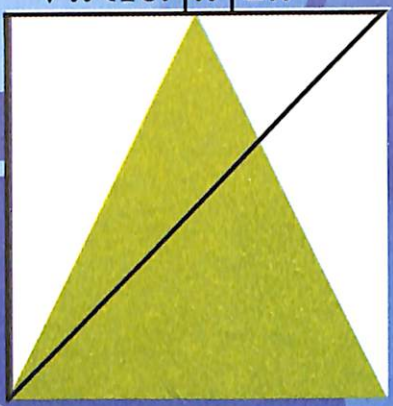
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ALGEBRA VA MATEMATIK ANALIZ ASOSLARIDAN MASALALAR TO'PLAMI

$$A \cap B = \{x \mid x \ A \text{ va } x \in B\}$$

$$\forall x \in R: |x| \geq x$$



$$a \equiv b (mod\ m) \Rightarrow (a-b) \vdots m$$

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O‘RTA MAXSUS, KASB-HUNAR TA’LIMI MARKAZI

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ALGEBRA VA MATEMATIK ANALIZ ASOSLARIDAN MASALALAR TO‘PLAMI

I q i s m

Akademik litseylar va kasb-hunar kollejlari uchun o‘quv qo‘llanma

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SO‘Z BOSHI

Respublikamizda ta’lim sohasida ulkan o‘zgarishlar amalga oshirilayotgan hozirgi davrda akademik litseylarning matematika fani chuqur o‘rganiladigan guruhlari uchun amaldagi o‘quv dasturiga to‘liq mos keladigan va dasturdagi mavzular bo‘yicha turli xil qiyinlik darajasiga ega bo‘lgan misol va masalalarni qamrab oladigan masalalar to‘plamining mavjud emasligi ushbu «Algebra va matematika analiz asoslaridan masalalar to‘plami»ning yaratilishiga sabab bo‘ldi.

O‘quv qo‘llanma O‘zbekiston Respublikasi Oliy va o‘rta maxsus ta’lim vazirligi tomonidan tasdiqlangan va 2000—2001-o‘quv yilidan boshlab amalga kiritilgan o‘quv dasturiga qat’iy amal qilingan holda yozildi.

Qo‘llanma asosan akademik litseylarning o‘quvchilari uchun mo‘ljallangan bo‘lib, undan kasb-hunar kollejlari o‘quvchilari, umumta’lim maktablarining o‘qituvchilari, shuningdek, matematikani mustaqil o‘rganuvchilar ham foydalanishlari mumkin.

Qo‘llanma yetti bobdan iborat bo‘lib, har bir bob paragraflarga bo‘lingan va u quyidagi mavzularni o‘z ichiga oladi:

1. To‘plamlar nazariyasi va matematik mantiq elementlari.
2. Haqiqiy sonlar.
3. Kompleks sonlar.
4. Ko‘phadlar.
5. Algebraik ifodalar.
6. Algebraik tenglamalar va tengsizliklar.
7. Funksiyalar va grafiklar.

Mualliflar zarur deb hisoblagan o‘rinlarda misol va masalalarning yechimlari, yechishga doir ko‘rsatmalar keltirilgan.

Mualliflar o‘quv qo‘llanmasining yaratilishi va uning sifatini yaxshilashga yaqindan yordam bergan SamDU akademik litseyi o‘qituvchilari, O‘zbekiston Respublikasida xizmat ko‘rsatgan yoshlar murabbiysi R. G‘ulomovga, fizika-matematika fanlari nomzodi, dots. A. Umarovga minnatdorchilik bildirishni o‘z burchlari deb hisoblaydilar, shuningdek, kitobni Pentium kompyuterida sahifalagan V. A. Mamedov va I. H. Nasimovlarga samimiy tashakkur bildiradilar.

Masalalar to‘plamida ba’zi bir kamchiliklar uchrashi ehtimoldan xoli emas. Kamchiliklar haqida fikr va mulohazalar bildirgan hamkasblarga mualliflar oldindan samimiy tashakkur izhor etadilar.

I b o b. TO‘PLAMLAR NAZARIYASI VA MATEMATIK MANTIQ ELEMENTLARI

1-§. TO‘PLAM VA UNING ELEMENTLARI. BO‘SH TO‘PLAM

To‘plam tushunchasi matematikaning ta’riflanmaydigan tushunchalaridan biridir.

To‘plamni tashkil etgan narsalar uning elementlari deyiladi. Masalan, 5 dan kichik bo‘lgan natural sonlar to‘plami quyidagi elementlardan tashkil topadi: 1,2,3,4.

To‘plamlar lotin alifbosining bosh harflari bilan, uning elementlari esa shu alifboning kichik harflari bilan belgilanadi. Masalan, $A=\{a,b,c,d\}$ yozuvi A to‘plam *à, b, c, d* elementlardan tashkil topganligini bildiradi.

Agar x element X to‘plamning elementi bo‘lsà, $x\in X$ shaklda yoziladi. $x\notin X$ yozuvi x element X to‘plamning elementi emasligini bildiradi.

Masalan, agar N-natural sonlar to‘plami bo‘lsa, u holda $4\in N$, $5\in N$, $\frac{3}{4}\in N$, $\pi\in N$.

Birorta ham elementga ega bo‘lmagan to‘plam bo‘sh to‘plam deyiladi va $\varnothing$ belgi bilan belgilanadi.

To‘plamga tegishli bo‘lgan elementlargina qanoatlantiradigan shartlar sistemasini shu to‘plamning xarakteristik xossasi deb atash qabul qilingan.

Misol. $A=\{x|x\in N, x<7\}$ to‘plam elementlarini ko‘rsating.

Yechish. A to‘plam 7 dan kichik bo‘lgan barcha natural sonlardan tuzilgan, ya’ni $A=\{1,2,3,4,5,6\}$.

1.1. O‘zbekiston Respublikasidagi barcha viloyatlar va Qoraqalpog‘iston Respublikasi nomlari to‘plamini tuzing.

1.2. O‘zbekiston Respublikasi davlat madhiyasida qatnashgan harflar to‘plamini tuzing.

1.3. O‘zbekiston Respublikasining davlat gerbi qabul qilingan yilda qatnashgan raqamlar to‘plamini tuzing.

1.4. $A=\{10;\ 12\frac{3}{4};\ 17{,}3;\ -7;\ 136\}$ to‘plam berilgan. Qaysi

natural sonlar bu to‘plamga kiradi? Shu to‘plamga tegishli bo‘lmagan uchta son ayting. $\in$, $\notin$ belgilari yordamida qo‘yilgan savollarga javob yozing.

1.5. S to‘plam $-3;-2;-1;4$ elementlardan tuzilgan. Shu to‘plamni yozing. Shu sonlarga qarama-qarshi sonlarning S_1 to‘plamini tuzing.

1.6. «Bo‘sh vaqtdan unumli foydalan» jumlasidagi harflar to‘plamini tuzing.

1.7. Quyidagi yozuvlarni o‘qing va har bir to‘plamning elementlarini ko‘rsating:

a) $E=\{x|\ x\in N,\ -1<x<5\}$;
b) $F=\{x|\ 5x=x-7\}$;
d) $Q=\{x|\ x(x+12)=0\}$;
e) $U=\{x|\ x\in R,\ x^2=2\}$;
f) $V=\{x|\ x\in N,\ x^2<9\}$;
g) $W=\{x|\ x\in N,\ x^2\le 9\}$.

1.8. Quyidagi to‘plamlarni son o‘qida belgilang:

a) $\{x|\ x\in N,\ x\le 3\}$;
b) $\{x|\ x\in Z,\ -2\le x\le 2\}$;
d) $\{x|\ x\in R,\ x>4.1\}$;
e) $\{x|\ x\in R,\ -2{,}7\le x\le 1\}$;
f) $\{x|\ x\in R,\ x<6\}$;
g) $\{x|\ x\in R,\ 3{,}4<x\le 8$;
h) $\{x|\ x\in R,\ -3\frac{1}{4}\le x\le -1\}$;
i) $\{x|\ x^2=4\}$;
j) $\{x|\ (x^2-1)(x^2-4)=0\}$.

1.9. Quyidagi to‘plam elementlarini toping:

a) 1 va 3 bilangina yoziladigan barcha uch xonali sonlar to‘plami;

b) 1,3,5 raqamlaridan (faqat bir marta) foydalanib yoziladigan barcha uch xonali sonlar to‘plami;

d) Raqamlarining yig‘indisi 5 ga teng bo‘lgan uch xonali sonlar to‘plami;

e) 100 dan kichik va oxirgi raqami 1 bo‘lgan barcha natural sonlar to‘plami.

1.10. Quyidagi to‘plamlardan qaysilari bo‘sh to‘plam:

a) Simmetriya markaziga ega bo‘lmagan kvadratlar to‘plami;
b) $\{x|\ x^2+1=0\}$;
d) $\{x|\ x\in R,\ |x|=3\}$;
e) $\{x|\ x\in R,\ x^3=1\}$;
f) $\{x|\ x^4-16=3\}$.

1.11. Quyidagi to‘plamning bo‘sh to‘plam ekanligini isbotlang:

a) $\{x|\ x\in N,\ x<-1\}$;
b) $\{x|\ x\in N,\ 15<x<16\}$;
d) $\{x|\ x\in N,\ x=\frac{3}{5}\}$;
e) $\{x|\ x>7,\ x<5\}$.

1.12. Tenglamaning haqiqiy ildizlari to‘plamini toping. Bu to‘plamlarning qaysilari bo‘sh to‘plam ekanligini aniqlang:

a) $3x+15=4(x-8)$; b) $2x+4=4$; d) $2(x-5)=3x$;
e) $x^2-4=0$; f) $x^2+16=0$; g) $(2x+7)(x-2)=0$.

1.13. Quyidagi to'plam elementlarini ko'rsating:
a) $\{l, f, g\}$; b) $\{a\}$; d) $\{\{a\}\}$; e) $\varnothing$;
f) $\{\varnothing\}$; g) $\{\{a;b\},\{c;d\}\}$; h) $\{\{a,b,c\},a\}$.

1.14. 5 ta elementga ega bo'lgan to'plam tuzing.

1.15. 5 ta natural son qatnashgan sonli to'plam tuzing.

2-§. QISM TO'PLAM. TENG TO'PLAMLAR

Agar B to'plamning har bir elementi A to'plamning ham elementi bo'lsa B to'plam A to'plamning *qism to'plami* deyiladi va $B \subset A$ ko'rinishida belgilanadi. Bunda $\varnothing \subset A$, $A \subset A$ deb hisoblanadi. Bu qism to'plamlar *xosmas qism to'plamlar* deyiladi. A to'plamning qolgan barcha qism to'plamlari *xos qism to'plamlar* deyiladi. n ta elementdan tuzilgan to'plamning barcha qism to'plamlari soni 2^n ga teng.

Agar $A \subset B$, $B \subset A$ bo'lsa, $A=B$ deyiladi.

1 - m i s o l. A — ikki xonali sonlar to'plami, B — ikki xonali juft sonlar to'plami bo'lsin. Har bir ikki xonali juft son A to'plamda ham mavjud. Demak $B \subset A$.

2 - m i s o l. $A=\{1,2,3\}$, $B=\{x \mid x \in N, x<4\}$ to'plamlar berilgan bo'lsin. B to'plam 4 dan kichik bo'lgan natural sonlar to'plamidir, ya'ni $B=\{1,2,3\}$. A va B to'plamlar ayni bir xil elementlardan tashkil topgan. Demak, $A=B$.

2.1. $A=\{a, b, c, d, e, f, g, k\}$, $B=\{a, l, k\}$, $C=\{b, d, g, k, t\}$, $D=\{a, l\}$, $E=\{e, f, k, g\}$ to'plamlar berilgan.

a) Ularning qaysilari A to'plamning xos qism to'plami bo'ladi?

b) D to'plam C to'plamning qism to'plamimi?

d) B to'plam qaysi to'plamning qism to'plami bo'ladi?

2.2. $C=\{213,45,324,732,136\}$ to'plam berilgan. C to'plamning

a) 3 ga bo'linadigan; b) 9 ga bo'linadigan;
d) 4 ga bo'linmaydigan; e) 5 ga bo'linmaydigan;

f) 3 ga bo'linmaydigan sonlaridan tuzilgan qism to'plamlarini toping.

2.3. $A=\{3,6,9,12\}$ to'plamning barcha qism to'plamlarini hosil qiling.

2.4. To'plamlar jufti berilgan:

a) A={Navoiy, Bobir, Furqat, Nodirabegim} va B — barcha shoir va shoiralar to'plami;

b) C — qavariq to'rtburchaklar to'plami va D to'rtburchaklar to'plami;

d) E — toshkentlik olimlar to'plami, F — O'zbekiston olimlari to'plami;

e) K — barcha tub sonlar to'plami, M — manfiy sonlar to'plami.

Juftlikdagi to'plamlardan qaysi biri ikkinchisining qism to'plami bo'lishini aniqlang.

2.5. To'rtburchaklar to'plami T va uning quyidagi qism to'plamlari berilgan:

A — parallelogrammlar to'plami;
B — romblar to'plami;
C — trapetsiyalar to'plami;
D — to'g'ri to'rtburchaklar to'plami;
E — kvadratlar to'plami.

Bu qism to'plamlarning har birini qanday xarakteristik xossalar bilan aniqlash mumkin?

2.6. Quyidagi to'plamlar uchun $A \subset B$ yoki $B \subset A$ munosabatlardan qaysi biri o'rinli:

a) $A=\{a,b,c,d\}$, $B=\{a,c,d\}$; b) $A=\{a,b\}$, $B=\{a,c,d\}$;
d) $A=\varnothing$, $B=\varnothing$; e) $A=\varnothing$, $B=\{a,b,c\}$;
f) $A=\varnothing$, $B=\{\varnothing\}$; g) $A=\{\{a\},a,\varnothing\}$, $B=\{a\}$;
h) $A=\{\{a,b\},\{c,d\},c,d\}$, $B=\{\{a,b\},c\}$;
i) $A=\{\{0\},0\}$, $B=\{\varnothing,\{\{0\},0\}\}$.

2.7. Tasdiq to'g'ri yoki noto'g'ri ekanligini aniqlang:

a) $\{1;2\} \subset \{\{1;2;3\};\{1;3\};1;2\}$;
b) $\{1;2\} \in \{\{1;2;3\};\{1;3\};1;2\}$;
d) $\{1;3\} \subset \{\{1;2;3\};\{1;3\};1;2\}$;
e) $\{1;3\} \in \{\{1;2;3\};\{1;3\};1;2\}$.

2.8. Quyidagi to'plamlar tengmi:

a) $A=\{2;4;6\}$ va $B=\{6;4;2\}$;
b) $A=\{1;2;3\}$ va $B=\{1;11;111\}$;
d) $A=\{\{1;2\},\{2;3\}\}$ va $B=\{2;3;1\}$;
e) $A=\{\sqrt{256};\ \sqrt{81};\ \sqrt{16}\}$ va $B=\{2^2;3^2;4^2\}$?

2.9. A — natural sonlar to'plami, B — juft natural sonlar to'plami, C — toq natural sonlar to'plami, D — 2 ga ham, 3 ga ham bo'linadigan sonlar to'plami, E — o'nli yozuvi 0 bilan tugaydigan sonlar to'plami, F — 6 ga karrali sonlar to'plami, M — 2 ga ham, 5 ga ham karrali bo'lgan sonlar to'plami bo'lsin. Qaysi to'plam qaysi to'plamning qism to'plami bo'lishini aniqlang. Berilgan to'plamlar orasida teng to'plamlar mavjudmi?

2.10. Qaysi to'plamlar juftligidagi to'plamlar teng:

a) $X=\{3,5,7,9\}$, Y — 2 dan katta, lekin 10 dan kichik toq sonlar to'plami;

b) $X=\{4,6,8\}$, Y — 1 dan katta, lekin 9 dan kichik juft sonlar to'plami;

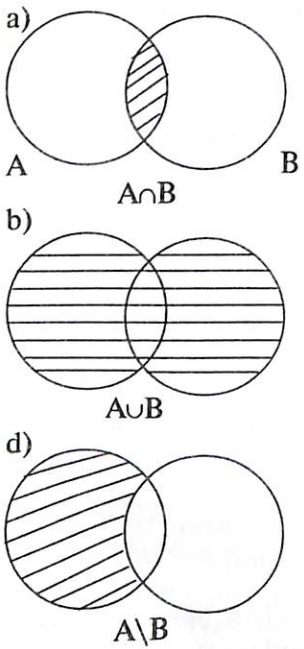
d) X — ikkita toq sonlarning yig'indisi bo'lgan sonlar to'plami, Y — juft sonlar to'plami;

e) X — tekislikda M va K nuqtalardan bir xil uzoqlashgan nuqtalar to'plami, Y — MK kesmaning o'rta perpendikulyaridagi nuqtalar to'plami.

3-§. TO'PLAMLAR USTIDA AMALLAR

Ikki to'plamning kesishmasi, birlashmasi va ayirmasiga beriladigan ta'riflar ayoniy bo'lishi uchun Eyler-Venn diagrammalaridan ham foydalanamiz.

A va B to'plamlarning har birida mavjud bo'lgan x element shu to'plamlarning umumiy elementi deyiladi. A va V to'plamlarning *kesishmasi* deb, ularning hamma umumiy elementlaridan tuzilgan to'plamga aytiladi. A va B to'plamlarning kesishma $A\cap B$ ko'rinishda belgilanadi (1-a rasm), $A\cap B=\{x \mid x\in A$ ва $x\in B\}$.



1-rasm.

A va B to'plamlarning *birlashmasi* deb, ularning kamida bittasida mavjud bo'lgan barcha elementlardan tuzilgan to'plamga aytiladi. A va B to'plamlarning birlashmasi $A\cup B$ ko'rinishida belgilanadi (1-b rasm):

$A\cup B=\{x \mid x\in A$ yoki $x\in B\}$.

A va B to'plamlarning ayirmasi deb, A ning B da mavjud bo'lmagan barcha elementlaridan tuzilgan to'plamga aytiladi. A va B to'plamlarning ayirmasi $A\setminus B$ ko'rinishda belgilanadi (1-d rasm):

$$A\setminus B=\{x \mid x\in A \text{ ва } x\notin B\}.$$

Agar $B\subset A$ bo'lsa, $A\setminus B$ to'plam B to'plamning *to'ldiruvchisi* deyiladi va B' bilan belgilanadi.

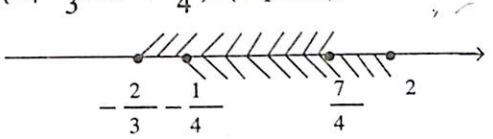
1 - m i s o l. $A=\{a,b,c,d,e,f\}$ va $B=\{b,d,e,g,h\}$ to'plamlar berilgan. Ularning kesishmasini toping.

Y e c h i s h. b,d,e elementlargina A va B to'plamlarning umumiy elementlaridir.

Shuning uchun, $A\cap B=\{b,d,e\}$.

2 - m i s o l. $A=\{x \mid -\frac{2}{3}\le x\le\frac{7}{4}\}$, $B=\{x \mid -\frac{1}{4}\le x\le 2\}$ to'plamlarning kesishmasi, birlashmasi va ayirmasini toping (2-rasm).

Y e c h i s h. $A\cap B=\{x \mid -\frac{1}{4}\le x\le\frac{7}{4}\}$; $A\cup B=\{x \mid -\frac{2}{3}\le x\le 2\}$; $A\setminus B=\{x \mid -\frac{2}{3}\le x<-\frac{1}{4}\}$. (2-pasm).



2-rasm.

3 - m i s o l. Agar $A\subset B$ bo'lsa, $A\cup B=B$ bo'ladi. Isbotlang.

Isbot. $A\subset B$ bo'lsin. a) $A\cup B\subset B$ ni ko'rsatamiz. $x\in A\cup B$ bo'lsin. U holda $x\in A$ yoki $x\in B$ bo'ladi. Agar $x\in A$ bo'lsa, $A\subset B$ ekanidan $x\in B$ ekani kelib chiqadi, ikkita holda ham $A\cup B$ ning har qanday elementi B ning ham elementidir. Demak, $A\cup B\subset B$.

b) $B\subset A\cup B$ ni ko'rsatamiz. $x\in B$ bo'lsin. U holda, to'plamlar birlashmasining ta'rifiga ko'ra $x\in A\cup B$ bo'ladi. Demak, B ning har qanday elementi $A\cup B$ ning ham elementi bo'ladi, ya'ni $B\subset A\cup B$.

Shunday qilib, $A\cup B\subset B$, $B\subset A\cup B$. Bular esa $B=A\cup B$ ekanini tasdiqlaydi. Isbot bo'ldi.

3.1. $M=\{36;29;15;68;27\}$; $P=\{4;15;27;47;36;90\}$; $Q=\{90;4;47\}$ to'plamlar berilgan. $M\cap P$, $M\cap Q$, $P\cap Q$, $M\cap P\cap Q$ larni toping.

3.2. A — 18 ning hamma natural bo'luvchilari to'plami, B — 24 ning hamma natural bo'luvchilari to'plami, $A\cap B$ to'plam elementlarini ko'rsating (Eyler-Venn diagrammasidan foydalaning).

3.3. P ikki xonali natural sonlar to'plami, S barcha toq natural sonlar to'plami bo'lsa, $K=P\cap S$ to'plamga qaysi sonlar kiradi:

a) $21\in K$; b) $32\in K$; d) $7\notin K$; e) $17\notin K$ deyish to'g'rimi?

3.4. «Matematika» va «grammatika» so'zlaridagi harflar to'plamini tuzing. Bu to'plamlar kesishmasini toping.

3.5. [1;5] va [3;7] kesmalarning kesishmasini toping.

3.6. $P=\{a,b,d,e,f,g\}$ va $E=\{a,h,i,j,k\}$ to'plamlar birlashmasini toping.

3.7. $A=\{n|\ n\in N, n<5\}$ va $B=\{n|\ n\in N, n>7\}$ to'plamlar birlashmasini toping: a) $4\in A\cup B$; b) $-3\in A\cup B$; v) $6\in A\cup B$ deyish to'g'rimi?

3.8. Agar a) $A=\{x|\ x=8k,\ k\in Z\}$, $B=\{x|\ x=8l-4,\ l\in Z\}$;

b) $A=\{x|\ x=6k-1,\ k\in Z\}$, $B=\{x|\ x=6l+4,\ l\in Z\}$ bo'lsa $A\cup B$ ni toping.

3.9. $A=\{2;4;6;8;\cdots 40\}$, $B=\{1;3;5;7;\cdots 37\}$, $C=\{\{a;b\}, \{c;d\},\ \{e;f\},\ \{g,h\}\}$ to'plamlarning har biridagi elementlar sonini aniqlang. $A\cup B$ da nechta element mavjud?

3.10. $A=\{2;3;4;5;7;10\}$, $B=\{3;5;7;9\}$, $C=\{4;9;11\}$ bo'lsin. Quyidagi to'plamlarda nechtadan element mavjud:

a) $A\cup(B\cup C)$; b) $(C\cup B)\cup(A)$; d) $A\cap(B\cup C)$;

e) $A\cup(B\cap C)$; f) $A\cap(B\cap C)$; g) $B\cap(A\cup C)\cap$?

3.11. $A=\{x|\ -5\le x\le 10\}$, $B=\{x|\ x\in N,\ 3\le x\le 15\}$ bo'lsin. $A\setminus B$ ва $B\setminus A$ to'plam elementlarini toping.

3.12. P — ikki xonali natural sonlar to'plami, Q — juft natural sonlar to'plami bo'lsin. $P\setminus Q$ va $Q\setminus P$ to'plamlarni tuzing.

3.13. C va Д kesishuvchi to'plamlar bo'lsin. Eyler-Venn diagrammalari yordamida $C\setminus Д$, $Д\setminus C$, $(C\setminus Д)\cup(Д\setminus C)$ larni tasvirlang.

3.14. N′ bilan natural sonlar to'plami N ning butun sonlar to'plami Z ga to'ldiruvchisini belgilaymiz. Quyidagilar to'g'rimi:

a) $-4\in N'$; b) $0\in N'$; d) $13\in N$; e) $-8\notin N'$; f) $-5,3\notin N'$; g) $0\notin N'$?

3.15. $A=\{x|\ x=2k+1,\ k\in Z\}$ to'plamning Z to'plamga to'ldiruvchisini toping.

3.16. $A=\{x|\ x=3k,\ k\in Z\}$ to'plamning Z to'plamga to'ldiruvchisini toping.

3.17. Agar $A\subset U$, $B\subset U$ bo'lsa, quyidagi tengliklar o'rinli bo'lishini isbotlang:

a) $(A\cup B)'=A'\cap B'$; b) $(A\cap B)'=A'\cup B'$. (Izoh: U ga to'ldirish deb tushunilsin).

3.18. Agar A to'plam $x^2-7x+6=0$ tenglamaning yechimlari to'plami va $B=\{1;6\}$ bo'lsa, $A=B$ bo'lishini isbotlang.

3.19. $A\setminus B=A\setminus(A\cap B)$ tenglikni isbotlang.

3.20. $A\cap(B\setminus A)=\varnothing$ tenglikni isbotlang.

4-§. TO'PLAM ELEMENTLARINING SONI BILAN BOG'LIQ AYRIM MASALALAR

$n(A)$ bilan A to'plam elementlarining sonini belgilaymiz. Har qanday A va B chekli to'plamlar uchun $n(A\cup B)=n(A)+n(B)-n(A\cap B)$ tenglik to'g'ri.

M i s o l. 50 o'quvchining 37 tasi ingliz tilini, 17 tasi esa nemis tilini o'rganayapti. Agar 50 o'quvchining har biri shu ikki tilning kamida bittasini o'rganayotgan bo'lsa, necha o'quvchi ikkala tilni ham o'rganayapti?

Yechish. A — barcha o'quvchilar to'plami, B — ingliz tilini o'rganayotgan o'quvchilar to'plami, C esa nemis tilini o'rganayotgan o'quvchilar to'plami bo'lsin. $n(A)=50$, $n(B)=37$, $n(C)=17$ larga egamiz. Masala mazmunidan, $n(B\cap C)$ ni topish lozimligini ko'ramiz. Tushunarliki, $A=B\cup C$. $n(B\cup C)=n(B)+n(C-n(B\cap C)$ tenglikdan $50=37+17-n(B\cap C)$ yoki $n(B\cap C)=4$ ni topamiz. Shunday qilib, 4 o'quvchi ikkala tilni ham o'rganayotgan ekan.

4.1. Sinfdagi bir necha o'quvchi marka yig'dilar. 15 o'quvchi O'zbekiston markalarini, 11 kishi chet el markalarini, 6 kishi ham O'zbekiston markalarini, ham chet el markalarini yig'di. Sinfda necha o'quvchi marka yig'gan?

4.2. 32 o'quvchining 12 tasi voleybol seksiyasiga, 15 tasi basketbol seksiyasiga, 8 kishi esa ikkala seksiyaga ham qatnashadi. Sinfdagi necha o'quvchi hech bir seksiyaga qatnashmaydi?

4.3. 30 o'quvchidan 18 tasi matematikaga, 17 tasi esa fizikaga qiziqadi. Ikkala fanga ham qiziqadigan o'quvchilar soni nechta bo'lishi mumkin? (Ko'rsatma: Ikkala fanga ham qiziqmaydigan o'quvchilar soni $k\in\{0,1,2,3,\cdots,12\}$).

4.4. 100 odamdan iborat turistlar guruhida 10 kishi nemis tilini ham, fransuz tilini ham bilmaydi, 75 tasi nemis tilini, 83 tasi esa fransuz tilini biladi. Ikkala tilni ham biladigan turistlar sonini toping.

4.5. 26 o'quvchining 14 tasi shaxmatga, 16 tasi shashkaga qiziqadi. Agar har bir o'quvchi shaxmatga yoki shashkaga qiziqsa, ham shashkaga, ham shaxmatga qiziqadigan o'quvchilar nechta?

5-§. TO'PLAMLAR USTIDA BARCHA AMALLARGA DOIR MASALALAR

5.1. To'plamlar kesishmasini va birlashmasini toping. Eyler-Venn diagrammasi yordamida grafik talqin qiling:

a) $A=\{5,6,7,8,9,10\}$, $B=\{8,9,10,11\}$;

b) $A=\{x \mid x=2n, n\in N\}$, $B=\{x \mid x=\frac{n+1}{2}, n\in N\}$;

d) $A=\{x \mid x=5n, n\in N\}$, $B=\{x \mid x=2n, n\in N\}$;

e) $A=\{x \mid x=\frac{1}{n}, n\in N\}$, $B=\{x \mid x=\frac{2}{n}, n\in N\}$;

5.2. P va Q to'plamlar kesishmasi va birlashmasini son to'g'ri chizig'ida tasvirlang:

a) $P=\{x \mid \frac{10}{3} < x < \sqrt{8}\}$, $Q=\{x \mid \frac{26}{27} < x < 3.2\}$;

b) $P=\{x \mid -\frac{1}{3} < x < \frac{5}{3}\}$, $Q=\{x \mid \sqrt{2} < x \le \frac{40}{27}\}$;

d) $P=\{x \mid \frac{11}{4} \le x \le \frac{19}{3}\}$, $Q=\{x \mid \frac{19}{7} < x \le \frac{32}{5}\}$;

e) $P=\{x \mid \frac{4}{11} \le x < \frac{18}{5}\}$, $Q=\{x \mid \sqrt{2} < x < 10\}$.

5.3. Quyidagi tengliklarni isbotlang:

a) $A\cap B=B\cap A$; b) $(A\cap B)\cap C=A\cap(B\cap C)$;

d) Agar $A\subset B$ bo'lsa, $A\cap B=A$; e) $A\cap\varnothing=\varnothing$;

f) $A\cap A=A$.

5.4. Quyidagi tengliklarni isbotlang:

a) $A\cup B=B\cup A$; b) $(A\cup B)\cup C=A\cup(B\cup C)$;

d) $A\subset B$ bo'lsa, $A\cup B=B$; e) $A\cup\varnothing=A$;

f) $A\cup A=A$.

5.5. Isbotlang:

a) $(A\cup B)\cap C=(A\cap C)\cup(B\cap C)$;

b) $(A\cap B)\cup C=(A\cap C)\cap(B\cup C)$.

5.6. Ayirish va to'ldirish amallarining quyidagi xossalarini isbotlang ($A\subset B$, $B\subset C$, $C\subset U$ deb hisoblang):

a) $AF\cap A=\varnothing$; f) $\varnothing'=U$;

b) $AF\cup A=U$; g) $UF=\varnothing$;

d) $(AF\cap BF)=AF\cup BF$; h) $(A\setminus B)\setminus C=A\setminus(B\cup C)$;

e) $(A\cup B)F=AF\cap BF$; i) $(A\setminus B)\cup B=A\cup B$.

6-§. MATEMATIK MANTIQ ELEMENTLARI

1) $\Rightarrow$ — agar ..... bo'lsa, u holda ..... bo'ladi. $P\Rightarrow Q$ — agar P bo'lsa, Q bo'ladi (P dan Q kelib chiqadi);

2) $\Leftrightarrow$ — teng kuchlilik $R\Leftrightarrow Q$, R va Q teng kuchli (R dan Q kelib chiqadi va aksincha);

3) $\vee$ — diz'yunksiya («yoki» amali);

4) $\wedge$ — kon'yuksiya («va» amali);

5) $\forall$ — ixtiyoriy, barcha;

6) $\exists$ — shunday mavjud;

7) $\nexists$ — mavjud emas.

1 - m i s o l. Agar $a>b$ va $b>c$ bo'lsa, $a>c$ bo'ladi.

$$(a>b)\wedge(b>c)\Rightarrow(a>c).$$

2 - m i s o l. $a>b$ bo'lsa, $a+c>b+c$ bo'ladi.

$$(a>b)\Rightarrow(a+c>b+c).$$

3 - m i s o l. $a=0$ yoki $b=0$ bo'lsa, $ab=0$ bo'ladi va aksincha $ab=0$ bo'lsa, $a=0$ yoki $b=0$ bo'ladi:

$$(ab=0)\Leftrightarrow((a=0)\vee(b=0)).$$

4 - m i s o l. $a>0$ va $b>0$ bo'lsa, $ab>0$ bo'ladi.

$$(a>0)\wedge(b>0)\Rightarrow(ab>0).$$

5 - m i s o l. Ixtiyoriy x haqiqiy soni son uchun $|x|\ge x$.

$$\forall x\in R:\ |x|\ge x.$$

6 - m i s o l. Ixtiyoriy $a\ge 0$ son uchun, shunday $x\in R$ son mavjudki, $x^2=a$ bo'ladi. $\forall a\ge 0$, $\exists x\in R$: $x^2=a$.

Quyidagi jumlalarni yuqoridagi belgilar yordamida yozing (6.1.– 6.10):

6.1. Ixtiyoriy $a\ge 0$ uchun, $\sqrt{a}=x$ tenglik o'rinli bo'ladigan x haqiqiy son mavjud.

6.2. $a<0$ va $b>0$ bo'lsa, $ab<0$ bo'ladi.

6.3. Har qanday a, b haqiqiy sonlar uchun $a+b=b+a$ bo'ladi.

6.4. Agar a butun son 9 ga bo'linsa, u holda bu son 3 ga ham bo'linadi.

6.5. 2 ham, 3 ga ham bo'linadigan butun son 6 ga ham bo'linadi va aksincha 6 ga bo'linadigan butun son 2 ga ham, 3 ga ham bo'linadi.

6.6. Agar $a^2+b^2+c^2=0$ bo'lsa, $a=b=c=0$ bo'ladi va aksincha, $a=b=c=0$ bo'lsa, $a^2+b^2+c^2=0$ bo'ladi.

6.7. Har qanday n natural sonni olmaylik, $n=2k-1$ yoki $n=2k$ bo'ladigan k natural son mavjud.

6.8. Ixtiyoriy n, k natural sonlar uchun, $n^2+k^3\in N$ bo'ladi.

6.9. Ixtiyoriy n, k natural sonlar uchun, n^2-k^3 butun son bo'ladi.

6.10. $a<0$ bo'lsa, $x^2=a$ tenglik to'g'ri bo'ladigan haqiqiy x son mavjud emas.

II b o b. HAQIQIY SONLAR

1-§. NATURAL SONLAR

Hisoblang:

1.1. $78\cdot29+6573:313-408$.

1.2. $477\cdot85-7784:56+10809$.

1.3. $927:103+(247-82):5-1$.

1.4. $(395\cdot52-603)\cdot25-960\cdot64$.

1.5. $25\cdot(28\cdot105+7236:18):6\cdot25$.

1.6. $1092322:574+152\cdot93-(96\cdot125-82215:9)$.

1.7. $79348-64\cdot84:28+6539:13-11005$.

1.8. $3121350-(15125:25+302\cdot804-(3044+2056):17)\cdot9$.

1.9. $(110292:14:101+4129-3127)\cdot(1237-23138:23)$.

1.10. $4097\cdot7-7659+64\cdot105-6992:38:23$.

Bo'linish alomatlarining tadbiqiga doir misollar

1.11. 1 dan 25 gacha bo'lgan natural sonlar qatoridagi 6 ga bo'linmaydigan natural sonlar to'plamini tuzing.

1.12. 1 dan 25 gacha bo'lgan natural sonlar qatoridagi 7 ga bo'linadigan natural sonlar to'plamini tuzing.

1.13. 15121, 117342, 1897524, 2134579, 31445698 sonlari orasidan 6 ga bo'linadigan natural sonlar to'plamini tuzing.

$k\in N$ soniga bo'linadigan barcha natural sonlar to'plamini A_k bilan belgilaymiz (1.14—1.20.).

1.14. Tasdiq to'g'rimi:

a) $2\in A_3$; f) $25\notin A_5$; j) $15342749\in A_9$;

b) $2\in A_4$; g) $36\in A_2$; k) $15342724\in A_4$;

d) $6\notin A_5$; h) $41\in A_3$; l) $15342824\in A_8$;

e) $11\in A_9$; i) $422\notin A_9$; m) $4343242\in A_{11}$?

1.15. $11\cdot12\cdot13\cdot14\cdot15\cdot16$ soni A_2, A_3, A_4, A_5, A_6, A_7, A_8, A_9, A_{10}, A_{11} to'plamlarning qaysilariga tegishli?

1.16.* $1\cdot2\cdot3\cdot4\cdots8\cdot9\notin A_k$ bo'lsa, $k=2431$ bo'lishi mumkinmi? $k\in\{15;18\}$ bo'lishi mumkinmi?

1.17.* $3\cdot5\cdot7\in A_k$ bo'lsa, k ning qabul qilishi mumkin bo'lgan barcha qiymatlarini toping.

1.18. $A_2\cap A_6$, $A_2\cap A_3$, $A_3\cap A_5$ larni toping.

1.19. $A_2\cup A_3=A_6$ tenglik to'g'rimi?

1.20. $a\in A_3$, $b\in A_4$ bo'lsa, $a+b\notin A_7$ bo'lishi mumkinmi?

1.21. Sonlarni tub ko'paytuvchilarga ajrating: 10; 100; 1000; 10000; 100000; 1000000. Qanday xulosaga kelish mumkin?

1.22. Sonlarni tub ko'paytuvchilarga ajrating: 250; 300; 340; 3700; 48950; 4725000.

1.23. Sonlarni kanonik shaklda yozing:

a) 36; f) 125; j) 946; n) 13860;

b) 72; g) 36; k) 1001; o) 2431;

d) 81; h) 512; l) 3125; p) 6783;

e) 96; i) 680; m) 4500; q) 36363.

1.24. Sonlarni kanonik shaklda yozing:

a) $2\cdot3^2\cdot2^4\cdot6^2$; f) $18\cdot18\cdot15\cdot5$; j) $15^2\cdot17\cdot21^3$;

b) $4\cdot5\cdot7\cdot9$; g) $17\cdot19\cdot25$; k) $27^3\cdot11\cdot3^4$;

d) $3\cdot5\cdot7\cdot11$; h) $3^4\cdot4^3\cdot53$; l) $33\cdot34\cdot43^2$;

e) $13\cdot13\cdot27$; i) $31^2\cdot33\cdot37^2\cdot39$; m) $117\cdot118\cdot119^2$.

Tub ko'paytuvchilarga ajratishning tadbiqlariga doir misollar

1.25. Sonning bo'luvchilarini toping:

a) 209; b) 143; d) 2431; e) 2717.

1.26. Sonlarning umumiy bo'luvchilarini toping: a) 209 va 143; b) 209 va 2431; d) 143 va 2717; e) 2431 va 2717.

1.27. Sonlarning eng katta umumiy bo'luvchisini toping:

a) 40 va 45; f) 50, 75 va 100; j) 63, 130, 143 va 1001;

b) 130 va 160; g) 74, 45 va 60; k) 74, 60, 84 va 480;

d) 121 va143; h) 84, 63 va 42; l) 750, 800, 865 va 1431;

e) 31 va 93; i) 72, 48va 36; m) 143, 209, 1431 va 2717.

1.28. Quyidagi sonlar o'zaro tubmi:

a) 15 va 95; f) 121 va 143; j) 169 va 1443;

b) 144 va 169; g) 11, 12 va 25; k) 111 va 121;

d) 143 va 144; h) 14, 16 va 19; l) n, $n+1$ va $n+2$ ($n\in N$);

e) 250 va 131; i) 63, 130 va 800; m) n, $n+2$ va $n+4$ ($n\in N$)?

1.29. Sonlarning eng kichik umumiy karralisini toping:

a) 84, 42 va 21 ; f) 50, 125 va 175; j) 33, 36 va 48;
b) 70, 80 va 90; g) 48, 92 va 75 ; k) 100, 150 va 250;
d) 17, 51 va 289; h) 10, 21 va 3600; l) 80, 240 va 360;
e) 11, 12 va 13 ; i) 18, 19 va 24 ; m) 34, 51 va 65.

1.30. Sonlarning eng katta umumiy bo'luvchisini va eng kichik umumiy karralisini toping (natijani kanonik ko'rinishda yozing):

a) 2^3, 3^2 va 15 ; f) $7^2 \cdot 3$; 46 va 15 ;
b) 2^3, 3^4 va 7 ; g) $3^2 \cdot 4$; $3 \cdot 6$ va $7 \cdot 9$;
d) 8, 13^2 va 5^2 ; h) 3^4, 11^2 va 13^3 ;
e) 12^2, 15 va1 ; i) 11^4, 13^5 va 100^4 .

1.31. $\tau(a)$ bilan $a \in N$ ning hamma natural bo'luvchilari sonini belgilaymiz. $a = p_1^{a1} \cdot p_2^{a2} \cdot p_3^{a3} \cdot p_n^{an}$ bo'lsa, $\tau(a) = (\alpha_1+1)(\alpha_2+1)\cdots(\alpha_n+1)$ bo'ladi. Quyidagilarni toping:

a) $\tau(81)$; f) $\tau(400)$; j) $\tau(2^3 \cdot 6 \cdot 7)$; n) $\tau(11 \cdot 13 \cdot 17)$;
b) $\tau(91)$; g) $\tau(680)$; k) $\tau(2^3 \cdot 3^2 \cdot 5)$; o) $\tau(19^2 \cdot 23 \cdot 29)$;
d) $\tau(512)$; h) $\tau(13860)$; l) $\tau(4^2 \cdot 6 \cdot 15)$; p) $\tau(121 \cdot 11^2)$;
e) $\tau(1001)$; i) $\tau(13800)$; m) $\tau(13 \cdot 100 \cdot 55)$; q) $\tau(144 \cdot 11^3)$.

1.32. Sonlarning umumiy bo'luvchisi nechta:

a) 18 va 54; f) 63 va 72; j) 150 va 180;
b) 42 va 56; g) 120 va 96; k) 12, 18 va 30;
d) 96 va 92; h) 102 va 170; l) 54, 90 va 162;
e) 84 va 120; i) 26, 65 va 130; m) 40, 60 va 100?

1.33. Sonlarning umumiy bo'luvchilarini toping:

a) $13 \cdot 17$ va $13^2 \cdot 17 \cdot 19$; b) $17 \cdot 19 \cdot 23$ va $17^4 \cdot 19 \cdot 23^8 \cdot 1849$.

1.34. A={100,15,200,300} va A={150,300,450} to'plamlar umumiy elementlarining umumiy bo'luvchilari nechta?

1.35. Hisoblang:

a) $\tau(\tau(\text{EKUB}(\text{EKUK}(250;500);100)))$;
B) $\text{EKUB}((100); \tau(\text{EKUB}(25;5)) + \tau(\text{EKUK}(10;35))$;
v) $\text{EKUK}(\text{EKUK}(\tau(144);51);18) - \tau(42)$;
g) $t(18 \cdot 91+15(ÝÊÓÁ(10;21))) \cdot \tau(142)$.

Yevklid algoritmini tadbiq etishga doir misollar

1.36. Sonlarning eng katta umumiy bo'luvchisini toping:

a) 8104 va 5602; f) 187 va 180; j) 795 va 2585;
b) 5555 va 11110; g) 2165 va 3556; k) 42628 va 33124;
d) 980 va 100; h) 5400 va 8400; l) 71004 va 154452;
e) 5345 va 4856; i) 78999 va 80000; m) 1000 va 999.

1.37. Quyidagi sonlar o'zaro tubmi:

a) 60 va 72; b) 55 va 71; d) 732 va 648; e) 111 va 11?

1.38. $\text{EKUB}(a;b) \cdot \text{EKUK}(a;b) = a \cdot b$ $(a \in N,\ b \in N)$ tenglikdan foydalanib, quyidagi sonlarning eng kichik umumiy karralisini toping:

a) 821 va 934 ; f) 28 va 947 ; j) 75 va 1853 ;
b) 743 va 907 ; g) 56 va 953 ; k) 23 va 1785 ;
d) 109 va 1005 ; h) 419 va 854 ; l) 113 va 9881;
e) 827 va 953 ; i) 887 va 6663 ; m) 875 va 1346.

1.39. Sonlarning o'zaro tub ekanligini isbotlang:

a) 911 va 130177 ; b) 811 va 10403.

1.40. Hisoblang: $\tau(\text{EKUB}(991;659;647+367))$.

2-§. BUTUN SONLAR

Hisoblang:

2.1. $143+(-42+85-52) \cdot 9-124$;
2.2. $-56 \cdot ((-43+54)-65:5-82)$;
2.3. $-53 \cdot (44+86-200 \cdot 5+300:(-6))$;
2.4. $660:(-88+44+92:2)+840:(-3)$;
2.5. $48 \cdot (-86 \cdot 2-95)+(-842) \cdot 31$.

Qoldiqli bo'lishga doir misollar

2.6. a ni b ga qoldiqli bo'ling:

a) $a=70$, $b=3$; d) $a=200$, $b=17$;
b) $a=180$, $b=9$; e) $a=76$, $b=9$.

2.7. a ni b ga qoldiqli bo'ling:

a) $a=5$, $b=9$; d) $a=9$, $b=18$;
b) $a=11$, $b=23$; e) $a=4$, $b=75$.

2.8. a ni b ga qoldiqli bo'ling:

a) $a=-81$, $b=75$; f) $a=-33$, $b=7$; j) $a=15$, $b=43$;
b) $a=-5$, $b=9$; g) $a=-48$, $b=6$; k) $a=27$, $b=9$;
d) $a=-41$, $b=7$; h) $a=-6$, $b=48$; l) $a=33$, $b=32$;
e) $a=-35$, $b=7$; i) $a=-8$, $b=24$; m) $a=108$, $b=36$.

2.9. $a \in N$, $b \in N$ bo'lib, $a=bq+r$ $(q \in Z,\ r \in N,\ 0 \le r < b)$ bo'lsin. $-a$ ni b ga bo'lishda hosil bo'ladigan to'liqsiz bo'linma q_1 ni va qoldiq r_1 ni toping.

2.10. a ni b ga bo'lishdagi qoldiqni toping:

a) $a=81932$, $b=9$; f) $a=4341$, $b=3$; j) $a=111$, $b=11$;
b) $a=25$, $b=75$; g) $a=144$, $b=6$; k) $a=-11$, $b=111$;
d) $a=-4$, $b=49$; h) $a=-15$, $b=11$; l) $a=-9$, $b=3$;
e) $a=-49$, $b=4$; i) $a=-13$, $b=35$; m) $a=-3$, $b=9$.

