\cos 0{,}7466$.

№ 22. *a)* $\sin 0{,}3425$; *b)* $\cos 1{,}0252$.

№ 23. *a)* $\sin 0{,}1648$; *b)* $\cos 1{,}1462$.

№ 24. *a)* $\sin 0{,}5438$; *b)* $\cos 0{,}9656$.

№ 25. *a)* $\sin 0{,}2642$; *b)* $\cos 1{,}0665$.

№ 26. *a)* $\mathrm{tg}\, 0{,}3654$; *b)* $\cos 0{,}9035$.

№ 27. *a)* $\mathrm{tg}\, 0{,}3083$; *b)* $\cos 0{,}8235$.

№ 28. *a)* $\sin 0{,}0236$; *b)* $\cos 0{,}2267$.

№ 29. *a)* $\sin 1{,}1438$; *b)* $\cos 0{,}7672$.

№ 30. *a)* $\sin 0{,}9057$;   *b)* $\cos 0{,}2632$.

2-misolga variantlar

1-jadval

x	u	Variantlar №	Argumentning qiymatlari	
			x_1	x_2
1,675	9,5618	1	1,6763	1,6787
1,676	9,4703	2	1,6778	1,6792
1,677	9,3804	3	1,6785	1,6762
1,678	9,2923	4	1,6794	1,6776
1,679	9,2057	5	1,6801	1,6786
1,680	9,1208	6	1,6816	1,6803
1,681	9,0373	7	1,6822	1,6808
1,682	8,9554	8	1,6837	1,6814
1,683	8,8749	9	1,6849	1,6823
1,684	8,7959	10	1,6853	1,6838
1,685	8,7182	11	1,6868	1,6843
1,686	8,6418	12	1,6773	1,6798
1,687	8,5668	13	1,6788	1,6802
1,688	8,4931	14	1,6813	1,6797
		15	1,6845	1,6821

2-jadval

x	y	Variantlar №	Argumentning qiymatlari	
			x_1	x_2
1,520	19,670	16	1,5223	1,5237
1,521	20,065	17	1,5228	1,5243
1,522	20,477	18	1,5239	1,5214
1,523	20,906	19	1,5241	1,5257
1,524	21,354	20	1,5256	1,5233
1,525	21,821	21	1,5267	1,5244
1,526	22,308	22	1,5272	1,5257
1,527	22,818	23	1,5284	1,5268
1,528	23,352	24	1,5295	1,5273
1,529	23,911	25	1,5303	1,5287

1,530	24,498	26	1,5318	1,5292
1,531	25,115	27	1,5242	1,5276
1,532	25,763	28	1,5263	1,5286
1,533	26,445	29	1,5288	1,5313
		30	1,5293	1,5308

5.5 -turdagi topshiriq.

Topshiriqlarni bajarish uchun namuna.

Topshiriq sharti. Argumentning quyidagi qiymatlarida $y = f(x)$ funksiya-ning qiymatlarini Gauss, Stirling va Bessel interpolyasion formulasidan foydalanib toping: a) *x*=0,168; b) *x*=0,192; v) *x*=0,204; g) *x*=0,175.

$y = f(x)$ funksiya jadval ko‘rinishda berilgan.

x	$y(x)$	x	$y(x)$
0,12	6,278	0,20	6,436
0,14	6,405	0,22	6,259
0,16	6,487	0,24	5,954
0,18	6.505		

Yechish:

$f(x)$ funksiya chekli ayirmalarining diogonal jadvalini tuzamiz:

x	$y(x)$	Δy_i	$\Delta^2 y_i$	$\Delta^3 y_i$
$x_{-3} = 0{,}12$	$y_{-3} = 6{,}278$			
		126		
$x_{-2} = 0{,}14$	$y_{-2} = 6{,}404$		-43	
		83		
$x_{-1} = 0{,}16$	$y_{-1} = 6{,}487$		-65	-22
		18		
$x_0 = 0{,}18$	$y_0 = 6{,}505$		-87	-22
		-69		
$x_1 = 0{,}20$	$y_1 = 6{,}436$		-108	-21
		-177		
$x_2 = 0{,}22$	$y_2 = 6{,}259$		-128	-20
		-305		
$x_3 = 0{,}24$	$y_3 = 6{,}954$			

Jadval uchinchi tartibli ayirmada to‘xtaydi, chunki uchinchi tartibli ayirma diyarli bir xil.