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2.11. Quyidagi tenglik qoldiqli bo'lishni ifodalaydimi:

a) $21=3\cdot4+9$; f) $26=4\cdot5+6$; j) $81=81\cdot0+81$;
b) $-18=9\cdot2-36$; g) $-15=11\cdot(-2)+7$; k) $-40=4\cdot(-11)+4$;
d) $35=2\cdot17+1$; h) $-49=7\cdot8+(-7)$; l) $-35=(-7)\cdot8+21$;
e) $11=2\cdot4+3$; i) $84=2\cdot42$; m) $49=4\cdot11+5$?

Taqqoslama va uning ayrim tadbiqlariga doir misollar

$a\in Z$, $b\in Z$, $m\in N$ bo'lsin. a va b larni m ga bo'lishda bir xil qoldiq hosil bo'lsa, a va b sonlari m modul bo'yicha taqqoslanadi (m modul bo'yicha teng qoldiqli) deyiladi.

a va b lar m modul bo'yicha taqqoslanuvchi ekanligi quyidagicha belgilanadi:

$$a\equiv b \pmod m. \qquad (1)$$

(1) munosabat taqqoslama deb ataladi.

1 - t e o r e m a. $\boldsymbol{a\equiv b \pmod m \Rightarrow (a-b)\ \vdots\ m}$.

2 - t e o r e m a. $\boldsymbol{(a-b)\ \vdots\ m \Rightarrow a\equiv b \pmod m}$.

3 - t e o r e m a. **Agar $a\equiv b \pmod m$ va $c\equiv d \pmod m$ bo'lsa, quyidagi taqqoslamalar to'g'ri bo'ladi:**

$$a+c\equiv b+d \pmod m;$$
$$a-c\equiv b+d \pmod m;$$
$$a\cdot c\equiv b\cdot d \pmod m.$$

Natijalar: 1. $a=a \pmod m$;

2. $a\equiv b \pmod m \Rightarrow a^n\equiv b^n \pmod m$, $n\in N$;
3. $a\equiv b \pmod m \Rightarrow c\cdot a\equiv c\cdot b \pmod m$, $c\in Z$
4. Agar $a\equiv b \pmod m$ va $c\equiv d \pmod m$ bo'lsa, ixtiyoriy n_1, $n_2\in Z$ uchun $n_1a+n_2c = n_1b+n_2d \pmod m$ bo'ladi.

1- m i s o l. 2222^{5555} sonini 7 ga bo'lishda hosil bo'ladigan qoldiqni toping.

Yechish. 2222 ni 7 ga qoldiqli bo'lamiz: $2222=7\cdot317+3$. Bundan, $2222\equiv3 \pmod 7$ ni olamiz. Hosil bo'lgan taqqoslamaning har ikki tomonini 5555 — darajaga ko'taramiz (2-natija):

$$2222^{5555}\equiv3^{5555} \pmod 7.$$

Bu taqqoslama izlanayotgan qoldiq 3^{5555} ni 7 ga bo'lishdan hosil bo'ladigan qoldiq bilan bir xil ekanligini ko'rsatadi. 3^{5555} ni 7 ga bo'lishda hosil bo'ladigan qoldiqni topamiz. Buning uchun 3 ning dastlabki bir nechta darajalarini 7 ga bo'lishda qanday qoldiqlar hosil bo'lishini kuzataylik:

$3^1\equiv3 \pmod 7$; $3^2\equiv3\cdot3\equiv9\equiv2 \pmod 7$; $3^3\equiv2\cdot3\equiv6 \pmod 7$;
$3^4\equiv6\cdot3\equiv18\equiv4 \pmod 7$; $3^5\equiv4\cdot3\equiv12\equiv5 \pmod 7$;
$3^6\equiv5\cdot3\equiv15\equiv1 \pmod 7$.

$3^6\equiv1 \pmod 7$ ga ega bo'dik. Bundan, $3^{6k}\equiv1^k \pmod 7$ $k\in N$ (2) ni olamiz.

Endi 5555 ni 6 ga bo'lamiz: $5555=6\cdot925+5$.

U holda $3^{5555}=3^{6\cdot925+5}=3^{6\cdot925}\cdot3^5\equiv1\cdot3^5\equiv5 \pmod 7$.

Shunday qilib, izlanayotgan qoldiq 5 ga teng.

2 - m i s o l. $2^{60}+7^{30}$ soni 13 ga bo'linadi. Shuni isbotlang.

I s b o t. $2^4=13+3$ va $7^2=49=13\cdot4-3$ bo'lgani uchun $2^4\equiv3 \pmod{13}$, $7^2\equiv-3 \pmod{13}$ larga egamiz. Oxirgi har bir taqqoslamani 15 darajaga ko'tarib, ularni hadma-had qo'shamiz: $2^{60}+7^{30}=0 \pmod{13}$.

Demak, $2^{60}+7^{30}$ soni 13 ga bo'linadi.

2.12. Taqqoslama to'g'rimi:

a) $125=-35 \pmod 4$; f) $113=13 \pmod{100}$;
b) $44=-32 \pmod{25}$; g) $842=42 \pmod{-5}$;
d) $-58=11 \pmod 5$; h) $31=-20 \pmod{17}$;
e) $111=13 \pmod{}$; i) $1=18 \pmod 0$?

2.13. $n\in\{3,5,9\}$ bo'lsin. n ning qaysi qiymatlarida taqqoslama to'g'ri bo'ladi:

a) $33=3 \pmod n$; f) $43\equiv-2 \pmod n$;
b) $134=-25 \pmod n$; g) $-121\equiv13 \pmod n$;
d) $-223=41 \pmod n$; h) $155\equiv11 \pmod n$;
e) $34=72 \pmod n$; i) $-48\equiv11 \pmod n$?

2.14. 5^{20} ni 24 ga bo'lishda hosil bo'ladigan qoldiqni toping.

2.15. 3333^{6666} ni 5 ga bo'lishda hosil bo'ladigan qoldiqni toping.

2.16. 7^{77} ning oxirgi raqamini toping.

Y e c h i s h. 7 ning dastlabki bir nechta darajalarining oxirgi raqamini kuzatamiz:

$7^1=7$; $7^5=*7$; Takrorlanish sodir bo'ldi.
$7^2=49$; $7^6=*9$; (qadam 4 ga teng).
$7^3=*3$; $7^7=*3$; Kuzatuv quyidagi xulosani chiqarishga
$7^4=*1$; $7^8=*1$. imkon beradi.

$$7^n=\begin{cases}*7, \text{ agar } n=1 \pmod 4.,\\ *9, \text{ agar } n=2 \pmod 4.,\\ *3, \text{ agar } n=3 \pmod 4.,\\ *1, \text{ agar } n=0 \pmod 4.,\end{cases} \qquad (3)$$

Endi $n=7^{77}$ ni 4 ga bo'lishda hosil bo'ladigan qoldiqni aniqlaymiz:

$7^1=3(\text{mod } 4)$; $7^2\equiv 3\cdot 7\equiv 1(\text{mod } 4)$; $7^{2k}\equiv 1(\text{mod } 4)$;

$7^{77}=7^{2\cdot 38+1}=7^{2\cdot 38}\cdot 7\equiv 1\cdot 7\equiv 3(\text{mod } 4)$.

$7^{77}=3(\text{mod } 4)$ bo'lgani uchun, (3) ga asosan $7^{7^{77}}=*3$.

Shunday qilib, oxirgi raqam 3 ekan.

2.17. Sonning oxirgi raqamini toping:

a) $8^{8^{87}}$; f) $555^{222^{22}}$; j) 10001^{9^n}, $n\in N$;

b) $113^{8^{91}}$; g) $333^{444^{555}}$; k) 1005^{1005^n}, $n\in Z$;

d) $144^{5^{555}}$; h) $1111^{9^{99}}$; l) $8^{8^{95}}$;

e) $2002^{9^{95}}$; i) $999^{2^{888^{999}}}$; m) $6^{7^{89}}$;

2.18. Ixtiyoriy n natural son uchun n^5-n soni 5 ga bo'linishini isbotlang.

Isbot: n - ixtiyoriy natural son bo'lsin. n ni 5 ga bo'lamiz.

Agar $n\equiv 0(\text{mod } 5)$ bo'lsa, $n^5-n\equiv 0^5-0\equiv 0(\text{mod } 5)$ bo'ladi.

Agar $n\equiv 1(\text{mod } 5)$ bo'lsa, $n^5-n\equiv 1^5-1\equiv 0(\text{mod } 5)$ bo'ladi.

Agar $n\equiv 2(\text{mod } 5)$ bo'lsa, $n^5-n\equiv 2^5-2\equiv 30\equiv 0$ (mod 5) bo'ladi.

Agar $n\equiv 3(\text{mod } 5)$ bo'lsa, $n^5-n=3^5-3=240=0$ (mod 5) bo'ladi.

Agar $n\equiv 4(\text{mod } 5)$ bo'lsa, $n^5-n\equiv 4^5-4=1020=0$ (mod 5) bo'ladi. n ning har qanday qiymatida, $n^5-n=0(\text{mod } 5)$ ekanini ko'ramiz. Demak, $\forall n\in N$ uchun $(n^2-n)5$ ga qoldiqsiz bo'linadi.

2.19. n ning barcha butun qiymatlarida (n^3+11n) soni 6 ga qoldiqsiz bo'linishini isbotlang.

2.20. n ning barcha butun qiymatlarida n^2-n soni 3 ga qoldiqsiz bo'linishini isbotlang.

2.21. n^2+1 soni n ning ixtiyoriy butun qiymatida 3 ga bo'linmasligini isbotlang.

2.22. n ning barcha natural qiymatlarida $n(n^2+1)$ soni 7 ga bo'linishini isbotlang.

2.23. $12^{2n+1}+11^{2n+2}$ soni n ning har qanday natural qaymatida 133 ga bo'linishini isbotlang.

2.24. p soni 3 dan katta tub soni bo'lsa, p^2-1 soni 24 ga bo'linadi. Isbotlang.

2.25. p va q sonlari 3 dan katta tub sonlar bo'lsa, p^2-q^2 soni 24 ga bo'linadi. Isbotlang.

Matematik induksiya metodi yordamida sonlarning bo'linishini isbotlashga doir misollar

M i s o l. n ning barcha natural qiymatlari $4^n+15n-1$ soni 9 ga bo'linadi. Isbotlang.

I s b o t. $n=1$ da $4^n+15n-1=18$ soni 9 ga bo'linadi.

$4^n+15n-1$ soni $n=k$ da 9 ga bo'linadi deb faraz qilamiz va $n=k+1$ bo'lganda ham $4^n+15n-1$ soni 9 ga bo'linishini ko'rsatamiz:

$n=k+1$ bo'lsa, $4^n+15n-1=4^{k+1}+15(k+1)-1=4\cdot 4^k+15k+14=$
$=4(4^k+15k-1)-60k+4+15k+14=4(4^k+15k-1)-45k+18=$
$=4(4^k+15k-1)+9(2-5k)$ ga ega bo'lamiz.

Birinchi qo'shiluvchi qilingan farazga ko'ra 9 ga bo'linadi. Ikkinchi qo'shiluvchi ham 9 ga bo'lingani uchun ularning yig'indisi ham 9 ga bo'linadi. Demak $4^n+15n-1$ soni n ning barcha natural qiymatlarida 9 ga bo'linadi. Shu bilan da'vo isbot bo'ldi.

2.26. $4^n+15n-1$ soni n ning barcha natural qiymatlarida 3 ga bo'linishini isbotlang.

2.27. n^3+5n soni ixtiyoriy natural n da 6 ga bo'linishini isbotlang.

2.28. 7^n+3n-1 ning 9 ga bo'linishini isbotlang, bunda $n\in N$.

2.29. $6^{2n}+19^n-2^{n+1}$ ning 17 ga bo'linishini isbotlang, bunda $n\in N$.

2.30. Barcha $n\in N$ lar uchun $(2n-1)^3-(2n-1)$ sonining 24 ga bo'linishini isbotlang.

2.31. n^3+11n soni ixtiyoriy $n\in N$ da 6 ga bo'linishini isbotlang.

2.32. $n^2(n^2+1)$ conining 4 ga bo'linishini isbotlang, bunda $n\in N$.

2.33. $n(2n+1)(7n+1)$ soni 6 ga bo'linishini isbotlang $(n\in N)$.

2.34. 2^n+2^{n+1} soni 6 ga bo'linishini isbotlang $(n\in N)$.

2.35. $n^2(n^2-1)$ soni 12 ga bo'linishini isbotlang $(n\in N)$.

Endi $n=7^{77}$ ni 4 ga bo'lishda hosil bo'ladigan qoldiqni aniqlaymiz:

$7^1=3(\text{mod } 4)$; $7^2\equiv 3\cdot 7\equiv 1(\text{mod } 4)$; $7^{2k}\equiv 1(\text{mod } 4)$;

$7^{77}=7^{2\cdot 38+1}=7^{2\cdot 38}\cdot 7\equiv 1\cdot 7\equiv 3(\text{mod } 4)$.

$7^{77}=3(\text{mod } 4)$ bo'lgani uchun, (3) ga asosan $7^{7^{77}}=*3$.

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2.22. n ning barcha natural qiymatlarida $n(n^2+1)$ soni 7 ga bo'linishini isbotlang.

2.23. $12^{2n+1}+11^{2n+2}$ soni n ning har qanday natural qaymatida 133 ga bo'linishini isbotlang.

2.24. p soni 3 dan katta tub soni bo'lsa, p^2-1 soni 24 ga bo'linadi. Isbotlang.

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$n=k+1$ bo'lsa, $4^n+15n-1=4^{k+1}+15(k+1)-1=4\cdot 4^k+15k+14=$
$=4(4^k+15k-1)-60k+4+15k+14=4(4^k+15k-1)-45k+18=$
$=4(4^k+15k-1)+9(2-5k)$ ga ega bo'lamiz.

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2.35. $n^2(n^2-1)$ soni 12 ga bo'linishini isbotlang $(n\in N)$.

3-§. RATSIONAL SONLAR

Oddiy kasrlar va ular ustida amallar

3.1. Amallarni bajaring:

a) $\frac{8}{45}+\frac{16}{45}$; g) $\frac{17}{18}+\frac{13}{36}$; l) $\frac{8}{15}\cdot\frac{19}{151}$;

b) $\frac{17}{48}-\frac{7}{48}$; h) $\frac{32}{15}-\frac{17}{148}$; m) $\frac{12}{121}\cdot\frac{11}{144}$;

d) $\frac{17}{35}+\frac{18}{35}$; i) $\frac{15}{17}-\frac{7}{18}$; n) $\frac{9}{113}\cdot\frac{15}{101}$;

e) $\frac{18}{69}+\frac{59}{69}$; j) $\frac{37}{113}-\frac{9}{131}$; o) $\frac{19}{38}:\frac{15}{49}$;

f) $\frac{1112}{150}-\frac{338}{150}$; k) $\frac{1}{151}+\frac{9}{153}$; p) $\frac{121}{49}:\frac{11}{7}$.

3.2. Ifodaning qiymatini toping:

a) $\left(45\frac{1}{2}-2\frac{3}{8}\right)-\left(5\frac{5}{6}+6\frac{3}{4}\right)+\left(10\frac{2}{3}-5\frac{5}{8}\right)$;

b) $\left(36\frac{4}{5}-12\frac{3}{10}\right)-\left(4\frac{2}{15}+1\frac{1}{30}\right)+\left(20\frac{11}{12}-10\frac{3}{8}-\frac{3}{16}-3\frac{1}{48}\right)$;

d) $\left(12\frac{1}{2}-3\frac{5}{6}\right)-\left(2\frac{8}{9}+1\frac{4}{5}\right)-\left(5\frac{5}{8}-4\frac{3}{4}\right)-\left(6\frac{9}{40}-5\frac{11}{90}\right)$;

e) $56\frac{2}{21}-\left\{\left(1\frac{5}{6}+2\frac{13}{14}\right)+\left[27\frac{13}{30}-\left(15\frac{5}{12}-12\frac{13}{20}\right)\right]\right\}$;

f) $\frac{4}{5}\cdot\frac{3}{8}\cdot\frac{3}{5}\cdot\frac{2}{3}$; g) $3\frac{1}{3}\cdot3\frac{13}{53}\cdot3\frac{1}{88}$;

h) $5\frac{1}{4}:1\frac{2}{7}:5\frac{1}{2}\cdot\frac{3}{22}$; i) $\left(1\frac{11}{24}+1\frac{13}{56}\right)\cdot9:1\frac{2}{5}$;

j) $\dfrac{8\frac{1}{2}}{15:\frac{5}{17}}$; k) $\dfrac{\frac{28}{29}:\frac{7}{29}}{\frac{7}{9}:\frac{1}{9}}$; l) $\dfrac{4\frac{4}{5}:\frac{4}{17}}{3\frac{2}{5}}$;

m) $8\frac{13}{16}\cdot\frac{47}{64}:1\frac{1}{35}:3\frac{1}{2}$.

Amallarni bajaring:

3.3. a) $2:\frac{3}{5}+\frac{3}{5}:2+1\frac{1}{2}:6+6:\frac{1}{2}$;

b) $6\frac{1}{4}\cdot8-3\frac{2}{3}\cdot5\frac{1}{2}+2\frac{2}{5}\cdot4\frac{7}{12}$;

d) $2\frac{1}{2}\cdot48-3\frac{3}{8}:\frac{1}{18}+5\frac{5}{12}:\frac{7}{36}$;

e) $13\frac{1}{2}:1\frac{1}{3}+16\frac{1}{2}\cdot1\frac{5}{11}+19\frac{1}{4}:\frac{4}{25}$.

3.4. a) $\left(3\frac{1}{2}-2\frac{2}{3}+5\frac{5}{6}+4\frac{3}{5}\right)\cdot24$;

b) $\left(5\frac{5}{8}+18\frac{1}{2}-7\frac{5}{24}\right):16\frac{2}{3}$;

d) $\left(12\frac{5}{12}+1\frac{2}{3}-3\frac{5}{6}+2\frac{3}{4}\right):\left(2\frac{1}{2}\cdot\frac{2}{5}-\frac{7}{9}\right)$;

e) $48\frac{3}{8}\cdot6\frac{3}{4}\cdot\frac{5}{12}-2\frac{5}{6}+1\frac{75}{94}\cdot\left(1\frac{1}{2}\cdot\frac{1}{3}-13:26\right)$.

3.5. a) $\left(\frac{5}{7}\cdot2\frac{1}{3}\cdot\frac{5}{6}-1\right):\left(1-\frac{7}{8}\cdot1\frac{3}{5}\cdot\frac{3}{14}\right)$;

b) $\left(8\frac{7}{15}-3\frac{3}{4}+4\frac{2}{3}-8\frac{7}{60}\right):\left(4\frac{1}{4}-2\frac{3}{4}\right)$;

d) $\left(1\frac{8}{13}\cdot\frac{13}{42}+5\frac{5}{7}:\frac{8}{21}\right):\left(8\frac{1}{8}+3\frac{1}{3}\right)$;

e) $2\frac{3}{5}:6\frac{1}{15}+1\frac{1}{14}-1\frac{39}{73}\cdot\left(5\frac{5}{7}-5\frac{5}{16}\right)$.

3.6. a) $\dfrac{12\frac{4}{5}\cdot3\frac{3}{4}-4\frac{4}{11}\cdot4\frac{1}{8}}{11\frac{2}{3}:4\frac{4}{7}}$;

b) $\dfrac{28\frac{4}{5}:13\frac{5}{7}+6\frac{3}{5}:\frac{2}{3}}{1\frac{11}{16}:2\frac{1}{44}}$;

d) $\dfrac{2\frac{3}{8}:\frac{3}{4}+24\frac{7}{9}}{7\frac{1}{8}-175\frac{4}{5}:24}$;

e) $\dfrac{\left(1\frac{1}{8}+2\frac{2}{3}+3\frac{3}{4}\right)\cdot3\frac{3}{5}}{14-15\frac{1}{8}:2\frac{1}{5}}$;

3-§. RATSIONAL SONLAR

Oddiy kasrlar va ular ustida amallar

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d) $\frac{17}{35}+\frac{18}{35}$; i) $\frac{15}{17}-\frac{7}{18}$; n) $\frac{9}{113}\cdot\frac{15}{101}$;

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d) $\left(12\frac{1}{2}-3\frac{5}{6}\right)-\left(2\frac{8}{9}+1\frac{4}{5}\right)-\left(5\frac{5}{8}-4\frac{3}{4}\right)-\left(6\frac{9}{40}-5\frac{11}{90}\right)$;

e) $56\frac{2}{21}-\left\{\left(1\frac{5}{6}+2\frac{13}{14}\right)+\left[27\frac{13}{30}-\left(15\frac{5}{12}-12\frac{13}{20}\right)\right]\right\}$;

f) $\frac{4}{5}\cdot\frac{3}{8}\cdot\frac{3}{5}\cdot\frac{2}{3}$; g) $3\frac{1}{3}\cdot3\frac{13}{53}\cdot3\frac{1}{88}$;

h) $5\frac{1}{4}:1\frac{2}{7}:5\frac{1}{2}\cdot\frac{3}{22}$; i) $\left(1\frac{11}{24}+1\frac{13}{56}\right)\cdot9:1\frac{2}{5}$;

j) $\dfrac{8\frac{1}{2}}{15:\frac{5}{17}}$; k) $\dfrac{\frac{28}{29}:\frac{7}{29}}{\frac{7}{9}:\frac{1}{9}}$; l) $\dfrac{4\frac{4}{5}:\frac{4}{17}}{3\frac{2}{5}}$;

m) $8\frac{13}{16}\cdot\frac{47}{64}:1\frac{1}{35}:3\frac{1}{2}$.

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3.3. a) $2:\frac{3}{5}+\frac{3}{5}:2+1\frac{1}{2}:6+6:\frac{1}{2}$;

b) $6\frac{1}{4}\cdot8-3\frac{2}{3}\cdot5\frac{1}{2}+2\frac{2}{5}\cdot4\frac{7}{12}$;

d) $2\frac{1}{2}\cdot48-3\frac{3}{8}:\frac{1}{18}+5\frac{5}{12}:\frac{7}{36}$;

e) $13\frac{1}{2}:1\frac{1}{3}+16\frac{1}{2}\cdot1\frac{5}{11}+19\frac{1}{4}:\frac{4}{25}$.

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b) $\left(5\frac{5}{8}+18\frac{1}{2}-7\frac{5}{24}\right):16\frac{2}{3}$;

d) $\left(12\frac{5}{12}+1\frac{2}{3}-3\frac{5}{6}+2\frac{3}{4}\right):\left(2\frac{1}{2}\cdot\frac{2}{5}-\frac{7}{9}\right)$;

e) $48\frac{3}{8}\cdot6\frac{3}{4}\cdot\frac{5}{12}-2\frac{5}{6}+1\frac{75}{94}\cdot\left(1\frac{1}{2}\cdot\frac{1}{3}-13:26\right)$.

3.5. a) $\left(\frac{5}{7}\cdot2\frac{1}{3}\cdot\frac{5}{6}-1\right):\left(1-\frac{7}{8}\cdot1\frac{3}{5}\cdot\frac{3}{14}\right)$;

b) $\left(8\frac{7}{15}-3\frac{3}{4}+4\frac{2}{3}-8\frac{7}{60}\right):\left(4\frac{1}{4}-2\frac{3}{4}\right)$;

d) $\left(1\frac{8}{13}\cdot\frac{13}{42}+5\frac{5}{7}:\frac{8}{21}\right):\left(8\frac{1}{8}+3\frac{1}{3}\right)$;

e) $2\frac{3}{5}:6\frac{1}{15}+1\frac{1}{14}-1\frac{39}{73}\cdot\left(5\frac{5}{7}-5\frac{5}{16}\right)$.

3.6. a) $\dfrac{12\frac{4}{5}\cdot3\frac{3}{4}-4\frac{4}{11}\cdot4\frac{1}{8}}{11\frac{2}{3}:4\frac{4}{7}}$;

b) $\dfrac{28\frac{4}{5}:13\frac{5}{7}+6\frac{3}{5}:\frac{2}{3}}{1\frac{11}{16}:2\frac{1}{44}}$;

d) $\dfrac{2\frac{3}{8}:\frac{3}{4}+24\frac{7}{9}}{7\frac{1}{8}-175\frac{4}{5}:24}$;

e) $\dfrac{\left(1\frac{1}{8}+2\frac{2}{3}+3\frac{3}{4}\right)\cdot3\frac{3}{5}}{14-15\frac{1}{8}:2\frac{1}{5}}$;

f) $\dfrac{14\frac{4}{5}-6\frac{11}{12}+12\frac{3}{4}-7\frac{2}{15}}{10\frac{2}{3}-3\frac{11}{12}}+2\frac{2}{3}\cdot 3\frac{3}{4}$;

g) $\dfrac{1\frac{9}{16}\cdot 3\frac{1}{5}+16\frac{2}{3}-9:2\frac{2}{5}}{17\frac{7}{12}-6\frac{1}{3}}+\dfrac{12\frac{2}{3}-61\frac{1}{2}:6\frac{3}{4}}{2\frac{2}{3}}$.

Quyidagi masalalarni tenglama tuzib yeching:

3.7. a) Ikki sonning yig'indisi $7\frac{1}{2}$ ga teng. Sonlardan biri ikkinchisidan $4\frac{4}{5}$ ta ortiq. Shu sonlarni toping;

b) Uchta sonning yig'indisi $35\frac{2}{3}$ ga teng. Birinchi son ikkinchisidan $5\frac{1}{3}$ ta, uchinchisidan esa $3\frac{5}{6}$ ta ortiq. Shu sonlarni toping.

3.8. Uy uchta xonadan iborat. Birinchi xonaning yuzasi $24\frac{3}{8}$ m^2 bo'lib, uy yuzasining $\frac{13}{36}$ qismini tashkil etadi. Ikkinchi xonaning yuzasi uchinchi xona yuzasiga qaraganda $8\frac{1}{8}$ m^2 ortiq. Ikkinchi xonaning yuzasini toping.

3.9. Uch bo'lak temirning umumiy og'irligi. $17\frac{1}{4}$ kg. Agar birinchi bo'lakning og'irligini $1\frac{1}{2}$ kg, ikkinchi bo'lakning og'irligini esa $2\frac{1}{4}$ kg kamaytirsak, uchta bo'lak temirning hammasi bir xil og'irlikda bo'lib qoladi. Har bir bo'lakning dastlabki og'irligini toping.

3.10. a) Ikki sonning yig'indisi $8\frac{11}{14}$ ga, ayirmasi esa $2\frac{3}{7}$ ga teng. Shu sonlarni toping;

b) motorli qayiq daryo oqimi bo'ylab $15\frac{1}{2}$ km/soat tezlik bilan, oqimga qarshi esa $8\frac{1}{4}$ km/soat bilan yuradi. Daryo oqimining tezligini toping.

3.11. Ota o'g'lidan 24 yosh katta. O'g'lining yoshi otasi yoshining $\frac{5}{13}$ qismiga teng. Ota necha yoshda? O'g'il-chi?

3.12. Kasrning maxraji uning suratidan 11 ta ortiq. Agar kasrning maxraji uning suratidan $3\frac{3}{4}$ marta ortiq bo'lsa, shu kasrni toping.

3.13. Ikki sonning yig'indisi 16 ga teng. Agar ikkinchi sonning $\frac{1}{3}$ qismi birinchi sonning $\frac{1}{5}$ qismiga teng bo'lsa, shu sonlarni toping.

3.14. Belgilangan ishni birinchi brigada 36 kunda, ikkinchi brigada esa 45 kunda bajaradi. Ikkita brigada birga ishlasa, shu ishni necha kunda bajaradi?

3.15. Ikki shahar orasidagi masofani yo'lovchi poezdi 10 soatda, yuk poezdi esa 15 soatda bosib o'tadi. Ular bir vaqtda bir-biriga qarab yo'lga chiqsa, necha soatdan keyin uchrashadi?

3.16. Birinchi quvur basseynni 5 soatda to'ldiradi. Ikkinchi quvur to'la basseynni 6 soatda bo'shatadi. Agar ikkala quvur, bir vaqtda ochilsa, basseyn necha soatdan keyin to'ladi (Basseyn bo'sh edi, deb hisoblansin).

O'nli kasrlar va ular ustida amallar

Amallarni bajaring:

3.17. a) 4,735:0,5+14,95:1,3–2,121:0,7;
b) 589,72:16–18,305:7+0,0567:4 ;
d) 3,006-0,3417:34–0,875:125 ;
e) 22,5:3,75+208,45+2,5:0,004 .

3.18. a) (0,1955+0,187):0,085 ;
b) 15,76267:(100,6+42,697) ;
d) (86,9+667,6):(37,1+13,2) ;
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3.19. a) (0,008+0,992)·(5·0,6–1,4);
b) (0,93+0,07)·(0,93–0,805);
d) (50000–1397,3):(20,4+33,603);
e) (2779,6+8024,4):(1,98+2,02).

3.20. a) $\dfrac{4{,}06\cdot 0{,}0058+3{,}3044895-(0{,}7584:2{,}37+0{,}0003:8)}{0{,}03625\cdot 80-2{,}43}$;

b) $\dfrac{2{,}045\cdot 0{,}033+10{,}518395-0{,}464774:0{,}0562}{0{,}003092:0{,}0001-5{,}188}$;

d) $\dfrac{57{,}24\cdot 3{,}55+430{,}728}{2{,}7\cdot 1{,}88-1{,}336}+\dfrac{127{,}18\cdot 4{,}35+14{,}067}{18+2{,}1492:3{,}582}$;

e) $52:\dfrac{6:(0{,}4-0{,}2)}{2{,}5\cdot(0{,}8+1{,}2)}+\dfrac{(34{,}06-33{,}81)\cdot 4}{6{,}48:(28{,}57-25{,}15)}-8.$

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e) $52 : \dfrac{6 : (0{,}4 - 0{,}2)}{2{,}5 \cdot (0{,}8 + 1{,}2)} + \dfrac{(34{,}06 - 33{,}81) \cdot 4}{6{,}48 : (28{,}57 - 25{,}15)} - 8.$

3.21. Ikki sonning o'rta arifmetigi 36,4. Bu sonlarning biri 36,8. Ikkinchi sonni toping.

3.22. Ikkita kema 3500 t yukni mo'ljalga yetkazishdi. Agar birinchi kema ikkinchisiga qaraganda 1,5 marta ortiq yukni mo'ljalga yetkazgan bo'lsa, har bir kema necha tonna yukni mo'ljalga yetkazgan?

3.23. Motorli qayiq oqim bo'yicha 14,5 km/soat tezlik bilan, oqimga qarshi esa 9,5 km/soat tezlik bilan harakat qiladi. Motorli qayiqning turg'un suvdagi tezligini va oqimning tezligini toping.

3.24. Kema oqim bo'yicha 4 soatda 85,6 km, oqimga qarshi 3 soatda 46,2 km yurdi. Kemaning turg'un suvdagi tezligini va oqimning tezligini toping.

3.25. Oralaridagi masofa 32,4 km bo'lgan ikkita aholi punktidan bir vaqtda bir-biriga qarab mototsiklchi va velosipedchi yo'lga chiqdi. Agar mototsiklchining tezligi velosipedchining tezligidan 4 marta ortiq bo'lsa, ular uchrashguncha qanchadan yo'l bosadi?

3.26. Ikkita kema oralaridagi masofa 501,9 km bo'lgan ikkita portdan bir-biriga qarab bir vaqtda yo'lga chiqishdi. Agar birinchi kemaning tezligi 25,5 km/soat, ikkinchisiniki esa 22,3 km/soat bo'lsa, ular necha soatdan keyin uchrashadi?

D a v r i y k a s r l a r

3.27. Oddiy kasr maxrajini tub ko'paytuvchilarga ajratish bilan uni o'nli kasrga aylantiring:

$$\frac{1}{2};\ \frac{1}{5};\ \frac{1}{4};\ \frac{3}{4};\ \frac{1}{8};\ \frac{5}{16};\ \frac{7}{25};\ \frac{23}{25};\ \frac{6}{125};\ 3\frac{9}{40};\ 11\frac{7}{80};$$

$$4\frac{3}{200};\ 4\frac{31}{500}.$$

3.28. Oddiy kasr suratini uning maxrajiga bo'lish yordamida oddiy kasrni o'nli kasrga aylantiring:

a) $\frac{9}{15};\ \frac{18}{252};\ \frac{11}{28};\ \frac{39}{65};\ \frac{30}{75};\ \frac{6}{48};\ 2\frac{3}{48};\ 5\frac{192}{575};\ 12\frac{177}{1500};$

b) $\frac{8}{5};\ \frac{25}{16};\ \frac{47}{32};\ \frac{263}{250};\ \frac{312}{125};\ 1\frac{711}{625};\ 5\frac{2541}{2000};$

$$4\frac{7359}{5000};\ 3\frac{23}{25000}.$$

3.29. Quyidagi sonlar berilgan:

$$\frac{1}{3};\ \frac{1}{4};\ \frac{1}{6};\ \frac{1}{12};\ \frac{3}{32};\ \frac{4}{21};\ \frac{5}{54};\ \frac{11}{90};\ 12\frac{7}{50};\ \frac{3}{6};\ \frac{15}{45};\ \frac{9}{27}.$$

a) Chekli o'nli kasrga aylanadigan sonlar to'plamini tuzing;

b) Cheksiz o'nli kasrga aylanadigan sonlar to'plamini tuzing.

3.30. Quyidagi sonlarni davriy o'nli kasr ko'rinishida yozing:

$$1;\ 1{,}4;\ \frac{7}{8};\ \frac{13}{26};\ \frac{81}{243};\ \frac{15}{43};\ \frac{71}{16};\ \frac{1}{25};\ \frac{15}{39};\ \frac{41}{43};\ 19.$$

3.31. Davriy o'nli kasrni oddiy kasrga aylantiring:

a) 0,(3) ; f) 13,0(48) ; j) 2,(123) ;
b) 0,3(2) ; g) 0,(4) ; k) 2,333(45) ;
d) 0,71(23) ; h) 0,(45) ; l) 41,8519(504) ;
e) 11,(75) ; i) 3,1(44) ; m) 35,73(4845) .

3.32. Ifodaning qiymatini toping:

a) $\dfrac{0{,}8333\ldots - 0{,}4(6)}{1\frac{5}{6}} \cdot \dfrac{1{,}125 + 1{,}75 - 0{,}41(6)}{0{,}59};$

b) $\dfrac{\left(\frac{5}{8} + 2{,}708333\ldots\right) : 2{,}5}{(1{,}3 + 0{,}7(6) + 0{,}(36)) \cdot \frac{110}{401}} \cdot \dfrac{1}{2};$

d) $\dfrac{\left(2\frac{38}{45} - \frac{1}{15}\right) : 13\frac{8}{9} + 3\frac{3}{65} \cdot 0{,}(26)}{(18{,}5 - 13{,}777\ldots) \cdot \frac{1}{85}} \cdot 0{,}5;$

e) $\dfrac{\frac{3}{4} + 0{,}8(5) \cdot \frac{1}{2}}{9 : (0{,}9(23) - 0{,}7(9))} + \dfrac{41}{43}.$

4-§. IRRATSIONAL SONLAR

Ta'rif. Cheksiz davriy bo'lmagan o'nli kasrlar irratsional sonlar deyiladi.

1 - m i s o l. 0,1010010001000010000001.... irratsional son ekanini isbotlang. (Birinchi birdan keyin bitta nol, ikkinchi birdan keyin ikkita nol va hokazo).

I s b o t. Berilgan kasr davriy va uning davri n ta raqamdan iborat deb faraz qilaylik (teskari faraz). 2n+1 inchi 1 ni tanlaymiz. Bu birdan keyin 2n+1 ta ketma-ket nollar keladi:

$$...\underbrace{100...0}_{\text{n ta}}\ \boxed{0}\ \underbrace{0...001}_{\text{n ta}}...$$

Shu o'rtada turgan 0 ni qaraymiz. Bu nol biror davrning yo boshida, yoki ichida, yoki oxirida keladi. Bu hollarning hammasida bu davr ajratilgan nollardan tuzilgan «kesma»da to'la joylashadi. Demak, davr faqat nollardan tuzilgan. Bunday bo'lishi esa sonning tuzilishiga zid. Faraz noto'g'ri.

2 - m i s o l. $\sqrt{2}$ soni irratsional son ekanini isbotlang.

I s b o t. $\sqrt{2}$ ratsional son deb faraz qilaylik. U holda uni qisqarmas oddiy kasr ko'rinishida yozish mumkin:

$$\sqrt{2}=\frac{m}{n} \qquad (*)$$

$$(*)\text{ dan } 2=\frac{m^2}{n^2}\text{ ni yoki } m^2=2n^2 \qquad (**)$$

ni olamiz. Bu yerdan m soni juft son ekanligi kelib chiqadi: $m=2k$, $k\in N$. Buni (**) ga qo'yamiz: $(2k)^2=2n^2$. Bundan $n^2=2k^2$ ni olamiz. Demak, n ham juft ekan. Bu esa, $\frac{m}{n}$ ning qisqarmas kasr ekanligiga zid. Farazimiz noto'g'ri. $\sqrt{2}$— irratsional son.

4.1. $\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}-2\sqrt{6}$ ning irratsional son emasligini isbotlang.

4.2. $\sqrt[3]{7+\sqrt{50}}+\sqrt[3]{7-5\sqrt{2}}$ irratsional sonmi?

4.3 $\frac{\sqrt{2}}{\sqrt{2\sqrt{2}+3}}-\frac{\sqrt{6-4\sqrt{2}}}{2\sqrt{2}-3}$ sonini irratsionallikka tekshiring.

4.4. 0,1234567891011121314... sonning irratsional son ekanligini isbotlang (verguldan keyin hamma natural sonlar ketma-ket yoziladi).

4.5. Sonlarning irratsional son ekanligini isbotlang:

a) $\sqrt{13}$; b) $\sqrt{17}$; d) $3\sqrt[3]{12}$; e) $\sqrt{2}+\sqrt{3}$.

4.6. a) a va b sonlar ratsional sonlar;

b) a va b sonlar irratsional sonlar;

d) a ratsional son, b irratsional son bo'lsa, $a+b$ sonning ratsional yoki irratsional ekanligi haqida nima deyish mumkin?

4.7. a) a va b sonlar ratsional sonlar;

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4.8. Kasr maxrajidagi irratsionallikni yo'qoting:

a) $\frac{\sqrt{2}}{\sqrt{3}+\sqrt{5}}$; d) $\frac{\sqrt{2}}{\sqrt[3]{3}-\sqrt[3]{2}}$; f) $\frac{2\sqrt{2}}{1+\sqrt{2}+\sqrt{3}}$;

b) $\frac{\sqrt{3}}{\sqrt{5}-\sqrt{2}}$; e) $\frac{\sqrt{3}}{\sqrt[3]{5}+\sqrt[3]{2}}$; g) $\frac{1}{\sqrt[3]{2}+\sqrt{3}}$;

h) $\frac{1}{\sqrt[3]{2}-\sqrt{3}}$; i) $\frac{2\sqrt{2}}{\sqrt[3]{5}-\sqrt[3]{2}}$.

4.9. Ifodani soddalashtiring:

a) $\sqrt{2+\sqrt{9+4\sqrt{2}}}$; b) $\sqrt{3-\sqrt{5-\sqrt{13+\sqrt{48}}}}$;

d) $\left(4\sqrt[3]{1}+2\sqrt{3}-\sqrt{13}+4\sqrt[6]{3}\right)\cdot\sqrt[3]{\frac{2\sqrt{3}-1}{11}}$.

4.10. Sonlarni taqqoslang:

a) $\sqrt{2}+\sqrt{3}$ va $\sqrt{11}$; d) $\sqrt{6}+2\sqrt{7}$ va $\sqrt{10}+\sqrt{21}$;

b) $\sqrt[3]{3}+\sqrt{7}$ va $2\sqrt{5}$; e) $\sqrt{11}$ va $5-\sqrt[3]{5}$.

4.11. Agar $z=\sqrt[3]{\sqrt{3}+\sqrt{2}}+\sqrt{\sqrt[3]{3}-\sqrt{2}}$,

$x=\sqrt[3]{\sqrt{5}+\sqrt{2}}+\sqrt[3]{\sqrt{5}-\sqrt{2}}$, bo'lsa, $\frac{z^3}{3}-z$ va x^3+x ifodalarning qiymatlarini taqqoslang.

4.12. $A=\sqrt[3]{38+\sqrt{1445}}+\sqrt[3]{38-3\sqrt{1445}}$,

$$B=\frac{\sqrt{7+4\sqrt{3}}\sqrt{19-8\sqrt{3}}}{4-\sqrt{3}}-\sqrt{3}$$

sonlarni taqqoslang.

4.13. a va b lar irratsional sonlar bo'lsin. $c\in(a;b)$ shartni qanoatlantiruvchi c irratsional son mavjudligini isbotlang.

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a) $\sqrt{2}+\sqrt{3}$ va $\sqrt{11}$; d) $\sqrt{6}+2\sqrt{7}$ va $\sqrt{10}+\sqrt{21}$;

b) $\sqrt[3]{3}+\sqrt{7}$ va $2\sqrt{5}$; e) $\sqrt{11}$ va $5-\sqrt[3]{5}$.

4.11. Agar $z=\sqrt[3]{\sqrt{3}+\sqrt{2}}+\sqrt{\sqrt[3]{3}-\sqrt{2}}$, $x=\sqrt[3]{\sqrt{5}+\sqrt{2}}+\sqrt[3]{\sqrt{5}-\sqrt{2}}$, bo'lsa, $\frac{z^3}{3}-z$ va x^3+x ifodalarning qiymatlarini taqqoslang.

4.12. $A=\sqrt[3]{38+\sqrt{1445}}+\sqrt[3]{38-3\sqrt{1445}}$,

$$B=\frac{\sqrt{7+4\sqrt{3}}\sqrt{19-8\sqrt{3}}}{4-\sqrt{3}}-\sqrt{3}$$

sonlarni taqqoslang.

4.13. a va b lar irratsional sonlar bo'lsin. $c\in(a;b)$ shartni qanoatlantiruvchi c irratsional son mavjudligini isbotlang.

4.14. a) Agar p, q — butun sonlar uchun $p+q\sqrt{3}=0$ bo'lsa, $p=q=0$ bo'lishini isbotlang;

b) agar p, q — butun sonlari uchun $p^2-9q^2=6q$ bo'lsa, $p=q=0$ bo'lishini isbotlang;

d) agar p, q — butun sonlari uchun $p^2-4q^2=4pq$ bo'lsa, $p=q=0$ bo'lishini isbotlang;

e) a, b, c ratsional sonlar uchun $a+b\sqrt[3]{2}+c\sqrt[3]{4}=0$ bo'lsa, $a=b=c=0$ bo'lishini isbotlang.

4.15. α, β lar irratsional sonlar, r esa ratsional son bo'lsin. Quyidagi sonlarning qaysilari ratsional son bo'lib qolishi mumkin:

a) $\alpha+\beta$; b) $\alpha+r$; d) $\sqrt{\alpha}$; e) $\sqrt{r}$; f) $\alpha\cdot\beta$;

g) $\sqrt{\alpha+r}$; h) $\sqrt{\alpha}+\sqrt{r}$?

5-§. HAQIQIY SONLAR

Sonning moduli

Haqiqiy son a ning moduli $|a|$ bilan belgilanadi va quyidagicha aniqlanadi:

$$|a|=\begin{cases} a, & \text{agar } a\geq 0 \text{ bo'lsa,}\\ -a, & \text{agar } a<0 \text{ bo'lsa,}\end{cases}$$

5.1. Haqiqiy son a ning moduli nomanfiy son ekanini isbotlang.

5.2. Taqqoslang:

a) $|8,7|$ va 8;
b) $|0|$ va 0;
d) $|-15,2|$ va $15,2$;
e) $|-6\frac{3}{4}|$ va $-6\frac{3}{4}$;
f) $-|-3,2|$ va $-3,2$;
g) $|a|$ va 0;
h) -5 $|a|$ va 0;
i) $|a|$ va a.

5.3. Harflarning ko'rsatilgan qiymatlarida ifodaning qiymatini hisoblang:

a) $|a|+2|b|$ $a=-3$, $b=5$;
b) $|-a|-2|b|$ $a=-1$, $b=-2$;
d) $\frac{-1-|-3a|+4|b|}{2|a|+|b|}$ $a=-4$, $b=0$;
e) $\frac{4-|a|+2|b+1|}{|-a|\cdot|b+3|\cdot|b+1|}$ $a=2$, $b=-4$;
f) $(-|-a|)^3+2|-b|^3$ $a=1$, $b=2$.

5.4. Agar a) $|a|=b$, b) $|a|=-b$ bo'lsa, b soni haqida nima deyish mumkin?

5.5. Agar a) $|a|=|b|$, b) $|a|=a$ d) $|b|=-b$ bo'lsa, a va b sonlar haqida nima deyish mumkin?

5.6. Modulning quyidagi xossalarini isbotlang:

a) $a\leq|a|$;
b) $-a\leq|a|$;
d) $|-a|=|a|$;
e) $-|a|\leq a\leq|a|$;
f) $|a+b|\leq|a|+|b|$;
g) $|a-b|\leq|a|+|b|$;
h) $|a+b|\geq|a|-|b|$;
i) $|a-b|\geq||a|-|b||$.

5.7. Tenglikni isbotlang:

a) $|a\cdot b|=|a|\cdot|b|$;
b) $\frac{|a|}{|b|}=\frac{|a|}{|b|}$ $(b\neq 0)$;
d) $|a^2|=|a|^2=a^2$;
e) $|a^{2n}|=|a|^{2n}=a^{2n}$ $n\in N$.

5.8. Ifodani modul belgisisiz yozing:

a) $|x-2|$;
b) $|x+2|$;
d) $|-x+3|$;
e) $|-x-4|$;
f) $|3x+7|$;
g) $|-3x+7|$;
h) $|-3x-9|$;
i) $|4x|$;
j) $a+|a|$;
k) $2x+|a-1|$;
l) $3|xy|+a$;
m) $2|x-y|+y$.

5.9. Ifodani modul belgisisiz yozing:

a) $|x+1|+|x-1|$;
b) $|x-1|-2|x+2|$;
d) $|2x-1|-|x-2|$;
e) $|3x-7|+|4x-5|$;
f) $|4x-8|+|x-2|+|x|$;
g) $|7x-5|+|2x-1|+|x-2|$;
h) $|7x+5|-|3x-2|+|x-3|$;
i) $|3x-6|+|8x-4|-|13x-20|$.

5.10.* Ifodani modul belgisisiz yozing:

a) $||x|-2|$;
b) $||x-3|-x|$;
d) $|x-3-|x||$;
e) $||x-3|-|x||$;
f) $||6x-1|-|4x+1||$;
g) $||x-3|-|x||-|x-1|$;
h) $|x^2-|x|^2+|x|-|x-3||$;
i) $||3x+1|-|x||-|x-2|$.

5.11. a, b, c, d haqiqiy sonlar bir vaqtda nolga teng emasligini modul belgisidan foydalanib qanday yozish mumkin?

5.12. a, b, c sonlaridan kamida ikkitasi o'zaro teng emasligini modul belgisi yordamida qanday yozish mumkin?

5.13. a, b, c lar o'zaro teng ekanini modul qatnashgan tengsizlik bilan ifodalang.

5.14. $A(a)$ va $B(b)$ nuqtalar orasidagi masofa $|a-b|$ ga teng ekanini isbotlang.

5.15. Tengsizliklarni yeching:

a) $|x-2|<3$; b) $|x+2|<3$; d) $|3x-1|<4$; e) $|4x+3|\leq 3$.

Sonning butun qismi va kasr qismi. a ning *butun qismi* deb, a dan katta bo'lmagan eng katta butun songa aytiladi. a ning butun qismi $[a]$ bilan belgilanadi.

$\{a\}=a-[a]$ sonni a ning *kasr qismi* deyiladi. $\{a\}\in[0;1)$ munosabat o'rinli.

1 - m i s o l. $\left[\frac{4-x}{5}\right]=6$ tenglamani yeching.

Y e c h i s h. Sonning butun qismi ta'rifiga ko'ra $6 \le \frac{4-x}{5} < 7$ bo'lishi lozim. Bundan $30 \le 4-x < 35$ yoki $-31 < x \le -26$ ni olamiz. Hosil qilingan tengsizliklardan ixtiyoriy bittasi yordamida qolganlarini hosil qilish mumkin bo'lganligi sababli, $-31 < x \le -26$ shartni qanoatlantiruvchi barcha $x\in R$ lar berilgan tenglamaning yechimi bo'la oladi.

J a v o b: $(-31;-26]$.

2 - m i s o l. $\left[\frac{2x+1}{2}\right]=3x$ tenglamani yeching.

Y e c h i s h. x^* son $\left[\frac{2x^*+1}{2}\right]=3x^*$ tenglamaning yechimi bo'lsin.

U holda, $3x^*$ soni butun son bo'ladi. Sonning butun qismi ta'rifiga ko'ra, $3x^* \le \frac{2x^*+1}{2} < 3x^*+1$ tengsizlikka yoki $-\frac{3}{4} < 3x^* \le -\frac{3}{4}$ tengsizlikka ega bo'lamiz. Oxirgi tengsizlikni qanoatlantiruvchi butun son 0 sonidir, ya'ni $3x^*=0$. Demak, tenglama $x^*=0$ dan iborat yagona yechimga ega.

5.16. Sonning butun qismini toping:

a) $[2,8]$; d) $[0]$; f) $[-1,5]$; h) $[\pi]$; j) $[\sqrt{15}]$;

b) $[2]$; e) $[0,9]$; g) $[-0,2]$; i) $[-\pi]$; k) $\left[\frac{100}{7}\right]$.

5.17. Hisoblang:

a) $100\cdot\left[\frac{1}{7}\right]$; e) $\left[12\frac{2}{7}\right]+5\frac{6}{7}$; h) $\left[\frac{100}{7^2}\right]\cdot 7$;

b) $\left[12\frac{2}{7}+5\frac{3}{7}\right]$; f) $8\cdot\left[3\frac{2}{3}\right]$; i) $\left[\frac{490}{100}\right]^2$;

d) $\left[12\frac{2}{7}\right]+\left[5\frac{6}{7}\right]$; g) $\left[\frac{100}{7}\right]\cdot 7$.

5.18. Tenglamani yeching:

a) $\left[\frac{3x-1}{4}\right]=5$; d) $[2x+4]=-5$;

b) $\left[\frac{3x}{4}-1\right]=15$; e) $[3x-1]=-4$.

5.19. Tenglamani yeching:

a) $\left[\frac{x-1}{2}\right]=x$; d) $\left[\frac{2x-1}{3}\right]=2x$;

b) $\left[\frac{3x+1}{2}\right]=-x$; e) $[3x+1]=\frac{x}{4}$.

5.20. Agar n nomanfiy butun son bo'lsa, $[nx] \ge n[x]$ bo'lishini isbotlang.

N i s b a t. P r o p o r s i y a. F o i z.

5.21. Quyidagi nisbatlardan proporsiya tuzish mumkinmi:

a) 42:14 va 72:24; d) 3,5:21 va $2\frac{1}{4}:13\frac{1}{2}$;

b) 78:13 va 60:12; e) 0,1:0,02 va 4:0,8 ?

5.22. Proporsiyaning noma'lum hadini toping:

a) $x:12=4\frac{3}{4}:7\frac{1}{8}$; f) $3\frac{1}{2}:0,4=x:1\frac{1}{7}$;

b) $x:1\frac{3}{7}=1\frac{3}{15}:1\frac{1}{3}$; g) $10,4:3\frac{5}{7}=x:\frac{5}{11}$;

d) $6\frac{1}{2}:x=6\frac{5}{6}:4,1$; h) $15,6:2,88=2,6:x$;

e) $0,38:x=4\frac{3}{4}:1\frac{7}{8}$; i) $1,25:1,4=0,75:x$.

5.23. Proporsiyadan x ni toping:

a) $7x:42=45:27$; h) $4x:31=44:11$;

b) $84:6x=28:14$; i) $85:17x=105:84$;

d) $21:7=2\frac{1}{2}x:5$; j) $\frac{1}{6}:2\frac{1}{3}=3\frac{1}{4}x:1,3$;

e) $13\frac{1}{3}:1\frac{1}{3}=26:0,2x$; k) $3,3:7\frac{1}{3}=4\frac{2}{7}:1\frac{3}{7}x$;

f) $3\frac{1}{3}x:1,5=4\frac{2}{7}:\frac{3}{14}$; l) $3\frac{7}{19}:1\frac{1}{2}=2\frac{3}{8}:0,8x$;

g) $11\frac{1}{3}:1\frac{8}{9}=5\frac{1}{3}x:\frac{5}{8}$; m) $6\frac{2}{3}:1\frac{7}{9}x=0,48:1,2$.

5.24. Quyidagi tengliklar yordamida proporsiyalar tuzing:

a) $15 \cdot 42 = 35 \cdot 18$; d) $2{,}5 \cdot 0{,}018 = 0{,}15 \cdot 0{,}3$;

b) $54 \cdot 55 = 66 \cdot 45$; e) $2\frac{1}{2} \cdot 1\frac{2}{7} = \frac{5}{7} \cdot 4\frac{1}{2}$.

5.25. Proporsiyadan x ni toping:

a) $\dfrac{\left(4 - 3{,}5\left(2\frac{1}{7} - 1\frac{1}{5}\right)\right) : 0{,}16}{x} = \dfrac{3\frac{2}{7} - \frac{3}{14} : \frac{1}{6}}{41\frac{23}{84} - 40\frac{49}{60}}$;

b) $\dfrac{1{,}2 : 0{,}3775 - 0{,}2}{6\frac{4}{25} : 15\frac{2}{5} + 0{,}8} = \dfrac{0{,}16 : 0{,}12 + 0{,}7}{x}$;

d) $\dfrac{0{,}125x}{\left(\frac{19}{24} - \frac{21}{40}\right) \cdot 8\frac{7}{16}} = \dfrac{\left(1\frac{28}{63} - \frac{17}{21}\right) \cdot 0{,}7}{0{,}675 \cdot 2{,}4 - 0{,}02}$;

e) $\dfrac{x}{10{,}5 \cdot 0{,}24 - 15{,}15 : 7{,}5} = \dfrac{9 \cdot \left(1\frac{11}{20} - 0{,}945 : 0{,}9\right)}{1\frac{3}{40} - 4\frac{3}{8} : 7}$.

5.26. Kasr ko‘rinishida ifodalang:

a) 7% ; f) 6,8% ; j) $1\frac{1}{4}\%$;

b) 0,75% ; g) 0,48% ; k) $4\frac{3}{7}\%$;

d) 255% ; h) 29% ; l) $225\frac{3}{4}\%$;

e) 300% ; i) $4\frac{3}{7}\%$; m) 0,099% .

5.27. Foizlarda ifodalang:

a) 0,5 ; f) $4\frac{3}{7}$; j) 15,2 ;

b) 2,15 ; g) $14\frac{1}{5}$; k) $4\frac{17}{43}$;

d) 1,75 ; h) 43 ; l) $8\frac{5}{9}$;

e) 3 ; i) 5,7 ; m) 0,79 .

5.28. a) 1 ning 4 ga ; f) 3,2 ning 1,28 ga ;

b) 3 ning 5 ga ; g) 15 ning 18 ga ;

d) 5 ning 2 ga ; h) 0,43 ning 5 ga ;

e) 12,5 ning 50 ga ; i) $\frac{1}{7}$ ning $\frac{3}{8}$ ga

protsent nisbatini toping.

5.29. a ning p % ini toping:

a) $a = 75$ $p = 4$; d) $a = 330$ $p = 18\frac{1}{3}$;

b) $a = 84$ $p = 15$; e) $a = 82{,}25$; $p = 160$.

5.30. p % i a ga teng bo‘lgan sonni toping:

a) $p = 1{,}25$ $a = 55$; d) $p = 0{,}8$ $a = 1{,}84$;

b) $p = 40$ $a = 12$; e) $p = 15$ $a = 1{,}35$.

5.31. Pol sirtining 72 % ini bo‘yash uchun 4,5 kg bo‘yoq ketdi. Polning qolgan qismini bo‘yash uchun qancha bo‘yoq kerak bo‘ladi?

5.32. To‘g‘ri to‘rtburchakning eni 20 % uzaytirildi, bo‘yi esa 20 % qisqartirildi. Uning yuzasi o‘zgaradimi? Agar o‘zgarsa, qanday o‘zgaradi?

5.33. Ishchi ish kunida 360 ta detal tayyorladi va kunlik rejani 150 % ga bajardi. Ishchi reja bo‘yicha bir kunda nechta detal tayyorlashi kerak edi?

5.34. Meva quritilganda o‘z og‘irligining 82 % ini yo‘qotadi. 36 kg quritilgan meva olish uchun necha kg ho‘l meva olish kerak?

5.35. 10 % ga arzonlashtirilgan tovar 18 so‘mga sotildi. Tovarning dastlabki narxini toping.

5.36. Zavod bir oyda 3360 ta mashina ishlab chiqib, rejani 140 % ga bajardi. Zavod rejaga nisbatan nechta ortiq mashina ishlab chiqargan?

5.37. To‘g‘ri to‘rtburchak va kvadrat teng perimetrga ega. To‘g‘ri to‘rtburchakning uzunligi 120 sm, eni esa bo‘yining 35 % iga teng. Kvadratning tomonini toping.

5.38. To‘g‘ri to‘rtburchakning eni 180 mm bo‘lib, bo‘yining $\frac{3}{4}$ qismini tashkil etadi. Uchburchakning tomoni to‘g‘ri to‘rtburchak bo‘yining 20 % iga teng, yuzi esa to‘g‘ri to‘rtburchak yuzining $\frac{2}{5}$ qismiga teng. Uchburchakning shu tomonga mos balandligini toping.

5.39. Shaxmat turnirida 16 o‘yinchi ishtirok etdi va har bir o‘yinchilar juftligi faqat bir partiya shaxmat o‘ynadi. O‘ynalgan partiyalarning 40 % ida durang qayd etildi. Nechta partiyada g‘alaba qayd etilgan?

5.40. Mahsulotlar narxi p % ga arzonlashtirilsa, aholining sotib olish quvvati necha % ortadi?

5.41. Uzunligi 19,8 m bo‘lgan arqon ikki bo‘lakka bo‘lindi. Bo‘laklardan birining uzunligi ikkinchisinikidan 20 % ortiq bo‘lsa, har bir bo‘lakning uzunligini toping.

5.42. Uzunligi 19,8 m bo'lgan arqon ikki bo'lakka bo'lindi. Bo'laklardan birining uzunligi ikkinchisinikidan 20 % kam bo'lsa, bo'laklarning uzunligini toping.

5.43. Tomonlari 9 sm va 7 sm bo'lgan to'g'ri to'rtburchakning tomonlari 10 % ga orttirilsa, to'g'ri to'rtburchakning yuzi necha foizga ortadi?

5.44. To'g'ri to'rtburchakning tomonlari 10 % ga orttirilsa, uning yuzi necha foizga ortadi?

5.45. To'g'ri to'rtburchakning tomonlari 10% ga kamaytirilsa, uning yuzi necha foiz kamayadi?

5.46. To'g'ri to'rtburchakning katta tomoni 10 % ga kamaytirilib, kichik tomoni 10 % ga orttirilsa, to'g'ri to'rtburchakning yuzi qanday o'zgaradi?

1 - m i s o l. O'zgaruvchi miqdorining boshlang'ich vaqt momenti $t_0=0$ dagi qiymati A_0 ga teng. Agar A miqdorning qiymati t vaqt oralig'ida p % ga ortib turishi ma'lum bo'lsa, A ning nt vaqt momentidagi qiymatini toping (bu yerda $n \geq 0$).

Y e ch i sh. A ning nt vaqt momentidagi qiymatini A_n bilan belgilaylik. Boshlang'ich vaqt momenti $t_0=0$ da A ning qiymati A_0 ga tengligidan foydalanib, vaqtning $t_1=1\cdot t$ momentida A ning qiymati quyidagiga tengligini topamiz:

$$A_1 = A_0 + \frac{A}{100}\cdot p = A_0\left(1+\frac{p}{100}\right).$$

U holda A ning $t_2=2t$ vaqt momentidagi qiymati

$$A_4 = A_1 + \frac{A}{100}\cdot p = A_0\left(1+\frac{p}{100}\right)^2$$

ga teng bo'ladi.

Shu tarzda davom etib, A ning $t_n=n\cdot t$ vaqt momentidagi qiymati

$$A_n = A_0\left(1+\frac{p}{100}\right)^n \qquad (*)$$

ga teng bo'lishligini topamiz.

(*) formula *murakkab foiz formulasi* deyiladi.

2 - m i s o l. Omonatchi bankga 20000 so'm pul qo'ydi. Oradan to'rt yil o'tgach, u o'ziga tegishli bo'lgan hamma pulni qaytarib oldi. Agar bank yiliga 3 % foyda to'lasa, omonatchi bankdan necha so'm pul olgan va qancha foyda ko'rgan?

Ye ch i sh. Bu misolda A o'zgaruvchining qiymatlari pul miqdoridir. t vaqt oralig'i 1 yilga teng. p esa 3 ga teng.

A ning boshlang'ich vaqt momentidagi qiymati $A_0=20000$ co'mga teng. Bizdan A ning $4t$ vaqt momentidagi qiymati A_4 ni va A_4-A_0 ni (foydani) topish talab qilinmoqda.

Murakkab foiz formulasiga ko'ra,

$$A_4 = A_0\left(1+\frac{p}{100}\right)^4 = 20000\cdot\left(1+\frac{3}{100}\right)^4 = \frac{2\cdot 103^4}{10000} \approx 22510.$$

$$A_4 - A_0 = 22510 - 20000 = 2510.$$

J a v o b: Omonatchi 22510 so'm pul olgan. Foyda 2510 so'm.

5.47. Xalq banki yiliga 20 % foyda to'laydi. Omonatchi kassaga 15000 so'm qo'ydi. Ikki yildan keyin uning kassadagi puli necha so'm bo'ladi?

5.48. Xalq banki yiliga 30% foyda to'laydi. Omonatga qo'yilgan pul necha yildan keyin 1,69 marta ko'payadi?

5.49. Ma'lum bir ishni ikkita zavod birgalikda 12 kunda bajaradi. Ular ikki kun birga ishlagach, birinchi zavod ishlamay qo'ydi. Agar ikkinchi zavodning ish unumdorligi birinchi zavod ish unumdorligining $66\frac{2}{3}$ % ini tashkil qilsa, ikkinchi zavod ishni necha kundan keyin tugatadi?

5.50. Sayyoh mehmonxonadan vokzalga qarab yo'lga chiqib, birinchi soatda 3 km yo'l bosdi. Shu tezlikda yursa poezdga 40 minut kechikib qolishini tushunib yetgach, o'z tezligini $33\frac{1}{3}$ % ga orttirdi. Natijada u vokzalga poezd jo'nashidan 45 minut oldin yetib keldi. Mehmonxonadan vokzalgacha bo'lgan yo'lni (masofani) va sayyoh shu yo'lni necha soatda bosib o'tganini aniqlang.

5.51. 16 bilan noma'lum sonning ayirmasi ular yig'indisining 60 % iga teng. Noma'lum sonni toping.

Biror sonni berilgan sonlarga proporsional bo'lgan bo'laklarga bo'lish uchun berilgan sonni shu sonlar yig'indisiga bo'lish, natijani esa berilgan sonlarning har biriga ko'paytirish kerak.

3 - m i s o l. 24 ni 3:4:5 nisbatda bo'ling.

Ye ch i s h. $\frac{24}{3+4+5}=2$, $2\cdot 3=6$, $2\cdot 4=8$, $2\cdot 5=10$.

J a v o b: 6; 8; 10.

Biror sonni berilgan sonlarga teskari proporsional bo'lgan bo'laklarga bo'lish uchun, shu sonni berilgan sonlarga teskari sonlarga to'g'ri proporsional bo'lgan bo'laklarga bo'lish yetarli.

5.42. Uzunligi 19,8 m bo'lgan arqon ikki bo'lakka bo'lindi. Bo'laklardan birining uzunligi ikkinchisinikidan 20 % kam bo'lsa, bo'laklarning uzunligini toping.

5.43. Tomonlari 9 sm va 7 sm bo'lgan to'g'ri to'rtburchakning tomonlari 10 % ga orttirilsa, to'g'ri to'rtburchakning yuzi necha foizga ortadi?

5.44. To'g'ri to'rtburchakning tomonlari 10 % ga orttirilsa, uning yuzi necha foizga ortadi?

5.45. To'g'ri to'rtburchakning tomonlari 10% ga kamaytirilsa, uning yuzi necha foiz kamayadi?

5.46. To'g'ri to'rtburchakning katta tomoni 10 % ga kamaytirilib, kichik tomoni 10 % ga orttirilsa, to'g'ri to'rtburchakning yuzi qanday o'zgaradi?

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U holda A ning $t_2=2t$ vaqt momentidagi qiymati

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ga teng bo'ladi.

Shu tarzda davom etib, A ning $t_n=n\cdot t$ vaqt momentidagi qiymati

$$A_n = A_0\left(1 + \frac{p}{100}\right)^n \qquad (*)$$

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(*) formula *murakkab foiz formulasi* deyiladi.

2 - m i s o l. Omonatchi bankga 20000 so'm pul qo'ydi. Oradan to'rt yil o'tgach, u o'ziga tegishli bo'lgan hamma pulni qaytarib oldi. Agar bank yiliga 3 % foyda to'lasa, omonatchi bankdan necha so'm pul olgan va qancha foyda ko'rgan?

Ye ch i sh. Bu misolda A o'zgaruvchining qiymatlari pul miqdoridir. t vaqt oralig'i 1 yilga teng. p esa 3 ga teng.

A ning boshlang'ich vaqt momentidagi qiymati $A_0=20000$ co'mga teng. Bizdan A ning $4t$ vaqt momentidagi qiymati A_4 ni va A_4-A_0 ni (foydani) topish talab qilinmoqda.

Murakkab foiz formulasiga ko'ra,

$$A_4 = A_0\left(1 + \frac{p}{100}\right)^4 = 20000\cdot\left(1 + \frac{3}{100}\right)^4 = \frac{2\cdot 103^4}{10000} \approx 22510.$$

$$A_4 - A_0 = 22510 - 20000 = 2510.$$

J a v o b: Omonatchi 22510 so'm pul olgan. Foyda 2510 so'm.

5.47. Xalq banki yiliga 20 % foyda to'laydi. Omonatchi kassaga 15000 so'm qo'ydi. Ikki yildan keyin uning kassadagi puli necha so'm bo'ladi?

5.48. Xalq banki yiliga 30% foyda to'laydi. Omonatga qo'yilgan pul necha yildan keyin 1,69 marta ko'payadi?

5.49. Ma'lum bir ishni ikkita zavod birgalikda 12 kunda bajaradi. Ular ikki kun birga ishlagach, birinchi zavod ishlamay qo'ydi. Agar ikkinchi zavodning ish unumdorligi birinchi zavod ish unumdorligining $66\frac{2}{3}$ % ini tashkil qilsa, ikkinchi zavod ishni necha kundan keyin tugatadi?

5.50. Sayyoh mehmonxonadan vokzalga qarab yo'lga chiqib, birinchi soatda 3 km yo'l bosdi. Shu tezlikda yursa poezdga 40 minut kechikib qolishini tushunib yetgach, o'z tezligini $33\frac{1}{3}$ % ga orttirdi. Natijada u vokzalga poezd jo'nashidan 45 minut oldin yetib keldi. Mehmonxonadan vokzalgacha bo'lgan yo'lni (masofani) va sayyoh shu yo'lni necha soatda bosib o'tganini aniqlang.

5.51. 16 bilan noma'lum sonning ayirmasi ular yig'indisining 60 % iga teng. Noma'lum sonni toping.

Biror sonni berilgan sonlarga proporsional bo'lgan bo'laklarga bo'lish uchun berilgan sonni shu sonlar yig'indisiga bo'lish, natijani esa berilgan sonlarning har biriga ko'paytirish kerak.

3 - m i s o l. 24 ni 3:4:5 nisbatda bo'ling.

Y e ch i s h. $\frac{24}{3+4+5} = 2$, $2\cdot 3=6$, $2\cdot 4=8$, $2\cdot 5=10$.

J a v o b: 6; 8; 10.

Biror sonni berilgan sonlarga teskari proporsional bo'lgan bo'laklarga bo'lish uchun, shu sonni berilgan sonlarga teskari sonlarga to'g'ri proporsional bo'lgan bo'laklarga bo'lish yetarli.

4 - m i s o l. 24 ni 3 va 4 sonlariga teskari proporsional bo'laklarga bo'ling.

Ye ch i sh. 3 va 4 ga teskari sonlar: $\frac{1}{3}$ va $\frac{1}{4}$. 24 ni $\frac{1}{3}:\frac{1}{4}$ nisbatda bo'lamiz.

$\frac{1}{3}+\frac{1}{4}=\frac{7}{12}$ va $\frac{\frac{24}{7}}{12}=\frac{24\cdot 12}{7}$ bo'lgani uchun quyidagilarga ega bo'lamiz: $\frac{1}{3}\cdot\frac{24\cdot 12}{7}=\frac{96}{7}=13\frac{5}{7}$,

$\frac{1}{4}\cdot\frac{24\cdot 12}{7}=\frac{72}{7}=10\frac{2}{7}$.

J a v o b: $13\frac{5}{7}$ va $10\frac{2}{7}$.

5.52. 150 soni 2:3:5 nisbatda bo'ling. Eng katta bo'lakning eng kichik bo'lakka nisbatining 10 % i nimaga teng?

5.53. 1800 sonini 2:3:5 sonlarga teskari proporsional nisbatda bo'ling.

5.54. 1554 soni 1:2 va 7:2 nisbatlarda bo'ling. Hosil bo'lgan barcha bo'laklar yig'indisi nimaga teng? Shu javobni og'zaki topish mumkinmi?

III b o b. KOMPLEKS SONLAR

1-§. ALGEBRAIK SHAKLDAGI KOMPLEKS SONLAR VA ULAR USTIDA AMALLAR

$$z = a + bi \qquad (1)$$

ko'rinishidagi son kompleks son deyiladi, bu yerda a, $b \in R$, i esa $i^2=-1$ tenglik bilan aniqlanadigan *mavhum birlikdir*. a soni z kompleks sonning *haqiqiy* qismi, b esa z kompleks sonining *mavhum* qismi deb ataladi va mos ravishda $a=\mathrm{Re}(z)$, $b=\mathrm{Im}(z)$ ko'rinishda belgilanadi. Kompleks sonning (1) ko'rinishdagi yozuvi uning algebraik shakli deyiladi.

Agar ikki $z_1=a_1+b_1i$ va $z_2=a_2+b_2i$ kompleks sonning haqiqiy va mavhum qismlari mos ravishda teng, ya'ni $a_1=a_2$, $b_1=b_2$ bo'lsa, ular teng deyiladi.

Mavhum qismlarining ishorasi bilangina bir-biridan farq qiladigan $z_1=a+bi$ va $z_2=a-bi$ kompleks sonlar *qo'shma kompleks sonlar* deyiladi.

z kompleks songa *qo'shma kompleks* son $\bar{z}$ bilan belgilanadi.

Algebraik shaklda berilgan kompleks sonlar ustida amallar quyidagi qoidalar bo'yicha bajariladi:

$$(a_1 + b_1i) \pm (a_2 + b_2i) = (a_1 \pm a_2) + (b_1 \pm b_2)i; \qquad (2)$$

$$(a_1 + b_1i)\cdot(a_2 + b_2i) = (a_1a_2 - b_1b_2) + (a_1b_2 + a_2b_1)i; \qquad (3)$$

$$\frac{a_1 + b_1i}{a_2 + b_2i} = \frac{(a_1 + b_1i)\cdot(a_2 - b_2i)}{(a_2 + b_2i)\cdot(a_2 - b_2i)} = \frac{a_1a_2 + b_1b_2}{a_2^2 + b_2^2} + \frac{a_2b_1 - a_1b_2}{a_2^2 + b_2^2}\,i \qquad (4)$$

1 - m i s o l. Kompleks sonlarning haqiqiy va mavhum qismlarini toping:

a) $z_1=3+0{,}5i$; b) $z_2=2-4i$; d) $z_3=-9i$; e) $z_4=8$.

Javob: a) $\mathrm{Re}(z_1)=3$, $\mathrm{Im}(z_1)=0{,}5$;
b) $\mathrm{Re}(z_2)=2$, $\mathrm{Im}(z_2)=-4$;
d) $\mathrm{Re}(z_3)=0$, $\mathrm{Im}(z_3)=-9$;
e) $\mathrm{Re}(z_4)=8$, $\mathrm{Im}(z_4)=0$.

2 - m i s o l. Quyidagi kompleks sonlar o'zaro tengmi:

a) $z_1=\frac{1}{3}+\sqrt{9}\,i$ ва $z_2=-\frac{1}{3}+3i$;

b) $z_1=\frac{1}{4}-i\sqrt[4]{81}$ ва $z_2=0{,}25-3\,i$?

Ye c h i s h. a) $\mathrm{Re}(z_1)=\frac{1}{3}$ va $\mathrm{Re}(z_2)=-\frac{1}{3}$ larga egamiz. $\mathrm{Re}(z_1)\neq\mathrm{Re}(z_2)$ bo'lgani uchun $z_1\neq z_2$;

b) $\mathrm{Re}(z_1)=\frac{1}{4}=0{,}25=\mathrm{Re}(z_2)$ ва $\mathrm{Im}(z_1)=-\sqrt[3]{81}=-3=\mathrm{Im}(z_2)$ bo'lgani uchun $z_1=z_2$ bo'ladi.

J a v o b: à) teng emas; b) teng.

3 - m i s o l. $z_1=3-2i$ va $z_2=1+3i$ kompleks sonlarning

a) yig'indisini;
b) ayirmasini;
d) ko'paytmasini;
e) bo'linmasini toping.

Ye c h i s h. a) $z_1+z_2=(3-2i)+(1+3i)=(3+1)+(-2+3)i=4+i$;

b) $z_1-z_2=(3-2i)-(1+3i)=(3-1)+(-2-3)i=2-5i$;

d) $z_1\cdot z_2$ ni topishda (3) formuladan foydalanish zarur, ammo (3) formulani yodda saqlashda biroz qiyinchilik tug'ilishi mumkin. Shu sababli, $z_1\cdot z_2$ ni topishda $i^2=-1$ ekanini e'tiborga olib, ko'phadlarni ko'paytirish qoidasidan foydalanish mumkin.

$$z_1\cdot z_2=(3-2i)(1+3i)=3\cdot 1+3\cdot(3i)-2i\cdot 1-2i\cdot(3i)=$$
$$=3+9i-2i-6i^2=3+7i-6\cdot(-1)=9+7i;$$

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$$\frac{a_1+b_1i}{a_2+b_2i}=\frac{(a_1+b_1i)\cdot(a_2-b_2i)}{(a_2+b_2i)\cdot(a_2-b_2i)}=\frac{a_1a_2+b_1b_2}{a_2^2+b_2^2}+\frac{a_2b_1-a_1b_2}{a_2^2+b_2^2;}\,i \qquad (4)$$

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$$z_1\cdot z_2=(3-2i)(1+3i)=3\cdot 1+3\cdot(3i)-2i\cdot 1-2i\cdot(3i)=$$
$$=3+9i-2i-6i^2=3+7i-6\cdot(-1)=9+7i;$$

e) $\frac{z_1}{z_2}$ ni topishda (4) formuladan foydalanamiz:

$$\frac{z_1}{z_2}=\frac{3-2i}{1+3i}=\frac{(3-2i)(1-3i)}{(1+3i)(1-3i)}=\frac{3-9i-2i-6}{1^2+3^2}=\frac{-3-11i}{10}=-0{,}3-1{,}1i$$

1.1. Kompleks son z ning haqiqiy qismi Re(z) ni va mavhum qismi Im(z) ni toping:

a) $z=-5+8i$; f) $z=0{,}5+3i$; j) $8i$;

b) $z=6+\frac{1}{2}i$; g) $z=2+0{,}3i$; k) 4 ;

d) $z=-15+2i$; h) $z=-4{,}1+2i$; l) 0 ;

e) $z=\frac{1}{2}+\frac{3}{2}i$; i) $z=-3-4i$; m) $-3i$.

1.2. Agar:

a) Re(z)= –4, Im(z)=8 ;
b) Re(z)=0, Im(z)=1,2 ;
d) Re(z)=1,2 , Im(z)=0 ;
e) Re(z)=0, Im(z)=0 .

bo'lsa, z kompleks sonini algebraik shaklda yozing.

1.3. Teng kompleks sonlarni toping:

a) $\frac{1}{2}+\frac{1}{3}i$; b) $0{,}5+3i$; d) $\frac{1}{4}+\frac{2}{6}i$; e) $\sqrt{9-4i}$;

f) $\sqrt{9}-\sqrt{81i}$; g) $3-4i$.

1.4. a) Kompleks sonlardan qaysilari teng:

a) $3i$; б) $-4+5i$; b) $\frac{1}{3}+i$; d) $-\frac{1}{4}-8i$; e) $0{,}(3)+i$;

f) $-\frac{2}{8}-\sqrt{64i}$; g) $\sqrt[4]{81i}$?

b) $(4x-3y)+(3x+5y)i=10-(3x-2y-30)i$ bo'lsa, x va y larni toping.

1.5. Agar:

a) $z=-3+5i$; f) $z=-3i$; j) $z=\frac{1}{3}+3{,}4i$;

b) $z=3-5i$; g) $z=4{,}2$; k) $z=0$;

d) $z=-3-5i$; h) $z=4i$; l) $z=\sqrt{81}+4i$;

e) $z=3+5i$; i) $z=4{,}(3)$; m) $z=-0{,}(3)-2{,}(3)i$

bo'lsa, z ni toping.

1.6. Yig'indini toping:

a) $(-3+2i)+(4-i)$; f) $(1{,}4-3i)+(2{,}6-4i)$; j) $8i+(4-6i)$;

b) $(4+5i)+(4-5i)$; g) $(3+8i)+(3-8i)$; k) $-15i+(-4+5i)$;

d) $(5+2i)+(-5-2i)$; h) $(-7+3i)+(7-3i)$; l) $(14+2i)+8i$;

e) $4+(-3+i)$; i) $4{,}3+(1{,}7-9i)$; m) $81+(43-17i)$.

1.7. Yig'indini toping:

a) $\left(\frac{1-\sqrt{2}}{2}+\frac{1+\sqrt{2}}{3}i\right)+\left(\frac{1+\sqrt{2}}{2}+\frac{1-\sqrt{2}}{3}\right)i$;

b) $(\cos^2\alpha+i\sin^2\alpha)+(\sin^2\alpha+i\cos^2\alpha)$ $(\alpha\in R)$;

d) $(0{,}(3)+i\cdot 1{,}(5))+(0{,}(6)+i\cdot 1{,}(55))$;

e) $(\mathrm{Re}(1+2i)+15i)+(3-i\cdot\mathrm{Im}(1+2i))$.

1.8. Ayirmani toping:

a) $(-5+2i)-(8-9i)$; f) $(32+4{,}(5)i)-(32+i)$;

b) $(5+21i)-(9i+8)$; g) $\left(\frac{1-\sqrt{2}}{2}+\frac{1-\sqrt{2}}{2}i\right)-(1+i)$;

d) $4-(42-3i)$; h) $4{,}8-\left(\frac{1-\sqrt{2}}{3}-i\right)$;

e) $(14+3i)-(21+3i)$; i) $i-(3i+8)$.

1.9. Ko'paytmani hisoblang:

a) $(3+5i)(2+3i)$; f) $(\frac{1}{2}+i)(\frac{1}{4}-i)$; j) $(5-2i)(2i+5)$;

b) $(4+7i)(2-i)$; g) $(\frac{4}{7}+3i)(\frac{7}{4}+4{,}7i)$; k) $(-3+i)(3-i)$;

d) $(5-3i)(2-5i)$; h) $(2+3i)(2-3i)$; l) $0\cdot(4{,}5-i)$;

e) $(-2+i)(7-3i)$; i) $4\cdot(8{,}3-i)$; m) $(\frac{1}{3}-0{,}3)\cdot i$.

1.10. Ikki kompleks sonning bo'linmasini toping:

a) $\frac{1+i}{1-i}$; f) $\frac{5-4i}{-3+2i}$; j) $\frac{51}{4-i}$; n) $\frac{0}{3i}$;

b) $\frac{3-4i}{2+i}$; g) $\frac{-7+2i}{5-4i}$; k) $\frac{4-i}{51}$; o) $\frac{1+4i}{1-5i}$;

d) $\frac{2+3i}{2-3i}$; h) $\frac{3-4i}{-3+2i}$; l) $\frac{31i}{17+i}$; p) $\frac{1}{1+5i}$;

e) $\frac{1+2i}{3-2i}$; i) $\frac{14-3i}{3i+2}$; m) $\frac{14+i}{31i}$; q) $\frac{1}{1-5i}$.

1.11. Qo'shma kompleks sonlarning ko'paytmasi shaklida yozing (bu yerda $a, b \in R$):

a) a^2+4b^2 ; f) $3a^2+45b^4$; j) $a^{2n}+33b^{2n}$ $(n\in N)$;

b) $9a^2+25b^2$; g) $10a^2+56b^4$; k) $a^{2n}+b^{2k}$ $(k, n\in N)$;

d) $8a^2+16b^2$; h) $11a^2+48b^6$; l) $\sqrt{3}a^2+b^{18}$;

e) $81a^2+5b^2$; i) $13a^4+29b^8$; m) $9a^2+\sqrt{5}b^{20}$.

1.12. Mavhum birlik i ning quyidagi darajalarini hisoblang va xulosa chiqaring:

a) i^1 ; d) i^3 ; f) i^5 ; h) i^7 ; j) i^9 ; l) i^{11} ;

b) i^2 ; e) i^4 ; g) i^6 ; i) i^8 ; k) i^{10} ; m) i^{12} .

1.13. Amallarni bajaring:

a) $-3i+5+8i(3-i)$; f) $(5-3i)(4+i)+15i$; j) $3+5i+2i^{1999}$;
b) $(4+2i)(-1-3i)+5-8i$; g) $16-(15-i)(1+i)$; k) $35-i^{2000}+i^{1997}$;
d) $3i(1+i)+3i(3-i)$; h) $4(0{,}5-2{,}5i)(3+i)+5i$; l) $i^{2001}(3+5i^4)$;
e) $i(5-2i)+i(9-8i)$; i) $4{,}2(3-i)(1+i)+2+3i$; m) $i^{2002}-i^{2001}-i^{1999}$.

1.14. Hisoblang:

a) $\frac{(2-3i)(3-2i)}{1+i}$; f) $\frac{11}{1-2i}-\frac{13}{2-i}$; j) $\frac{i^{18}+i^{19}}{2-3i}+\frac{1}{3+4i}$;
b) $\frac{(3-i)(1+3i)}{2-i}$; g) $\frac{3-5}{3+i}+\frac{2+3i}{2-i}$; k) $\frac{2-3i}{2+3i}\cdot i^{18}+\frac{i}{1+i}$;
d) $\frac{3-4i}{(1+i)(2-i)}$; h) $\frac{13}{1-4i}+\frac{11}{1+4i}$; l) $\frac{4i^8}{9}+i\,(1+i^9)$;
e) $\frac{2-3i}{(1-i)(3+i)}$; i) $\frac{1-i}{1+i}+\frac{3-i}{3+i}$; m) $i^3\,(1-i^4)+i^{21}$.

1.15. Amallarni bajaring:

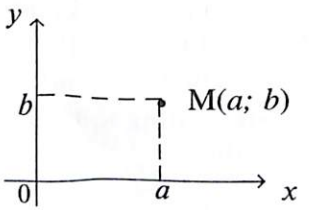
a) $(3-2i)^2$; f) $(3+2i)^2-(3-2i)$;
b) $(4+3i)^2$; g) $-3+5i)+(-3-5i)$;
d) $\left(\frac{1-2i}{1+i}\right)^2$; h) $\left(\frac{i^6+1}{i^8-1}\right)^2$;
e) $\left(\frac{1+i}{1-i}\right)^2$; i) $\left(\frac{4+i^7}{3-i^4}\right)^2$.

2-§. KOMPLEKS SONNING GEOMETRIK TASVIRI VA TRIGONOMETRIK SHAKLI

$z=a+bi$ kompleks son ikki xil usul bilan geometrik tasvirlanishi mumkin:

1. $z=a+bi$ kompleks songa xOy dekart koordinatalar sistemasidagi $(a;b)$ nuqtani mos qo'yish mumkin.

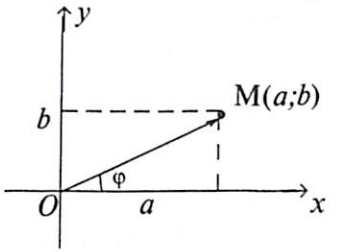
Har bir kompleks son xOy tekislikning faqat bitta nuqtasi mos keladi va aksincha, xOy tekislikning $M(a;\ b)$ nuqtasi bittagina $z=a+bi$ kompleks sonning geometrik tasviri bo'ladi (3-rasm).



3-rasm.

Shu munosabat bilan, xOy dekart koordinatalar sistemasini *kompleks tekislik* deb, Ox o'qni *haqiqiy o'q*, Oy o'qni esa *mavhum o'q* deb atash qabul qilingan.

2. $z=a+bi$ kompleks son xOy dekart koordinatalar sistemasida boshi koordinatalar boshida, oxiri esa $M(a;b)$ nuqtada bo'lgan vektor bilan tasvirlanadi (4-rasm):



4-rasm

Bu vektor z kompleks sonning radius-vektori deb aytiladi. Uning uzunligi z kompleks sonining *moduli* deyiladi $|z|$ yoki r bilan belgilanadi:

$$|z|=r=\sqrt{a^2+b^2}. \qquad (5)$$

Kompleks sonning moduli uchun quyidagi tengliklar o'rinli:

$$|z_1\cdot z_2|=|z_1|\cdot|z_2| \quad \left|\frac{z_1}{z_2}\right|=\frac{|z_1|}{|z_2|},\ (z^2\neq 0)$$

z kompleks son radius — vektorining $0x$ haqiqiy o'qning musbat yo'nalishi bilan hosil qilgan burchagi z kompleks sonining argumenti deyiladi. Kompleks sonning argumentlari cheksiz ko'p bo'lib, ular bir-biridan 2π ga karrali son bilan farq qiladi. Biz kompleks sonning argumenti deyilganda, argumentning $[0;2\pi]$ oraliqqa tegishli bo'lgan qiymatini nazarda tutamiz va bu qiymatni arg(z) yoki φ bilan belgilaymiz. arg(z) ni topishda uning ta'rifidan va

$$\begin{cases}\sin\varphi=\left|\frac{b}{z}\right| \\ \cos\varphi=\left|\frac{a}{z}\right| \\ \varphi\in[0;2\pi]\end{cases}, \quad \text{yoki} \quad \begin{cases}\operatorname{tg}(\arg(z))=\operatorname{tg}\varphi=\frac{b}{a} \\ \varphi\in[0;2\pi]\end{cases}, \qquad (6)$$

ko'rinishdagi sistemadan foydalaniladi (4-misol va 5-misolga qarang).

$z=a+bi$ kompleks sonning trigonometrik shakli quyidagi ko'rinishga ega:

$$z=r\,(\cos\varphi+i\sin\varphi) \qquad (7)$$

1.13. Amallarni bajaring:

a) $-3i+5+8i(3-i)$; f) $(5-3i)(4+i)+15i$; j) $3+5i+2i^{1999}$;

b) $(4+2i)(-1-3i)+5-8i$; g) $16-(15-i)(1+i)$; k) $35-i^{2000}+i^{1997}$;

d) $3i(1+i)+3i(3-i)$; h) $4(0{,}5-2{,}5i)(3+i)+5i$; l) $i^{2001}(3+5i^4)$;

e) $i(5-2i)+i(9-8i)$; i) $4{,}2(3-i)(1+i)+2+3i$; m) $i^{2002}-i^{2001}-i^{1999}$.

1.14. Hisoblang:

a) $\frac{(2-3i)(3-2i)}{1+i}$; f) $\frac{11}{1-2i}-\frac{13}{2-i}$; j) $\frac{i^{18}+i^{19}}{2-3i}+\frac{1}{3+4i}$;

b) $\frac{(3-i)(1+3i)}{2-i}$; g) $\frac{3-5}{3+i}+\frac{2+3i}{2-i}$; k) $\frac{2-3i}{2+3i}\cdot i^{18}+\frac{i}{1+i}$;

d) $\frac{3-4i}{(1+i)(2-i)}$; h) $\frac{13}{1-4i}+\frac{11}{1+4i}$; l) $\frac{4i^8}{9}+i\,(1+i^9)$;

e) $\frac{2-3i}{(1-i)(3+i)}$; i) $\frac{1-i}{1+i}+\frac{3-i}{3+i}$; m) $i^3\,(1-i^4)+i^{21}$.

1.15. Amallarni bajaring:

a) $(3-2i)^2$; f) $(3+2i)^2-(3-2i)$;

b) $(4+3i)^2$; g) $-3+5i)+(-3-5i)$;

d) $\left(\frac{1-2i}{1+i}\right)^2$; h) $\left(\frac{i^5+1}{i^8-1}\right)^2$;

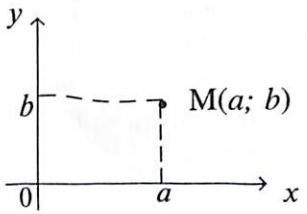
e) $\left(\frac{1+i}{1-i}\right)^2$; i) $\left(\frac{4+i^7}{3-i^4}\right)^2$.

2-§. KOMPLEKS SONNING GEOMETRIK TASVIRI VA TRIGONOMETRIK SHAKLI

$z=a+bi$ kompleks son ikki xil usul bilan geometrik tasvirlanishi mumkin:

1. $z=a+bi$ kompleks songa xOy dekart koordinatalar sistemasidagi $(a;b)$ nuqtani mos qo'yish mumkin.

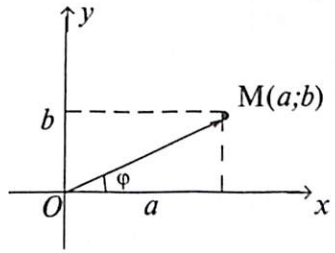
Har bir kompleks son xOy tekislikning faqat bitta nuqtasi mos keladi va aksincha, xOy tekislikning $M(a;\ b)$ nuqtasi bittagina $z=a+bi$ kompleks sonning geometrik tasviri bo'ladi (3-rasm).



3-rasm.

Shu munosabat bilan, xOy dekart koordinatalar sistemasini *kompleks tekislik* deb, Ox o'qni *haqiqiy o'q*, Oy o'qni esa *mavhum o'q* deb atash qabul qilingan.

2. $z=a+bi$ kompleks son xOy dekart koordinatalar sistemasida boshi koordinatalar boshida, oxiri esa $M(a;b)$ nuqtada bo'lgan vektor bilan tasvirlanadi (4-rasm):



4-rasm

Bu vektor z kompleks sonning radius-vektori deb aytiladi. Uning uzunligi z kompleks sonining *moduli* deyiladi $|z|$ yoki r bilan belgilanadi:

$$|z| = r = \sqrt{a^2+b^2}. \qquad (5)$$

Kompleks sonning moduli uchun quyidagi tengliklar o'rinli:

$$|z_1 \cdot z_2| = |z_1| \cdot |z_2| \quad \left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}, \ (z^2 \neq 0)$$

z kompleks son radius — vektorining $0x$ haqiqiy o'qning musbat yo'nalishi bilan hosil qilgan burchagi z kompleks sonining argumenti deyiladi. Kompleks sonning argumentlari cheksiz ko'p bo'lib, ular bir-biridan 2π ga karrali son bilan farq qiladi. Biz kompleks sonning argumenti deyilganda, argumentning $[0;2\pi]$ oraliqqa tegishli bo'lgan qiymatini nazarda tutamiz va bu qiymatni arg(z) yoki φ bilan belgilaymiz. arg(z) ni topishda uning ta'rifidan va

$$\begin{cases} \sin\varphi = \left|\frac{b}{z}\right| \\ \cos\varphi = \left|\frac{a}{z}\right| \\ \varphi \in [0;2\pi] \end{cases}, \quad \text{yoki} \quad \begin{cases} \operatorname{tg}(\arg(z)) = \operatorname{tg}\varphi = \frac{b}{a} \\ \varphi \in [0;2\pi] \end{cases}, \qquad (6)$$

ko'rinishdagi sistemadan foydalaniladi (4-misol va 5-misolga qarang).