1) $y(0,168)$ qiymatni aniqlash uchun $x_0 = 0,16$ deb olamiz, u holda

$$t = (x - x_0)/h = (0,168 - 0,16)/0,02 = 0,4$$

Gaussning birinchi formulasidan foydalanamiz:

$$y(x) \approx P(x) = y_0 + t\Delta y_0 + \frac{t(t-1)}{2!}\Delta^2 y_{-1} + \frac{(t+1)t(t-1)}{2!}\Delta^3 y_{-1}\,.$$

Quyidagiga egamiz

$$y(0,168) \approx 6,487 + 0,4 \cdot 0,018 + \frac{0,4(-0,6)}{2} \cdot (-0,065) + \frac{1,4 \cdot 0,4(-0,6)}{6} \times$$
$$\times (-0,022) \approx 6,487 + 0,0072 + 0,0078 + 0,0012 = 6,5032 \approx 6,503.$$

2) $y(0,192)$ qiymatni aniqlash uchun $x_0 = 0,18$ deb olamiz, u holda

$$t = (x - x_0)/h = (0,192 - 0,18)/0,02 = 0,6\,.$$

Bessel formulasidan foydalanamiz:

$$y(x) \approx P(x) = \frac{y_0 + y_{-1}}{2} + \left(t - \frac{1}{2}\right) \cdot \Delta y_0 + \frac{t(t-1)}{2!} \cdot \frac{\Delta^2 y_{-1} + \Delta^2 y_0}{2} +$$
$$+ \frac{\left(t - \frac{1}{2}\right) t(t-1)}{3!}\Delta^3 y_{-1} + \cdots\,.$$

Bundan

$$y(0,192) \approx \frac{6b505 + 6,436}{2} + (0,6 - 0,5) \cdot (-0,069) + \frac{0,6 \cdot (-0,4)}{2} \times$$
$$\times \frac{-0,087 - 0,108}{2} + \frac{(0,6 - 0,5) \cdot 0,6 \cdot (-0,4)}{6} \cdot (-0,021) \approx$$
$$\approx 6,4705 - 0,0069 + 0,0117 + 0,0001 = 6,4754 \approx 6,475\,.$$

3) $y(0,204)$ qiymatni aniqlash uchun $x_0 = 0,20$ deb olamiz, u holda

$$t = (x - x_0)/h = (0,204 - 0,20)/0,02 = 0,2\,.$$

Stirling formulasidan foydalanamiz:

$$y(x) \approx P(x) = y_0 + t \cdot \frac{\Delta y_{-1} + \Delta y_0}{2} + \frac{t^2}{2} \cdot \Delta^2 y_{-1} + \frac{t(t^2-1)}{6} \cdot \frac{\Delta^3 y_{-2} + \Delta^2 y_{-1}}{2}.$$

Bundan

$$y(0{,}204) \approx 6{,}436 + 0{,}2 \cdot \frac{-0{,}069 - 0{,}177}{2} + \frac{0{,}04}{2} \cdot (-0{,}108) +$$

$$+ \frac{0{,}2 \cdot (0{,}04 - 1)}{2} \cdot \frac{-0{,}021 - 0{,}020}{6} \approx 6{,}436 - 0{,}0246 - 0{,}0022 + 0{,}0007 =$$

$$= 6{,}4099 \approx 6{,}410.$$

4) $y(0{,}175)$ qiymatni aniqlash uchun $x_0 = 0{,}18$ deb olamiz, u holda

$$t = (x - x_0)/h = (0{,}175 - 0{,}18)/0{,}02 = -0{,}25$$

Gaussning ikkinchi formulasidan foydalanamiz:

$$y(x) \approx P(x) = y_0 + t\Delta y_{-1} + \frac{(t+1)t}{2!}\Delta^2 y_{-1} + \frac{(t+1)t(t-1)}{3!}\Delta^3 y_{-2}.$$

Bundan quyidagiga egamiz

$$y(0{,}175) \approx 6{,}505 + (-0{,}25) \cdot 0{,}018 + \frac{0{,}75 \cdot (-0{,}25)}{2} \cdot (-0{,}087) +$$

$$+ \frac{0{,}7 \cdot (-0{,}25) \cdot (-1{,}25)}{6} \cdot (0{,}022) \approx 6{,}505 - 0{,}0045 + 0{,}0082 - 0{,}0009 =$$

$$= 6{,}5078 \approx 6{,}508.$$

Mustaqil yechish uchun:

Quyidagi amallarni va misollarni mos ravishda bajaring:

Topshiriqlar. Argumentning quyidagi qiymatlarida $y = f(x)$ funksiya-ning taqribiy qiymatini Gauss, Stirling va Bessel interpolyasion formulalaridan foydalanib toping: a) $x = 0{,}60 + 0{,}006n$; $b)$ $x = 1{,}725 + 0{,}002n$; $v)$ $x = 1{,}83 + 0{,}003n$; $g)$ $x = 2 - 0{,}013n$ $(n = 1, 2, 3, \cdots, 30)$.

$y = f(x)$ funksiya jadval ko‘rinishda berilgan:

x	$y(x)$	x	$y(x)$
1,50	15,132	1,85	43,189
1,55	17,422	1,90	48,689
1,60	20,393	1,95	54,225

1,65	23,994	2,00	59,653
1,70	28,160	2,05	64,817
1,75	32,812	2,10	69,550
1,80	37,857		

5.6 – turdagi topshiriq.

Topshiriqlarni bajarish uchun namuna.

Topshiriq sharti: Teng taqsimlanmagan tugunlar uchun Nyuton interpolyasion formulasidan foydalanib argumentning berilgan qiymatlarida funksiyaning qiymatini toping.

Argumentning 1) $x_1 = 0{,}112$; 2) $x_2 = 0{,}133$ qiymatlarida $y(x)$ funksiyaning qiymatini toping.

x	y
0,235	1,20800
0,240	1,21256
0,250	1,22169
0,255	1,22628
0,265	1,23547
0,280	1,24933
0,295	1,26328
0,300	1,26795
0,305	1,27263

yoki

x_1	x_2	x	y
0,112	0,133	0,235	1,20800
		0,240	1,21256
		0,250	1,22169
		0,255	1,22628
		0,265	1,23547
		0,280	1,24933
		0,295	1,26328
		0,300	1,26795
		0,305	1,27263

Yechish. Hisoblashlarni quyidagi formula bilan bajaramiz

$$y(x) \approx\ y_0 + f(x_0, x_1) \cdot (x - x_0) + f(x_0, x_1, x_2) \cdot (x - x_1),$$

Bunda

$$f(x_0, x_1) = \frac{f(x) - f(x_0)}{x_1 - x_0}; \quad f(x_0, x_1, x_2) = \frac{\big(f(x_1, x_2) - f(x_0, x_1)\big)}{x_2 - x_0}.$$

Zarur bo‘lgan “bo‘lingan ayirmalar” qiymatini oldindan hisoblaymiz.

x_i	y_i	$f(x_i, y_{i+1})$	$f(x_i, x_{i+1}, x_{i+2})$
0,103	2,01284	4,116	-18,238166
0,108	2,03342	3,896142	-16,761833
0,115	2,06070	3,696	-14,788461
0,120	2,07918	3,503750	-13,281250
0,128	2,10721	3,291250	-11,942307
0,136	2,13354	3,136	-
0,141	2,14922	-	-

1) $f(0{,}112)$ ning qiymatini ikki usulda topamiz. Birinchi x_0 sifatida 0,103 ni olamiz, keyin esa 0,108 ni olamiz: $x_0 = 0{,}103$

$$f(0{,}112) \approx 2{,}01284 + 4{,}116 \cdot (0{,}112 - 0{,}103) + (-18{,}238166) \cdot (0{,}112 - 0{,}103) \cdot (0{,}112 - 0{,}106) = 2{,}01284 + 0{,}037044 - 0{,}000657 = 2{,}04923,$$

$x_0 = 0{,}108$

$$f(0{,}112) \approx 2{,}03342 + 3{,}897142 \cdot (0{,}112 - 0{,}108) + (-16{,}761833) \times \times (0{,}112 - 0{,}108) \cdot (0{,}112\text{-}0{,}115) = 2{,}03342 + 0{,}015589 + 0{,}000201 = 2{,}04921.$$

Bundan $f(0{,}112) \approx 2{,}04921$ desak bo‘ladi.