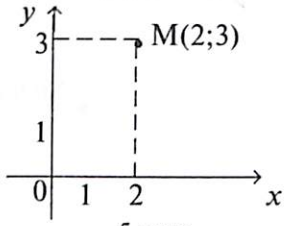
$z=a+bi$ kompleks sonning trigonometrik shakli quyidagi ko'rinishga ega:

$$z = r\,(\cos\varphi + i\sin\varphi) \qquad (7)$$

(7) da $r = \sqrt{a^2 + b^2}$ (z ning moduli) va φ — kompleks sonning argumenti.

1 - m i s o l. Kompleks tekislikning z kompleks songa mos keluvchi nuqtasini yasang: $z=2+3i$.

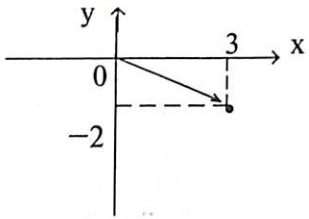
E ch i sh. a) $Re(z)=2$, $Jm(z)=3$ bo'lgani uchun bu songa kompleks tekislikning M(2;3) nuqtasi mos keladi (5-rasm):



5-rasm.

2 - m i s o l. $z = 3 - 2i$ kompleks songa mos keluvchi vektorni yasang.

E c h i sh z. $= 3 - 2i$ kovpleks songa mos keluvchi nuqtani belgilab, koordinatalar boshini M $(3; -2)$ nuqta bilan tutashtiruvchi vektorni yasash kifoya (6-rasm):



6-rasm.

3 - m i s o l. Kompleks son z ning modulini toping:
a) $z=3-4i$; b) $z=1-3i$; d) $z=\cos^2\alpha+i\sin\alpha$ ($\alpha\in R$); e) $z=3$.

Y e c h i s h.
a) $|z| = \sqrt{3^2 + (-4)^2} = 5$; b) $|z| = \sqrt{1^2 + (-3)^2} = \sqrt{10}$;
d) $|z| = \sqrt{(\cos^2\alpha)^2 + \sin^2\alpha} = \sqrt{\cos^4\alpha + \sin^2\alpha}$;
e) $|z| = \sqrt{3^2 + 0^2} = 3$.

4 - m i so l. Kompleks son z ning argumenti φ ni toping:
a) $z = 100$;
b) $z = 100 + 100i$;
d) $z = 100i$;
e) $z = -\frac{9\sqrt{3}}{2} + \frac{9}{2}i$;
f) $z = -100$

Y e ch i sh. (Kompleks sonning argumentini aniqlashda, dastlab shu son radius-vektorini sxematik yasab olish tavsiya etiladi).

a) Kompleks son argumentining ta'rifiga ko'ra, $\varphi=0$ (7-a rasm).

b) 1-usul. OAB to'g'ri burchakli uchburchakning teng yonli uchburchak (7-b rasm) ekanligidan foydalansak, $\varphi = \frac{\pi}{4}$ ekani kelib chiqadi.

2-usul. $|z| = \sqrt{100^2 + 100^2} = 100\sqrt{2}$.

$$\left.\begin{array}{l} \sin\varphi = \dfrac{100}{100\sqrt{2}} = \dfrac{\sqrt{2}}{2} \\ \cos\varphi = \dfrac{100}{100\sqrt{2}} = \dfrac{\sqrt{2}}{2} \\ \varphi \in \left[0; \dfrac{\pi}{4}\right] \subset [0;2\pi] \end{array}\right\} \Rightarrow \varphi = \frac{\pi}{4}.$$

3-usul.
$$\left.\begin{array}{l} \operatorname{tg}\varphi = \dfrac{100}{100} = 1 \\ \varphi \in \left[0; \dfrac{\pi}{2}\right] \subset [0;2\pi] \end{array}\right\} \Rightarrow \varphi = \frac{\pi}{4}.$$

d) Kompleks son argumentining ta'rifiga ko'ra, $\varphi = \frac{\pi}{2}$ (7-d rasm).

e) $\varphi = \pi - \theta$ ekanidan foydalanamiz.

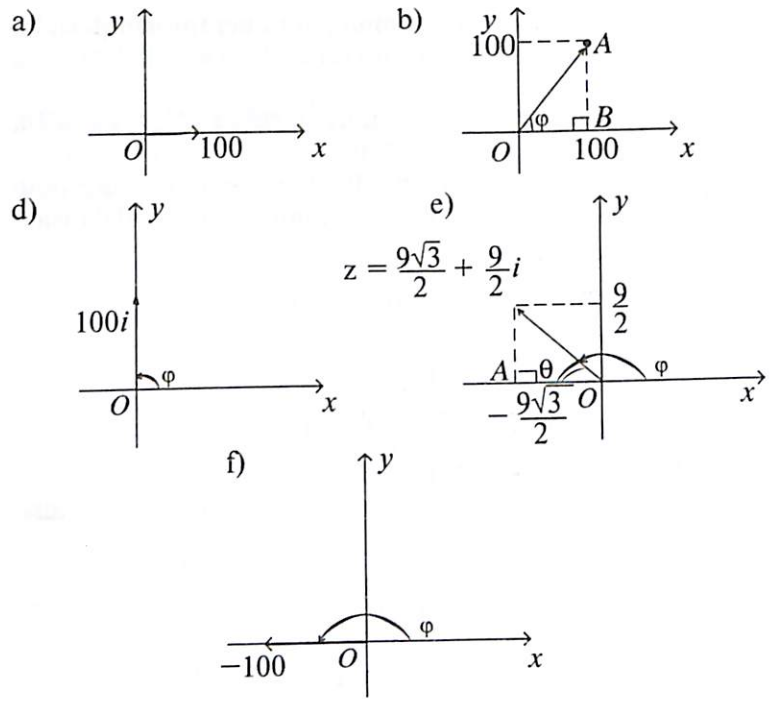
$$|z| = \left|-\frac{9\sqrt{3}}{2} + \frac{9}{2}i\right| = \frac{9}{2}|\sqrt{3} + i| = \frac{9}{2} \cdot \sqrt{(\sqrt{3})^2 + 1^2} = 9$$

bo'lgani uchun OZA to'g'ri burchakli uchburchakdan (7-e rasm):

$$\left.\begin{array}{l} \sin\theta = \dfrac{ZA}{OZ} = \dfrac{\frac{9}{2}}{9} = \dfrac{1}{2} \\ \cos\theta = \dfrac{OA}{OZ} = \dfrac{\frac{9\sqrt{3}}{2}}{9} = \dfrac{\sqrt{3}}{2} \\ \theta \in \left[0; \dfrac{\pi}{2}\right] \end{array}\right\} \Rightarrow \varphi = \frac{\pi}{6}$$

Demak, $\varphi = \pi - \theta = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$

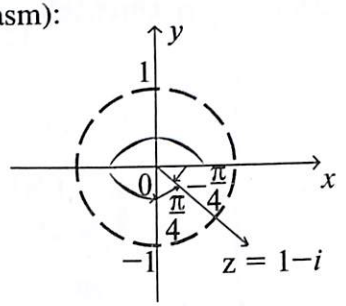
f) Kompleks son argumentining ta'rifiga ko'ra, $\varphi=\pi$ (7-f rasm).



7-rasm.

5 - m i s o l. $z = 1 - i$ ning argumentini toping.

Y e c h i s h. Bu sonning argumenti φ deylik. (6) ga ko'ra $\operatorname{tg}\varphi = \frac{-1}{1} = -1$ va $\varphi \in [0;2\pi]$ ga egamiz. $\operatorname{tg}\varphi = -1$, $\varphi \in [0;2\pi]$ shartlar o'rinli bo'ladigan φ ni rasmdan foydalanib topamiz (8-rasm):



8-rasm.

$\varphi = \frac{3\pi}{2} + \frac{\pi}{4} = \frac{7\pi}{4}$ (yoki $\varphi = 2\pi - \frac{\pi}{4} = \frac{7\pi}{4}$).

Javob: $\frac{7\pi}{4}$.

6 - m i s o l. Sonlarni trigonometrik shaklda yozing:

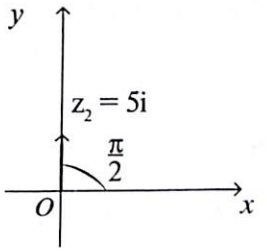
a) $z_1 = 3 + \sqrt{3}i$; b) $z_2 = 5i$.

Y e c h i s h. a) z_1 conining modulini va argumenti φ ni topamiz. $\operatorname{Re}(z_1)=\sqrt{3}$, $\operatorname{Im}(z_1)=\sqrt{3}$ bo'lgani uchun $|z_1| = \sqrt{3^2 + \sqrt{3})^2} = 2\sqrt{3}$.

$\begin{cases} \operatorname{tg}\varphi = \frac{\sqrt{3}}{3}, \\ \varphi \in [0;2\pi] \end{cases}$ sistemadan $\varphi = \frac{\pi}{6}$ ni topamiz.

Demak, $z_1 = 2\sqrt{3}(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6})$.

b) $z_2 = 5i$ ninig moduli $|z_1| = |5i|\sqrt{0^2 + 5^2} = 5$ ga teng. $z_2 = 5i$ ning radius vektori mavhum o'qning musbat qismida yotgani uchun $\varphi = \frac{\pi}{2}$ bo'ladi (9-rasm).



1-rasm.

Shu cababli $z_2 = 5\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)$,

2.1. Kompleks tekislikning z kompleks songa mos keluvchi nuqtasini yasang:

a) $z=1+2i$; f) $z=2i$; j) $z=0$; n) $z=2+3i(1+2i)$;
b) $z=-1+2i$; g) $z=1$; k) $z=3-2i$; o) $z=i-4i(1+i)$;
d) $z=-1-2i$; h) $z=-2i$; l) $z=-3+2i$; p) $z=i^4+i^5$;
e) $z=1-2i$; i) $z=-1$; m) $z=\frac{\sqrt{2}}{2}$; q) $z=\cos\frac{\pi}{3}+i\sin\frac{\pi}{2}$.

2.2. z kompleks songa mos keluvchi vektorni yasang:

a) $z=2+3i$; f) $z=3i$; j) $z=0$; n) $z=\frac{1+i}{1-i}$;
b) $z=2-3i$; g) $z=-4i$; k) $z=-3+2i$; o) $z=(1+i)(1+2i)$;
d) $z=-2+3i$; h) $z=2$; l) $z=3-i$; p) $z=(1-i)(1+i)$;
e) $z=-2-3i$; i) $z=-2$; m) $z=\sqrt{4}$; q) $z=i^3-4i$.

2.3. z kompleks sonning modulini toping:

a) $z=3+4i$; g) $z=3+3i$; l) $z=\cos\alpha+i\sin\alpha$ $(\alpha\in R)$;
b) $z=-3-4i$; h) $z=1+2\sqrt{3}i$; m) $z=1+i\cos^2\alpha$ $(\alpha\in R)$;
d) $z=1+\sqrt{8}i$; i) $z=1+i$; n) $z=(2+3i)(3-4i)$;
e) $z=2\sqrt{2}+i$; j) $z=\sqrt{2}+i$; o) $z=4\sqrt{81}+3\sqrt{2}i$;
f) $z=-4$; k) $z=bi$, $b\in R$; p) $z=i$; q) $z=0$.

2.4. z kompleks sonning argumentini toping:

a) $z=\frac{1}{\sqrt{2}}+i\frac{1}{\sqrt{2}}$; f) $z=\frac{\sqrt{33}}{2}+i\frac{\sqrt{11}}{2}$; j) $z=1$;
b) $z=\frac{\sqrt{2}}{2}+i\frac{\sqrt{6}}{2}$; g) $z=-2\sqrt{3}i$; k) $z=i$;
d) $z=3i$; h) $z=-\sqrt{6}-\sqrt{6}i$; l) $z=-1$;
e) $z=3$; i) $z=\frac{\sqrt{3}}{2}-\frac{1}{2}i$; m) $z=-i$.

2.5. Kompleks sonni trigonometrik shaklda yozing:

a) $z=-1-i$; f) $z=-2$; j) $z=1+i$; n) $z=2i$;
b) $z=1-i$; g) $z=i$; k) $z=-\frac{1}{2}+i\frac{\sqrt{3}}{2}$; o) $z=\frac{1}{\sqrt{2}}+i\frac{1}{\sqrt{2}}$;
d) $z=\sqrt{3}+i$; h) $z=1$; l) $z=\frac{\sqrt{33}}{2}+i\frac{\sqrt{11}}{2}$; p) $z=-i$;
e) $z=-1+\sqrt{3}i$; i) $z=-i$; m) $z=\frac{\sqrt{3}}{2}-\frac{1}{2}i$; q) $z=-\sqrt{6}-\sqrt{6}i$.

2.6. $z=-3-4i$ ni trigonometrik shaklda yozing.

2.7. $z=\cos\frac{7\pi}{4}-2i\sin\frac{7\pi}{4}$ ni trigonometrik shaklda yozing.

2.8. $z=-\cos\frac{\pi}{17}+i\sin\frac{\pi}{17}$ ni trigonometrik shaklda yozing.

2.9. $z=2+\sqrt{3}+i$ ni trigonometrik shaklda yozing.

2.10. $z=1+\cos\varphi+i\sin\varphi$ $(-\pi\le\varphi\le\pi)$ ni trigonometrik shaklda yozing.

3-§. TRIGONOMETRIK SHAKLDA BERILGAN KOMPLEKS SONLAR USTIDA AMALLAR

Agar $z_1=r_1(\cos\varphi_1+i\sin\varphi_1)$ va $z_2=r_2(\cos\varphi_2+i\sin\varphi_2)$ lar trigonometrik shaklda yozilgan kompleks sonlar bo'lsa, quyidagi tengliklar o'rinli bo'ladi:

$$z_1\cdot z_2=r_1\cdot r_2(\cos(\varphi_1+\varphi_2)+i\sin(\varphi_1+\varphi_2));$$

$$\frac{r_1}{r_2}=\frac{z_1}{z_2}(\cos(\varphi_1-\varphi_2)+i\sin(\varphi_1-\varphi_2)),\quad (z_2\neq 0).$$

Agar $z=r(\cos\varphi+i\sin\varphi)$ trigonometrik shakldagi kompleks son bo'lsa, $z^n=r^n(\cos n\varphi+i\sin n\varphi)$,

$$\sqrt[n]{z}=\sqrt[n]{r}\left(\cos\frac{\varphi+2k\pi}{n}+i\sin\frac{\varphi+2k\pi}{n}\right)\quad k=0,1,2,\cdots,n-1$$

tengliklar o'rinli bo'ladi.

Agar darajaga ko'tarish formulasida $r=1$ bo'lsa,

$(\cos\varphi+i\sin\varphi)^n=\cos n\varphi+i\sin n\varphi$ Muavr formulasi hosil bo'ladi.

1 - m i s o l. Kompleks sonning trigonometrik shaklidan foydalanib, quyidagi amallarni bajaring:

a) $(1-i)\cdot(\sqrt{3}+i)$; b) $\frac{1-i}{\sqrt{3}+i}$.

Y e c h i s h. $z_1=1-i$ va $z_2=\sqrt{3}+i$ sonlarni trigonometrik shaklda yozib olamiz.

$|z_1|=|1-i|=\sqrt{1^2+(-1)^2}=\sqrt{2}$; $\varphi_1=\frac{7\pi}{4}$ bo'lgani uchun $z_1=\sqrt{2}(\cos\frac{7\pi}{4}+i\sin\frac{7\pi}{4})$.

$|z_2|=|\sqrt{3}+i|=\sqrt{(\sqrt{3})^2+1^2}=2$; $\varphi_2=\frac{\pi}{6}$ bo'lgani uchun $z_2=2(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6})$ bo'ladi. U holda,

$$\text{a) } (1-i)\cdot(\sqrt{3}+i)=\left(\sqrt{2}\left(\cos\frac{7\pi}{4}+i\sin\frac{7\pi}{4}\right)\right)\left(2\left(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}\right)\right)=$$

$$=(\sqrt{2}\cdot 2)\left(\cos\left(\frac{7\pi}{4}+\frac{\pi}{6}\right)+i\sin\left(\frac{7\pi}{4}+\frac{\pi}{6}\right)\right)=$$

$$=2\sqrt{2}\left(\cos\frac{23\pi}{12}+i\sin\frac{23\pi}{12}\right);$$

$$\text{b) } \frac{1-i}{\sqrt{3}+i}=\frac{\sqrt{2}\left(\cos\frac{7\pi}{4}+i\sin\frac{7\pi}{4}\right)}{2\left(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}\right)}=\frac{\sqrt{2}}{2}\left(\cos\left(\frac{7\pi}{4}-\frac{\pi}{6}\right)+\right.$$

$$\left.+i\sin\left(\frac{7\pi}{4}-\frac{\pi}{6}\right)\right)=\frac{\sqrt{2}}{2}\left(\cos\frac{19\pi}{12}+i\sin\frac{19\pi}{12}\right).$$

Javob: a) $(1-i)\cdot(\sqrt{3}+i)=2\sqrt{2}\left(\cos\frac{23\pi}{12}+i\sin\frac{23\pi}{12}\right)$;

b) $\frac{1-i}{\sqrt{3}+i}=\frac{\sqrt{2}}{2}\left(\cos\frac{19\pi}{12}+i\sin\frac{19\pi}{12}\right)$.

2 - m i s o l. $(1-i)^3$ ni hisoblang.

Yechish. $1-i=\sqrt{2}\left(\cos\frac{7\pi}{4}+i\sin\frac{7\pi}{4}\right)$ bo'lgani uchun (1-misol) darajaga ko'tarish formulasiga ko'ra,

$(1-i)^3 = \left(\sqrt{2}\left(\cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4}\right)\right)^3 = (\sqrt{2})^3 \cdot \left(\cos\left(3\cdot\frac{7\pi}{4}\right) + i\sin\left(3\cdot\frac{7\pi}{4}\right)\right) = 2\sqrt{2}\left(\cos\frac{21\pi}{4} + i\sin\frac{21\pi}{4}\right) = 2\sqrt{2}\left(\cos\frac{21\pi}{4} + i\sin\frac{21\pi}{4}\right)$ ga ega bo'lamiz.

3 - m i s o l. $\sqrt[3]{1-i}$ ni hisoblang.

Yechish. $z = 1 - i = \sqrt{2}\left(\cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4}\right)$ bo'lgani uchun ildiz chiqarish formulasiga ko'ra,

$\sqrt[3]{1-i} = \sqrt[3]{\sqrt{2}}\left(\cos\frac{7\pi+2\pi k}{4} + i\sin\frac{7\pi+2\pi k}{4}\right)$; (k=0,1,2).

Shu sababli quyidagilarni topamiz:

k=0 da, $\sqrt[3]{z} = \sqrt[6]{2}\left(\cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4}\right)$;

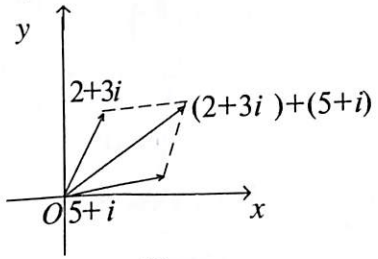
k=1 da, $\sqrt[3]{z} = \sqrt[6]{2}\left(\cos\frac{7\pi+2\pi}{4} + i\sin\frac{7\pi+2\pi}{4}\right) = \sqrt[6]{2}\left(\cos\frac{9\pi}{4} + \sin\frac{9\pi}{4}\right) = \sqrt[6]{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$;

k=2 da, $\sqrt[3]{z} = \sqrt[6]{2}\left(\cos\frac{7\pi+4\pi}{4} + i\sin\frac{7\pi+4\pi}{4}\right) = \sqrt[6]{2}\cos\left(\frac{11\pi}{4} + i\sin\frac{11\pi}{4}\right) = \sqrt[6]{2}\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$,

4 - m i s o l. Daraja asosini trigonometrik shaklda yozmasdan, $(1+i)^{45}$ darajani hisoblang.

Yechish. $(1+i)^2 = 1+2i-1 = 2i$ bo'lgani uchun, $(1+i)^{45} = (1+i)^{44}\cdot(1+i) = ((1+i)^2)^{22}\cdot(1+i) = (2i)^{22}\cdot(1+i) = 2^{22}\cdot(i^2)^{11}\cdot(1+i) = 2^{22}\cdot(-1)(1+i) = -2^{22}-2^{22}i$.

5 - m i s o l. $(2+3i)+(5+i)$ yig'indining radius-vektorini toping.

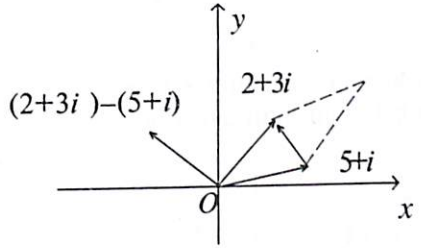


10-rasm.

Y e c h i s h. Qo'shiluvchilar radius-vektorlarida parallelogramm yasaymiz. Uning katta diagonali yig'indining radius-vektoridir (10-rasm).

6-misol. $(2+3i)-(5+i)$ ayirmaning radius vektorini toping.

Y e c h i s h. $(2+3i)$ va $(5+i)$ sonlarning radius-vektorlaridan parallelogramm yasaymiz. So'ngra boshi ayriluvchi radius-vektorning oxirida, oxiri esa kamayuvchi radius-vektorning oxirida bo'lgan vektorini yasaymiz. Bu vektorni uning boshi koordinatalar boshi bilan ustma-ust tushadigan qilib, o'z-o'ziga parallel ko'chiramiz va izlangan radius-vektorga ega bo'lamiz (11-rasm):



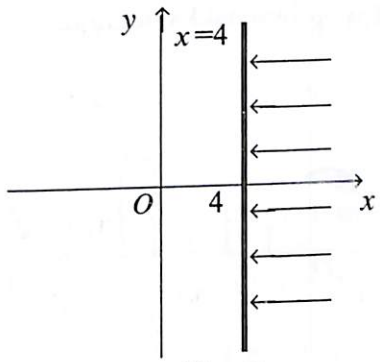
11-rasm.

7 - m i s o l. Kompleks tekislikning quyidagi shartlarni qanoatlantiruvchi nuqtalarining geometrik o'rnini shtrixlab ko'rsating:

a) $\mathrm{Re}(z)>4$; b) $\mathrm{Re}(z)\le 1$; d) $\mathrm{Jm}(z)<4$, $\mathrm{Re}(z)>2$;

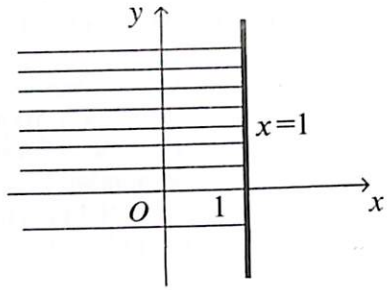
e) $0<\arg(z)\le\frac{\pi}{6}$.

Y e c h i s h. a) $z=x+iy$ nuqta uchun $\mathrm{Re}(z)>4$, yani $x>4$ bo'lsin. Abstsissasi 4 dan katta bo'lgan nuqtalar $x=4$ to'g'ri chiziqdan o'ng tomonda joylashgan nuqtalardan iborat (12-rasm).



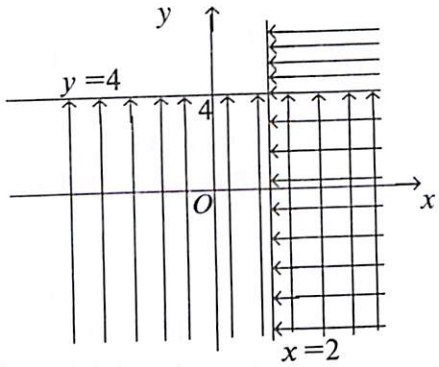
12-rasm.

b) $z=x+iy$ nuqta uchun $\mathrm{Re}(z)\le 1$, yani $x\le 1$ bo'lsin. U holda, a) holdagi o'xshash mulohaza yuritib, quyidagi shaklni hosil qilamiz (13-rasm).



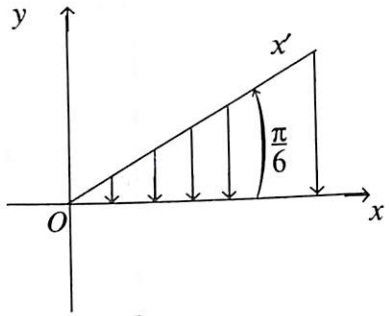
1-rasm.

b) $z=x+yi$ nuqta uchun $\mathrm{Im}(z)<4$, $\mathrm{Re}(z)>2$ bo'lsa, $y<4$, $x>2$ tengsizliklar bilan aniqlangan sohaga ega bo'lamiz (14-rasm):



14-rasm.

g) Ox o'qni $\varphi=\frac{\pi}{6}$ burchakka buramiz:



15-rasm.

xOx' burchakdagi barcha nuqtalar uchun (Ox o'q ustidagi nuqtalar bundan mustasno) $0<\arg(z)\le\frac{\pi}{6}$ shart bajariladi.

3.1. Trigonometrik shaklda berilgan sonlarning ko'paytmasini toping:

a) $z_1=\frac{\sqrt{2}}{2}\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right.$ va $z_2=\cos\frac{\pi}{8}+i\sin\frac{\pi}{8}$;

b) $z_1=\frac{1}{2}\left(\cos\frac{\pi}{15}+i\sin\frac{\pi}{15}\right)$ va $z_2=4\left(\cos\frac{\pi}{9}+i\sin\frac{\pi}{9}\right)$;

d) $z_1=\sqrt{3}\left(\cos\frac{\pi}{24}+i\sin\frac{\pi}{24}\right)$ va $z_2=3\left(\cos\frac{\pi}{12}+i\sin\frac{\pi}{12}\right)$;

e) $z_1=5(\cos\pi+i\sin\pi)$ va $z_2=\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}$.

3.2. $\frac{z_1}{z_2}$ ni hisoblang:

a) $z_1=\sqrt{3}\left(\cos\frac{\pi}{19}+i\sin\frac{\pi}{19}\right)$, $z_2=2\left(\cos\frac{\pi}{21}+i\sin\frac{\pi}{21}\right)$;

b) $z_1=6\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)$, $z_2=9\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)$;

d) $z_1=\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}$, $z_2=\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}$;

e) $z_1=\frac{1}{3}\left(\cos\frac{3\pi}{2}+i\sin\frac{3\pi}{2}\right)$, $z_2=\frac{1}{3}\left(\cos\frac{5\pi}{6}+i\sin\frac{5\pi}{6}\right)$.

3.3. Darajani hisoblang:

a) $\left(\cos\frac{3\pi}{2}+i\sin\frac{3\pi}{2}\right)^{20}$;

b) $\left(\cos\frac{\pi}{8}+i\sin\frac{\pi}{8}\right)^{16}$;

d) $\left(\cos\frac{\pi}{9}+i\sin\frac{\pi}{9}\right)^{15}$;

e) $\left(\cos\frac{\pi}{7}+i\sin\frac{\pi}{7}\right)^{17}$;

f) $\left(2\left(\cos\frac{\pi}{21}+i\sin\frac{\pi}{21}\right)\right)^{7}$;

g) $\left(\sqrt{3}\left(\cos\frac{\pi}{9}+i\sin\frac{\pi}{9}\right)\right)^{18}$;

h) $\left(\sqrt{4}\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)\right)^{6}$;

i) $\left(3\left(\cos\frac{\pi}{13}+i\sin\frac{\pi}{13}\right)\right)^{2}$.

3.4. $\sqrt{z}$ ni hisoblang:

a) $z=\frac{\sqrt{2}}{2}\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)$; d) $z=\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}$;

b) $z=\frac{1}{2}\left(\cos\frac{\pi}{15}+i\sin\frac{\pi}{15}\right)$; e) $z=\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}$.

3.5. $z=16\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)$ sonning uchinchi darajali va to'rtinchi darajali ildizlarini toping.

4-§. KOMPLEKS SONLAR USTIDA BARCHA AMALLARGA DOIR MISOLLAR

4.1. Hisoblang:

a) $(2+3i)(4-5i)+(2-3i)(4+5i)$;

b) $(x-1-i)(x-1+i)(x+1+i)(x+1-i)$, $x\in R$;

d) $\dfrac{(1+2i)^2}{1-3i}$;

e) $(1-4i)-(i(3-4i)+3i)$;

f) $(1+4i)^2-(3+i^9)$;

g) $3+8i+9i^2+10i^3$;

h) $8-4(i^{15}-1)+13i$;

i) $21i^4+23i^{91}-17i^{17}$.

4.2. Tenglamani yeching (bunda $x\in R$):

a) $-2x+4i=3x(\frac{1}{3}+i^2)+2i-2i^3$;

b) $3+xi=(\frac{18}{9}+x)+1+i$;

d) $5+(3+x)i=3x+2+4i$;

e) $x+5-(3+x^2)i=7-7i$.

4.3. Agar $(5x-3y)+(x-2y)i=6+(8-x+y)i$ bo'lsa, x, y haqiqiy sonlarni toping.

4.4. Daraja asosini trigonometrik shaklda yozmasdan darajani hisoblang:

a) $(1+i)^{20}$; b) $(1-i)^{21}$.

4.5. Quyidagilarni $\sin x$ va $\cos x$ orqali ifodalang:

a) $\sin 3x$; b) $\cos 3x$; d) $\sin 4x$; e) $\cos 4x$; f) $\sin 5x$; g) $\cos 5x$; h) $\sin 2x$.

N a m u n a:

h) $\cos 2x+i\sin 2x=(\cos x+i\sin x)^2=\cos^2 x+2i\sin x\cos x-\sin^2 x=(\cos^2 x-\sin^2 x)+(2\sin x\cos x)i \Rightarrow \begin{cases}\cos x=\cos^2 x-\sin^2 x\\ \sin 2x=2\sin x\cos x\end{cases}$

$\sin 2x=2\sin x\cos x$.

4.6. Kompleks sonlarni trigonometrik shaklda yozib, hisoblashlarni bajaring:

a) $(1+i)^{26}$;

b) $\left(\dfrac{1+i\sqrt{3}}{1-i}\right)^{20}$;

d) $1\left(-\dfrac{\sqrt{3}-i}{2}\right)^{24}$;

e) $\dfrac{(-1+i\sqrt{3})^{20}}{(1-i)^{20}}$;

f) $(1+i)^9(1-i)^{15}$;

g) $(1+2i)^8(2+3i)^3$;

h) $(2+i)^{26}(2+3i)^9$;

i) $\dfrac{(-1-i\sqrt{3})^{15}}{(1+i)^{21}}$.

4.7. $\sqrt[n]{z}$ ni hisoblang:

a) $z=1$, $n=3$;

b) $z=-1$, $n=4$;

d) $z=-4+\sqrt{48}i$, $n=3$;

e) $z=1+i$, $n=8$;

f) $z=i$, $n=3$;

g) $z=-i$, $n=3$;

h) $z=-9$, $n=3$;

i) $z=-15$, $n=4$;

j) $z=\dfrac{\sqrt{2}}{2}+\dfrac{\sqrt{2}}{2}i$, $n=3$;

k) $z=1-i$, $n=6$;

l) $z=5i$, $n=2$;

m) $z=-9i$, $n=2$.

4.8. Tenglamani yeching:

a) $z^4=-1$; b) $z^3=1+i$; d) $z^2=-9$; e) $z^2=16$.

4.9. a) $ax^2+bx+c=0$ $(a\neq 0)$ tenglamada $b^2-4ac<0$. Tenglamani kompleks sonlar to'plamida yeching;

b) $z^4+z^2+1=0$ tenglamani yeching.

4.10. Hisoblang:

a) $\sqrt[3]{\dfrac{1-i}{\sqrt{3}+i}}$; b) $\sqrt[4]{\dfrac{1+i}{\sqrt{3}-i}}$; d) $\sqrt[5]{\dfrac{1-i}{1+i\sqrt{3}}}$; e) $\sqrt[6]{\dfrac{1+i}{1-i\sqrt{3}}}$.

4.11. Tenglamadan x va y ni toping: $(x\in R,\ y\in R)$

a) $(x-y)+(3x+y)i=3-3i$;

b) $(5x+3yi)+(2y-xi)=3-i$;

d) $(\frac{3}{4}x-2yi)-(\frac{1}{3}y+6xi)=21i$;

e) $(2-3i)(x+yi)=-1-5i$.

4.12. Berilgan kompleks sonlarni qo'shing. Qo'shiluvchilarning va yig'indining geometrik tasvirini yasang:

a) $(2+3i)+(4+2i)$;

b) $(-4+5i)+(3-2i)$;

d) $(-7+6i)+(-3-8i)$;

e) $(-5-2i)+(-6+8i)$;

f) $(-4-7i)+(4+7i)$;

g) $(-3+2i)+(3-2i)$;

h) $3i+(4-5i)$;

i) $4i+(-8i)$.

4.13. Ayirishni bajaring. Kamayuvchi, ayriluvchi va ayirmaning geometrik tasvirini yasang:

a) $(3+2i)-(2-2i)$;

b) $i-5i$;

d) $(4+3i)-(2-3i)$;

e) $(4-2i)-(3+3i)$;

f) $8-(4-3i)$;

g) $i-(2-3i)$.

4.14. Bo'lish amalini bajaring:

a) $6(\cos 70^\circ+i\sin 70^\circ) : 3(\cos 25^\circ+i\sin 25^\circ)$;

b) $2(\cos 120^\circ+i\sin 120^\circ) : 4(\cos 90^\circ+i\sin 90^\circ)$;

d) $\sqrt{6}(\cos 160^\circ+i\sin 160^\circ) : \sqrt{3}(\cos 40^\circ+i\sin 40^\circ)$;

e) $4(\cos 75^\circ+i\sin 75^\circ) : -(\cos(-15^\circ)+i\sin(-15^\circ))$;

f) $8i:(1+\sqrt{3}i)$;

g) $(6-6i):3(\cos 15^\circ+i\sin 15^\circ)$;

h) $-6i:(-4-4i)$;

i) $(2+2\sqrt{3}i) : (4-4i)$.

4.15. Ko'paytuvchilarga ajrating:

a) x^2+4; b) x^4-16; d) x^2+3-4i; e) $7+\sqrt{5}$.

4.16. Tenglikni tekshiring:

a) $\left(\frac{-1+i\sqrt{3}}{2}\right)^4+\left(\frac{-1-i\sqrt{3}}{2}\right)^4=1$; d) $\left(\frac{-\sqrt{3}+i}{2}\right)^5+\frac{-\sqrt{3}-i}{2}^5=\sqrt{3}$;

b) $\left(\frac{1-i}{\sqrt{2}}\right)^5+\left(\frac{1+i}{\sqrt{2}}\right)^5=-2$; e) $\left(\frac{1+i}{1-i}\right)^3-\left(\frac{1-i}{1+i}\right)^3=2$.

4.17. Kompleks tekislikda quyidagi shartni qanoatlantiruvchi nuqtalarning geometrik o'rnini shtrixlang:

a) $\mathrm{Re}(z)<5$; f) $\mathrm{Re}(z)<0$; j) $|z-4|<2$;

b) $-<\frac{\pi}{4}\arg(z)<\frac{\pi}{3}$; g) $\mathrm{Re}(z)+\mathrm{Im}(z)=0$; k) $|z+2i|\geq 4$;

d) $\mathrm{Re}(z)=2$; h) $|z|>5$; l) $|z+1-i|<2$;

e) $\mathrm{Im}(z)=-2$; i) $1<|z|<3$; m) $|z-i|<|z-1|$.

4.18. $z=(p+qi)(-qi)$ kompleks sonning modulini toping ($p\in R$, $q\in R$).

4.19. $z_1=-2+\sqrt{3}i$ va $z_2=1-i$ sonlarni trigonometrik shaklga keltirib, quyidagi ifodalarni hisoblang:

a) $z_1\cdot z_2$; d) $\frac{z_1^3}{z_2}$; f) $\sqrt[4]{z_1}$; h) $z_1^2\cdot z_2$;

b) $\frac{z_2}{z_2}$; e) z_2^6; g) $\sqrt[3]{z_2}$; i) $z_1\cdot z_2^2$.

4.20. Quyidagi tengliklarni isbotlang:

a) $z\cdot\overline{z}=|z|^2$; v) $z+\overline{z}=2Re(z)$;

b) $z_1+z_2=\overline{z}_1+\overline{z}_2$; g) $z-\overline{z}=2Im(z)\cdot i$.

IV bob. KO'PHADLAR

1-§. BIRHADLAR VA KO'PHADLAR

Natural ko'rsatkichli daraja va uning xossalari

Ta'rif:

$$a^n=\underbrace{a\cdot a\ldots\cdot a}_{n\text{ marta}}\quad (n\geq 2, n\in N),\quad a^1=a.$$

Natural ko'rsatkichli daraja quyidagi xossalarga ega:

1°. $a^m\cdot a^n=a^{m+n}\cdot m, n\in N$;
2°. $a^m:a^n=a^{m-n}, m, n\in N$;

3°. $(a^m)^n=a^{mn}, m, n\in N$;
4°. $(ab)^n=a^n\cdot b^n\cdot m, n\in N$;
5°. $\left(\frac{a}{b}\right)^n=\frac{a^n}{b^n}$, $a,b\in R$, $b\neq 0$, $n\in N$.

1.1. Ifodani x asosli daraja ko'rinishida yozing:

a) $x^3\cdot x^5$; f) $(x^2)^3$; j) $x^3\cdot x^a$;
b) $x^4\cdot x^5\cdot x^6$; g) $(x^3)^2$; k) $(x^2\cdot x^3)^a$;
d) $-x^3\cdot x^4$; h) $(x^2\cdot x^4)^3$; l) $x^2\cdot(x^3)^4$;
e) $-x^3\cdot x^3$; i) $((x^3)^4)^5$; m) $(x^4)^2\cdot(x^2)^4$.

1.2. Ifodaning qiymatini toping:

a) $\frac{2^5\cdot 11^8}{22^{10}}\cdot\frac{34^4\cdot 2^{10}}{17^5\cdot 8^4}$; f) $\frac{12^5}{2^3\cdot 3^4}:\frac{10^5}{2^6\cdot 5^7}$;

b) $\frac{2^8\cdot 7^9}{14^{10}}:\frac{26^5}{13^6\cdot 8^4}$; g) $\frac{12^5}{2^3\cdot 3^4}\cdot\frac{10^5}{2^6\cdot 5^7}$;

d) $\frac{14^{10}}{2^8\cdot 7^9}:\frac{13^6\cdot 8^4}{26^5}$; h) $\frac{10^5}{2^6\cdot 5^7}:\frac{12^5}{2^3\cdot 3^4}$;

e) $\frac{12^5}{2^3\cdot 4^4}$; i) $\frac{10^5}{2^7\cdot 5^6}\cdot\frac{2^4\cdot 3^3}{12^5}$.

1.3. Birhadning darajasini aniqlang:

a) $3x^4xy^5$; f) $3xy^9z$; j) 15;
b) $-31xy^4$; g) $14x^2y^3z^4$; k) x^4y^2z;
d) $0{,}8x^2y^2$; h) $13yz^{15}$; l) $x\cdot x^2\cdot\ldots\cdot x^9$;
e) 15; i) $43x^2y^3z^{19}$; m) $xyx^2y^2x^4y^4x^6y^6\ldots x^{20}y^{20}$.

1.4. Birhadni standart shaklga keltiring:

a) $13xy\cdot 14x^2y^3$; f) $3xy(-1{,}5)y^3$;
b) $x^2y^2xzy^4$; g) $-ax^2y^2\cdot 6{,}5x^3$;
d) $3x^2z^2y^2\cdot xz^5$; h) $a\cdot xy^2z\cdot y^4\cdot x^5$;
e) $11x^2y\cdot 13x^3y^4$; i) $a(x^2)^3yz^2x^3$.

1.5. A^n ni toping:

a) $A=3x^2yz$, $n=3$; f) $A=2x^2yz^2$, $n=4$;
b) $A=13xy^2$, $n=2$; g) $A=3xz^4$, $n=5$;
d) $A=x^2y^4z$, $n=14$; h) $A=4y^2z^3$, $n=4$;
e) $A=41xyz^2$, $n=3$; i) $A=14xy^3z^3$, $n=2$.

1.6. Birhadning koeffitsiyentini aniqlang:

a) $1{,}5xy^2\left(\frac{2}{3}\right)x^2$; f) $1{,}(51)x^2yz^2\cdot\frac{3}{4}xy$;

b) $\frac{4}{7}xz\cdot\frac{13}{8}x^2y$; g) $1\frac{3}{7}xy^2\cdot\frac{4}{10}z^2$;

d) $\frac{14}{15}x\cdot\frac{15}{28}y\cdot 2y^3$; h) $\frac{11}{13}x^2y^3z$;

e) $0{,}(3)xy\cdot\frac{1}{9}z$; i) $\frac{13}{14}xy\cdot\frac{17}{13}z^2$.

4.16. Tenglikni tekshiring:

a) $\left(\frac{-1+i\sqrt{3}}{2}\right)^4+\left(\frac{-1-i\sqrt{3}}{2}\right)^4=1$; d) $\left(\frac{-\sqrt{3}+i}{2}\right)^5+\frac{-\sqrt{3}-i}{2}^5=\sqrt{3}$;

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4°. $(ab)^n=a^n\cdot b^n\cdot m, n\in N$;
5°. $\left(\frac{a}{b}\right)^n=\frac{a^n}{b^n}$, $a,b\in R$, $b\neq 0$, $n\in N$.

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a) $x^3\cdot x^5$; f) $(x^2)^3$; j) $x^3\cdot x^a$;
b) $x^4\cdot x^5\cdot x^6$; g) $(x^3)^2$; k) $(x^2\cdot x^3)^a$;
d) $-x^3\cdot x^4$; h) $(x^2\cdot x^4)^3$; l) $x^2\cdot(x^3)^4$;
e) $-x^3\cdot x^3$; i) $((x^3)^4)^5$; m) $(x^4)^2\cdot(x^2)^4$.

1.2. Ifodaning qiymatini toping:

a) $\frac{2^5\cdot 11^8}{22^{10}}\cdot\frac{34^4\cdot 2^{10}}{17^5\cdot 8^4}$; f) $\frac{12^5}{2^3\cdot 3^4}:\frac{10^5}{2^6\cdot 5^7}$;

b) $\frac{2^8\cdot 7^9}{14^{10}}:\frac{26^5}{13^6\cdot 8^4}$; g) $\frac{12^5}{2^3\cdot 3^4}\cdot\frac{10^5}{2^6\cdot 5^7}$;

d) $\frac{14^{10}}{2^8\cdot 7^9}:\frac{13^6\cdot 8^4}{26^5}$; h) $\frac{10^5}{2^6\cdot 5^7}:\frac{12^5}{2^3\cdot 3^4}$;

e) $\frac{12^5}{2^3\cdot 4^4}$; i) $\frac{10^5}{2^7\cdot 5^6}\cdot\frac{2^4\cdot 3^3}{12^5}$.

1.3. Birhadning darajasini aniqlang:

a) $3x^4xy^5$; f) $3xy^9z$; j) 15;
b) $-31xy^4$; g) $14x^2y^3z^4$; k) x^4y^2z;
d) $0{,}8x^2y^2$; h) $13yz^{15}$; l) $x\cdot x^2\cdot\ldots\cdot x^9$;
e) 15; i) $43x^2y^3z^{19}$; m) $xyx^2y^2x^4y^4x^6y^6\cdots x^{20}y^{20}$.

1.4. Birhadni standart shaklga keltiring:

a) $13xy\cdot 14x^2y^3$; f) $3xy(-1{,}5)y^3$;
b) $x^2y^2xzy^4$; g) $-ax^2y^2\cdot 6{,}5x^3$;
d) $3x^2z^2y^2\cdot xz^5$; h) $a\cdot xy^2z\cdot y^4\cdot x^5$;
e) $11x^2y\cdot 13x^3y^4$; i) $a(x^2)^3yz^2x^3$.

1.5. A^n ni toping:

a) $A=3x^2yz$, $n=3$; f) $A=2x^2yz^2$, $n=4$;
b) $A=13xy^2$, $n=2$; g) $A=3xz^4$, $n=5$;
d) $A=x^2y^4z$, $n=14$; h) $A=4y^2z^3$, $n=4$;
e) $A=41xyz^2$, $n=3$; i) $A=14xy^3z^3$, $n=2$.

1.6. Birhadning koeffitsiyentini aniqlang:

a) $1{,}5xy^2\left(\frac{2}{3}\right)x^2$; f) $1{,}(51)x^2yz^2\cdot\frac{3}{4}xy$;

b) $\frac{4}{7}xz\cdot\frac{13}{8}x^2y$; g) $1\frac{3}{7}xy^2\cdot\frac{4}{10}z^2$;

d) $\frac{14}{15}x\cdot\frac{15}{28}y\cdot 2y^3$; h) $\frac{11}{13}x^2y^3z$;

e) $0{,}(3)xy\cdot\frac{1}{9}z$; i) $\frac{13}{14}xy\cdot\frac{17}{13}z^2$.

1.7. Ifodani soddalashtiring:

a) $(13a+15b)-(14a-7b)$;
b) $(11x^3-12x^2)+(x^3-x^2+x^4)$; g) $(7a^2-5ax-x^2)+(-2a^2+ax-2x^2)$;
d) $(3a^2x-11x^2)-(3a^2x+6x^2)$; h) $(13x^2-8xy+y^2)+(-11x^2-9xy)$;
e) $(4x^2y+8xy)-(3x^2y-5xy)$; i) $(11xy+13y^2)-(9xy+x^2)$.
f) $(23x-11y+10a)-(-15x+10y-15a)$;

1.8. Amallarni bajaring:

a) $a(a^2+x)-x(a-x)$; f) $-3(a^2-x^2)-2(a^2+x^2)$;
b) $13(x^2+y)+5(x^2-y)$; g) $-(3a-2x)+5(a-2x)$;
d) $2(a-3x)+3(a-2x)$; h) $17(x^2-y^2)-15(y^2-x^2)$;
e) $13(2a-3x)+11(a+x)$; i) $19(x^3y-xz^2)+17(-x^3y+3xz^2)$.

1.9. Ifodani soddalashtiring va o'zgaruvchining ko'rsatilgan qiymatida ifoda qiymatini toping:

a) $(a-4)(a-2)-(a-1)(a-3)$, $a=1{,}75$;
b) $(2a-5)(a+1)-(a+2)(a-3)$, $a=-2{,}6$;
d) $(a-5)(a-1)+(a-2)(a-3)$, $a=1{,}3$;
e) $(x+1)(x+2)+(x+3)(x+4)$, $x=-0{,}4$.

1.10. Ko'phadni ko'paytuvchilarga ajrating:

a) $7ax+14ay$; f) $x(a-c)+y(c-a)$; j) $5x^{a+2}+10x^2$;
b) $3a^2x+6a^4x^3$; g) $a(x-y)-c(y-x)$; k) $a^{3x}-a^{2x}$;
d) $ax+bx+x$; h) $2y(x-3)-5c(3-x)$; l) $a^cx^{2c}+a^cx^c$;
e) a^3-2a^2-a ; i) $5(x-3)-a(3-x)$; m) $15x^{2c+3}-25x^{c+1}$.

1.11. Qisqa ko'paytirish formulalarini isbotlang:

a) $(a-b)(a+b)=a^2-b^2$; b) $(a+b)^2=a^2+2ab+b^2$;
d) $(a-b)^2=a^2-2ab+b^2$; e) $(a+b)(a^2-ab+b^2)=a^3+b^3$;
f) $(a-b)(a^2+ab+b^2)=a^3-b^3$;
g) $(a+b)^3=a^3+3a^2b+3ab^2+b^3$;
h) $(a-b)^3=a^3-3a^2b+3ab^2-b^3$;
i) $(a+b+c)^2=a^2+b^2+c^2+2ab+2ac+2bc$.

1.12. Kasrning qiymatini toping:

a) $\dfrac{35^2-18^2}{72^2-16^2}$; f) $\dfrac{63^2-23^2}{71^2-15^2+86\cdot 24}$;

b) $\dfrac{39{,}5^2-3{,}5^2}{57{,}5^2-14{,}5^2}$; g) $\dfrac{(4^{k+1}+6\cdot 4^k)^3}{(8^{k+2}+2\cdot 8^k)^2}$, $k\in N$;

d) $\dfrac{856^2-44^2}{406}$; h) $\dfrac{(8^{k+1}+8^k)^2}{(4^k-4^{k-1})^3}$, $k\in N$;

e) $\dfrac{71^2-23^2+94\cdot 42}{62^2-32^2}$; i) $\dfrac{(13^2-11^2)(13^2+11^2)}{36^2-12^2}$.

1.13. Ko'paytuvchilarga ajrating:

a) x^2-y^2-x-y ; f) ax^2-a-x^2+x; j) $(x+y)(x^2+y^2)-x^3-y^3$;
b) $x^2-2xy+y^2-c^2$; g) $x^3+y^3+2xy(x+y)$; k) $36a^2-(a^2+9)^2$;
d) $(x-5)^2-16$; h) $x^3-y^3-5x(x^2+xy+y^2)$; l) $8x^3-27y^{18}$;
e) $2x^2-4x+2$; i) $a^4+ax^2-a^3x-x^4$;
m) $(x-y)(x^3+y^3)(x^2+xy+y^2)-(x^6-y^6)$.

1.14. k ning istalgan natural qiymatida

a) $(k+1)^2-(k-1)^2$ ning qiymati 4 ga;
b) $(2k+3)^2-(2k-1)^2$ ning qiymati 8 gà;
d) k^3-k ning qiymati 6 ga;
e) $(3k+1)^2-(3k-1)^2$ ning qiymati 12ga bo'linishini isbotlang.

1.15. Agar $a+b+c=0$ bo'lsa, $a^3+b^3+c^3=3abc$ bo'lishini isbotlang.

1.16. Sonlarni taqqoslang:

a) 45^2-31^2 va 44^2-30^2 ; b) $297\cdot 299$ va 298^2 ;
d) 26^3-24^3 va $(26-24)^3$; e) $(17+13)^2$ va 17^3+13^3 .

1.17. $ab=0$ bo'lsa, $|a+b|$ ning qiymati nimaga teng bo'lishi mumkin? ($\sqrt{x^2}=|x|$ dan foydalaning).

1.18. $|a|^2+|b|^2+|c|^2=$bo'lsa, $(a+b+c)^2$ ning qiymatini toping.

1.19. $(x+y+z)^2-2xy-2xz$ ni soddalashtiring.

1.20. $(x-y-z)^2$ ni ko'phadga aylantiring.

2-§. BIR O'ZGARUVCHILI KO'PHADLAR

$f(x)=a_0x^n+a_1x^{n-1}+\ \ldots\ +a_{n-1}x+a_n$ $(a_0\neq 0)$ ifoda *bir o'zgaruvchili n–darajali ko'phad* deyiladi. $a_0, a_1, \ldots, a_n$ lar uning *koeffitsiyentlaridir.* Ularni haqiqiy sonlar deb hisoblaymiz. x esa o'zgaruvchi bo'lib, kompleks qiymatlar ham qabul qilishi mumkin.

Agar bir o'zgaruvchili ko'phadning ifodasida $x=0$ bo'lsa, ozod had hosil bo'ladi; $x=1$ bo'lsa, barcha koeffitsiyentlar yig'indisi hosil bo'ladi.

$$P(x)=a_0x^n+a_1x^{n-1}+\ \ldots\ +a_{n-1}x+a_n \quad (a_0\neq 0)$$

va

$$D(x)=b_0x^m+b_1x^{m-1}+\ \ldots\ +b_{m-1}x+b_m \quad (b\neq 0)$$

ko'phadlar berilgan bo'lib, n>m bo'lsin.

1 – t e o r e m a. **P(x) va D(x) ko'phadlar uchun $P(x)=Q(x)D(x)+R(x)$ tenglik o'rinli bo'ladigan $Q(x)$**

1.7. Ifodani soddalashtiring:

a) $(13a+15b)-(14a-7b)$;
b) $(11x^3-12x^2)+(x^3-x^2+x^4)$; g) $(7a^2-5ax-x^2)+(-2a^2+ax-2x^2)$;
d) $(3a^2x-11x^2)-(3a^2x+6x^2)$; h) $(13x^2-8xy+y^2)+(-11x^2-9xy)$;
e) $(4x^2y+8xy)-(3x^2y-5xy)$; i) $(11xy+13y^2)-(9xy+x^2)$.
f) $(23x-11y+10a)-(-15x+10y-15a)$;

1.8. Amallarni bajaring:

a) $a(a^2+x)-x(a-x)$; f) $-3(a^2-x^2)-2(a^2+x^2)$;
b) $13(x^2+y)+5(x^2-y)$; g) $-(3a-2x)+5(a-2x)$;
d) $2(a-3x)+3(a-2x)$; h) $17(x^2-y^2)-15(y^2-x^2)$;
e) $13(2a-3x)+11(a+x)$; i) $19(x^3y-xz^2)+17(-x^3y+3xz^2)$.

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d) $ax+bx+x$; h) $2y(x-3)-5c(3-x)$; l) $a^cx^{2c}+a^cx^c$;
e) a^3-2a^2-a ; i) $5(x-3)-a(3-x)$; m) $15x^{2c+3}-25x^{c+1}$.

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f) $(a-b)(a^2+ab+b^2)=a^3-b^3$;
g) $(a+b)^3=a^3+3a^2b+3ab^2+b^3$;
h) $(a-b)^3=a^3-3a^2b+3ab^2-b^3$;
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e) $2x^2-4x+2$; i) $a^4+ax^2-a^3x-x^4$;
m) $(x-y)(x^3+y^3)(x^2+xy+y^2)-(x^6-y^6)$.

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2-§. BIR O'ZGARUVCHILI KO'PHADLAR

$f(x)=a_0x^n+a_1x^{n-1}+\ldots+a_{n-1}x+a_n$ $(a_0\neq 0)$ ifoda *bir o'zgaruvchili n–darajali ko'phad* deyiladi. $a_0, a_1, \ldots, a_n$ lar uning *koeffitsiyentlaridir.* Ularni haqiqiy sonlar deb hisoblaymiz. x esa o'zgaruvchi bo'lib, kompleks qiymatlar ham qabul qilishi mumkin.

Agar bir o'zgaruvchili ko'phadning ifodasida $x=0$ bo'lsa, ozod had hosil bo'ladi; $x=1$ bo'lsa, barcha koeffitsiyentlar yig'indisi hosil bo'ladi.

$$P(x)=a_0x^n+a_1x^{n-1}+\ldots+a_{n-1}x+a_n \quad (a_0\neq 0)$$

va

$$D(x)=b_0x^m+b_1x^{m-1}+\ldots+b_{m-1}x+b_m \quad (b\neq 0)$$

ko'phadlar berilgan bo'lib, n>m bo'lsin.

1 – t e o r e m a. **P(x) va D(x) ko'phadlar uchun $P(x)=Q(x)D(x)+R(x)$ tenglik o'rinli bo'ladigan $Q(x)$**

va $R(x)$ ko'phadlar mavjud va yagonadir, bunda $R(x)$ ning darajasi $D(x)$ darajasidan kichik.

Bu teoryema $P(x)$ ko'phadni $D(x)$ ko'phadga qoldiqli bo'lishni ifodalovchi teoryemadir.

Aytilgan, $Q(x)$ va $R(x)$ ko'phadlarni topishning amaliy usullarini misollarda ko'rsatamiz.

1 – m i s o l. $P(x)=4x^{10}+x^9+5x^7-20x^4-x^3+x^2-25x+5$ ko'phadni $D(x)=x^7-5x^2+1$ ko'phadga qoldiqli bo'lishni bajaring.

Y e c h i s h. «Burchakli bo'lish» usulidan foydalanamiz:

$$\begin{array}{l|l}
\boxed{4x^{10}}+x^9+5x^7 20x^4x^3+x^2 25x+5 & \boxed{x^7}\ 5x^2+11 \\
- & \\
\underline{4x^{10} \qquad -20x^5+4x^3} & \underline{\qquad\qquad 4x^3+x^2+5=Q(x)} \\
\boxed{x^9}+5x^7+20x^5-20x^4-5x^3+x^2-25x+5 & \\
- & \\
\underline{x^9 \qquad -5x^4 \quad +x^2} & \\
\boxed{5x^7}+20x^5-15x^4-5x^3 \quad -25x+5 & \\
- & \\
\underline{5x^7 \qquad\qquad -25x^2+5} & \\
20x^5-15x^4-5x^3+25x^2-25x=R(x). &
\end{array}$$

Bosh hadni bosh hadga bo'lish jarayoni darajasi bo'luvchining darajasidan kichik bo'lgan $R(x)$ ko'phad hosil qilinguncha davom ettiriladi.

2-m i s o l. $P(x)=x^5+6x^4+11x^3+5x^2-2x$ ko'phadni $D(x)=x^3+3x^2+x-1$ ko'phadga qoldiqli bo'lishni bajaring.

Y e c h i s h. «Aniqmas koeffitsiyentlar» usulidan foydalanamiz.

$R(x)$ ning darajasi 5, $D(x)$ ning darajasi esa 3 bo'lgani uchun $Q(x)$ ning darajasi 2 ga, $R(x)$ ning darajasi esa ko'pi bilan 2 ga teng bo'ladi. Shu sababli, $Q(x)$ va $R(x)$ larni $Q(x)=ax^2+bx+c$, $R(x)=dx^2+ex+m$ ko'rinishda izlaymiz, bu yerda a, b, c, d, e, m lar aniqlanishi lozim bo'lgan noma'lum koeffitsiyentlar.

$x^5+6x^4+11x^3+5x^2-2x=(x^3+3x^2+x-1)(ax^2+bx+c)+dx^2+ex+m$ tenglik o'rinli bo'lsin. Bu tenglikning o'ng tomonida ko'rsatilgan amallarni bajarib, o'xshash qo'shiluvchilarni ixchamlasak, quyidagi tenglik hosil bo'ladi:

$$x^5+6x^4+11x^3+5x^2-2^x=ax^5+(3a+b)x^4+(a+3b+c)x^3+$$
$$+(-a+b+3c+d)x^2+(-b+c+e)x+(-c+m).$$

Ko'phadlarning tenglik shartidan foydalanib quyidagi sistemani tuzamiz:

$$\begin{cases} a=1, \\ 3a+b=6, \\ a+3b+c=11, \\ -a+b+3c+d=5, \\ c-b+e=-2, \\ -c+m=0. \end{cases}$$

Bu sistemani yechib, a=1, b=3, c=1, d=0, e=0, m=1 larni topamiz. Demak, $Q(x)=x^2+3x+1$, $R(x)=1$.

3 – m i s o l. $x^4-2x^3+3x^2+4x+1$ ni x^2+x-2 ga bo'lishdan chiqqan qoldiqni toping.

Y e c h i s h. Bo'linuvchining darajasi 4 ga, bo'luvchining darajasi 2 ga teng bo'lgani uchun to'liqsiz bo'linmaning darajasi 2 ga teng bo'ladi. Qoldiq esa birinchi darajali ko'phad yoki o'zgarmas son bo'lishi mumkin:

$$x^4-2x^3+3x^2+4x+1=(x^2+x-2)(ax^2+bx+c)+(d+r).$$

Bu tenglik x ning istalgan qiymatida, jumladan $x^2+x-2=0$ bo'ladigan qiymatlarda ham to'g'ridir. $x^2+x-2=0$ dan $x=-2$, $x=1$ larni topamiz.

Yuqoridagi tenglikda, dastlab $x=-2$, so'ngra $x=1$ desak, d va r larni topish imkonini beruvchi quyidagi sistema hosil bo'ladi:

$$\begin{cases} 37=-2d+r \\ 7=d+r \end{cases}$$

Bundan, $d=-10$, $r=17$ larni topamiz.

Shunday qilib, ko'phadlarni bo'lishdagi qoldiq $-10x+17$ dan iborat ekan.

4 – m i s o l. $P(x)=3(9x^2-7x)^{99}+7(x^5-1)^{100}-x^2+x-7$ ko'phad barcha koeffitsiyentlarining yig'indisini toping.

Y e c h i s h. $P(x)=ax^n+bx^{n-1}+\ldots$ ko'phadda $x=1$ desak, koeffitsiyentlar yig'indisiga ega bo'lamiz. Bizning misolda $P(1)=3\cdot(9\cdot1^2-7\cdot1)^{99}+7(1^5-1)^{100}-1^2+1-7=3\cdot2^{99}-7$.

Javob: $3\cdot2^{99}-7$.

2.1. $f(x)=x^2-3x^2+2x-1$ ko'phad berilgan. Quyidagilarni hisoblang:

a) $f(2)$; b) $f(i)$; d) $f(i+1)$; e) $f(\sqrt{2})$;
f) $f(-i)$; g) $f(i+1)$; h) $f(\sqrt{3}-i)$; i) $f(\sqrt{3}-1)$;
j) $f(x-1)$; k) $f(a)$; l) $f(2^n)$; m) $f(\frac{1}{\sqrt{3}})$.

2.2. Ko'phad koeffitsientlarining yig'indisini toping:

a) $f(x)=(4x-1)^{1999}(2x-1)^{2000}+(8x-1)^2(4x-1)$;
b) $f(x)=(3x-2)^{2000}(3x-1)^{199}+(8x+1)^2+2$;
d) $f(x)=(x-2)^{200}(2-x)+(4-x)^{99}(x-1)^{20}+3$;
e) $f(x)=(x-1)(x-2)^{20}+(4-4x)^{18}(x+3)^2+17$.

2.3. $f(x)$ ko'phad koeffitsientlarining yig'indisi m ga teng. a ni toping:

a) $f(x)=x^3+ax^2+3x+1$, $m=5$;
b) $f(x)=7x^3+2x^2+ax+2$, $m=4$;
d) $f(x)=12x^4+2x^3+ax^2+1$, $m=12$;
e) $f(x)=ax^5+4x^4+8x+1$, $m=-4$.

2.4. Ko'phadning ozod hadini toping:

a) $f(x)=(3x^2-1)^{20}(4x+1)^{15}-x^{20}+15$;
b) $f(x)=(3x-4)^{18}(13x-1)^{16}+x^{17}-15$;
d) $f(x)=(2x+1)^{15}(3x^2+2)^4+(x-2)^2+17$;
e) $f(x)=(3x+1)^2(3x+4)^3(x+1)^{200}+(x-1)^{20}+19$.

2.5. $f(x)$, $g(x)$ lar teng ko'phadlar bo'lsa, a, b larni toping:

a) $f(x)=ax^7+3x^6+x^2+1$, $g(x)=3x^6+bx^2+1$;
b) $f(x)=ax^3+bx^2+3x+2$, $g(x)=x^3+bx^2+3x+2$;
d) $f(x)=ax^3+2x+3$, $g(x)=4x^3+bx+$;
e) $f(x)=ax^8+bx^3+9$, $g(x)=ax^{10}+4x^3+ax^2+9$.

2.6. $x+5=a(x-2)(x-3)+b(x-1)(x-3)+c(x-1)(x-2)$ tenglik ayniyat bo'lsa, a, b, c larni toping.

2.7. Ko'phadlar yig'indisini toping:

a) $f(x)=x^{88}+3x^{77}+4x^2+1$, $g(x)=4x^{88}+3x^{65}+15$;
b) $f(x)=x^4-5x^3+4x^2-1$, $g(x)=-x^4+6x^3+x+2$;
d) $f(x)=x^6+5x^2+11x+4$, $g(x)=2x^6+x^4+3x^3+5$;
e) $f(x)=x^7+x^6+5x^4+12$, $g(x)=7x^3+8x^2-11$.

2.8. Ko'phadlar yig'indisining darajasini toping:

a) $f(x)=(x-1)^7(x-2)^5+3x$, $g(x)=(2x-4)^{12}+4x^2$;
b) $f(x)=(2x+5)^{15}+3x^4+4$, $g(x)=(2x+3)^{16}-4x^3+x+1$;
d) $f(x)=(3x+5)^{15}+31x^5+2$, $g(x)=-(3x+11)^{15}+33x^6+4$;
e) $f(x)=x^7+x^6+3x^2+x+3$, $g(x)=-x^7+2x^6+4x^5+2$.

2.9. 2.7 – misoldagi ko'phadlar uchun f(x)–g(x) ni toping.

2.10. Ko'phadlarni ko'paytiring:

a) $f(x)=5x^4+4x^2+x+2$, $g(x)=4x$;
b) $f(x)=4x^4+3x^3+2$, $g(x)=4x^3+7x+1$;
d) $f(x)=11x^4+3x^2+3x+5$, $g(x)=5x^6+7x^2+4x+2$;
e) $f(x)=13x^3+4x^2+x+2$, $g(x)=2x^2+5x+6$.

2.11. $P(x)$ ni $D(x)$ ga qoldiqli bo'lishni bajaring:

a) $P(x)=x^3+5x^2+5x+3$, $D(x)=x^2+4x+1$;
b) $P(x)=x^3+5x^2+5x+3$, $D(x)=x+1$;
d) $P(x)=x^4+5x^3+9x^2+11x+6$, $D(x)=x^2+3x+1$;
e) $P(x)=x^4+5x^3+9x^2+11x+6$, $D(x)=x^2+2x+1$;
f) $P(x)=3x^5+2x^4-10x^3+5x^2+x+10$, $D(x)=x^3-x^2+x-1$;
g) $P(x)=3x^5+2x^4-10x^3+5x^2+x+10$, $D(x)=x^2+3x-4$;
h) $P(x)=4x^6+3x^5-15x^2+4x+5$, $D(x)=x^3+4x^2-1$;
i) $P(x)=4x^6+3x^5-15x^2+4x+5$, $D(x)=x^4-4x+2$;
j) $P(x)=3x^4+3x^2+5x+4$, $D(x)=x^2+3x+2$;
k) $P(x)=x^5+3x^4+9x^3+12x^2+20x$, $D(x)=x^3+4x$;
l) $P(x)=x^5+3x^4+9x^3+12x^2+20x$, $D(x)=x^2+3x+5$;
m) $P(x)=4x^4+5x^2+6x+11$, $D(x)=x^2+5x-4$.

2.12. Ko'phadlarning eng katta umumiy bo'luvchisini Yevklid algoritmi yordamida toping:

a) $x^4+x^3-3x^2-4x-1$, x^3+x^2-x-1 ;
b) $x^5+x^4-x^3-2x-1$, $3x^4+2x^3+x^2+2x-2$;
d) $x^6-7x^4+8x^3-7x+7$, $3x^5-7x^3+3x^2-7$;
e) $x^5-2x^4+x^3+7x^2-12x+10$, $3x^4-6x^3+5x^2+2x-2$;
f) $x^6+2x^4-4x^3-3x^2+8x-5$, x^5+x^2-x+1;
g) $x^5+3x^4-12x^3-52x^2-52x-12$, $x^4+3x^3-6x^2-22x-12$;
h) $x^5+x^4-x^3-3x^2-3x-1$, $x^4-2x^3-x^2-2x+1$;
i) x^4-4x^3+1; x^3-3x^2+1.

V b o b. ALGEBRAIK IFODALAR

1-§. RATSIONAL ALGEBRAIK IFODALAR VA ULAR USTIDA SHAKL ALMASHTIRISHLAR

1.1. O'zgaruvchining ifoda ma'noga ega bo'lmaydigan barcha qiymatlari to'plamini toping:

a) $\frac{5-x}{x-2}$; f) $\frac{3a}{3+2a}$; j) $\frac{3x}{x(x+2)}$; n) x^2+x+2 ;

b) $\frac{x^2+3}{x^2+4}$; g) $\frac{a-4}{5}$; k) $\frac{x-2}{a^2-x^2}$; o) $\frac{x-1}{x}+\frac{7}{x-3}$;

d) $\frac{x+3}{(x-1)(x-2)}$; h) $\frac{a^2-5}{a-4,5}$; l) $\frac{x}{x^2-16}$; p) $\frac{4x}{x+5}+\frac{8x^2}{x-9}$;

e) $\frac{x^2-4}{x^2-9}$; i) $\frac{13a+2}{26-2a}$; m) $\frac{y}{3y(y-5)}$; q) $\frac{31x^2}{9x-9}+x^2-x$.

1.2. O'zgaruvchining ifoda ma'noga ega bo'ladigan barcha haqiqiy qiymatlari to'plamini tuzing:

a) $\frac{3}{x+2}$; g) $\frac{a+5}{4-a}$; l) $\frac{x+2}{7x-7}+\frac{13}{x-7}$;

b) $\frac{x^3+13}{x^2+5}$; h) $\frac{3a+13}{4a^2-1}$; m) $\frac{x^2+x-3}{x^2-5x}+\frac{1}{x}$;

d) $\frac{x+5}{x^2-9}$; i) $\frac{17a}{(a-1)(a-2)(a-3)}$; n) x^2-x-1;

e) $\frac{3x+5}{4x^2-9}$; j) $\frac{x+4}{x-3}+\frac{1}{x+2}$; o) $\frac{x-2}{x^2-a^2}$;

f) $\frac{11a}{13-a^2}$; k) $\frac{7x-4}{x^2-16}+x+2$; p) $\frac{x}{x^2+x+1}+x^2$;

q) $x^2-\frac{1}{(x-1)(x-4)}$;

1.3. Ifodaning aniqlanish sohasini toping:

a) $\frac{2x-y}{x(x-y)}$; f) $\frac{x}{x-2}+\frac{y}{y(x-3)}$; j) $x+y+\frac{x}{y-4}$;

b) $\frac{x}{x^2-y^2}$; g) $\frac{x-1}{x}+\frac{y}{3x-y}$; k) $xy+x^2y-\frac{y}{x+3}$;

d) $\frac{x+y}{x^2-y}$; h) $\frac{y}{x-y}-\frac{x}{x+y}$; l) $1+x^3y+x^4y^2$;

e) $\frac{x-2y}{x^2-y}$; i) $\frac{3x+y}{x^3-y^3}-\frac{y}{3x-3}$; m) $13-2x^2+(x-y)^2$.

1.4. Kasrni qisqartiring:

a) $\frac{21a^3-6a^2b}{12ab-42a^2}$; h) $\frac{a^2-3a}{a^2+3a-18}$;

b) $\frac{6m^3-3mn^2}{2m^3n+mn^2}$; i) $\frac{4x^2-8x+3}{4x^2-1}$;

d) $\frac{x^2-2mx+3x-6m}{x^2+2mx+3x+6m}$; j) $\frac{m^2+4m-5}{m^2+7m+10}$;

e) $\frac{8ab+2a-20b-5}{4ab-8b^2+a-2b}$; k) $\frac{x^2+10x+25}{(x+5)^3}$;

f) $\frac{16a^2-8ab+b^2}{16a^2-b^2}$; l) $\frac{(x-2)^2}{(2-x)^2}$;

g) $\frac{9x^2-25y^2}{9x^2+30xy+25y^2}$; m) $\frac{x^6+x^4}{x^4+x^2}$.

1.5. Quyida keltirilgan ifodalar orasidagi butun ratsional ifodalar to'plamini tuzing:

$3x^2+a$; $3x^2+\frac{1}{y}$; $3x^2+\frac{1}{2}$; $4a^2-x(a-3x)$; $\frac{x^2}{x-4}$; $\frac{x^3}{4}$;

$6x-\frac{1}{2}$; $\frac{x^2+y}{1\frac{1}{2}-0,(5)x}$; $\frac{xyz-\frac{1}{z}}{3-1\frac{1}{4}}$; $xy+\sqrt{z}-\frac{z^2}{14}$.

Amallarni bajaring (1.6 – 1.8.):

1.6. a) $\frac{a-2}{2}-1-\frac{a-3}{3}$; f) $c-\frac{(x+c)^2}{2x}$;

b) $\frac{a+x}{4}-a+x$; g) $a+x-\frac{a^2+x^2}{a-x}$;

d) $4a-\frac{a-1}{4}-\frac{a+2}{3}$; h) $\frac{a}{4x}+\frac{5}{12y}-\frac{c}{9xy^2}$;

e) $\frac{(a-x)^2}{2a}+x$; i) $1-\frac{x}{x-y}-\frac{1}{x+y}$;

1.7. a) $\frac{a^2}{ax-x^2}+\frac{x}{x-a}$; f) $\frac{x-25}{5x-25}-\frac{3x+5}{5x-x^2}$;

b) $\frac{x^2-4xy}{2y^2-xy}-\frac{4y}{x-2y}$; g) $\frac{12-y}{6y-36}+\frac{6}{6y-y^2}$;

d) $\frac{x}{2a^2-ax}-\frac{4a}{2ax-x^2}$; h) $3x-\frac{x-y}{2-x}+\frac{x+y}{4}$;

e) $\frac{4y}{3x^2+2xy}-\frac{9x}{3xy+2x^2}$; i) $\frac{x-12a}{x^2-16a^2}+\frac{4x}{4ax-x^2}$.

1.8. a) $\frac{a^2+3a}{ax-5x+8a-40}$; d) $\frac{x}{3ax-2-x+6a}-\frac{x}{3a-1}$;

b) $\frac{y}{3x-2}-\frac{3y}{6xy+9x-4y-6}$; e) $\frac{3x}{2y+3}+\frac{x^2+3x}{4xy-3-2y+6x}$.

1.9. Kasr ko'rinishida ifodalang:

a) $\frac{x^2-xy}{y}\cdot\frac{y^2}{x^3}$; h) $\frac{kx+k^2}{x^2}\cdot\frac{x}{x+k}$;

b) $\frac{3a}{b^2}\cdot\frac{ab+b^2}{9}$; i) $\frac{ax+ay}{xy^2}\cdot\frac{x^2y}{3x+3y}$;

d) $\frac{x-y}{xy}\cdot\frac{2xy}{xy-y^2}$; j) $\frac{xy}{a^2+a^3}\cdot\frac{a+a^2}{x^2y^2}$;

e) $\frac{4ab}{cx+bx}\cdot\frac{ax+bx}{2ab}$; k) $\frac{6a}{x^2-x}\cdot\frac{2x-2}{3ax}$;

f) $\frac{xa-xy}{3c^2}\cdot\frac{2x}{cy-ca}$; l) $\frac{x^2-y^2}{2xy}\cdot\frac{2x}{x+y}$;

g) $\frac{ax-ay}{5x^2y^2}\cdot\frac{5xy}{by-bx}$; m) $\frac{4x^2}{x^2-9}\cdot\frac{3a-ax}{4x}$.

1.10. Soddalashtiring:

a) $\frac{x^2-4x}{x^2+7x}:\frac{24-6x}{49-x^2}$; f) $\frac{(x+3)^2}{2x-4}:\frac{3x+9}{x^2-4}$;

b) $\frac{y^3-16y}{2y+18}:\frac{4-y}{y^2+9y}$; g) $\frac{(x-3)^2}{x-8}:\frac{4x-12}{3x-24}$;

d) $\frac{(a+b)^2-2ab}{4a^2}:\frac{a^2+b^2}{ab}$; h) $\frac{a+b}{(a-b)^2}:\frac{(a+b)^2}{(a-b)^3}$;

e) $\frac{5c^3-5}{c+2}:\frac{(c+1)^2-c}{13c+26}$; i) $\frac{(3c-b)^2}{3c+b}:\frac{3c-b}{(3c+b)^2}$.

1.11. Ifodani soddalashtiring:

a) $\left(\frac{7(m-2)}{m^3-8}-\frac{m+2}{m^2+2m+4}\right)\cdot\frac{2m^2+4m+8}{m-3}$;

b) $\frac{a+5}{a^2-9}:\left(\frac{a+2}{a^2-3a+9}-\frac{2(a+8)}{a^3+27}\right)$;

d) $\left(\frac{x+2}{3x}-\frac{2}{x-2}-\frac{x-14}{3x^2-6x}\right):\frac{x+2}{6x}\cdot\frac{1}{x-5}$;

e) $\frac{1}{2}+\left(\frac{3m}{1-3m}+\frac{2m}{3m+1}\right)\cdot\frac{9m^2-6m+1}{6m^2+10m}$;

f) $\left(\frac{1}{x+y}-\frac{y^2}{xy^2-x^3}\right):\left(\frac{x-y}{x^2+xy}-\frac{x}{x^2+xy}\right)-\frac{x}{x-y}$;

g) $\frac{2a+3}{2a-3}\cdot\left(\frac{2a^2+3a}{4a^2+12a+9}-\frac{3a+2}{2a+3}\right)+\frac{4a-1}{2a-3}-\frac{a-1}{a}$;

h) $\left(\frac{a+3}{a^2+2a+1}+\frac{a-1}{a^2-2a-3}\right)\cdot\frac{a^2-2a-3}{a+2}-1$;

i) $\frac{3(m+3)}{m^2+3m+9}+\frac{m^2-3m}{(m+3)^2}\cdot\left(\frac{3m}{m^3-27}+\frac{1}{m-3}\right)$.

1.12. Ifodani soddalashtiring:

a) $\left(\frac{a}{a-b}-\frac{b}{a+b}\right):\left(\frac{a+b}{b}-\frac{a-b}{a}\right)$;

b) $\left(2x+1-\frac{1}{1-2x}\right):\left(2x-\frac{4x^2}{2x-1}\right)$;

d) $\left(p-q+\frac{4q^2-p^2}{p+q}\right):\left(\frac{p}{p^2-q^2}+\frac{2}{q-p}+\frac{1}{p+q}\right)$;

e) $\left(\frac{2}{2x+y}-\frac{1}{2x-y}-\frac{3y}{y^2-4x^2}\right)\cdot\left(\frac{y^2}{8x^2}-\frac{1}{2}\right)$;

f) $\left(\frac{5x+y}{x^2-5xy}+\frac{5x-y}{x^2+5xy}\right)\cdot\frac{x^2-25y^2}{x^2+y^2}$;

g) $\frac{9a^2-16b^2}{7a}\cdot\left(\frac{3b-4a}{4b^2-3ab}-\frac{3b+4a}{4b^2+3ab}\right)$;

h) $\frac{4xy}{y^2-x^2}:\left(\frac{1}{y^2-x^2}+\frac{1}{x^2+2xy+y^2}\right)$;

i) $\frac{a-2}{a^2+2a}:\left(\frac{a}{a^2-2a}-\frac{a^2+4}{a^3-4a}-\frac{1}{a^2+2a}\right)$;

j) $\frac{4a-5}{a^2-9}+\frac{9(a-3)}{15-27a-4a^2}\cdot\frac{4a^2-17a+15}{a-2}-\frac{7}{a+3}$;

k) $(a^2-y^2-x^2+2xy):\frac{a+y-x}{a+y+x}$;

l) $\frac{a^2-1}{x^2+ax}\cdot\left(\frac{x}{x-1}-1\right)\cdot\frac{a-ax^3-x^4+x}{1-a^2}$ $(x=-1)$;

m) $\frac{x}{ax-2a^2}-\frac{2}{x^2+x-2ax-2a}\cdot\left(1+\frac{3x+x^2}{x+3}\right)$.

1.13. Kasrni qisqartiring:

a) $\frac{x^2-x+1}{x^4+x^2+1}$;

b) $\frac{x^{14}-x^7+1}{x^{21}+1}$;

d) $\frac{x(y-a)-y(x-a)}{x(y-a)^2-y(x-a)^2}$;

e) $\frac{x^{33}-1}{x^{33}+x^{22}+x^{11}}$.

1.14. k ning qanday qiymatlarida $\frac{(k-3)^2}{k}$ ifoda natural qiymatlar qabul qiladi?

1.15. Ifodani soddalashtiring va o'zgaruvchilarning ko'rsatilgan qiymatlarida ifodaning qiymatini hisoblang:

a) $\left(\frac{x-2y}{x^3+y^3}+\frac{y}{x^3-x^2y+xy^2}\right)\cdot\frac{x^3-xy^2}{x^2+y^2}+\frac{2y^2}{x^3+x^2y+xy^2+y^3}$;

$x=0{,}2;\ y=0{,}8;$

b) $\frac{1}{a(a-b)(a-c)}+\frac{1}{b(b-a)(b-c)}+\frac{1}{c(c-a)(c-b)}$;

$a=\frac{1}{3};\ b=\sqrt{3};\ c=\frac{\sqrt{3}}{2}$.

2-§. IRRATSIONAL IFODALAR VA ULAR USTIDA SHAKL ALMASHTIRISHLAR. n-DARAJALI ILDIZ VA UNING XOSSALARI

2.1. Ifoda ma'noga egami:

a) $\sqrt[3]{-9}$; f) $\sqrt[6]{-0{,}25}$; j) $\sqrt[3]{i}$; n) $\sqrt[8]{x-y}$, bunda $x<y$;
b) $\sqrt{-9}$; g) $\sqrt{0{,}25}$; k) $\sqrt[3]{-i}$; o) $\sqrt[7]{x-y}$, bunda $x\le y$;
d) $\sqrt[3]{9}$; h) $\sqrt[4]{-81}$; l) $\sqrt[4]{i}$; p) $\sqrt[8]{y-x}$, bunda $x\le y$;
e) $\sqrt{9}$; i) $\sqrt[7]{-2}$; m) $\sqrt[4]{-i}$; q) $\sqrt[9]{y-x}$, bunda $x\ge y$?

2.2. Ifoda o'zgaruvchining qanday qiymatlarida ma'noga ega:

a) $\sqrt{-x}$; f) $\sqrt[3]{x-1}$; j) $\sqrt[4]{-x^2}+\sqrt[4]{x^2-1}$;
b) $\sqrt[4]{x^2}$; g) $\sqrt[5]{(x+1)^2}$; k) $\sqrt{x^2-6x+9}$;
d) $\sqrt[6]{x^2+4}$; h) $\sqrt[7]{16x}$; l) $\sqrt{x^2+2x+2}$;
e) $\sqrt[8]{(x+4)^2}$; i) $\sqrt[3]{-x+2}$; m) $\sqrt[6]{-(x-3)^2}$?

2.3. Tenglik o'zgaruvchining qanday qiymatlarida to'g'ri:

a) $\sqrt{(x-2)^2}=2-x$; f) $\sqrt[3]{x-3}=\sqrt[3]{3-x}$; j) $\sqrt[3]{-x}=2$;
b) $\sqrt{(x+3)^2}=x+3$; g) $\sqrt[3]{x-3}=0$; k) $\sqrt[3]{-x}=-2$;
d) $\sqrt{(x-3)^2}=x-3$; h) $\sqrt{x^2-1}=-1$; l) $\sqrt{x^2-6x+9}=1$;
e) $\sqrt{(x-4)^2}=4-x$; i) $\sqrt{x}=1$; m) $\sqrt[3]{x-2}=1$?

2.4. Ko'paytmadan ildiz chiqaring:

a) $\sqrt{16 \cdot 121}$;
b) $\sqrt[3]{-125 \cdot 27}$;
d) $\sqrt[4]{16 \cdot 81}$;
e) $\sqrt[5]{32 \cdot 243}$;
f) $\sqrt{9 \cdot 25 \cdot 26 \cdot 49}$;
g) $\sqrt[3]{8 \cdot 27 \cdot 64 \cdot 125}$;
h) $\sqrt[4]{81 \cdot 625 \cdot 256}$;
i) $\sqrt{0{,}01 \cdot 0{,}09 \cdot 0{,}25}$.

2.5. Bo'linmadan ildiz chiqaring:

a) $\sqrt{\frac{36}{49}}$; b) $\sqrt[3]{-\frac{64}{27}}$; d) $\sqrt[4]{\frac{16}{81}}$; e) $\sqrt[5]{\frac{243}{32}}$;
f) $\sqrt{\frac{25}{64}}$; g) $\sqrt[3]{\frac{64}{125}}$; h) $\sqrt[4]{\frac{81}{625}}$; i) $\sqrt{\frac{0{,}01}{0{,}09}}$.

2.7. Darajadan ildiz chiqaring:

a) $\sqrt[4]{158}$; b) $\sqrt[4]{(-15)^8}$; d) $\sqrt[3]{-5^6}$; e) $\sqrt{\left(\frac{1}{3}\right)^4}$;
f) $\sqrt[4]{x^4}$, bunda $x \le 0$;
g) $\sqrt[3]{x^6}$, bunda $x \in R$;
h) $\sqrt{(x^2+1)^2}$, bunda $x \in R$;
i) $\sqrt{x^6}$, bunda $x \ge 0$.

2.8. Ildizdan ildiz chiqaring:

a) $\sqrt[3]{\sqrt[3]{16}}$; b) $\sqrt[4]{\sqrt[3]{76}}$; d) $\sqrt[5]{\sqrt[3]{4}}$; e) $\sqrt[7]{\sqrt[3]{25}}$;
f) $\sqrt[7]{\sqrt[3]{x^2}}$, bunda $x \le 0$;
g) $\sqrt[3]{\sqrt{x}}$, bunda $x \ge 0$;
h) $\sqrt[3]{\sqrt[4]{x}}$, bunda $x \ge 0$;
i) $\sqrt[3]{\sqrt[3]{x}}$, bunda $x \in R$.

2.9. Ildizni darajaga ko'taring:

a) $(\sqrt[4]{2})^3$; b) $(\sqrt[6]{16})^3$; d) $(\sqrt[3]{-2})^5$; e) $(\sqrt[4]{4})^2$;
f) $(\sqrt[4]{x})^3$; g) $(\sqrt[4]{x^2})^6$; h) $(\sqrt[4]{x}+2)^5$; i) $(\sqrt[3]{x^4})^6$.

2.10. Berilgan ildizni bir xil ko'rsatkichli ildizga aylantiring.

a) $\sqrt{3}$ va $\sqrt[3]{4}$;
b) $\sqrt[3]{2}$ va $\sqrt[4]{4}$;
d) $\sqrt{5}$ va $\sqrt[4]{6}$;
e) $\sqrt[5]{2}$ va $\sqrt[3]{3}$;
f) $\sqrt{x}$ va $\sqrt[8]{y}$;
g) $\sqrt[3]{x+1}$ va $\sqrt[7]{y}$;
h) $\sqrt{x^2+1}$ va $\sqrt[6]{y^2-1}$;
i) $\sqrt[5]{x-y}$ va $\sqrt[4]{y}$.

Ratsional ko'rsatkichli daraja

2.11. Ifoda ma'noga egami:

a) $3^{-\frac{4}{3}}$; b) $(-3)^{-\frac{1}{3}}$; d) $4^{\frac{1}{9}}$; e) $(-3)^{-\frac{2}{3}}$; f) $(\sqrt[3]{-4})^{\frac{1}{2}}$;
g) $(\sqrt{4})^{\frac{2}{5}}$; h) $(x-1)^{\frac{1}{3}}$, $(x<1)$; i) $(x+2)^{\frac{1}{4}}$, $(x \ge -2)$?

2.12. O'zgaruvchining ifoda ma'noga ega bo'ladigan barcha qiymatlarini toping:

a) $4{,}5^{\frac{x}{2}}$, bunda $x \in Q$;
b) $(-4{,}5)^{\frac{x}{2}}$, bunda $x \in Q$;
d) $(3+x)^{\frac{1}{5}}$;
e) $(x^2+1)^{\frac{1}{3}}$;
f) $\left(\frac{x}{2}\right)^{-\frac{1}{4}}$;
g) $(|x|+1)^{\frac{2}{3}}$;
h) $(1-|x|)^{\frac{4}{5}}$;
i) $(1-|x|)^{-3}$.

2.13. Hisoblang:

a) $49^{\frac{1}{2}}$; f) $9^{2\frac{1}{2}}$; j) $9^{-1.5}$; n) $27^{-\frac{5}{6}} \cdot 3^{2.5}$;
b) $1000^{\frac{1}{3}}$; g) $0{,}16^{-1\frac{1}{6}}$; k) $\left(\frac{1}{8}\right)^{-\frac{4}{3}}$; o) $\left(\frac{1}{8}\right)^{-\frac{4}{3}}$;
d) $4^{-\frac{1}{2}}$; h) $0{,}008^{1\frac{1}{3}}$; l) $\left(\frac{1}{64}\right)^{-\frac{4}{3}}$; p) $\left(\frac{1}{8}\right)^{\frac{3}{2}}$;
e) $8^{-\frac{1}{3}}$; i) $\left(3\frac{3}{8}\right)^{\frac{4}{3}}$; m) $(25)^{-\frac{3}{2}}$; q) $\left(\frac{4}{9}\right)$.

2.14. Ifodaning qiymatini toping:

a) $\left(\left(\frac{3}{4}\right)^0\right)^{-0.5} - 7{,}5 \cdot 4^{-\frac{2}{3}} - (-2)^{-4} + 81^{0.25}$;

b) $0{,}027^{-\frac{1}{3}} - \left(-\frac{1}{6}\right)^{-2} + 256^{0.75} - 3^{-1} + (5{,}5)^0$;

d) $\left(\frac{9}{16}\right)^{-\frac{1}{10}} : \left(\frac{25}{36}\right)^{-\frac{3}{2}} - \left(\left(\frac{4}{3}\right)^{-\frac{1}{2}}\right)^{-\frac{2}{5}} \cdot \left(\frac{6}{5}\right)^{-3}$;

e) $\left(9^{-\frac{2}{3}}\right)^{\frac{3}{4}} - (25^{2.5})^{-0.1} + \left(\left(\frac{3}{4}\right)^{-1} \cdot \left(\frac{2}{9}\right)^{\frac{6}{7}}\right)^0 : 36^{\frac{1}{2}} + \frac{1}{\sqrt{5}}$;

f) $\left(4^{-\frac{1}{4}} + \left(\frac{1}{2^{-\frac{1}{2}}}\right)^{-\frac{4}{3}}\right) \cdot \left(4^{-0.25} - (2\sqrt{2})^{-\frac{4}{3}}\right)$;

g) $(0{,}004)^{-1.5} \cdot (0{,}125)^{-\frac{4}{3}} - \left(\frac{1}{121}\right)^{-\frac{1}{2}}$;

h) $\dfrac{2 \cdot 4^{-2} + \left(81^{-\frac{1}{2}}\right)^3 \cdot \left(\frac{1}{9}\right)^{-3}}{125^{-\frac{1}{3}} \cdot \left(\frac{1}{5}\right)^{-2} + (\sqrt{3})^0 \cdot \left(\frac{1}{2}\right)^{-2}}$.

2.4. Ko'paytmadan ildiz chiqaring:

a) $\sqrt{16 \cdot 121}$;
b) $\sqrt[3]{-125 \cdot 27}$;
d) $\sqrt[4]{16 \cdot 81}$;
e) $\sqrt[5]{32 \cdot 243}$;
f) $\sqrt{9 \cdot 25 \cdot 26 \cdot 49}$;
g) $\sqrt[3]{8 \cdot 27 \cdot 64 \cdot 125}$;
h) $\sqrt[4]{81 \cdot 625 \cdot 256}$;
i) $\sqrt{0{,}01 \cdot 0{,}09 \cdot 0{,}25}$.

2.5. Bo'linmadan ildiz chiqaring:

a) $\sqrt{\frac{36}{49}}$; b) $\sqrt[3]{-\frac{64}{27}}$; d) $\sqrt[4]{\frac{16}{81}}$; e) $\sqrt[5]{\frac{243}{32}}$;
f) $\sqrt{\frac{25}{64}}$; g) $\sqrt[3]{\frac{64}{125}}$; h) $\sqrt[4]{\frac{81}{625}}$; i) $\sqrt{\frac{0{,}01}{0{,}09}}$.