2) $f(0{,}133)$ ning qiymatini ham ikki usulda topamiz. Birinchi x_0 sifatida 0,120 ni olamiz, keyin esa 0,128 ni olamiz:

$x_0 = 0{,}120$

$$f(0{,}133) \approx 2{,}07918 + 3{,}50375 \cdot (0{,}133 - 0{,}120) + (-13{,}28125) \times \times (0{,}133 - 0{,}120) \cdot (0{,}133 - 0{,}128) = 2{,}07918 + 0{,}045549 + 0{,}000863 = = 2{,}12387.$$

$x_0 = 0{,}128$

$$f(0{,}133) \approx 2{,}10721 + 3{,}29125 \cdot (0{,}133 - 0{,}128) + (-11{,}942307) \times \times (0{,}133 - 0{,}128) \cdot (0{,}133 - 0{,}136) = 2{,}10721 + 0{,}016456 + 0{,}000179 = = 2{,}12385.$$

Bundan $f(0{,}112) \approx 2{,}12386$ desak bo‘ladi.

Mustaqil yechish uchun:

Quyidagi amallarni va misollarni mos ravishda bajaring:

Topshiriqlar: Teng taqsimlanmagan tugunlar uchun Nyuton interpolyasion formulasidan foydalanib argumentning berilgan qiymatlarida funksiyaning qiymatini toping.

1-jadval

№ Varianta	x_1	x_2	x	y
1	0,308	0,335	0,298	3,25578
7	0,314	0,337	0,303	3,17639
13	0,325	0,303	0,310	3,12180
19	0,312	0,304	0,317	3,04819
25	0,321	0,336	0,323	2,98755
			0,330	2,91950
			0,339	2,83598

2-jadval

№ Varianta	x_1	x_2	x	y
	0,608	0,630	0,593	0,532050
	0,615	0,594	0,598	0,535625
	0,622	0,596	0,605	0,540598
	0,603	0,631	0,613	0,546235
	0,610	0,628	0,619	0,550431
			0,627	0,555983
			0,632	0,559428

3-jadval

№ Varianta	x_1	x_2	x	y
3	0,720	0,775	0,698	2,22336
9	0,740	0,705	0,706	2,24382
15	0,750	0,777	0,714	2,26446
21	0,765	0,700	0,727	2,29841
27	0,755	0,704	0,736	2,32221
			0,747	2,35164
			0,760	2,38690
			0,769	2,41162
			0,782	2,44777

4-jadval

№ Varianta	x_1	x_2	x	y
4	0,115	0,160	0,100	1,12128
10	0,124	0,162	0,108	1,13160
16	0,130	0,164	0,119	1,14594
22	0,140	0,104	0,127	1,15648
28	0,150	0,102	0,135	1,16712
			0,146	1,18191
			0,157	1,19689
			0,169	1,21344

5-jadval

№ Varianta	x_1	x_2	x	y
5	0,238	0,257	0,235	1,20800
11	0,261	0,298	0,240	1,21256
17	0,244	0,272	0,250	1,22169
23	0,275	0,303	0,255	1,22628
29	0,268	0,292	0,265	1,23547
			0,280	1,24933
			0,295	1,26328
			0,300	1,26795

6-jadval

№ Varianta	x_1	x_2	x	y
6	0,105	0,114	0,095	1,09131
12	0,103	0,117	0,102	1,23490
18	0,109	0,115	0,104	1,27994
26	0,108	0,100	0,107	1,35142
30	0,111	0,118	0,110	1,42815
			0,112	1,48256
			0,116	1,60033
			0,120	1,73205

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2. htt://lib.mexmat.ru.
3. htt://texlibrary.ru.

MUNDARIJA

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HISOBLASH USULLARI FANIDAN AMALIY MASHG'ULOTLAR

(na'muna uchun yechimlar ko'rsatilgan)

O'quv qo'llanma

Litsenziya raqami AA № 0048. 18.03.2020.
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Bichimi 60x84 $^{1}/_{16}$. Ofset qog'ozi.
Ofset bosma usulida bosildi.
Times New Roman garniturasi. Shartli bosma taboq 10,23.
Adadi 100 nusxa.

«AVTO-NAShR» bosmaxonasida chop etildi.
Toshkent shahar. Navoiy ko'chasi, 30.