2.7. Darajadan ildiz chiqaring:

a) $\sqrt[4]{158}$; b) $\sqrt[4]{(-15)^8}$; d) $\sqrt[3]{-5^6}$; e) $\sqrt{\left(\frac{1}{3}\right)^4}$;
f) $\sqrt[4]{x^4}$, bunda $x \le 0$;
g) $\sqrt[3]{x^6}$, bunda $x \in R$;
h) $\sqrt{(x^2+1)^2}$, bunda $x \in R$;
i) $\sqrt{x^6}$, bunda $x \ge 0$.

2.8. Ildizdan ildiz chiqaring:

a) $\sqrt[3]{\sqrt{16}}$; b) $\sqrt[4]{\sqrt[3]{76}}$; d) $\sqrt[5]{\sqrt{4}}$; e) $\sqrt[7]{\sqrt[3]{25}}$;
f) $\sqrt[7]{\sqrt[3]{x^2}}$, bunda $x \le 0$;
g) $\sqrt[3]{\sqrt{x}}$, bunda $x \ge 0$;
h) $\sqrt[3]{\sqrt{x}}$, bunda $x \ge 0$;
i) $\sqrt[3]{\sqrt{x}}$, bunda $x \in R$.

2.9. Ildizni darajaga ko'taring:

a) $(\sqrt[4]{2})^3$; b) $(\sqrt[6]{16})^3$; d) $(\sqrt[3]{-2})^5$; e) $(\sqrt[4]{4})^2$;
f) $(\sqrt[4]{x})^3$; g) $(\sqrt[4]{x^2})^6$; h) $(\sqrt[4]{x}+2)^5$; i) $(\sqrt[3]{x^4})^6$.

2.10. Berilgan ildizni bir xil ko'rsatkichli ildizga aylantiring.

a) $\sqrt{3}$ va $\sqrt[3]{4}$;
b) $\sqrt[3]{2}$ va $\sqrt[4]{4}$;
d) $\sqrt{5}$ va $\sqrt[4]{6}$;
e) $\sqrt[5]{2}$ va $\sqrt[3]{3}$;
f) $\sqrt{x}$ va $\sqrt[8]{y}$;
g) $\sqrt[3]{x+1}$ va $\sqrt[7]{y}$;
h) $\sqrt{x^2+1}$ va $\sqrt[6]{y^2-1}$;
i) $\sqrt[5]{x-y}$ va $\sqrt[4]{y}$.

Ratsional ko'rsatkichli daraja

2.11. Ifoda ma'noga egami:

a) $3^{-\frac{4}{3}}$; b) $(-3)^{-\frac{1}{3}}$; d) $4^{\frac{1}{9}}$; e) $(-3)^{-\frac{2}{3}}$; f) $(\sqrt[3]{-4})^{\frac{1}{2}}$;
g) $(\sqrt{4})^{\frac{2}{5}}$; h) $(x-1)^{\frac{1}{3}}$, $(x<1)$; i) $(x+2)^{\frac{1}{4}}$, $(x \ge -2)$?

2.12. O'zgaruvchining ifoda ma'noga ega bo'ladigan barcha qiymatlarini toping:

a) $4{,}5^{\frac{x}{2}}$, bunda $x \in Q$;
b) $(-4{,}5)^{\frac{x}{2}}$, bunda $x \in Q$;
d) $(3+x)^{\frac{1}{5}}$;
e) $(x^2+1)^{\frac{1}{3}}$;
f) $\left(\frac{x}{2}\right)^{-\frac{1}{4}}$;
g) $(|x|+1)^{\frac{2}{3}}$;
h) $(1-|x|)^{\frac{4}{5}}$;
i) $(1-|x|)^{-3}$.

2.13. Hisoblang:

a) $49^{\frac{1}{2}}$; f) $9^{2\frac{1}{2}}$; j) $9^{-1.5}$; n) $27^{-\frac{5}{6}} \cdot 3^{2.5}$;
b) $1000^{\frac{1}{3}}$; g) $0{,}16^{-1\frac{1}{6}}$; k) $\left(\frac{1}{8}\right)^{-\frac{4}{3}}$; o) $\left(\frac{1}{8}\right)^{-\frac{4}{3}}$;
d) $4^{-\frac{1}{2}}$; h) $0{,}008^{1\frac{1}{3}}$; l) $\left(\frac{1}{64}\right)^{-\frac{4}{3}}$; p) $\left(\frac{1}{8}\right)^{\frac{3}{2}}$;
e) $8^{-\frac{1}{3}}$; i) $\left(3\frac{3}{8}\right)^{\frac{4}{3}}$; m) $(25)^{-\frac{3}{2}}$; q) $\left(\frac{4}{9}\right)$.

2.14. Ifodaning qiymatini toping:

a) $\left(\left(\frac{3}{4}\right)^0\right)^{-0.5} - 7{,}5 \cdot 4^{-\frac{2}{3}} - (-2)^{-4} + 81^{0.25}$;

b) $0{,}027^{-\frac{1}{3}} - \left(-\frac{1}{6}\right)^{-2} + 256^{0.75} - 3^{-1} + (5{,}5)^0$;

d) $\left(\frac{9}{16}\right)^{-\frac{1}{10}} : \left(\frac{25}{36}\right)^{-\frac{3}{2}} - \left(\left(\frac{4}{3}\right)^{-\frac{1}{2}}\right)^{-\frac{2}{5}} \cdot \left(\frac{6}{5}\right)^{-3}$;

e) $\left(9^{-\frac{2}{3}}\right)^{\frac{3}{4}} - (25^{2.5})^{-0.1} + \left(\left(\frac{3}{4}\right)^{-1} \cdot \left(\frac{2}{9}\right)^{\frac{6}{7}}\right)^0 : 36^{\frac{1}{2}} + \frac{1}{\sqrt{5}}$;

f) $\left(4^{-\frac{1}{4}} + \left(\frac{1}{2^{-\frac{1}{2}}}\right)^{-\frac{4}{3}}\right) \cdot \left(4^{-0.25} - (2\sqrt{2})^{-\frac{4}{3}}\right)$;

g) $(0{,}004)^{-1.5} \cdot (0{,}125)^{-\frac{4}{3}} - \left(\frac{1}{121}\right)^{-\frac{1}{2}}$;

h) $\dfrac{2 \cdot 4^{-2} + \left(81^{-\frac{1}{2}}\right)^3 \cdot \left(\frac{1}{9}\right)^{-3}}{125^{-\frac{1}{3}} \cdot \left(\frac{1}{5}\right)^{-2} + (\sqrt{3})^0 \cdot \left(\frac{1}{2}\right)^{-2}}$.

2.25. Hisoblang:

a) $\frac{1}{\sqrt{3}+\sqrt{2}}+\frac{1}{\sqrt{4}+\sqrt{3}}+\frac{1}{\sqrt{5}+\sqrt{4}}+\cdots+\frac{1}{\sqrt{37}+\sqrt{36}}$;

b) $\frac{1}{\sqrt{7}+\sqrt{6}}+\frac{1}{\sqrt{8}+\sqrt{7}}+\frac{1}{\sqrt{9}+\sqrt{8}}+\cdots+\frac{1}{\sqrt{23}+\sqrt{22}}$;

d) $\frac{1}{\sqrt{3}-\sqrt{2}}+\frac{1}{\sqrt{2}-\sqrt{3}}-\sqrt{2}-\sqrt{23}$;

e) $\frac{3}{\sqrt{5}-\sqrt{2}}+\frac{5}{\sqrt{7}+\sqrt{2}}-\sqrt{7}+\sqrt{5}$.

2.26. Tenglik to'g'rimi:

a) $\frac{3}{\sqrt{6}-\sqrt{3}}+\frac{4}{\sqrt{7}+\sqrt{3}}=\frac{1}{\sqrt{7}-\sqrt{6}}$; b) $-\frac{2}{\sqrt{8}+\sqrt{6}}+\frac{5}{\sqrt{11}+\sqrt{6}}=-\frac{3}{\sqrt{8}+\sqrt{11}}$;

d) $-\frac{8\sqrt{7}}{\sqrt{5\sqrt{7}}-\sqrt{2\sqrt{7}}}+\frac{4\sqrt{7}}{\sqrt{5\sqrt{7}}+\sqrt{8\sqrt{7}}}=-4\sqrt[4]{175}$;

e) $\frac{4\sqrt{5}}{\sqrt{3\sqrt{5}}-\sqrt{2\sqrt{5}}}-\frac{5\sqrt{5}}{4\sqrt{2\sqrt{5}}-3\sqrt{3\sqrt{5}}}=\sqrt[4]{45}$?

Irratsional ifodalar ustida shakl almashtirishlar

2.27. Hisoblang:

$$\frac{2\cdot 4^{-2}+\left(81^{\frac{1}{2}}\right)^3\cdot\left(\frac{1}{9}\right)^{-3}}{125^{-\frac{1}{3}}\cdot\left(-\frac{1}{5}\right)^{-2}+(\sqrt{3})^0\cdot\left(\frac{1}{2}\right)^{-2}}.$$

2.28. Ifodani soddalashtiring:

a) $\frac{x^{\frac{1}{2}}\cdot y^{\frac{1}{2}}}{z^{\frac{1}{6}}}:\left(\frac{z^{-\frac{1}{2}}}{x^{-\frac{1}{3}}b^{-\frac{1}{3}}}\cdot\frac{x^{\frac{5}{6}}\cdot z^{\frac{2}{3}}}{y^{\frac{5}{6}}}\right)$; b) $\left(\frac{x^{-\frac{1}{2}}\cdot y^{-\frac{1}{3}}}{x^{-\frac{3}{4}}\cdot y^{-\frac{5}{6}}}:\sqrt[4]{x^{-3}\cdot y^{-5}}\right)^{\frac{2}{7}}$;

d) $ab\sqrt[3]{\frac{b}{a^2}}-ab\sqrt[3]{\frac{a}{b^2}}+\frac{a}{b}\sqrt[3]{ab^4}-\frac{b}{a}\sqrt[3]{a^4b}$;

e) $\sqrt{\frac{3a^{-\frac{7}{2}}\cdot b^8}{a^{\frac{2}{3}}\cdot b^{-\frac{1}{2}}}}\cdot\sqrt[4]{4a^{-10}\cdot b^6\cdot\frac{1}{\left(a^{-\frac{1}{2}}b\right)^3}}$;

f) $\left(\frac{\sqrt[4]{ab}-\sqrt{b}}{\sqrt{a}-\sqrt[4]{ab}}\right)^{-4}$;

g) $\sqrt{\frac{a}{b}\sqrt{\frac{a}{b}\sqrt[3]{\frac{a}{b}}}\cdot a^{\frac{1}{3}}\cdot b^{\frac{1}{3}}}$.

2.29. Murakkab ildiz formulalarini isbotlang:

a) $\sqrt{a}+\sqrt{b}=\sqrt{\frac{a+\sqrt{a^2-b}}{2}}+\sqrt{\frac{a-\sqrt{a^2-b}}{2}}$;

b) $\sqrt{a}-\sqrt{b}=\sqrt{\frac{a+\sqrt{a^2-b}}{2}}-\sqrt{\frac{a-\sqrt{a^2-b}}{2}}$.

2.30. Murakkab ildiz formulalaridan foydalanib, ifodalarni soddalashtiring:

a) $\sqrt{5+2\sqrt{6}}$;

b) $\sqrt{6-\sqrt{20}}$;

d) $\sqrt{10-2\sqrt{21}}$;

e) $\sqrt{4\sqrt{2}+2\sqrt{6}}$.

2.31. Darajaga ko'taring:

a) $\left(\frac{2+\sqrt{3}}{\sqrt{2}+\sqrt{2}+\sqrt{3}}+\frac{2-\sqrt{3}}{\sqrt{2}-\sqrt{2}-\sqrt{3}}\right)^2$.

2.32. Ifodani soddalashtiring:

a) $\left(\sqrt{ab}-\frac{ab}{a+\sqrt{ab}}\right):\frac{\sqrt[4]{ab}-\sqrt{b}}{a-b}$; b) $\left(\frac{(\sqrt{a}+1)^3-a\sqrt{a}+2}{(\sqrt{a}+1)^2-\frac{a-\sqrt{ax}}{\sqrt{a}-\sqrt{x}}}\right)^{-3}$;

d) $\left(\frac{\sqrt{a+1}}{\sqrt{1+a}-\sqrt{1-a}}+\frac{1-a}{\sqrt{1-a^2+a-1}}\right)\cdot\left(\sqrt{\frac{1}{a^2}-1}-\frac{1}{a}\right)$;

e) $\frac{(\sqrt{a}-\sqrt{b})^3+2a^2:\sqrt{a}+b\sqrt{b}}{a\sqrt{a}+b\sqrt{b}}+\frac{3\sqrt{ab}-3b}{a-b}$;

f) $\frac{\frac{a+x}{\sqrt[3]{a^2}-\sqrt[3]{x^2}}+\frac{\sqrt[3]{ax^2}-\sqrt[3]{a^2x}}{\sqrt[3]{a^2}-2\sqrt[3]{ax}+\sqrt[3]{x^2}}}{\sqrt[6]{a}-\sqrt[6]{x}}$;

g) $\left(\frac{4a-9a^{-1}}{2a^{\frac{1}{2}}-3a^{-\frac{1}{2}}}+\frac{a-4+\frac{3}{a}}{a^{\frac{1}{2}}-a^{-\frac{1}{2}}}\right)^2$;

h) $\left(\frac{3x^{-\frac{1}{3}}}{x^{\frac{2}{3}}-2x^{-\frac{1}{3}}}-\frac{x^{\frac{1}{3}}}{x^{\frac{4}{3}}-x^{\frac{1}{3}}}-\frac{1-2x}{3x-2}\right)^{-1}$;

i) $\left(a+b^{\frac{3}{2}}:\sqrt{a}\right)^{\frac{2}{3}}\cdot\left(\frac{\sqrt{a}-\sqrt{b}}{\sqrt{a}}+\frac{\sqrt{b}}{\sqrt{a}-\sqrt{b}}\right)^{-\frac{2}{3}}$.

2.33. $x=\frac{\sqrt{3}}{2}$ bo'lsa, $\frac{1+x}{1+\sqrt{1+x}}+\frac{1-x}{1-\sqrt{1-x}}$ ifodaninig qiymatini toping.

2.34. $x=13$, $y=5$ bo'lsa, $\left(x+y^{\frac{3}{2}}:\sqrt{x}\right)^{\frac{2}{3}}\cdot\left(\frac{\sqrt{x}-\sqrt{y}}{\sqrt{x}}+\frac{\sqrt{y}}{\sqrt{x}-\sqrt{y}}\right)^{\frac{2}{3}}$ ifodaning qiymatini toping.

2.35. Ayniyatni isbotlang:

a) $\frac{a^{\frac{1}{2}}+1}{a+a^{\frac{1}{2}}+1}:\frac{1}{a^{\frac{3}{2}}-1}-a=-1$;

b) $\left(\frac{(a+\sqrt[3]{a^2x}):(x+\sqrt[3]{ax^2})-1}{\sqrt[3]{a}+\sqrt[3]{x}}-\frac{1}{\sqrt[3]{x}}\right)^6=\frac{a^2}{x^4}$.

2.25. Hisoblang:

a) $\frac{1}{\sqrt{3}+\sqrt{2}}+\frac{1}{\sqrt{4}+\sqrt{3}}+\frac{1}{\sqrt{5}+\sqrt{4}}+\cdots+\frac{1}{\sqrt{37}+\sqrt{36}}$;

b) $\frac{1}{\sqrt{7}+\sqrt{6}}+\frac{1}{\sqrt{8}+\sqrt{7}}+\frac{1}{\sqrt{9}+\sqrt{8}}+\cdots+\frac{1}{\sqrt{23}+\sqrt{22}}$;

d) $\frac{1}{\sqrt{3}-\sqrt{2}}+\frac{1}{\sqrt{2}-\sqrt{3}}-\sqrt{2}-\sqrt{23}$;

e) $\frac{3}{\sqrt{5}-\sqrt{2}}+\frac{5}{\sqrt{7}+\sqrt{2}}-\sqrt{7}+\sqrt{5}$.

2.26. Tenglik to'g'rimi:

a) $\frac{3}{\sqrt{6}-\sqrt{3}}+\frac{4}{\sqrt{7}+\sqrt{3}}=\frac{1}{\sqrt{7}-\sqrt{6}}$; b) $-\frac{2}{\sqrt{8}+\sqrt{6}}+\frac{5}{\sqrt{11}+\sqrt{6}}=-\frac{3}{\sqrt{8}+\sqrt{11}}$;

d) $-\frac{8\sqrt{7}}{\sqrt{5\sqrt{7}}-\sqrt{2\sqrt{7}}}+\frac{4\sqrt{7}}{\sqrt{5\sqrt{7}}+\sqrt{8\sqrt{7}}}=-4\sqrt[4]{175}$;

e) $\frac{4\sqrt{5}}{\sqrt{3\sqrt{5}}-\sqrt{2\sqrt{5}}}-\frac{5\sqrt{5}}{4\sqrt{2\sqrt{5}}-3\sqrt{3\sqrt{5}}}=\sqrt[4]{45}$?

Irratsional ifodalar ustida shakl almashtirishlar

2.27. Hisoblang:

$$\frac{2\cdot 4^{-2}+\left(81^{\frac{1}{2}}\right)^3\cdot\left(\frac{1}{9}\right)^{-3}}{125^{-\frac{1}{3}}\cdot\left(-\frac{1}{5}\right)^{-2}+(\sqrt{3})^0\cdot\left(\frac{1}{2}\right)^{-2}}.$$

2.28. Ifodani soddalashtiring:

a) $\frac{x^{\frac{1}{2}}\cdot y^{\frac{1}{2}}}{z^{\frac{1}{6}}}:\left(\frac{z^{-\frac{1}{2}}}{x^{-\frac{1}{3}}b^{-\frac{1}{3}}}\cdot\frac{x^{-\frac{5}{6}}\cdot z^{-\frac{2}{3}}}{y^{\frac{5}{6}}}\right)$; b) $\left(\frac{x^{-\frac{1}{2}}\cdot y^{-\frac{1}{3}}}{x^{-\frac{3}{4}}\cdot y^{-\frac{5}{6}}}:\sqrt[4]{x^{-3}\cdot y^{-5}}\right)^{\frac{2}{7}}$;

d) $ab\sqrt[3]{\frac{b}{a^2}}-ab\sqrt[3]{\frac{a}{b^2}}+\frac{a}{b}\sqrt[3]{ab^4}-\frac{b}{a}\sqrt[3]{a^4b}$;

e) $\sqrt{\frac{3a^{-\frac{7}{2}}\cdot b^8}{a^{\frac{2}{3}}\cdot b^{-\frac{1}{2}}}}\cdot\sqrt[4]{4a^{-10}\cdot b^6\cdot\frac{1}{\left(a^{-\frac{1}{2}}b\right)^3}}$;

f) $\left(\frac{\sqrt[4]{ab}-\sqrt{b}}{\sqrt{a}-\sqrt[4]{ab}}\right)^{-4}$;

g) $\sqrt{\frac{a}{b}\sqrt{\frac{a}{b}\sqrt[3]{\frac{a}{b}\cdot a^{\frac{1}{3}}\cdot b^{\frac{1}{3}}}}}$.

2.29. Murakkab ildiz formulalarini isbotlang:

a) $\sqrt{a}+\sqrt{b}=\sqrt{\frac{a+\sqrt{a^2-b}}{2}}+\sqrt{\frac{a-\sqrt{a^2-b}}{2}}$;

b) $\sqrt{a}-\sqrt{b}=\sqrt{\frac{a+\sqrt{a^2-b}}{2}}-\sqrt{\frac{a-\sqrt{a^2-b}}{2}}$.

2.30. Murakkab ildiz formulalaridan foydalanib, ifodalarni soddalashtiring:

a) $\sqrt{5+2\sqrt{6}}$;

b) $\sqrt{6-\sqrt{20}}$;

d) $\sqrt{10-2\sqrt{21}}$;

e) $\sqrt{4\sqrt{2}+2\sqrt{6}}$.

2.31. Darajaga ko'taring:

a) $\left(\frac{2+\sqrt{3}}{\sqrt{2}+\sqrt{2}+\sqrt{3}}+\frac{2-\sqrt{3}}{\sqrt{2}-\sqrt{2}-\sqrt{3}}\right)^2$.

2.32. Ifodani soddalashtiring:

a) $\left(\sqrt{ab}-\frac{ab}{a+\sqrt{ab}}\right):\frac{\sqrt[4]{ab}-\sqrt{b}}{a-b}$; b) $\left(\frac{(\sqrt{a}+1)^3-a\sqrt{a}+2}{(\sqrt{a}+1)^2-\frac{a-\sqrt{ax}}{\sqrt{a}-\sqrt{x}}}\right)^{-3}$;

d) $\left(\frac{\sqrt{a+1}}{\sqrt{1+a}-\sqrt{1-a}}+\frac{1-a}{\sqrt{1-a^2}+a-1}\right)\cdot\left(\sqrt{\frac{1}{a^2}-1}-\frac{1}{a}\right)$;

e) $\frac{(\sqrt{a}-\sqrt{b})^3+2a^2:\sqrt{a}+b\sqrt{b}}{a\sqrt{a}+b\sqrt{b}}+\frac{3\sqrt{ab}-3b}{a-b}$;

f) $\frac{a+x}{\sqrt[3]{a^2}-\sqrt[3]{x^2}}+\frac{\sqrt[3]{ax^2}-\sqrt[3]{a^2x}}{\sqrt[3]{a^2}-2\sqrt[3]{ax}+\sqrt[3]{x^2}}:\frac{}{\sqrt[6]{a}-\sqrt[6]{x}}$;

g) $\left(\frac{4a-9a^{-1}}{2a^{\frac{1}{2}}-3a^{-\frac{1}{2}}}+\frac{a-4+\frac{3}{a}}{a^{\frac{1}{2}}-a^{-\frac{1}{2}}}\right)^2$;

h) $\left(\frac{3x^{-\frac{1}{3}}}{x^{\frac{2}{3}}-2x^{-\frac{1}{3}}}-\frac{x^{\frac{1}{3}}}{x^{\frac{4}{3}}-x^{\frac{1}{3}}}-\frac{1-2x}{3x-2}\right)^{-1}$;

i) $\left(a+b^{\frac{3}{2}}:\sqrt{a}\right)^{\frac{2}{3}}\cdot\left(\frac{\sqrt{a}-\sqrt{b}}{\sqrt{a}}+\frac{\sqrt{b}}{\sqrt{a}-\sqrt{b}}\right)^{-\frac{2}{3}}$.

2.33. $x=\frac{\sqrt{3}}{2}$ bo'lsa, $\frac{1+x}{1+\sqrt{1+x}}+\frac{1-x}{1-\sqrt{1-x}}$ ifodaninig qiymatini toping.

2.34. x=13, y=5 bo'lsa, $\left(x+y^{\frac{3}{2}}:\sqrt{x}\right)^{\frac{2}{3}}\cdot\left(\frac{\sqrt{x}-\sqrt{y}}{\sqrt{x}}+\frac{\sqrt{y}}{\sqrt{x}-\sqrt{y}}\right)^{\frac{2}{3}}$ ifodaning qiymatini toping.

2.35. Ayniyatni isbotlang:

a) $\frac{a^{\frac{1}{2}}+1}{a+a^{\frac{1}{2}}+1}:\frac{1}{a^{\frac{3}{2}}-1}-a=-1$;

b) $\left(\frac{(a+\sqrt[3]{a^2x}):(x+\sqrt[3]{ax^2})-1}{\sqrt[3]{a}+\sqrt[3]{x}}-\frac{1}{\sqrt[3]{x}}\right)^6=\frac{a^2}{x^4}$.

VI b o b. ALGEBRAIK TENGLAMALAR VA TENGSIZLIKLAR

1-§. CHIZIQLI TENGLAMALAR

1.1. Tenglamani x ga nisbatan yeching:

a) $3x+1=a$;
b) $5+x=ax$;
d) $4=ax$;
e) $x=a^2x$;
f) $ax-a^2=4-2x$;
g) $a+x=a^2x-1$;
h) $ax-b=1+x$;
i) $x=b-a^2x$;
j) $ax-b^2=7$;
k) $3-a^2x=x-b$.

1.2. $m\cdot x=n$ tenglama:

a) faqat bitta ildizga;
b) faqat ikkita har xil ildizga;
d) faqat 1000 ta har xil ildizga;
e) cheksiz ko'p har xil ildizga ega bo'lishi mumkinmi?

1.3. $ax=1+b^2$ tenglama cheksiz ko'p har xil ildizga ega bo'lishi mumkinmi?

1.4. $(a-1)x=a^2-3a+2$ tenglama ildizga ega bo'lmasligi mumkinmi?

1.5. Ota 45 yoshda, o'g'li 15 yoshda. Necha yildan keyin o'g'li otasidan ikki marta kichik bo'ladi?

1.6. Tenglamani yeching:

a) $1-|x|=0,5$;
b) $1+|x|=a$;
d) $|1-x|=0,5$;
e) $|1-x|=a$;
f) $1+x=|\sqrt{2-3}|$;
g) $|1+x|=-|\sqrt{2-3}|$;
h) $1+x=2-\sqrt{3}$;
i) $|-x|=1,7$.

1.7. Tenglamani yeching:

a) $3x(x-1)-17=x(1+3x)+1$; b) $2x-(x+2)\times(x-2)=5-(x-1)^2$;

d) $\frac{3x+1}{2}=\frac{2x-3}{5}$; e) $\frac{x-3}{6}+x=\frac{2x-1}{3}-\frac{4-x}{2}$.

1.8. m ning qanday qiymatlarida berilgan tenglamalar R da teng kuchli bo'ladi:

a) $2x+3=12$ va $2x+3=12(3m-\frac{1}{2})+15$;

b) $3x+5=12$ va $(3x+5)(3m-\frac{1}{2})=12$;

d) $4-3x=5$ va $-3x+4=3m-8$;

e) $10x-mx=1$ va $(10-m)x=0$?

1.9. Tenglamani yeching:

a) $(x+2)(a-1)+1=a^2$;
b) $x=a^2x$;
d) $ax-a^2=4-2x$;
e) $a+x=a^2x-1$;
f) $ax-b^2=7$;
g) $ax-b=1+x$.

1.10. Tenglamaning yechimlari to'plamini tuzing:

a) $\frac{3-2x}{15}=\frac{x-2}{3}+\frac{x}{5}$; b) $\frac{1-3x}{12}=\frac{5x-1}{3}+\frac{7x}{4}$;

d) $\frac{6x-5}{3}-\frac{11}{5}=\frac{4x+3}{5}-0,6$; e) $\frac{8x+1}{2}-\frac{9x}{5}=\frac{6x-1}{5}+0,1$;

f) $\frac{5x-2}{3}=\frac{2x+3}{2}-\frac{x+2}{3}$; g) $3(x+8)=4(7-x)$;

h) $(x+3)(x-6)=(x+2)(x+1)+4$; i) $(x-3)(x-4)=(x-5)(x-6)-7,5$.

2-§. KVADRAT TENGLAMALAR

2.1. Kvadrat uchhaddan to'la kvadrat ajrating:

a) $2x^2+4x-3$;
b) $\frac{1}{3}x^2-4x+16$;
d) $-5x^2+20x-13$;
e) $-0,5x^2-0,25x-2,25$;
f) x^2-6x+8;
g) $ax^2-4a^2x+4a^3+3$;
h) $6a^2x-9a^3-ax^2+a-1$;
i) $x^2+(a+b)x+ab$.

2.2. x ning barcha qiymatlarida x^2+x+1 kvadrat uchhad musbat qiymatlar qabul qilishini isbotlang.

2.3. x ning barcha qiymatlarida $-3x^2+12x-13$ kvadrat uchhad manfiy qiymatlar qabul qilishini isbotlang.

2.4. x, y, z larning barcha qiymatlarida $5x^2+5y^2+5z^2+6xy-8xz-8yz\geq 0$ bo'lishini isbotlang.

2.5. Tengsizlikni isbotlang:

$$x^2+2xy+3y^2+2x+6y+3\geq 0.$$

2.6. Tenglamani yeching:

a) $6x^2-x-1=0$;
b) $3x^2-5x+1=0$;
d) $x^2-x+1=0$;
e) $-x^2+8x-16=0$;
f) $2x^2-12x+12=0$;
g) $2x-x^2-6=0$;
h) $x^2-4x+5=0$;
i) $\frac{1}{3}x^2-12x+9=0$.

2.7. 15 sonini ko'paytmasi 70 ga teng bo'ladigan ikkita sonning yig'indisi ko'rinishida yozish mumkinmi?

2.8. Tenglamani eng qulay formula yordamida yeching:

a) $3x^2-5x+2=0$;
b) $3x^2-20x-52=0$;
d) $x^2-10x+24=0$;
e) $x^2+7x-30=0$;
f) $5x^2+9x-14=0$;
g) $4x^2-x+10=0$;
h) $5x^2-16x+3=0$;
i) $x^2+4x-12=0$.

2.9. Tenglamani og'zaki yeching:

a) $x^2-3x+2=0$;
b) $x^2+99x-100=0$;
d) $x^2+548x-549=0$;
e) $-x^2+6x-5=0$;
f) $-x^2-7x+8=0$;
g) $x^2-7x+12=0$;
h) $3x^2+x-2=0$;
i) $x^2-(a+b)x+ab=0$.

2.10. x_1 va x_2 lar $x^2-7x+10=0$ tenglamaning ildizlari bo'lsin. Bu ildizlarni topmasdan, quyidagilarni hisoblang:

a) $x_1^2+x_2^2$;
b) $x_1^3+x_2^3$;
d) $\frac{1}{x_1}+\frac{1}{x_2}$;
e) $\frac{1}{x_1^2}+\frac{1}{x_2^2}$;
f) $\frac{x_1}{x_2}+\frac{x_2}{x_1}$;
g) $x_1x_2-\frac{1}{x_1}-\frac{1}{x_2}$;
h) $(x_1x_2)^2-x_1^3-x_2^3$;
i) $x_1^2+x_2^2+2x_1x_2$.

2.11. 2.10-misoldagi tenglamani $-3x^2+x+24=0$ tenglama bilan almashtiring va hisoblashlarni bu tenglama uchun bajaring.

2.12. x_1,x_2 lar $ax^2+bx+a=0$ tenglamaning ildizlari bo'lsin, x_1 va x_2 sonlar o'zaro teskari sonlar ekanini isbotlang.

2.13. Berilgan tenglamani yechmay, uning ildizlari ishorasini aniqlang:

a) $x^2-4x+3=0$;
b) $x^2-6x+5=0$;
d) $x^2-x-42=0$;
e) $x^2-x-6=0$;
f) $x^2+x+1=0$;
g) $6x^2-x-1=0$;
h) $-20x^2-3x+2=0$;
i) $x^2-6x+10=0$;
j) $-3x^2+17=0$;
k) $-5x^2+x-7=0$.

2.14. Kvadrat uchhadning ildizlarini toping:

a) $10x^2+5x-5$;
b) $9x^2-9x+2$;
d) $0{,}2x^2+3x-20$;
e) $-2x^2-x-0{,}125$;
f) $0{,}1x^2+0{,}4$;
g) $-0{,}3x^2+1{,}5x$;
h) x^2+x-6;
i) x^2-2x-4.

2.15. Kvadrat uchhadni ko'paytuvchilarga ajrating:

a) $3x^2-24x+21$;
b) $5x^2+10x-15$;
d) $\frac{1}{6}x^2+\frac{1}{2}x+\frac{1}{3}$;
e) $x^2-12x+24$;
f) $-y^2+16y-15$;
g) $-x^2-8x+9$;
h) $2x^2-5x+3$;
i) $5y^2+2y-3$;
j) $-2x^2+5x+7$;
k) $2x^2-2x+\frac{1}{2}$.

2.16. Kasrni qisqartiring:

a) $\frac{4x+x}{3x^2+2x-1}$;
b) $\frac{2a^2-5a-3}{3a-9}$;
d) $\frac{16-b^2}{b^2-b-12}$;
e) $\frac{2y^2+7y+3}{y^2-9}$;
f) $\frac{p^2-11p+10}{20+8p-p^2}$;
g) $\frac{3x^2+16x-12}{10-13x-3x^2}$;
h) $\frac{x^2-11x+24}{x^2-64}$;
i) $\frac{2y^2+9y-5}{4y^2-1}$.

2.17. Ildizlari quyidagicha bo'lgan kvadrat tenglama tuzing:

a) 2 va –3; d) $\frac{1}{4}$ va $\frac{1}{6}$; f) 2 va 2; h) 0 va 5;

b) –1 va –5; e) $-\frac{1}{2}$ va $-\frac{1}{3}$; g) $\frac{1}{3}$ va $\frac{1}{3}$; i) α va β.

2.18. Ildizlari $\frac{5}{7}$ va $-\frac{1}{2}$ bo'lgan shunday kvadrat tenglama tuzingki, uning barcha koeffitsiyentlari butun sonlar bo'lib, ularning yig'indisi 6 ga teng bo'lsin.

2.19. Ildizlari 3 va –2 bo'lgan shunday kvadrat tenglama tuzingki, uning bosh koeffitsiyenti $\frac{1}{2}$ bo'lsin.

2.20. Ildizlaridan biri a) $2+\sqrt{3}$ ga, b) $3-\sqrt{2}$ ga, d) $2-\sqrt{5}$ ga, e) $3+\sqrt{5}$ ga teng bo'lgan butun koeffitsiyentli keltirilgan kvadrat tenglama tuzing.

3-§. KASR-RATSIONAL TENGLAMALAR

$$\frac{P(x)}{Q(x)}=0 \qquad (1)$$

ko'rinishdagi tenglama *kasr-ratsional tenglama* deyiladi, bu yerda $P(x)$ va $Q(x)$ lar ko'phadlar bo'lib, $Q(x)$ ning darajasi kamida 1 ga teng.

(1) tenglamani yechish uchun $P(x)=0$ tenglamaning $Q(x)\neq 0$ shartni qanoatlantiradigan yechimlarini topish kifoya, ya'ni (1) tenglama

$$\begin{cases} P(x)=0, \\ Q(x)\neq 0 \end{cases} \qquad (2)$$

sistemaga teng kuchlidir.

M i s o l. $\frac{1}{x+1}+\frac{2}{x-2}=1$ tenglamani yeching.

Ye c h i s h. Bu tenglamani (1) ko'rinishga keltirib olamiz:

$$\frac{x^2-4x-2}{(x+1)(x-2)}=0.$$

Oxirgi tenglama $\begin{cases} x^2-4x-2=0, \\ (x+1)(x-2)\neq 0 \end{cases}$ sistemaga teng kuchli. $x^2-4x-2=0$ tenglama $x_1=2+\sqrt{6}$, $x_2=2-\sqrt{6}$ ildizlarga ega bo'lib, bu ildizlar $(x+1)(x-2)=0$ tenglamaning ildizlari emas.

Shunday qilib, berilgan tenglama ikkita ildizga ega: $x_{1,2}=2\pm\sqrt{6}$.

Tenglamani yeching:

3.1. $\frac{5(x-2)}{x+2}-\frac{2(x-3)}{x+3}=3$. **3.2.** $\frac{x^2-1}{x}=x^2-\frac{1}{x}$

3.3. $\frac{y+5}{y^2-5y}-\frac{y-5}{2y^2-10y}=\frac{y+25}{2y^2-50}$ **3.4.** $\frac{x^2}{x+5}=\frac{25}{x+5}$

3.5. $\frac{3(9x-3)}{9x-6}=2+\frac{3x+1}{3x-2}$. **3.6.** $\frac{3-7x}{2x+4}=\frac{1,5-3,5x}{x+2}$.

3.7. $\frac{1+x}{1-x}=\frac{a}{c}$. **3.8.** $\frac{3ax-5}{(a-1)(x+3)}+\frac{3a-11}{a-1}=\frac{2x+7}{x+3}$.

Tenglamani yeching:

3.9. $\frac{5+2x}{4x-3}=\frac{3(x+1)}{7-x}$. **3.10.** $\frac{x+3}{x-3}+\frac{x-3}{x+3}=\frac{10}{3}+\frac{36}{x^2-9}$.

3.11. $\frac{30}{x^2-1}-\frac{13}{x^2+x+1}-\frac{18x+7}{x^3-1}=0$. **3.12.** $\frac{x^2}{x+3}=\frac{x}{x+3}$.

3.13. $\frac{x^2-6x}{x-5}=\frac{5}{5-x}$. **3.14.** $\frac{x^2-6x}{x-5}-\frac{5}{x-5}=0$.

3.15. $\frac{x^2-4}{x}=\frac{3+2x}{2}$. **3.16.** $\frac{8}{x}=3x+2$.

3.17. $\frac{3x+1}{x+2}=1+\frac{x-1}{x-2}$. **3.18.** $\frac{2x-2}{x+3}-\frac{x+3}{3-x}=5$.

3.19. $\frac{4}{9y^2-1}-\frac{4}{3y+1}=\frac{5}{1-3y}$.

3.20. $\frac{4}{x+3}+1=\frac{1}{x-3}+\frac{5}{3-x}$.

4-§. KO'PAYTUVCHILARGA AJRATISH USULI

1-misol. $x^4-4x^3-10x^2+37x-14=0$ tenglamani yeching.

Yechish. Tenglamaning chap tomonida 4-darajali ko'phad turibdi. Uni kvadrat uchhadlar ko'paytmasi shaklida tasvirlashga harakat qilamiz:

$$x^4-4x^3-10x^2+37x-14=(x^2+px+q)(x^2+bx+c).$$

Chap va o'ng tomonlarda turgan ko'phadlarning mos koeffitsiyentlarini tenglashtiramiz:

$$\begin{cases} p+b=-4, \\ c+q+pb=-10, \\ pc+qb=37, \\ qc=-14. \end{cases}$$

Oxirgi sistemaning biror butun qiymatli yechimini topamiz. $qc=-14$ dan q va c lar 14 ning bo'luvchilari ekanini ko'rish qiyin emas. Demak, ular uchun ±1, ±2, ±7, ±14 sonlarni sinab ko'rish kerak.

Agar $q=1$ bo'lsa, $s=-14$ bo'ladi. Ikkinchi va uchinchi tenglamalar $\begin{cases} pb=3, \\ -14p+b=37 \end{cases}$ sistemani beradi. Bu sistemadan b uchun $b^2-37b-42=0$ tenglama hosil bo'ladi. Bu tenglama esa yechimga ega emas.

Shuning uchun, $q=1$ da sistema butun yechimga ega emas.

Agar $q=2$ bo'lsa, $c=-7$ ga ega bo'lamiz. Bu holda sistema $q=2$, $c=-7$, $b=1$, $p=-5$ lardan tuzilgan butun yechimga ega bo'ladi (tekshirib ko'ring).

Shunday qilib,

$$x^4-4x^3-10x^2+37x-14=(x^2+5x+2)(x^2+x-7).$$

Demak, berilgan tenglama $x^2-5x+2=0$ va $x^2+x-7=0$ tenglamalarga ajraladi. Bu tenglamalarni yechib, berilgan tenglamaning ham yechimlari bo'ladigan $\frac{5\pm\sqrt{17}}{2}$, $\frac{-1\pm\sqrt{29}}{2}$ sonlarni topamiz.

2-misol. $(x^2+x+4)^2+3x(x^2+x+4)+2x^2=0$ tenglamani yeching.

Yechish. Tenglamaning chap tomonini $y=x^2+x+4$ ga nisbatan kvadrat uchhad sifatida qarab, bu kvadrat uchhadni odatdagi standart usulda ko'paytuvchilarga ajratamiz:

$$y^2+3xy+2x^2=(y+x)(y+2x).$$

Bundan $(x^2+2x+4)(x^2+3x+4)=0$ tenglama hosil bo'ladi. Oxirgi tenglama yechimga ega emas. Demak, berilgan tenglama ham yechimga ega emas.

Tenglamani yeching:

4.1*. $x^3-3x=a^3+\frac{1}{a^3}$ $(a\neq0)$.

4.2. $x^3-8x^2-x+8=0$.

4.3. $x^3-0,1x=0,3x^2$.

4.4. $9x^3-18x^2-x+2=0$.

4.5. $y^4-y^3-16y^2+16y=0$.

4.6. $x^3-x^2=x-1$.

4.7. $x^4-x^2=6x^3-6x$.

4.8. $3x^3-x^2+18x-6=0$.

4.9. $2x^4-18x^2=5x^3-45x$. **4.10.** $3y^2-2y=2y^3-3$.

4.11. $x^3-3x-2=0$. **4.12.** $(x^2+x+1)(x^2+x+2)-12=0$.

4.13.* $2(x^2+6x+1)^2+5(x^2+6x+1)(x^2+1)+2(x^2+1)^2=0$.

4.14.* $(x^2-x+1)^4-6x^2(x^2-x+1)^2+5x^4=0$.

4.15.* $\frac{x+6}{x-6}\cdot\left(\frac{x-4}{x+4}\right)^2+\frac{x-6}{x+6}\cdot\left(\frac{x+9}{x-9}\right)^2=2\cdot\frac{x^2+36}{x^2-36}$.

4.16. $x^3+7x^2+14x+8=0$. **4.17.** $x^3-5x+4=0$.

4.18. $x^3-8x^2+40=0$. **4.19.** $x^3-2x-1=0$.

4.20. $x^4-4x^2+x+2=0$.

5-§. YANGI O'ZGARUVCHI KIRITISH USULI

1 - m i s o l. $(x^2-3x+1)(x^2+3x+2)(x-9x+20)=-30$ tenglamani yeching.

Y e c h i s h. $(x^2+3x+2)(x^2-9x+20)=(x+1)(x+2)(x-4)(x-5)=$ $=[(x+1)(x-4)]\cdot[(x+2)(x-5)]=(x^2-3-4)\cdot(x^2-3x-10)$ bo'lgani uchun berilgan tenglamani quyidagicha yozib olish mumkin:

$$(x^2-3x+1)(x^2-3x-4)(x^2-3x-10)=-30.$$

Bu tenglamada $y=x^2-3x$ almashtirish orqali yangi o'zgaruvchi y ni kiritamiz:

$$(y+1)(y-4)(y-10)=-30.$$

Oxirgi tenglamadan $y_1=5$, $y_2=4+\sqrt{30}$, $y_3=4-\sqrt{30}$ larni topib, quyidagi uchta kvadrat tenglamaga ega bo'lamiz:

$$x^2-3x=5,\quad x^2-3x=4+\sqrt{30},\quad x^2-3x=4-\sqrt{30}.$$

Bu tenglamalarni yechsak, berilgan tenglamaning barcha ildizlari topiladi:

$$\frac{3\pm\sqrt{29}}{2},\ \frac{3\pm\sqrt{25+4\sqrt{30}}}{2},\ \frac{3\pm\sqrt{25-4\sqrt{30}}}{2}.$$

2 - m i s o l. $x^4-2\sqrt{2}x^2-x+2-\sqrt{2}=0$ tenglamani yeching.

Y e c h i s h. $\sqrt{2}=a$ deb, $x^4-2ax^2-x+a^2-a=0$ tenglamani hosil qilamiz. Bu tenglamani a ga nisbatan kvadrat tenglama sifatida qarab, uning $a=x^2-x$, $a=x^2+x+1$ ildizlarini topish mumkin. $a=\sqrt{2}$ ekanini e'tiborga olsak, quyidagi tenglamalarga ega bo'lamiz:

$$x^2-x=\sqrt{2},\quad x^2+x+1=\sqrt{2}.$$

Bu tenglamalar berilgan tenglamaning hamma ildizlarini aniqlash imkonini beradi:

$$x_{1,2}=\frac{1\pm\sqrt{1+4\sqrt{2}}}{2},\quad x_{3,4}=\frac{-1\pm\sqrt{4\sqrt{2}-3}}{2}.$$

3-misol. $\frac{4x}{x^2+x+3}+\frac{5x}{x^2-5x+3}=-\frac{3}{2}$ tenglamani yeching.

Y e c h i s h. $x=0$ tenglamaning yechimi emas. Shu sababli, berilgan tenglama quyidagi tenglamaga teng kuchli:

$$\frac{4}{x+\frac{3}{x}+1}+\frac{5}{x+\frac{3}{x}-5}=-\frac{3}{2},$$

$y=x+\frac{3}{x}$ desak, $\frac{4}{y+1}+\frac{5}{y-5}=-\frac{3}{2}$ tenglama hosil bo'ladi. Bu tenglama $y_1=-5$, $y_2=3$ ildizlarga ega bo'lgani uchun berilgan tenglama $x+\frac{3}{x}=-5$, $x+\frac{3}{x}=3$ tenglamalar majmuasiga teng kuchlidir. Ularni yechib, berilgan tenglamaning ildizlariga ega bo'lamiz:

$$x_{1,2}=\frac{-5\pm\sqrt{13}}{2}.$$

Yechilgan bu tenglama $\frac{Ax}{ax^2+b_1x+c}+\frac{Bx}{ax^2+b_2x+c}=D$ ko'rinishdagi tenglamaning xususiy holidir. Bunday ko'rinishdagi barcha tenglamalar, shuningdek

$$\frac{ax^2+b_1x+c}{ax^2+b_2x+c}\pm\frac{ax^2+b_3x+c}{ax^2+b_4x+c}=A \text{ va}$$

$$\frac{ax^2+b_1x+c}{ax^2+b_2x+c}=\frac{Ax}{ax^2+b_3x+c},\ A\neq 0$$

ko'rinishdagi (bu yerda $ac\neq 0$) tenglamalarni yechish sxemasi 3-misolni yechish sxemasi kabidir.

Tenglamani yeching:

5.1. $(x^2-5x+4)(x^2-5x+6)=120$. **5.2.** $(x^2+3)^2-1(x^2+3)+28=0$.

5.3. $t^4-2t^2-3=0$. **5.4.** $2x^4-9x^2+4=0$.

5.5. $5y^4-5y^2+2=0$. **5.6.** $x^4-4x^2+4=0$.

5.7. $(x^2-2x)^2-(x-1)^2+1=0$. **5.8.** $(x^2+2x)^2-(x+1)^2=55$.

5.9. $(x^2+x+1)(x^2+x+2)-12=0$. **5.10.** $(x^2-5x+7)-(x-2)(x-3)=0$.

5.11. $(x-2)(x+1)(x+4)(x+7)=19$. **5.12.** $2x^8+x^4-15=0$.

5.13. $(2x-1)^6+3(2x-1)^3=10$. **5.14.** $(x-2)^6-19(x-2)^3=216$.

5.15. $\frac{x-4}{x+5}+\frac{x+5}{x-4}=2$. **5.16.** $\frac{x-4}{x-5}+\frac{6x-30}{x-4}=5$.

4.9. $2x^4-18x^2=5x^3-45x$. **4.10.** $3y^2-2y=2y^3-3$.

4.11. $x^3-3x-2=0$. **4.12.** $(x^2+x+1)(x^2+x+2)-12=0$.

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4.14.* $(x^2-x+1)^4-6x^2(x^2-x+1)^2+5x^4=0$.

4.15.* $\frac{x+6}{x-6}\cdot\left(\frac{x-4}{x+4}\right)^2+\frac{x-6}{x+6}\cdot\left(\frac{x+9}{x-9}\right)^2=2\cdot\frac{x^2+36}{x^2-36}$.

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5-§. YANGI O'ZGARUVCHI KIRITISH USULI

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Y e c h i s h. $(x^2+3x+2)(x^2-9x+20)=(x+1)(x+2)(x-4)(x-5)=$ $=[(x+1)(x-4)]\cdot[(x+2)(x-5)]=(x^2-3-4)\cdot(x^2-3x-10)$ bo'lgani uchun berilgan tenglamani quyidagicha yozib olish mumkin:

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Bu tenglamada $y=x^2-3x$ almashtirish orqali yangi o'zgaruvchi y ni kiritamiz:

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Oxirgi tenglamadan $y_1=5$, $y_2=4+\sqrt{30}$, $y_3=4-\sqrt{30}$ larni topib, quyidagi uchta kvadrat tenglamaga ega bo'lamiz:

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Bu tenglamalarni yechsak, berilgan tenglamaning barcha ildizlari topiladi:

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2 - m i s o l. $x^4-2\sqrt{2}x^2-x+2-\sqrt{2}=0$ tenglamani yeching.

Y e c h i s h. $\sqrt{2}=a$ deb, $x^4-2ax^2-x+a^2-a=0$ tenglamani hosil qilamiz. Bu tenglamani a ga nisbatan kvadrat tenglama sifatida qarab, uning $a=x^2-x$, $a=x^2+x+1$ ildizlarini topish mumkin. $a=\sqrt{2}$ ekanini e'tiborga olsak, quyidagi tenglamalarga ega bo'lamiz:

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$y=x+\frac{3}{x}$ desak, $\frac{4}{y+1}+\frac{5}{y-5}=-\frac{3}{2}$ tenglama hosil bo'ladi. Bu tenglama $y_1=-5$, $y_2=3$ ildizlarga ega bo'lgani uchun berilgan tenglama $x+\frac{3}{x}=-5$, $x+\frac{3}{x}=3$ tenglamalar majmuasiga teng kuchlidir. Ularni yechib, berilgan tenglamaning ildizlariga ega bo'lamiz:

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Yechilgan bu tenglama $\frac{Ax}{ax^2+b_1x+c}+\frac{Bx}{ax^2+b_2x+c}=D$ ko'rinishdagi tenglamaning xususiy holidir. Bunday ko'rinishdagi barcha tenglamalar, shuningdek

$$\frac{ax^2+b_1x+c}{ax^2+b_2x+c}\pm\frac{ax^2+b_3x+c}{ax^2+b_4x+c}=A \text{ va}$$

$$\frac{ax^2+b_1x+c}{ax^2+b_2x+c}=\frac{Ax}{ax^2+b_3x+c},\ A\neq 0$$

ko'rinishdagi (bu yerda $ac\neq 0$) tenglamalarni yechish sxemasi 3-misolni yechish sxemasi kabidir.

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5.3. $t^4-2t^2-3=0$. **5.4.** $2x^4-9x^2+4=0$.

5.5. $5y^4-5y^2+2=0$. **5.6.** $x^4-4x^2+4=0$.

5.7. $(x^2-2x)^2-(x-1)^2+1=0$. **5.8.** $(x^2+2x)^2-(x+1)^2=55$.

5.9. $(x^2+x+1)(x^2+x+2)-12=0$. **5.10.** $(x^2-5x+7)-(x-2)(x-3)=0$.

5.11. $(x-2)(x+1)(x+4)(x+7)=19$. **5.12.** $2x^8+x^4-15=0$.

5.13. $(2x-1)^6+3(2x-1)^3=10$. **5.14.** $(x-2)^6-19(x-2)^3=216$.

5.15. $\frac{x-4}{x+5}+\frac{x+5}{x-4}=2$. **5.16.** $\frac{x-4}{x-5}+\frac{6x-30}{x-4}=5$.

5.17. $\frac{x^2+x-5}{x}+\frac{3x}{x^2+x-5}+4=0$. **5.18.** $x^4-\frac{50}{2x^4-7}=14$.

5.19. $\frac{1}{x(x+2)}-\frac{1}{(x+1)^2}=\frac{1}{12}$. **5.20.** $(x^2+2x)^2-(x+1)^2=55$.

5.21. $\frac{x^2-13x+15}{x^2-14x+15}-\frac{x^2-15x+15}{x^2-16x+15}=-\frac{1}{12}$.

5.22. $\frac{4x}{4x^2-8x+7}+\frac{3x}{4x^2-10x+7}=1$.

5.23. $\frac{1}{x^2-2x+2}+\frac{1}{x^2-2x+3}=\frac{9}{2(x^2-2x+4)}$.

5.24. $\frac{x^2-10x+15}{x^2-6x+15}=\frac{4x}{x^2-12x+15}$.

5.25. $\frac{2x}{3x^2-x+2}-\frac{7x}{3x^2+5x+2}=1$. **5.26.** $\frac{x-a}{x-b}+\frac{x-b}{x-a}=2$.

6-§. BEZU TEOREMASI. GORNER SXEMASI

Bezu teoremasi. ***$P_n(x)=a_0x^n+a_1x^{n-1}+\ldots+a_{n-1}x+a_n$ ($a_0\neq0$) ko'phadni $x-c$ ikki hadga bo'lishdan hosil bo'ladigan r qoldiq $P_n(x)$ ko'phadning $x=c$ nuqtadagi qiymatiga, ya'ni $P_n(c)$ ga teng: $r=P_n(c)$.***

1-misol. $P_4(x)=x^4+x^3+3x^2+2x+2$ ko'phadni $x-1$ ga bo'lishdan hosil bo'lgan qoldiqni toping.

Yechish. Bezu teoremasiga asosan: $r=P_4(1)=1+1+3+2+2=9$.

2-misol. $P_3(x)=x^3+2x^2+x-a^2$ ko'phadni $x-2$ ga bo'lishdan hosil bo'lgan qoldiq 8 ga teng bo'lsa, a ni toping.

Yechish. $P_3(2)=2^3+2\cdot2^2+2-a^2=8$ tenglikdan $a^2=10$ ni hosil qilamiz. Bundan $a\sqrt{10}$ yoki $a=-\sqrt{10}$.

Javob: $a=\pm\sqrt{10}$.

$P_n(x)=a_0x^n+a_1x^{n-1}+\ldots+a_{n-1}x+a_n$ ko'phadni $x-c$ ga qoldiqli bo'lishning amaliy usullaridan biri Gorner sxemasi (usuli)dir.

Bu usulning mohiyati quyidagicha: $P(x)$ ko'phadni $x-c$ ga qoldiqli bo'lishda $Q_{n-1}(x)=b_0x^{n-1}+b_1x^{n-2}+\ldots+b_{n-2}x+b_{n-1}$ ($b_0\neq0$) ko'phad va $r\in R$ qoldiq hosil bo'ladi. $b_0, b_1, \ldots, b_{n-2}, b_{n-1}, r$ sonlarni quyidagi sxema yordamida topish mumkin:

	a_0	a_1	a_2	$\ldots$	a_{n-2}	a_{n-1}	a_n
+	0	cb_0	cb_1	$\ldots$	cb_{n-3}	cb_{n-2}	cb_{n-1}
c	$b_0=a_0$	b_1	b_2	$\ldots$	b_{n-2}	b_{n-1}	r

3-misol. $P_5(x)=2x^5-x^4-3x^3+x-3$ ni $x-3$ ga qoldiqli bo'ling.

Yechish.

	2	–1	–3	0	1	–3
		3·2	3·5	3·12	3·36	3·109
$c=3$	2	5	12	36	109	$324=r$

Demak, $P_5(x)=(x-3)(2x^4+5x^3+12x^2+36x+109)+324$.

4-misol. $P_3(x)=2x^3-x^2+3x+2$ ni $x+1$ ga qoldiqli bo'ling.

Yechish.

	2	–1	3	2
		–1·2	–1·(–3)	–1·6
$c=-1$	2	–3	6	$-4=r$

$P_3(x)=(x+1)(2x^2-3x+6)-4$.

Bezu teoremasidan $P_n(x)$ ko'phadni $ax+b$ ko'rinishdagi ikkihadga bo'lishda hosil bo'ladigan r qoldiq $P_n\left(-\frac{b}{a}\right)$ ga teng bo'lishligi kelib chiqadi.

5-misol. $P_3(x)=x^3-3x^2+5x+7$ ni $2x+1$ ga bo'lishdan hosil bo'lgan qoldiqni toping.

Yechish. Qoldiq $r=P_3\left(-\frac{1}{2}\right)=\left(-\frac{1}{2}\right)^3-3\cdot\left(-\frac{1}{2}\right)^2+5\cdot\left(\frac{1}{2}\right)+7=\frac{29}{8}$ ga teng.

6.1. $P(x)$ ko'phad $D(x)$ ko'phadga bo'linadimi:
a) $P(x)=x^{100}-3x+2$, $D(x)=x-1$;
b) $P(x)=x^{100}-3x+2$, $D(x)=x+1$;
d) $P(x)=x^{100}-3x^2+2$, $D(x)=x^2-1$;
e) $P(x)=x^{100}-3x+2$, $D(x)=2x^2-1$?

6.2. $x^{2n-1}+a^{2n-1}$ ko'phad $x+a$ ga bo'linishini isbotlang, bunda $a\neq0$, $n\in N$.

6.3. x^n-a^n ko'phad $x-a$ ga bo'linishini isbotlang, bunda $a\neq0$, $n\in N$.

6.4. a) x^4-3x^2+1 ni $x-2$ ga; b) $x^5-4x^3+x^2$ ni $x-3$ ga;
d) $x^5-4x^3-x^2+1$ ni $2x-3$ ga; e) $x^4-3x^3+x^2-1$ ni $3x-4$ ga bo'lishdagi qoldiqni toping.

6.5. m ning qanday qiymatlarida $3x^4-2x^3-m^2x-2$ ko'phad $x-2$ ga qoldiqsiz bo'linadi?

6.6. m ning qanday qiymatlarida $3x^3-4x^2-mx-1$ ko'phad $x+1$ ga bo'linmaydi?

6.7. a va b ning qanday qiymatlari $2x^4+ax^3+bx-2$ ko'phad x^2-x-2 uchhadga qoldiqsiz bo'linadi?

6.8*. m va n ning qanday qiymatlarida x^3+mx+n ko'phad $x^2+3x+10$ uchhadga qoldiqsiz bo'linadi?

6.9. $P(x)$ ko'phadni $x-1$ ga bo'lishda qoldiq 3, $x-2$ ga bo'lishda esa qoldiq 5 hosil bo'ladi. $P(x)$ ni x^2-3x+2 ga bo'lishda hosil bo'ladigan qoldiqni toping.

6.10. $P(x)$ ko'phadni $x-a$ ga bo'lishda r_1 qoldiq, $x-b$ ga bo'lishda esa r_2 qoldiq hosil bo'ladi ($a\neq b$). $P(x)$ ni $x^2-(a+b)x+ab$ ga bo'lishda hosil bo'ladigan qoldiqni toping.

6.11. Gorner sxemasi yordamida $P(x)$ ko'phadni $D(x)$ ikkihadga qoldiqli bo'ling:

a) $P(x)=x^2-5x-7$, $D(x)=x-1$;

b) $P(x)=x^3-3x^2+5x-6$, $D(x)=x-2$;

d) $P(x)=2x^4-3x^2-5x+2$, $D(x)=x+1$;

e) $P(x)=3x^5-4x^3-x+1$, $D(x)=x+3$;

f) $P(x)=3x^6-4x^5-x^4+x^3-x^2-1$, $D(x)=x-3$;

g) $P(x)=x^5-x^2-5x-6$, $D(x)=x-2$;

h) $P(x)=x^4-x^3+2x^2-5x-42$, $D(x)=x+2$;

i) $P(x)=x^5-4x^2+5x-3$, $D(x)=x-3$;

j) $P(x)=x^4-3x^3+2x^2-4x-1$, $D(x)=x+4$;

k) $P(x)=x^5-4x^3-3x^2+1$, $D(x)=x-4$;

l) $P(x)=x^6-5x^4+3x^2-5x+6$, $D(x)=x+2$;

m) $P(x)=x^5-4x^3+2x^2-3$, $D(x)=x-1$.

6.12. Gorner sxemasidan foydalanib, $f(x)$ ko'phadning $x=a$ nuqtadagi qiymatini toping:

a) $f(x)=x^3-x^2+2$, $a=1$; b) $f(x)=x^4-3x^3-x+10$, $a=2$;

d) $f(x)=x^5-x^4+3x^2-x+1$, $a=-1$;

e) $f(x)=x^6-7x^2+3x^2-3$, $a=3$; f) $f(x)=x^6-5x^3-4x^2+8$, $a=4$;

g) $f(x)=x^8+7x^7+x^6+3x^5+3x^4+2x^3+x^2-x+1$, $a=5$.

6.13. Gorner sxemasidan foydalanib, $a^3+b^3+c^3-3abc$ ni ko'paytuvchilarga ajrating.

6.14. Agar $a\geq 0$, $b\geq 0$, $c\geq 0$ bo'lsa, $\frac{a^3+b^3+c^3}{3}\geq abc$ bo'ladi. Shuni 6.13-masala natijasidan foydalanib isbotlang.

7-§. ALGEBRANING ASOSIY TEOREMASI

Algebraning asosiy teoremasi (Gauss teoremasi).

***n*-darajali (bu yerda $n\geq 1$) har qanday ko'phad aqalli bitta kompleks ildizga ega.**

T e o r e m a. ***Agar $\alpha+\beta i$ ($\beta\neq 0$) kompleks son $P(z)$ ko'phadning ildizi bo'lsa, $\alpha-\beta i$ soni ham $P(z)$ ko'phadning ildizi bo'ladi.***

N a t i j a: ***n*-darajali $P_n(x)$ ko'phad $x-\alpha$ ko'rinishidagi ikkihadlar va x^2+px+q ko'rinishidagi manfiy diskriminantli kvadrat uchhadlar darajalarining ko'paytmasidan iborat:**

$$P_n(x)=a_0(x-\alpha)^k\cdot\ldots\cdot(x^2+px+q)^m\cdot\ldots$$

bu yerda $k\in\{0,1,2,\ldots\}$ $m\in\{0,1,2,\ldots\}$.

1 - m i s o l. $x^2+4x+15=0$ tenglamaning barcha kompleks ildizlarini toping.

Y e c h i s h. Algebraning asosiy teoremasidan bu tenglama ko'pi bilan ikkita kompleks ildizga egaligi kelib chiqadi. Bu ildizlar kvadrat tenglamani yechishning odatdagi usuli yordamida topiladi:

$$x^2+4x+15=0;\ D=4^2-4\cdot 15=-44;\ x_{1,2}=\frac{-4\pm 2\sqrt{-11}}{2}=-2\pm i\sqrt{11}.$$

2 - m i s o l. $x^2+4x+15$ uchhadni ko'paytuvchilarga ajrating.

Y e c h i s h. Kvadrat uchhadning ildizlarini topamiz: $x_{1,2}=-2\pm i\sqrt{11}$. Shuning uchun $x^2+4x+15=(x+2-i\sqrt{11})\cdot(x+2+i\sqrt{11})$.

3 - m i s o l. $x^4+4x^2+15=0$ tenglamaning barcha kompleks ildizlarini toping.

Y e c h i s h. $x^2=t$ deb, $t^2+4t+15=0$ kvadrat tenglamani hosil qilamiz. Uning ildizlari: $t_1=-2+i\sqrt{11}$, $t_2=-2-i\sqrt{11}$. $x^2=-2+i\sqrt{11}$, $x^2=-2-i\sqrt{11}$ tenglamalarga ega bo'ldik. $\sqrt{-2+i\sqrt{11}}$, $\sqrt{-2-i\sqrt{11}}$ ifodalarning qiymatlarini hisoblasak, berilgan tenglamaning 4 ta kompleks ildizlariga ega bo'lamiz (bu ildizlarni o'zingiz aniqlang).

4 - m i s o l. Ildizlaridan biri $1+3i$ bo'lgan haqiqiy koeffitsiyentli kvadrat tenglama tuzing.

Y e c h i s h. $1+3i$ son izlanayotgan kvadrat tenglamaning ildizi bo'lgani uchun $1-3i$ son ham uning ildizi bo'ladi. Demak, izlangan tenglama $a(x-(1+3i))(x-(1-3i))=0$ ko'rinishda bo'lib, bu yerda $a\in R$, $a\neq 0$. Qavslarni ochib,

o'xshash qo'shiluvchilar ixchamlansa, $ax^2-2ax+10a=0$ kvadrat tenglama hosil bo'ladi. Bu esa izlangan tenglamadir.

Javob: $ax^2-2ax+10a=0$, $(a\in R,\ a\neq 0)$.

7.1. Tenglamaning barcha kompleks yechimlarini toping.

a) $x^2-2x+2=0$; f) $x^2+2x+17=0$; j) $9x^2-12x+5=0$;
b) $x^2-4x+5=0$; g) $x^2-8x+41=0$; k) $16z^2-32z+17=0$;
d) $x^2+6x+13=0$; h) $9x^2+6x+10=0$; l) $z^2+4z+7=0$;
e) $x^2+4x+13=0$; i) $4x^2+4x+5=0$; m) $z^2-6z+11=0$.

7.2. Kvadrat uchhadni chiziqli ko'paytuvchilarga ajrating:

a) x^2+2x+5; d) $4z^2+8z+5$;
b) $x^2-3x+10$; e) $25z^2+50z+26$.

7.3. Tenglamani kompleks sonlar to'plamida yeching:

a) $z^4+5z^2-36=0$; f) $x^4+3x^2-18=0$;
b) $x^4-8x^2-9=0$; g) $x^4+4x^2-32=0$;
d) $y^4-y^2-6=0$; h) $z^4+z^2+1=0$;
e) $t^4+2t^2-15=0$; i) $z^6-2z^3+4=0$.

7.4. Ildizlaridan biri $2-3i$ bo'lgan haqiqiy koeffitsiyentli kvadrat tenglama tuzing.

7.5. Ildizlari $2-3i$, $2-i$ bo'lgan haqiqiy koeffitsiyentli to'rtinchi darajali tenglama tuzing.

7.6. Ildizlari 2, $2-3i$, $2-i$ bo'lgan haqiqiy koeffitsiyentli beshinchi darajali tenglama tuzing.

7.7. $x=1$ soni $x^{2n}-nx^{n+1}+nx^{n-1}-1$ ko'phadning necha karrali ildizi ekanini aniqlang.

7.8. Quyidagi ko'phadlarni chiziqli va kvadrat ko'paytuvchilar ko'paytmasi shaklida tasvirlang.

a) x^6+27; d) x^6+64;
b) x^4+16x^2; e) x^4+7x^2.

8-§. YUQORI DARAJALI TENGLAMALAR

Teorema: $\frac{p}{q}$ **qisqarmas kasr** $(p\in Z,\ q\in N)$ **bo'lsin.** $\frac{p}{q}$ **son** $P_n(x)=a_0x^n+a_1x^{n-1}+...+a_{n-1}x+a_n$ ***ko'phadning ildizi*** **bo'lishi uchun** p **son ozod had** a_n **ning,** q **soni esa bosh koeffitsiyent** a_0 **ning bo'luvchisi bo'lishi zarur.**

Natija: $p\in Z$ **soni** $P_n(x)$ **ko'phadning ildizi bo'lishi uchun** p **soni ozod had** a_n **ning bo'luvchisi bo'lishi zarur.**

1 - misol. $2x^3+x^2-4x-2=0$ tenglamaning ratsional ildizlarini toping.

Yechish. Ozod hadning barcha butun bo'luvchilari: $-2; -1; 1; 2$. Bosh koeffitsiyentning barcha natural bo'luvchilari: 1; 2.

Tenglamaning ratsional ildizlarini quyidagi sonlar orasidan izlaymiz:

$$-2;\ -1;\ -\frac{1}{2};\ \frac{1}{2};\ 1;\ 2.$$

Bu sonlarni berilgan tenglamaga bevosita qo'yib ko'rish bilan, ularning ildiz bo'lish yoki bo'lmasligini aniqlaymiz.

Tekshirish ko'rsatadiki, $-\frac{1}{2}$ soni berilgan tenglamaning ildizi bo'ladi, qolgan sonlar esa ildiz bo'lmaydi.

Shunday qilib, berilgan tenglama faqat bitta ratsional ildizga ega: $x=-\frac{1}{2}$.

Javob: $-\frac{1}{2}$.

2 - misol. Tenglamaning butun ildizlarini toping: $2x^4-x^3+2x^2+3x-2=0$.

Yechish. Ozod hadning barcha butun bo'luvchilari: $-2; -1; 1; 2$. Tenglamaning barcha butun ildizlarini quyidagi sonlar orasidan izlaymiz: $-2; -1; 1; 2$.

Bu sonlarning har birini tenglamaga qo'yib ko'rib, ular orasidan faqat -1 songina tenglamaning yechimi ekanini aniqlaymiz.

Demak, berilgan tenglama faqat bitta butun yechimga ega: $x=-1$.

Javob: $x=-1$.

3 - misol. $x^3+3x^2-1=0$ tenglamaning butun ildizlarini toping.

Yechish. Butun ildizlarni -1; 1 sonlar orasidan izlaymiz. Bu sonlarning ikkalasi ham tenglamaning ildizi emasligini ko'rish qiyin emas.

Javob: tenglama butun ildizga ega emas.

4 - misol. $2x^4-x^3+2x^2+3x-2=0$ $(x\in R)$ tenglamani yeching.

Yechish. Oldingi misollardan farqli o'laroq, bu yerda tenglamaning barcha haqiqiy ildizlarini topish talab qilinayapti.

Dastlab ratsional ildizlarini izlaymiz. Ratsional ildizlar esa $-2; -1; -\frac{1}{2}; \frac{1}{2}; 1; 2$ sonlar orasida bo'ladi (agar ular

mavjud bo'lsa). Ratsional ildizlar quyidagi sonlar ekanligiga ishonch hosil qilish mumkin: -1 va $\frac{1}{2}$.

Shuning uchun tenglamaning chap tomonidagi ko'phad $(x+1)(x-\frac{1}{2}) = x^2+\frac{1}{2}x-\frac{1}{2}$ ga qoldiqsiz bo'linadi. Bo'lishni bajarib,

$$2x^4 - x^3 + 2x^2 + 3x - 2 = \left(x^2 + \frac{1}{2}x - \frac{1}{2}\right) \cdot (2x^2 - 2x + 4)$$

ni hosil qilamiz.

Tenglamani quyidagi ko'rinishda yozib olamiz:

$$\left(x^2 + \frac{1}{2}x - \frac{1}{2}\right) \cdot (2x^2 - 2x + 4) = 0.$$

$2x^2 - 2x + 4 = 0$ tenglama yangi haqiqiy ildizlarni bermaydi.

Javob: $x_1=-1$; $x_2=\frac{1}{2}$.

8.1. Tenglamaning ratsional ildizlarini toping:

a) $3x^3-4x^2+5x-18=0$;
b) $x^3-4x^2-27x+90=0$;
d) $x^4-x^3+x+2=0$;
e) $2x^3-5x^2+8x-3=0$;
f) $4x^4+8x^3-3x^2-7x+3=0$;
g) $x^4+x^3+x^2+3x+2=0$;
h) $x^4-4x^3-13x^2+28x+12=0$;
i) $3x^4+4x^2+5x-12=0$.

8.2. Tenglamaning butun ildizlarini toping:

a) $x^4+2x^3+4x^2+3x-10=0$;
b) $x^3+7x^2+14x+8=0$;
d) $x^4-x^3+2x^2-x+1=0$;
e) $x^4+x^2+x+2=0$.

8.3. Tenglamani yeching ($x\in R$):

a) $3x^3-5x^2+3x+5=0$;
b) $4x^5+8x^4+5x^3+10x^2-3x-6=0$;
d) $3x^5-6x^4+4x^3-8x^2-3x+6=0$;
e) $2x^5+4x^4-5x^3-10x^2-7x-14=0$;
f) $3x^5-6x^4-8x^3+16x^2-16x+32=0$;
g) $2x^5+6x^4-7x^3-21x^2-4x-12=0$.

8.4. Tenglamaning barcha haqiqiy ildizlarini toping:

a) $2x^4+3x^3-8x^2-9x+6=0$;
b) $2x^4-5x^3-x^2+5x+2=0$;
d) $5x^4-3x^3-4x^2-3x+5=0$;
e) $4x^4-3x^3-8x^2+3x+4=0$;
f) $3x^4-4x^3-7x^2+4x+4=0$;
g) $2x^4-7x^3-5x^2+7x+3=0$.

8.5. Tenglamani yeching ($x\in R$):

a) $8x^4+6x^3-13x^2-x+3=0$;
b) $x^3+6x+4x^2+3=0$;
d) $2x^4-x^3-9x^2+13x-5=0$;
e)* $(x-1)^3+(2x+3)^3=27x^3+8$;
f) $x^3-(2a+1)x^2+(a^2+a)x-(a^2-a)=0$;
g) $x^4-4x^3-19x^2+106x-120=0$.

8.6. $(x+a)(x+b)(x+c)(x+d)=m$ (bu yerda $a+b=c+d$) ko'rinishdagi tenglamani yeching:

a) $(x+1)(x-2)(x+3)(x-4)=144$;
b) $(x-1)(x+2)(x-3)(x-6)=6$;
d) $(x-3)(x+2)(x-6)(x-1)=-56$;
e) $(x+2)(x-3)(x-4)(x+3)=9$;
f) $(x+3)(x-2)(x-6)(x+7)=-180$;
g) $(x+6)(x+2)(x-7)(x-3)=-180$.

8.7. Qaytma tenglamani yeching:

a) $x^4-3x^3+4x^2-3x+1=0$;
b) $x^4-3x^3+x^2+3x+1=0$;
d) $x^4-4x^3+x^2-4x+1=0$;
e) $2x^4-4x^3+2x^2-4x+2=0$;
f) $x^4+2x^3-x^2+2x+1=0$;
g) $x^4+2x^3+x^2-2x+1=0$.

8.8. Qaytma tenglamaning barcha haqiqiy ildizlarini toping:

a) $x^4+5x^3+2x^2+5x+1=0$;
b) $4x^4+2x^3+3x^2+x+1=0$;
d) $2x^4+3x^3-13x^2-6x+8=0$;
e) $3x^4-2x^3+x^2-6x+27=0$.

8.9. Tenglamani yeching:

a) $8x^3+36x^2+54x=98$.

Y e c h i s h. $8x^3+36x^2+54x=(2x+3)^3-27$ bo'lgani uchun, berilgan tenglama $(2x+3)^3-27=98$ tenglmaga teng kuchli. Bundan: $(2x+3)^3=125$; $2x+5=5$. Bundan $x=1$ ekanini topamiz.

J a v o b: 1.

b) $8x^3-36x^2+54x=28$;
d) $16x^4+32x^3+12x^2+8x-80=0$;
e) $x^4-8x^3+24x^2-8x=65$;
f) $(x^2+27)^2-5(x^2+27)(x^2+3)+6(x^2+3)^2=0$.
g) $(x^2-1)^2+5(x^4-1)-6(x^2+1)^2=0$;
h) $(x^2-3)^2-7(x^4-9)+6(x^2+3)^2=0$.
i) $(x-2)^2+(x-2)(x+1)+(x+1)^2=0$.

Namuna sifatida f) tenglamani yechib ko'rsatamiz.

E c h i s h. Bu tenglamaning hadlarini $(x^2+3)^2$ ga bo'lsak, u ushbu ko'rinishni oladi: $\frac{(x^2+27)^2}{(x^2+3)^2} - 5 \cdot \frac{x^2+27}{x^2+3} + 6 = 0$, $y=\frac{x^2+27}{x^2+3}$ deb belgilasak, $y^2-5y+6=0$ tenglama hosil bo'ladi.

Bundan $y_1=2$, $y_2=3$ larga egamiz.

$\frac{x^2+27}{x^2+3} = 2$, $\frac{x^2+27}{x^2+3} = 3$ tenglamalar mos ravishda $\pm\sqrt{21}$ va ± 3 ildizlarga ega.

8.10. $f[f(x)]=x$ ko'rinishidagi tenglamani yeching:

a) $(x^2-4x+6)^2-4(x^2-4x+6)+6=x$ (*).

Y e c h i s h. $x^2-4x+6=x$ tenglamani yechamiz:

$x^2-5x+6=0$; $x_1=2$, $x_2=3$;

$(x^2-4x+6)^2-4(x^2-4x+6)-x=0$ ko'phad $(x-2)(x-3)$ ga qoldiqsiz bo'linadi. Bo'lishni bajarib, x^2-3x+3 bo'linmani topamiz. (*) ni quyidagi ko'rinishda yozish mumkin: $(x^2-3x+3)(x-2)(x-3)=0$

Bu tenglama $x=2$, $x=3$ lardan boshqa haqiqiy ildizlarga ega emas. (*) tenglamaning hamma ildizlari: 2; 3.

b) $(x^2+2x-5)^2+2(x^2+2x-5)-5=x$;

v) $(x^2-x-3)^2-(x^2-x-3)-3=x$;

d) $(x^2-8x+18)^2-8(x^2-8x+18)+18=x$;

e) $(x^2-9x+16)^2-9(x^2-9x+16)+16=x$;

f) $(x^2-3x+3)^2-3(x^2-3x+3)+3=x$.

9-§. DETERMINANTLAR

1 - m i s o l. Tenglamani yeching: $\begin{vmatrix} x & 3 \\ 1 & x^2 \end{vmatrix} = 5$.

Y e c h i s h. $\begin{vmatrix} x & 3 \\ 1 & x^2 \end{vmatrix} = x \cdot x^2 - 1 \cdot 3 = x^3 - 3$ bo'lgani uchun tenglama $x^3-3=5$ yoki $x^3=8$ ko'rinishini oladi.

J a v o b: $x=2$.

2 - m i s o l. $\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix}$ ni hisoblang.

Y e c h i s h. I usul:

$$\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix} = 1\cdot5\cdot9+2\cdot6\cdot7+4\cdot8\cdot3-7\cdot5\cdot3-4\cdot2\cdot9-8\cdot6\cdot1=$$
$$=45+84+96-105-72-48=0.$$

II usul:

$$\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix} = (-1)^{1+1}\cdot1\cdot\begin{vmatrix} 5 & 6 \\ 8 & 9 \end{vmatrix} + (-1)^{1+2}\cdot2\cdot\begin{vmatrix} 4 & 6 \\ 7 & 9 \end{vmatrix} + (-1)^{1+3}\cdot3\cdot\begin{vmatrix} 4 & 5 \\ 7 & 8 \end{vmatrix} =$$
$$=1\cdot(45-48)-2(36-42)+3(32-35)=-3+12-9=0.$$

9.1. Determinantlarni hisoblang:

a) $\begin{vmatrix} -3 & 0 \\ 7 & 5 \end{vmatrix}$; d) $\begin{vmatrix} -5 & -7 \\ 13 & -6 \end{vmatrix}$; f) $\begin{vmatrix} 0 & 0 \\ 1 & -6 \end{vmatrix}$; h) $\begin{vmatrix} 1-a & -a \\ a & 1+a \end{vmatrix}$;

b) $\begin{vmatrix} 2 & -1 \\ 3 & 0 \end{vmatrix}$; e) $\begin{vmatrix} 1 & -\frac{3}{2} \\ -2 & 3 \end{vmatrix}$; g) $\begin{vmatrix} 1 & 0 \\ -3 & 0 \end{vmatrix}$; i) $\begin{vmatrix} x & 1 \\ x^2 & x^3 \end{vmatrix}$.

9.2. a ning qanday qiymatlarida determinantning satrlari proporsional bo'ladi:

a) $\begin{vmatrix} 1 & 3 \\ 2 & a \end{vmatrix}$; b) $\begin{vmatrix} a & -4 \\ 1 & 2 \end{vmatrix}$; d) $\begin{vmatrix} 7 & 5 \\ a & 3a \end{vmatrix}$; e) $\begin{vmatrix} 0 & 0 \\ 6 & a \end{vmatrix}$?

9.3. Tenglamani yeching:

a) $\begin{vmatrix} a & 2 \\ 2 & a \end{vmatrix}=0$; b) $\begin{vmatrix} a-1 & 3 \\ a^2 & 3a \end{vmatrix}=0$; d) $\begin{vmatrix} a & a-1 \\ a+2 & a \end{vmatrix}=0$.

9.4. Determinantlarni hisoblang:

a) $\begin{vmatrix} 2 & 3 & 4 \\ 5 & -2 & 1 \\ 1 & 2 & 3 \end{vmatrix}$; d) $\begin{vmatrix} a & 1 & a \\ -1 & a & 1 \\ a & -1 & a \end{vmatrix}$; f) $\begin{vmatrix} a & 1 & a \\ 0 & -a & -1 \\ a & 1 & -a \end{vmatrix}$;

b) $\begin{vmatrix} 1 & 2 & 5 \\ 3 & -4 & 7 \\ -3 & 12 & -15 \end{vmatrix}$; e) $\begin{vmatrix} 1 & b & 1 \\ 0 & b & 0 \\ b & 0 & b \end{vmatrix}$; g) $\begin{vmatrix} a & -a & a \\ a & a & -a \\ a & -a & -a \end{vmatrix}$.

9.5. Tenglamani yeching:

a) $\begin{vmatrix} x & 1 & 0 \\ 2 & 2 & 3 \\ 1 & 2 & x \end{vmatrix} = 0$; d) $\begin{vmatrix} 1 & 2 & 3 \\ 1 & 2 & x \\ 1 & 2 & 4 \end{vmatrix}=0$;

b) $\begin{vmatrix} x^2 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 2 & 3 \end{vmatrix}=0$; e) $\begin{vmatrix} 1 & 3 & 5 \\ 2 & 6 & 10 \\ x^4 & x & x \end{vmatrix}=0$.

9.6. Hisoblang:

a) $2\cdot\begin{vmatrix} 1 & 3 \\ 2 & 5 \end{vmatrix} - 2\begin{vmatrix} x & 2 \\ 1 & -1 \end{vmatrix}$, bunda $x=3,1(73)$;

b) $2,(7)\cdot\begin{vmatrix} x & 0 \\ 2 & 0 \end{vmatrix} + 3,(13)$, bunda $x=2,(71)$.

9.7. Determinantlarni hisoblang:

a) $\begin{vmatrix} 5 & 20 & 15 \\ 2 & 4 & 8 \\ 1 & 4 & 7 \end{vmatrix}$; b) $\begin{vmatrix} 1 & 0 & 0 \\ 6 & 2 & 0 \\ 5 & 4 & 3 \end{vmatrix}$; d) $\begin{vmatrix} 7 & 3 & 2 \\ 3 & 1 & 2 \\ 10 & 12 & 8 \end{vmatrix}$;

e) $\begin{vmatrix} 1 & 3 & 2 \\ 2 & 1 & 2 \\ 4 & 12 & 8 \end{vmatrix}$; f) $\begin{vmatrix} 7 & 1 & 2 \\ 3 & 2 & 2 \\ 10 & 4 & 8 \end{vmatrix}$; g) $\begin{vmatrix} 7 & 3 & 1 \\ 3 & 1 & 2 \\ 10 & 12 & 4 \end{vmatrix}$.

9.8. Tenglamani yeching:

a) $2\cdot\begin{vmatrix} x & 1 \\ 2 & 3 \end{vmatrix} + 3\cdot\begin{vmatrix} x & 1 & 0 \\ x^2 & x & 0 \\ 1 & 2 & 3 \end{vmatrix}=0$; b) $2\cdot\begin{vmatrix} x^2 & 1 & 0 \\ 1 & 1 & 0 \\ 3 & 4 & 1 \end{vmatrix} - 3\cdot\begin{vmatrix} x & 3 \\ 2 & 4 \end{vmatrix}=16$;

d) $\dfrac{\begin{vmatrix} x^2 & 1 & 0 \\ 1 & 1 & 0 \\ 3 & 4 & 1 \end{vmatrix}}{\begin{vmatrix} x & 3 \\ 2 & 4 \end{vmatrix}} - \dfrac{\begin{vmatrix} 4 & 2 \\ 3 & 6 \end{vmatrix}}{4x-6} = -\dfrac{67}{4}$; e) $\dfrac{3}{\begin{vmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 1 & 2 & 1 \end{vmatrix}} - x = 1.$

10-§. CHIZIQLI TENGLAMALAR SISTEMASI

$\begin{cases} a_1x + b_1y = c_1, \\ a_2x + b_2y = c_2 \end{cases}$ sistema ikki o'zgaruvchili chiziqli tenglamalar sistemasi deyila. Bu yerda $a_1, b_1, c_1, a_2, b_2, c_2$ lar haqiqiy sonlar bo'lib, ularning hammasi bir vaqtda nolga teng bo'lishi ham mumkin.

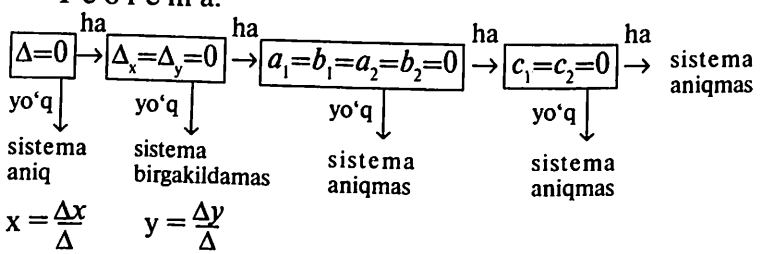
Yagona yechimga ega bo'lgan sistema *aniq sistema* deb, cheksiz ko'p yechimga ega bo'lgan sistema esa *aniqmas sistema* deb ataladi.

Echimga ega bo'lmagan sistema *birgalikda bo'lmagan sistema* deyiladi.

$\Delta = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$ ni sistemaning *asosiy determinanti* deb, quyidagi determinantlarni esa sistemaning *yordamchi determinantlari* deb ataymiz:

$$\Delta_x = \begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}; \quad \Delta_y = \begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}.$$

Teorema.



$x = \dfrac{\Delta x}{\Delta}$ $y = \dfrac{\Delta y}{\Delta}$

Misol 1. $\begin{cases} 2x + ay = a + 2, \\ (a+1)x + 2ay = 2a + 4 \end{cases}$ sistema a ning qanday qiymatlarida cheksiz ko'p yechimga ega bo'ladi?

Yechish. Teoremadan ko'rinadiki, sistema quyidagi hollardagina aniqmas sistema bo'la oladi:

1) $\Delta = \Delta_x = \Delta_y = 0$ va o'zgaruvchilar oldidagi koeffisiyentlardan kamida bittasi noldan farqli;

2) sistemadagi 6 ta koeffitsiyentning hammasi nolga teng.

Bizning sistema uchun 2-hol o'rinli emas. Shu sababli, 1-holni qarash yetarlidir.

Berilgan sistema aniqmas sistema bo'lishi uchun a parametr quyidagi sis-temaning yechimi bo'lishi kerak:

$$\begin{cases} \Delta = \begin{vmatrix} 2 & a \\ a+1 & 2a \end{vmatrix} = 0, \\ \Delta_x = \begin{vmatrix} a+2 & a \\ 2a+4 & 2a \end{vmatrix} = 0, \\ \Delta_y = \begin{vmatrix} 2 & a+2 \\ a+1 & 2a+4 \end{vmatrix} = 0. \end{cases}$$

Bu sistema yagona yechimga ega: $a=3$.

Javob: $a=3$.

10.1. Sistemani o'rniga qo'yish usuli bilan yeching:

a) $\begin{cases} x - y = 5, \\ 2x + 3y = 5; \end{cases}$ b) $\begin{cases} 2x - y = 1, \\ 3x + 4y = 5; \end{cases}$ d) $\begin{cases} x + y = 5, \\ 2x + 2y = 10; \end{cases}$

e) $\begin{cases} \frac{1}{2}x - y = 5, \\ x - 9y = 31; \end{cases}$ f) $\begin{cases} \frac{1}{2}x + \frac{1}{3}y = 6, \\ \frac{x}{5} + \frac{y}{4} = 3\frac{9}{20}; \end{cases}$ g) $\begin{cases} \frac{3}{4}x - \frac{5}{7}y = \frac{23}{168}, \\ 21x - 20y = 21; \end{cases}$

h) $\begin{cases} 0{,}3x - y = \frac{4}{7}, \\ 30x - 10y = \frac{40}{7}; \end{cases}$ i) $\begin{cases} 0{,}3 - 4y = \frac{1}{3}, \\ 0{,}7x - 7y = 43. \end{cases}$

10.2. Sistemani algebraik qo'shish usulida yeching:

a) $\begin{cases} x - y = -1, \\ 4x + y = 6; \end{cases}$ b) $\begin{cases} 2x + y = 2, \\ -2x - y = 3; \end{cases}$ d) $\begin{cases} 2x + 3y = 7, \\ -4x - 6y = -14; \end{cases}$

e) $\begin{cases} 2x + 3y = 2, \\ \frac{1}{2}x - 3y = -\frac{11}{8}; \end{cases}$ f) $\begin{cases} 2x + 3y = \frac{281}{143}, \\ 3x + 4y = \frac{405}{143}; \end{cases}$ g) $\begin{cases} 3{,}1x + \frac{1}{13}y = 1, \\ 3{,}1x + \frac{1}{11}y = 3. \end{cases}$

10.3. Sistemaning asosiy determinantini hisoblang:

a) $\begin{cases} 3x + 4y = 7, \\ 2x - 5y = 1; \end{cases}$; b) $\begin{cases} 1{,}2x - 4y = 3, \\ 3x - 5y = 7; \end{cases}$;

d) $\begin{cases} ax - y = 1, \\ 5x + 2y = 2; \end{cases}$; e) $\begin{cases} ax - by = 1, \\ 13x - 4y = 2. \end{cases}$

10.4. Sistemaning yordamchi determinantlarini hisoblang:

a) $\begin{cases} 2x - 3y = 1, \\ x - y - 7 = 0; \end{cases}$ b) $\begin{cases} 3x - 1{,}7y = 2, \\ 4x - 4{,}3y = 1; \end{cases}$

d) $\begin{cases} 3x - 5y = 2, \\ 4x - 4{,}3y = 5; \end{cases}$ e) $\begin{cases} 4x - 3y = 5, \\ 6x - 7y = 0. \end{cases}$

10.5. Sistemani Kramer formulalaridan foydalanib yeching:

a) $\begin{cases}2x+3y=-4,\\3x+8y=1;\end{cases}$ f) $\begin{cases}x-2y=0,\\4x-8y=5;\end{cases}$ j) $\begin{cases}x-y=1,\\3x-3y=-3;\end{cases}$

b) $\begin{cases}2x+11y=15,\\10x-11y=9;\end{cases}$ g) $\begin{cases}2x-y=3,\\x-0{,}5y=1;\end{cases}$ k) $\begin{cases}3x-5y=0,\\-15x+25y=0;\end{cases}$

d) $\begin{cases}2x-3y=-3,\\x+3y=21;\end{cases}$ h) $\begin{cases}-x+3y=-2,\\2x-6y=-1;\end{cases}$ l) $\begin{cases}2x-3y=-1,\\4x-6y=1;\end{cases}$

e) $\begin{cases}2x-3y=16,\\x+2y=1;\end{cases}$ i) $\begin{cases}\frac{3}{4}x-\frac{5}{7}x=\frac{23}{168},\\2x+6y=\frac{31}{165};\end{cases}$ m) $\begin{cases}7x-2y=16,\\3{,}5x-y=8.\end{cases}$

10.6. $\begin{cases}3x-5y=-7,\\4x+7y=18\end{cases}$ sistema berilgan:

a) Sistemaning har bir tenglamasi nechta yechimga ega?
b) Sistema nechta yechimga ega?

10.7. Sistemani yeching:

a) $\begin{cases}2x+ay=-6,\\ax+8y=12;\end{cases}$ b) $\begin{cases}ax-(a-1)y=0{,}5,\\(a-1)x-ay=a;\end{cases}$

d) $\begin{cases}3x-ay=6-a,\\-ax+3y=3-2a.\end{cases}$

10.8. $\begin{cases}a^2x-ay=a-1,\\bx+(3-2b)y=3+a\end{cases}$ sistema (1;1) dan iborat yagona yechimga ega. a va b larni toping.

10.9. a va b larning quyidagi sistema cheksiz ko'p yechimga ega bo'ladigan barcha qiymatlarini toping:

$\begin{cases}a^2x-by=a^2-b,\\bx-b^2y=2+4b.\end{cases}$

10.10. a ning qanday qiymatlarida quyidagi sistema yechimga ega bo'lmaydi:

$\begin{cases}ax-4y=a+1,\\2x+(a+6)y=a+3\end{cases}$

10.11. a ning qanday qiymatlarida

$\begin{cases}2x-ay=a+2,\\(a+1)y+2ay=2a+4\end{cases}$ sistema cheksiz ko'p yechimga ega bo'ladi?

10.12. Sistemani Gauss usulida yeching:

a) $\begin{cases}x+y+z=1,\\2x+3y-2z=7,\\3x+2y+5z=0;\end{cases}$ b) $\begin{cases}x+y-z=-1,\\3x-2y+4z=9,\\2x+3y+2z=1;\end{cases}$

d) $\begin{cases}x-y+z=-1,\\2x+3y+4z=5,\\3x-2y-2z=-7;\end{cases}$ e) $\begin{cases}x-y-z=-1,\\4x+5y-3z=6,\\2x+3y-2z=3;\end{cases}$

f) $\begin{cases}-x+y+z=-3,\\2x+2y-3z=3,\\3x+4y+5z=-6;\end{cases}$ g) $\begin{cases}-x-y+z=3,\\5x+2y+3z=-4,\\3x+4y-2z=-9.\end{cases}$

10.13. Sistemani Kramer formulalari yordamida yeching:

a) $\begin{cases}-x_1+2x_2+3x_3=0,\\x_1-4x_2-13x_3=0,\\-3x_1+5x_2+4x_3=0;\end{cases}$ b) $\begin{cases}2x-4y+z=1,\\x-2y+4z=3,\\3x-y+5z=2;\end{cases}$

d) $\begin{cases}x+2y+3z=1,\\2x+y-z=3,\\3x+3y+2z=10;\end{cases}$ e) $\begin{cases}x+2y+3z=4,\\2x+4y+6z=3,\\3x+y-z=1;\end{cases}$

f) $\begin{cases}2x-3y+z-2=0,\\x+5y-4z+5=0,\\4x+y-3z=-4;\end{cases}$ g) $\begin{cases}7x+3y+2z=1,\\3x+y+2z=2,\\10x+12y+8z=4.\end{cases}$

10.14. Sistemani yeching:

a) $\begin{cases}5x+2y+3z=-7,\\5x+2y+3z=4;\end{cases}$ b) $\begin{cases}5x-3y=7,\\-2x+9y=4,\\2x+4y=-2;\end{cases}$

d) $\begin{cases}4x+5z=6,\\y-6z=-2;\end{cases}$ e) $\begin{cases}x+2y=3,\\3y-2z=-1.\end{cases}$

11-§. CHIZIQLI BO'LMAGAN TENGLAMALAR SISTEMASI

1 - m i s o l. $\begin{cases}x-y=2,\\x^2+y^2=10\end{cases}$ sistemani yeching.

Y e c h i s h. Sistemaning tarkibida o'zgaruvchilarning birini ikkinchisi orqali chiziqli ifodalab olish imkonini beradigan tenglama mavjud. Bunday holda sistemani o'rniga qo'yish usuli bilan yechish mumkin. $x-y=2$ dan $x=y+2$ ni topib, ikkinchi tenglamada $x=y+2$ o'rniga qo'yishni bajaramiz: $(y+2)^2+y^2=10$.

Bu tenglama $y_1=-3$, $y_2=1$ ildizlarga ega. U holda $x_1=y_1+2=-3+2=-1$; $x_2=y_2+2=1+2=3$.

Javob: $(-1;-3)$ va $(3;1)$.

2 - m i s o l. $\begin{cases}x-y=1,\\x^2-y^2=3\end{cases}$ sistemani yeching.

Y e c h i s h. Bu sistemani ham o'rniga qo'yish usuli bilan yechish qulay. Ammo o'rniga qo'yishni boshqacha yo'l bilan amalga oshiramiz: $x^2-y^2=(x-y)(x+y)$ bo'lgani uchun siste-

mani $\begin{cases} x-y=1, \\ x+y=3 \end{cases}$ ko'rinishda yozib olamiz. Bu esa chiziqli tenglamalar sistemasidir. Uni yechib, (2;1) yechimni topamiz.

Javob: (2;1).

3 - misol. $\begin{cases} x+y=15, \\ xy=56 \end{cases}$ sistemani yeching.

Yechish. Bu sistemaga o'rniga qo'yish usulini qo'llasak, kvadrat tenglamani yechishdan iborat oraliq masalaga kelib qolamiz. Bu kvadrat tenglamani berilgan sistemaga o'rniga qo'yish usulini qo'llamay hosil qilish mumkin. Buning uchun Viet teoremasiga teskari teoremadan foydalanish zarur.

x, y lar uchun $x+y=15$, $xy=56$ bo'lsa, ular $t^2-15t+56=0$ kvadrat tenglamaning ildizlari bo'ladi. $t^2-15t+56=0$ tenglamani yechib, $t_1=7$, $t_2=8$ larni topamiz. Demak, $x_1=7$, $y_1=8$ va $x_2=8$, $y_2=7$.

Javob: (7;8) va (8;7).

3 - misolda qarab chiqilgan usul «yordamchi kvadrat tenglama tuzish usuli» deb atalishi mumkin.

4 - misol. $\begin{cases} x-y=1, \\ xy=12 \end{cases}$ sistemani yeching.

Echish. $z=-y$ deb olib, sistemani quyidagicha yozib olamiz:

$$\begin{cases} x+z=1, \\ xz=-12 \end{cases}$$

Yordamchi kvadrat tenglamani tuzamiz:

$$t^2-t-12=0.$$

Bundan $t_1=-3$, $t_2=4$ larni topamiz. U holda $x_1=-3$, $z_1=4$ va $z_2=4$, $z_2=-3$ bo'ladi. $z=-y$ ekanini e'tiborga olib, berilgan sistemaning yechimlarini aniqlaymiz: $x_1=-3$; $y_1=-4$; $x_2=4$; $y_2=3$.

Javob: (–3;–4); (4;3).

5 - misol. $\begin{cases} x^2+y^2=10, \\ xy=3 \end{cases}$ sistemani yeching.

Yechish. I usul. Ikkiga ko'paytirilgan ikkinchi tenglamani birinchi tenglamaga hadma-had qo'shib, $(x+y)^2=16$ tenglamani hosil qilamiz. Bundan $x+y=-4$ yoki $x+y=4$ ekanligi kelib chiqadi. Shuning uchun berilgan sistema ikkita sistemaga ajraladi:

$$\begin{cases} x+y=-4, \\ xy=3 \end{cases} \quad (1); \qquad \begin{cases} x+y=4, \\ xy=3 \end{cases} \quad (2).$$

(1) sistemani yechib, (–1;–3), (–3;–1) yechimlarni; (2) sistemani yechib, (1;3), (3;1) yechimlarni topamiz.

Shunday qilib, berilgan sistema (–1;–3), (–3;–1), (1;3) va (3;1) lardan iborat yechimlarga ega.

II usul. Berilgan sistema yordamida o'zgaruvchilardan birining ikkinchisiga nisbatini topishga harakat qilamiz. Tenglamalarning chap tomonlarida bir xil darajali hadlar turgani sababli bir jinsli tenglama hosil qilish mumkin. Buning uchun uchga ko'paytirilgan birinchi tenglamadan 10 ga ko'paytirilgan ikkinchi tenglamani ayiramiz: $3x^2-10xy+3y^2=0$.

Sistemadan ko'rinib turibdiki, $x\neq0$, $y\neq0$. Shy sababli, hosil qilingan bir jinsli tenglamadan $\frac{x}{y}$ ni topish mumkin:

$$3\left(\frac{x}{y}\right)^2-10\cdot\frac{x}{y}+3=0;$$

$$\frac{x}{y}=\frac{1}{3} \text{ yoki } \frac{x}{y}=3.$$

U holda berilgan sistema quyidagi sistemalarga ajraladi:

$$\begin{cases} 3x=y, \\ xy=3 \end{cases} \quad (3); \qquad \begin{cases} x=3y, \\ xy=3 \end{cases} \quad (4).$$

Bu sistemalar ham birinchi usulda topilgan yechimlarga olib keladi.

6 - misol. $\begin{cases} x(x+y)=12, \\ y(x+y)=4 \end{cases}$ sistemani yeching.

Yechish. 5-misoldagi yechishning ikkinchi usulida o'zgaruvchilardan birining ikkinchisiga nisbatini bir jinsli tenglama tuzib topdik. Bu misolda ham shunday qilish mumkin. Biroq, bu sistema uchun o'zgaruvchilar nisbatini boshqa usulda topish qulay.

$x+y\neq0$, $x\neq0$, $y\neq0$ ekani ravshan. Shuning uchun tenglamalarni hadma-had bo'lish mumkin:

$$\frac{x(x+y)}{y(x+y)}=\frac{12}{4} \text{ yoki } \frac{x}{y}=3.$$

Natijada $\begin{cases} y(x+y)=4, \\ x=3y \end{cases}$ sistemaga ega bo'lamiz. Bu sistema va demak, berilgan sistema ham (–3;–1), (3;1) yechimlarga ega.

Javob: (–3;–1) va (3;1).

7-misol. $\begin{cases} x+y+xy=7, \\ x^2+y^2+xy=13 \end{cases}$ sistemani yeching.

Yechish. Sistemada x ni y bilan, y ni esa x bilan almashtirsak, yana shu sistemaning o'zi hosil bo'ladi. Bunday sistemalar *simmetrik sistema* deb ataladi.

$x+y=u$, $xy=v$ deb olib, $\begin{cases} u+v=7, \\ u^2-v=13 \end{cases}$ sistemani hosil qilamiz.

Bundan $u_1=4$, $v_1=3$ va $u_2=-5$, $v_2=12$.

Eski o'zgaruvchilarga qaytsak,

$\begin{cases} x+y=4, \\ xy=3 \end{cases}$ va $\begin{cases} x+y=-5, \\ xy=12 \end{cases}$ sistemalar hosil bo'ladi.

Ikkinchi sistema yechimga ega emas, birinchi sistema esa (1;3), (3;1) yechimlarga ega.

Javob: (1;3), (3;1).

11.1. Sistemani yeching:

a) $\begin{cases} x-y=1, \\ x^2+y^2=1; \end{cases}$ b) $\begin{cases} x^2-3xy-2y^2=2, \\ x+2y=1; \end{cases}$ d) $\begin{cases} y-2x=2, \\ 5x^2-y=1; \end{cases}$

e) $\begin{cases} x-2y+1=0, \\ 5xy+y^2=16; \end{cases}$ f) $\begin{cases} x+y=4, \\ y+xy=6; \end{cases}$ g) $\begin{cases} 2x^2-xy=33, \\ 4x-y=17. \end{cases}$

11.2. Sistemani yeching:

a) $\begin{cases} x+y=5, \\ xy=6; \end{cases}$ b) $\begin{cases} x+y=3, \\ xy+4=0; \end{cases}$ d) $\begin{cases} x+y=7, \\ xy=12; \end{cases}$

e) $\begin{cases} x-y=5, \\ xy=-6; \end{cases}$ f) $\begin{cases} x-y=9, \\ xy=-20; \end{cases}$ g) $\begin{cases} x-y=10, \\ xy=-21. \end{cases}$

11.3. Sistemani yeching:

a) $\begin{cases} \frac{x}{25}+\frac{y}{9}=1, \\ x^2+y^2=1; \end{cases}$ b) $\begin{cases} 8x+7y=56, \\ x^2+y^2-4y=0; \end{cases}$

d) $\begin{cases} x+y=1, \\ x^2+xy+y=1; \end{cases}$ e) $\begin{cases} x-2y=-3, \\ -2y^2+xy+3y=0. \end{cases}$

11.4. Sistemani yeching:

a) $\begin{cases} x^2+y^2=20, \\ xy=8; \end{cases}$ f) $\begin{cases} y^2-xy=12, \\ x^2-xy=28; \end{cases}$

b) $\begin{cases} x^2+y^2=68, \\ xy=16; \end{cases}$ g) $\begin{cases} x^2+y^2=25-2xy, \\ y(x+y)=10; \end{cases}$

d) $\begin{cases} x(x+y)=9, \\ y(x+y)=16; \end{cases}$ h) $\begin{cases} 5(x+y)+2xy=-19, \\ 15xy+5(x+y)=-175; \end{cases}$

e) $\begin{cases} x^2+xy=15, \\ y^2+xy=10; \end{cases}$ i) $\begin{cases} 5(x+y)+2xy=-19, \\ 3xy+x+y=-35; \end{cases}$

d) $\begin{cases} x^2-xy=28, \\ y^2-xy=-12; \end{cases}$ k) $\begin{cases} 4x^2+y^2-2xy=7, \\ (2x-y)y=y. \end{cases}$

11.5. Sistemani yeching:

a) $\begin{cases} x+y+xy=5, \\ x^2+y^2+xy=7; \end{cases}$ f) $\begin{cases} xy+3y^2-x+4y-7=0, \\ 2xy+y^2-2x-2y+1=0; \end{cases}$

b) $\begin{cases} 2x^2-xy+3x^2\ 17, \\ y^2-x^2=16; \end{cases}$ g) $\begin{cases} 2xy+y^2-4x-3y+2=0, \\ xy+3y^2-2x-14y+16=0; \end{cases}$

d) $\begin{cases} x^2-xy+y^2=21, \\ y^2-2xy+15=0; \end{cases}$ h) $\begin{cases} 3x^2+xy-2x+y-5=0, \\ 2x^2-xy-3x-y-5=0; \end{cases}$

e) $\begin{cases} 2y^2+xy-x^2=0, \\ x^2-xy-y^2+3x+7y+3=0; \end{cases}$ i) $\begin{cases} 2x^2+y^2+3xy=12, \\ 2(x+y)^2-y^2=14. \end{cases}$

11.6. Sistemani yeching:

a) $\begin{cases} xy-x+y=1, \\ x^2y-xy^2=30; \end{cases}$ b) $\begin{cases} xy+x-y=3, \\ x^2y-xy^2=2; \end{cases}$

d) $\begin{cases} x^2+xy+x=10, \\ y^2+xy+y=20; \end{cases}$ e) $\begin{cases} x^2+xy+2y^2=37, \\ 2x^2+2xy+y^2=26. \end{cases}$

11.7. Sistemani yeching:

a) $\begin{cases} x^3+y^3=55, \\ x+y=5; \end{cases}$ f) $\begin{cases} x^4+y^4=82, \\ xy=3; \end{cases}$

b) $\begin{cases} x-y=1, \\ x^3-y^3=7; \end{cases}$ g) $\begin{cases} x^3+y^3=7, \\ x^3y^3=-8; \end{cases}$

d) $\begin{cases} x^3+y^3=7, \\ xy(x+y)=-2; \end{cases}$ h) $\begin{cases} (x^2+y^2)xy=78, \\ x^4+y^4=97; \end{cases}$

e) $\begin{cases} \frac{x^2}{y}+\frac{y^2}{x}=18, \\ x+y=12; \end{cases}$ i) $\begin{cases} x^3+y^3=19, \\ x-y=5. \end{cases}$

11.8. Sistemani yeching:

a) $\begin{cases} x+y+z=13, \\ x^2+y^2+z^2=91, \\ y^2=xz; \end{cases}$ b) $\begin{cases} \frac{xy}{x+y}=1, \\ \frac{xz}{x+z}=2, \\ \frac{yz}{y+z}=3; \end{cases}$

d) $\begin{cases} x^2+y^2+z^2=xy+yz+zx, \\ x^3+y^3+z^3=1; \end{cases}$ e) $\begin{cases} x+y+z=0, \\ x^2+y^2+z^2=1, \\ x^3+y^3+z^3=0; \end{cases}$

f) $\begin{cases} x+y+z=1, \\ x^2+y^2+z^2=1, \\ x^4+y^4+z^4=1; \end{cases}$ g) $\begin{cases} xy=2, \\ yz=3, \\ zx=6. \end{cases}$

12-§. MATNLI MASALALAR

1 - m a s a l a. Ikki ishchi birga ishlab smena davomida 72 ta detal tayyorladi. Ishlab chiqarish unumdorligini birinchi ishchi 15% ga, ikkinchi ishchi esa 25% ga oshirgach, ular smena davomida birgalikda 86 ta detal tayyorlay boshlashdi. Mehnat unumdorligi oshgach, har bir ishchi smena davomida nechtadan detal tayyorlagan?

Y e c h i s h. Mehnat unumdorligini oshirgunga qadar birinchi ishchi smena mobaynida x ta detal, ikkinchisi esa y ta detal tayyorlagan bo'lsin. U holda mehnat unumdorligi oshgandan so'ng, birinchi ishchi $x+0{,}15x$ ta detal, ikkinchi ishchi esa $y+0{,}25y$ ta detal tayyorlay boshlagan.

Quyidagi sistemaga ega bo'lamiz: $\begin{cases} x + y = 72, \\ 1{,}15x + 1{,}25y = 86. \end{cases}$

Bundan x=40, y=32 larni topamiz. Mehnat unumdorligi oshgach birinchi ishchi smena mobaynida $1{,}15x=1{,}15\cdot 40=46$ ta, ikkinchi ishchi esa $1{,}25y=1{,}25\cdot 32=40$ ta detal tayyorlagan.

J a v o b: 46 ta va 40 ta.

2 - m a s a l a. Ikki sonning yig'indisi 60 ga, nisbati esa 4 ga teng. Shu sonlarni toping.

Y e c h i s h. x va y izlangan sonlar bo'lib, $x>y$ bo'lsin. Quyidagi sistemaga egamiz:

$\begin{cases} x + y = 60, \\ x:y = 4. \end{cases}$ Bu sistemadan $x=48$, $y=12$ ni topamiz.

J a v o b: 48 va 12.

3-m a s a l a. Ikki ishchining ikkinchisi birinchisidan $1\frac{1}{2}$ kun keyin ishga tushsa, ular birgalikda bir ishni 7 kunda tamomlay oladilar. Agar bu ishni har qaysi ishchi yolg'iz o'zi bajarsa, u holda birinchi ishchi ikkinchi ishchiga qaraganda 3 kun ortiq ishlashi kerak bo'ladi. Har qaysi ishchining yolg'iz o'zi bu ishni necha kunda tamomlay oladi?

Y e c h i s h. Birinchi ishchi yolg'iz o'zi ishlab, ishni x kunda, ikkinchi ishchi esa yolg'iz o'zi ishlab, y kunda bajarsin. U holda birinchi ishchi bir kunda ishning $\frac{1}{x}$ qismini, ikkinchi ishchi bir kunda ishning $\frac{1}{y}$ qismini bajaradi.

Birinchi ishchi $1\frac{1}{2}$ kun ishlab, ishning $1\frac{1}{2}\cdot\frac{1}{x}=\frac{3}{2x}$ qismini bajargach, ikkinchi ishchi ishlashni boshladi. Ular birgalikda 7 kun ishlagan. Shu 7 kunda ishning $7\cdot\frac{1}{x}+7\cdot\frac{1}{y}=\frac{7x+7y}{xy}$ qismi bajarilgan. Shunga ko'ra $\frac{3}{2x}+\frac{7x+7y}{xy}=1$ tenglamaga ega bo'lamiz. Yolg'iz o'zi ishlagan birinchi ishchi ikkinchisiga qaraganda 3 kun ko'p ishlab, ishni tamomlaydi. Demak, $x-3=y$.

$\begin{cases} \frac{3}{2x}+\frac{7x+7y}{xy}=1, \\ x-3=y \end{cases}$ sistemani hosil qilamiz. Bu sistemani yechsak, $x=17$, $y=14$ ekani topiladi.

J a v o b: Birinchi ishchi 17 kunda, ikkinchi ishchi 14 kunda.

4 - m a s a l a. Oltin va kumushdan hosil qilingan ikki xil qotishmalarning birinchisida oltin va kumush 2:3 nisbatda, ikkinchisida esa 3:7 nisbatda ekanligi ma'lum. Oltin va kumush 5:11 nisbatda bo'ladigan yangi qotishma hosil qilish uchun ko'rsatilgan qotishmalarni qanday nisbatda olish kerak?

Y e c h i s h. Birinchi qotishmaning $\frac{2}{2+3}=\frac{2}{5}$ qismi oltin va $\frac{2}{2+3}=\frac{2}{5}$ qismi kumushdan iborat. Ikkinchi qotishmaning $\frac{3}{3+7}=\frac{3}{10}$ qismi oltin va $\frac{3}{3+7}=\frac{3}{10}$ qismi esa kumushdir.

Yangi qotishma hosil qilish uchun olingan birinchi qotishmaning miqdorini x bilan va ikkinchi qotishmaning miqdorini y bilan belgilaylik (x va y lar og'irlikni ifodalaydi).

x miqdordagi birinchi qotishmadagi oltinning va kumushning miqdori mos ravishda $\frac{2}{5}x$ va $\frac{3}{5}x$ ga teng. y miqdordagi ikkinchi qotishmadagi oltinning miqdori $\frac{3}{10}y$ ga, kumushning miqdori esa, $\frac{7}{10}y$ ga teng. Yangi qotishmaga $\frac{2}{5}x+\frac{3}{10}y$ miqdorda oltin va $\frac{3}{5}x+\frac{7}{10}y$ miqdorda kumush kiradi. Shartga ko'ra, $\frac{\frac{2}{5}x+\frac{3}{10}y}{\frac{3}{5}x+\frac{7}{10}y}=\frac{5}{11}$. Bu tenglik yordamida $\frac{x}{y}$ nisbatni topamiz:

$\frac{4x+3y}{6x+7y}=\frac{5}{11}\Rightarrow 44x+33y\Rightarrow 30x+35y\Rightarrow 14x=2y\Rightarrow\frac{x}{y}=\frac{1}{7}$.

J a v o b. Qotishmalarni 1:7 nisbatda olish kerak.

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$\begin{cases} \frac{3}{2x}+\frac{7x+7y}{xy}=1, \\ x-3=y \end{cases}$ sistemani hosil qilamiz. Bu sistemani yechsak, $x=17$, $y=14$ ekani topiladi.

J a v o b: Birinchi ishchi 17 kunda, ikkinchi ishchi 14 kunda.

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Shartga ko'ra, $\frac{\frac{2}{5}x+\frac{3}{10}y}{\frac{3}{5}x+\frac{7}{10}y}=\frac{5}{11}$. Bu tenglik yordamida $\frac{x}{y}$ nisbatni topamiz:

$\frac{4x+3y}{6x+7y}=\frac{5}{11}\Rightarrow 44x+33y\Rightarrow 30x+35y\Rightarrow 14x=2y\Rightarrow\frac{x}{y}=\frac{1}{7}$.

J a v o b. Qotishmalarni 1:7 nisbatda olish kerak.

5 - m a s a l a. Mahsulot dastlab 20 % ga arzonlashtirildi. Yangi narx yana 10 % kamaytirilgach, hosil bo'lgan keyingi narx yana 5% ga kamaytirildi. Mahsulotning dastlabki narxi necha foiz kamaytirildi?

Y e c h i s h. Mahsulotning dastlabki narxi x (so'm) bo'lsin. Bu narx 20% kamaytirilgach, mahsulotning narxi $x-0{,}20x=0{,}80x$ (so'm) bo'ladi. Bu narx 10 % kamaytirilsa, $0{,}80x-0{,}10\cdot 0{,}80x=0{,}72x$ so'mdan iborat bo'lgan yangi narx paydo bo'ladi. Bu narx 5 % kamaytirilsa, mahsulotning oxirgi narxi $0{,}72x-0{,}05\cdot 0{,}72x=0{,}684x$ so'm ekanligi kelib chiqadi.

Dastlabki narx x so'm, eng oxirgi narx $0{,}684x$ so'm bo'ldi. Mahsulot $x-0{,}684x=0{,}316x$ so'mga arzonlashtirildi. $0{,}316x$ so'm x so'mning necha foizini tashkil etishini topamiz.

Proporsiya tuzaylik: $\frac{x}{0{,}316x}=\frac{100}{p}$. Bundan $p=31{,}6$ ekani kelib chiqadi.

J a v o b. 31,6 %.

6 - m a s a l a. Ikki xonali noma'lum son raqamlarining yig'indisi 12 ga teng. Shu ikki xonali noma'lum songa 36 soni qo'shilsa, noma'lum sonning raqamlarini teskari tartibda yozishdan hosil bo'ladigan son kelib chiqadi. Noma'lum sonni toping.

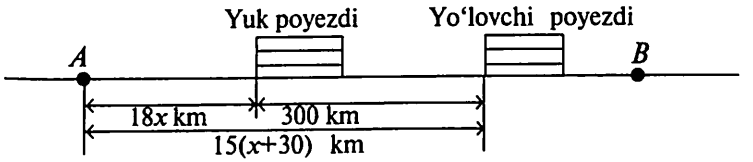
Y e c h i s h. Ikki xonali noma'lum sonning raqamlari x, y bo'lsin, ya'ni $xy = 10x + y$ izlangan son bo'lsin. Quyidagilarga egamiz:

$\begin{cases} x+y=12, \\ \overline{xy}+36=\overline{yx} \end{cases}$, yoki $\begin{cases} x+y=12, \\ 10x+y+36=10y+x. \end{cases}$ Bu sistemadan $x=4$, $y=8$ ekani kelib chiqadi. Demak, izlangan son 48 ekan.

J a v o b: 48.

7 - m a s a l a. Yuk poezdi A shahardan B shaharga qarab jo'nadi (16-rasm). Oradan 3 soat o'tgach, A shahardan V shaharga qarab, yo'lovchi poezdi yo'lga chiqdi va oradan 15 soat o'tgach yuk poezdidan 300 km o'zib ketdi. Agar yo'lovchi poezdining tezligi yuk poezdining tezligidan 30 km/soat ortiq bo'lsa, yuk poezdining tezligini toping.

Y e c h i s h. Yuk poezdining tezligi x km/soat bo'lsin. U holda yo'lovchi poezdining tezligi $x+30$ km/soat bo'ladi. Yo'lovchi poezdi 15 soat yurib, $15(x+30)$ km masofani bosib o'tadi. Yuk poezdi 18 soatda $18x$ km masofani bosib o'tgan.



16-rasm.

Shunga ko'ra $18x+300=15(x+30)$ tenglamaga ega bo'lamiz. Uni yechib, $x=50$ ekanini aniqlaymiz.

J a v o b. 50 km/soat.

Quyidagi masalalarni kvadrat tenglama tuzib yeching:

12.1. To'g'ri to'rtburchakning balandligi asosining 75% iga teng. Agar shu to'g'ri to'rtburchakning yuzi 48 m^2 bo'lsa, uning perimetrini toping.

12.2. 15 t sabzavotni tashish uchun ma'lum miqdorda yuk ortadigan bir necha mashina so'ralgan edi. Garajda tayyor turgan mashinalar bo'lmagani uchun, garaj so'ralgandan bitta ortiq, lekin 0,5 t kam yuk ortadigan mashinalar yubordi. Yuborilgan mashinalarning har biriga necha tonna sabzavot ortilgan?

12.3. Jamoa xo'jaligi 200 ga yerga ma'lum muddatda chigit ekib bo'lishi kerak edi, ammo u har kuni rejadagidan 5 ga ortiq ekib, ishni muddatidan 2 kun oldin tugatdi. Chigit ekish necha kunda tugatilgan?

12.4. Tomosha zalida 320 ta o'rin bor edi. Har bir qatordagi o'rinlar soni 4 ta orttirilib, yana bir qator qo'shilgandan so'ng 420 ta joy bo'ldi. Tomosha zalidagi joylar endi necha qator bo'ldi?

12.5. Kema oqimga qarshi 48 km va oqim bo'yicha ham shuncha yo'l bosdi, hamma yo'lga 5 soat vaqt sarf qildi. Daryo oqimining tezligi 4 km/soat bo'lsa, kemaning turg'un suvdagi tezligini toping.

12.6. Ikki pristan orasidagi masofa daryo yo'li bilan 80 km. Kema shu pristanlarning biridan ikkinchisiga borib kelish uchun 8 soat 20 minut vaqt sarf qiladi. Daryo oqimining tezligi 4 km/soat bo'lsa, kemaning turg'un suvdagi tezligini toping.

12.7. Qayiq daryo oqimiga qarshi 22,5 km, oqim bo'yicha esa 28,5 km yurib, butun yo'lga 8 soat vaqt sarfladi. Oqimning tezligi 2,5 km/soat. Qayiqning turg'un suvdagi tezligini toping.

12.8. Daryo yoqasidagi qishloqdan sol oqizildi. Oradan 5 soat 20 minut o'tgach, o'sha qishloqdan motorli qayiq jo'natildi. Motorli qayiq 20 km yo'l bosib, solga yetib oldi.

Agar motorli qayiqning tezligi solning tezligidan 12 km/soat ortiq bo'lsa, solning tezligini toping.

12.9. Suv ikkita quvurdan kelganda suv haydash qozoni 2 soat 55 minutda to'ladi. Birinchi quvurning yolg'iz o'zi suv haydash qozonini ikkinchisiga qaraganda 2 soat oldin to'ldira oladi. Har qaysi quvurning yolg'iz o'zi suv haydash qozonini qancha vaqtda to'ldiradi?

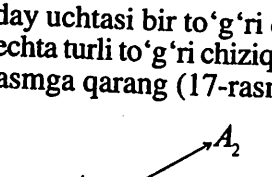
12.10. Ikki ishchi ayni bir ishni birgalashib ishlasa, 12 kunda tamom qiladi. Agar oldin bittasi ishlab, ishning yarmini tamom qilgandan keyin uning o'rniga ikkinchisi ishlasa, ish 25 kunda tamom bo'ladi. Shu ishni har qaysi ishchi yolg'iz o'zi ishlasa, necha kunda tamom qiladi?

12.11. Quvvatlari har xil ikkita traktor 4 kun birga ishlab jamoa xo'jaligi yerining $\frac{2}{3}$ qismini haydadi. Agar butun yerni birinchi traktor ikkinchisiga qaraganda 5 kun tezroq hayday olsa, butun yerni har qaysi traktor yolg'iz o'zi necha kunda hayday oladi?

12.12. Portdagi ikki kema bir vaqtda, biri shimolga qarab, ikkinchisi sharqqa qarab jo'nadi. 2 soatdan keyin ular orasidagi masofa 60 km bo'ldi. Bu kemalardan birining tezligi ikkinchisinikidan 6 km/soat ortiq. Har qaysi kemaning tezligini toping.

12.13. Har qanday uchtasi bir to'g'ri chiziqda yotmaydigan 7 ta nuqtadan nechta turli to'g'ri chiziq o'tkazish mumkin?

Y e c h i s h. Rasmga qarang (17-rasm):



17-rasm.

Boshi A_1 nuqtada bo'lgan 6 ta vektorga egamiz. Boshi qolgan nuqtalarda bo'lgan vektorlar ham 6 tadan bo'ladi. Hammasi bo'lib 7·6=42 ta turli vektorlar hosil bo'ladi. Bu vektorlar 21 juft qarama-qarshi vektorlardir. Qarama-qarshi vektorlar jufti bitta to'g'ri chiziqda yotadi (Bizning misolda).

Shunday qilib, aytilgan to'g'ri chiziqlar 42:2=21 ta ekan.

Topshiriq. Har qanday uchtasi bir to'g'ri chiziqda yotmaydigan n ta nuqta orqali o'tuvchi turli to'g'ri chiziqlar soni $\frac{n(n-1)}{2}$ ga tengligini isbotlang. Bu tasdiqdan foydalanib, 12.14–12.18-masalalarni yeching.

12.14. Futbol o'yini musobaqasida hammasi bo'lib 55 ta o'yin o'ynaldi. Bunda har bir komanda qolgan komandalar bilan faqat bir martadan o'ynadi. Musobaqada nechta komanda qatnashgan?

12.15. Shaxmat turnirida hammasi bo'lib 231 partiya shaxmat o'ynaldi. Agar har bir shaxmatchi qolgan shaxmatchilarning har biri bilan faqat bir partiya shaxmat o'ynagan bo'lsa, turnirda necha kishi qatnashgan?

12.16. Maktab bitiruvchilari bir-birlari bilan rasm almashtirdi. Agar 870 ta rasm almashtirilgan bo'lsa, maktabni necha o'quvchi bitirgan?

12.17. Qavariq ko'pburchakning 14 ta diagonali mavjud. Uning tomonlari nechta?

12.18. Qanday ko'pburchak diagonallarining soni tomonlarining sonidan 12 ta ortiq bo'ladi?

Tenglamalar sistemasi tuzib yechiladigan masalalar

12.19. Poezd yo'lda 6 minut to'xtab qoldi va 20 km yo'lda tezligini soatiga jadvaldagidan 10 km oshirib, kechikishni yo'qotdi. Poezd shu yo'lda jadvalga muvofiq qanday tezlik bilan yurishi kerak edi?

12.20. *A* va *B* stansiyalar orasidagi yo'lning o'rtasida poezd 10 minut to'xtab qoldi. *B* stansiyaga kechikmasdan borish uchun haydovchi poezdning dastlabki tezligini 6 km/soat oshirdi. Agar stansiyalar orasidagi masofa 60 km bo'lsa, poezdning dastlabki tezligini toping.

12.21. Perimetri 28 sm bo'lgan to'g'ri to'rtburchakning qo'shni tomonlariga tashqaridan yasalgan kvadratlar yuzlarining yig'indisi 116 sm^2 ga teng. To'g'ri to'rtburchakning tomonlarini toping.

12.22. Yuzi 120 sm^2, diagonali esa 17 sm bo'lgan to'g'ri to'rtburchakning tomonlarini toping.

12.23. To'g'ri burchakli uchburchakning gipotenuzasi 41 sm, yuzi 180 sm^2. Katetlarini toping.

12.24. To'g'ri burchakli uchburchakning perimetri 48 sm, yuzi 96 sm^2. Uchburchakning tomonlarini toping.

12.25. Ikki musbat sonning o'rta arifmetigi 20, o'rta geometrigi esa 12. Shu sonlarni toping.

12.26. Ikki shahar orasidagi masofa 480 km; shu masofani yo'lovchi poezdi yuk poezdiga qaraganda 4 soat tez bosadi.

Agar yo'lovchi poezdining tezligi 8 km/soat oshirilsa, yuk poezdining tezligi esa 2 km/soat oshirilsa, passajir poezdi shu masofani yuk poezdiga qaraganda 5 soat tez o'tadi. Har qaysi poezdning tezligini toping.

12.27. Oralaridagi masofa 180 km bo'lgan A va B shaharlardan ikki poezd bir vaqtda bir-biriga qarab yo'lga chiqdi. Ular uchrashgandan keyin A shahridan chiqqan poezd B shaharga 2 soatda yetib boradi, ikkinchisi esa A shaharga 4,5 soatda yetib boradi. Poezdlar tezligini toping.

12.28. Velosipedchilar poygasi uchun 6 km uzunlikdagi masofa belgilandi. Akmal Shavkatdan o'tib ketib, marraga 2 minut oldin keldi. Agar Akmal tezligini 0,1 km/minut kamaytirib, Shavkat tezligini 0,1 km/minutga oshirsa, unda Akmal marraga Shavkatdan 2 minut oldin yetib kelardi. Akmal va Shavkatlarning tezligini toping.

12.29. Ikki ekskavator birga ishlab, biror hajmdagi yer ishlarini 3 soat-u 45 minutda bajaradi. Bir ekskavator alohida ishlab, bu hajmdagi ishni ikkinchisiga qaraganda 4 soat tezroq bajaradi. Shunday hajmdagi yer ishlarini bajarish uchun har bir ekskavatorga alohida qancha vaqt kerak bo'ladi?

12.30. Bir kombaynchi maydondagi bug'doy hosilini ikkinchi kombaynchidan 24 soat tezroq o'rib olishi mumkin. Ikkala kombaynchi birgalikda ishlaganda esa hosilni 35 soatda o'rib olishadi. Har bir kombaynchi alohida ishlab, hosilni o'rib olishi uchun qancha vaqt kerak bo'ladi?

12.31. Ikkita musbat sonning yig'indisi ularning ayirmasidan 5 marta katta. Agar shu sonlar kvadratlari ayirmasi 180 ga teng bo'lsa, bu sonlarni toping.

12.32. Ikki xonali son o'zining raqamlari kvadratlarining yig'indisidan 11 ta kam va raqamlarining ikkilangan ko'paytmasidan 5 ta ortiq. Shu ikki xonali sonni toping.

12.33. Ikki xonali son raqamlari kvadratlarining yig'indisi 13 ga teng. Agar bu sondan 9 ni ayirsak, shu raqamlar bilan teskari tartibda yozilgan son hosil bo'ladi. Shu sonni toping.

12.34. Baliq og'irligi bo'yicha beshta qismga 14:12:11:9:15 kabi nisbatlarda bo'lingan. Ikkinchi bo'lakning og'irligi 11,2 g ekanligini bilgan holda baliqning butun og'irligini toping.

12.35. Ikkita metalldan ikki xil qotishma tayyorlangan. Birinchi qotishmada metallar 1:2 nisbatda, ikkinchi qotishmada esa 3:2 nisbatda. Metallar nisbati 8:7 bo'ladigan qilib yangi qotishma tayyorlash uchun metallarni qanday nisbatda olish kerak?

12.36. Tovar dastlab 20 % ga, so'ngra yana 15 % ga arzonlashgach, 2380 so'm deb baholandi. Tovarning dastlabki narxini toping.

13-§. CHIZIQLI TENGSIZLIKLAR. KVADRAT TENGSIZLIKLAR

Tengsizliklarni yeching:

13.1. $7x-3(2x+3)>2(x-4)$. **13.2.** $\frac{x+1}{4}<2\frac{1}{2}-\frac{1-2x}{3}$.

13.3. $\frac{6-5x}{5}+\frac{3x-1}{2}>5-x$. **13.4.** $\frac{7x}{4}<0.3(x+7)+2\frac{1}{5}$.

13.5. $-x(x-1)-6>5x-x^2$. **13.6.** $7x-6<x+12$.

13.7. $1-2x\geq 4-5x$. **13.8.** $1-x\geq 2x+3$.

13.9. $\frac{2}{3-x}<0$. **13.10.** $\frac{4}{2+x}\leq 0$.

13.11. $\frac{x^2}{3x+5}<0$. **13.12.** $3(x-2)+x<4x+1$.

13.13. $5(x+1)\geq 2(x-1)+3x+3$. **13.14.** $\frac{5x+3}{2}-1\geq 3x-\frac{x-7}{2}$.

13.15. $2-\frac{x-4}{3}<2x-\frac{7x-4}{3}$. **13.16.** $(x-1)^2+7>(x+4)^2$.

13.17. $(x+1)^2+3x^2<(2x-1)^2+7$.

13.18. $(x+3)(x-2)\geq(x+2)(x-3)$.

13.19. $(x+1)(x-4)+4\geq(x+2)(x-3)-x$.

13.20. $\frac{2}{3x+6}<0$. **13.21.** $\frac{3}{2x-4}>0$. **13.22.** $\frac{-1,7}{0,5x-2}>0$.

Parametr qatnashgan chiziqli tengsizliklarni yeching:

13.23. $(a^2+1)y>3$. **13.24.** $-(b^2+2)z<0$.

13.25. $ax>-3$. **13.26.** $ax<b$.

13.27. $(a-5)x>2$. **13.28.** $ax>b$.

13.29. $(2m+1)x>2n-7$. **13.30.** $a(x-1)>x-2$.

13.31. $(a-1)x<5a+1$. **13.32.** $ax>a(a-1)$.

13.33. $(2b-1)y<4$. **13.34.** $(2a+1)x<3a-2$.

13.35. y ning qanday qiymatlarida:

a) $\frac{7-2y}{6}$ kasrning qiymati $\frac{3y-7}{12}$ kasrning mos qiymatlaridan katta bo'ladi?

b) $\frac{4,5-2y}{5}$ kasrning qiymati $\frac{2-3y}{10}$ kasrning mos qiymatlaridan kichik bo'ladi?

d) $5y-1$ ikki hadning qiymati $\frac{3y-1}{4}$ kasrning mos qiymatidan katta bo'ladi?

e) $\frac{5-2y}{12}$ kasrning qiymati $1-6y$ ikkihadning mos qiymatidan kichik bo'ladi?

Tengsizlikni grafik usulda yeching:

13.36. $x^2-4x+3>0$. **3.37.** $x^2-6x+5\le 0$.

13.38. $-5x^2+3x+2\ge 0$. **13.38.** $-x^2+x>0$.

13.39. $x^2+x+1<0$. **13.40.** $x^2-x+1\ge 0$.

13.41. $x^2-6x+10\le 0$. **13.42.** $-3x^2+2x+1>0$.

13.44. a ning qanday qiymatlarida $(a-1)x^2-(a+1)x+(a+1)>0$ tengsizlik x ning barcha haqiqiy qiymatlari uchun bajariladi?

13.45. a ning qanday qiymatlarida $(2-a)x^2+2(3-2a)x-5a+6\le 0$ tengsizlik x ning hech bir qiymatida bajarilmaydi?

13.46. a ning $(a-3)x^2-2(3a-4)x+7a-6=0$ tenglama yechimga ega bo'ladigan barcha qiymatlarini toping.

Parametrli tengsizliklarni yeching:

13.47. $kx^2-x-1>0$. **13.48.** $kx^2+12x-5<0$.

13.49. $x^2+kx+3<0$. **13.50.** $x^2-2x+k>0$.

13.51. $kx^2+kx-5<0$. **13.52.** $x^2>a$.

13.53. $x^2+(2k+3)x+k^2+4k+3<0$. **13.54.** $kx^2+(2k)x+k+2>0$.

13.55. $(k+2)x^2+2(k+1)x+k-1>0$.

13.56. $\frac{x^2+x-6}{2k+1}>x+6(2k-1)$.

Tengsizlikni yeching:

13.57. $3x^2-7x+4\le 0$. **13.58.** $3x^2-7x+6<0$.

13.59. $3x^2-7x-6<0$. **13.60.** $x^2-3x+5>0$.

13.61. $x^2-14x-15>0$. **13.62.** $2-x-x^2\ge 0$.

14-§. RATSIONAL TENGSIZLIKLAR

Tengsizliklarni yeching:

14.1. $x^2-4x+45>0$. **14.2.** $x^2+2x>6x-15$.

14.3. $x^2-11x+30>0$. **14.4.** $3x^2-4x+3>0$.

14.5. $3x^2-5x-2>0$. **14.6.** $5x^2-7x+2<0$.

14.7. $3x^2-7x-6<0$. **14.8.** $3x^2-2x+5>0$.

14.9. $(x-2)(x-5)(x-12)>0$. **14.10.** $(x+7)(x+1)(x-4)<0$.

14.11. $x(x+1)(x+5)(x-8)>0$. **14.12.** $(x+48)(x-37)(x-42)>0$.

14.13. $(x+0,7)(x-2,8)(x-9,2)<0$. **14.14.** $(x^2-16)(x+17)>0$.

14.15. $\left(x-\frac{2}{3}\right)(x^2-121)<0$. **14.16.** $x^3-25x<0$.

14.17. $x^3-0,001>0$. **14.18.** $(x^2-9)(x^2-1)>0$.

14.19. $(x^2-1,5x)(x^2-36)<0$. **14.20.** $(x^2+17)(x-6)(x+2)<0$.

14.21. $x(2x^2+1)(x-4)>0$. **14.22.** $(x-1)^2(x-24)<0$.

14.23. $(x+7)(x-4)^2(x-21)>0$. **14.24.** $\frac{x-8}{x+4}>0$.

14.25. $\frac{x+16}{x-11}<0$. **14.26.** $\frac{x+1}{3-x}\ge 0$.

14.27 $\frac{6-x}{x-4}\le 0$. **14.28.** $(x-1)^2(x-2)^3(x-3)^4(x-4)^5>0$.

14.29. $(x-1)^2(x+1)^3(x-2)^4(x-4)^5\ge 0$.

14.30. $(x+2)^2(x-1)^3(x-2)^7\le 0$. **14.31.** $x^3(x+1)^2(x-4)^3\ge 0$.

14.32. $(x-1)^4(x+1)^2<0$. **14.33.** $(x-0,5)(x+0,5)^2(x-2)>0$.

14.34. $x^2(x^2-1)(x+1)\le 0$.

14.35. $\frac{(x-1)(x+2)^4(x-3)^2}{(x-4)^3}>0$. **14.36.** $\frac{(x-1)^4(x-2)^3(x+5)}{(x-7)^2}\ge 0$.

14.37. $\frac{(x-2)^4(x+2)^3(x-1)}{(x-3)^2}\le 0$. **14.38.** $\frac{(x-2)(x-3)^4(x-4)}{x+2}<0$.

14.39. $\frac{(1-x)(x-2)}{12-3x}>0$. **14.40.** $(11-x)^3(x-1,5)\ge 0$.

14.41. $(2-3x)(4x+5)\le 0$. **14.42.** $(2-3x)(4x+5)(3-4x)\ge 0$.

14.43. $(3-4x)(5-6x)(x-7)\le 0$. **14.44.** $(3-4x)^2(4-7x)^3(x+5)>0$.

14.45. $(13-9x)^3(11-8x)^4(5-x)\le 0$. **14.46.** $\frac{(3x-5)(7-4x)^3}{4x+7}>0$.

14.47. $\frac{(4x-7)(3-5x)^2}{(7x-4)^3}<0$. **14.48.** $\frac{(4,5x-9)^2}{7x-21}<0$.

14.49. $\frac{0,5}{x-x^2-1}<0$. **14.50.** $\frac{x^2-5x+6}{x^2+x+1}<0$.

14.51. $\frac{x^2+2x-3}{x^2+1}<0$.

14.52. $\frac{x^2+4x+4}{2x^2-x-1}>0$.

14.53. $x^4-5x^2+4<0$.

14.54. $x^4-2x^2-63\le 0$.

14.55. $\frac{3}{x-2}<1$.

14.56. $\frac{1}{x-1}\le 2$.

14.57. $\frac{4x+3}{2x-5}<6$.

14.58. $\frac{5x-6}{x+6}<1$.

14.59. $\frac{5x-1}{x^2+3}<1$.

14.60. $\frac{x-2}{x^2+1}<-\frac{1}{2}$.

14.61. $\frac{x+1}{(x-1)^2}<1$.

14.62. $\frac{x^2-7x+12}{2x^2+4x+5}>0$.

14.63. $\frac{x^2+6x-7}{x^2+1}\le 2$.

14.64. $\frac{x^2-5x+7}{-2x^2+3x+2}>0$.

14.65. $\frac{x+7}{x-5}+\frac{3x+1}{2}\ge 0$.

14.66. $2x^2+\frac{1}{x}>0$.

14.67. $\frac{x^2-x-6}{x^2+6x}\ge 0$.

14.68. $\frac{x^2-5x+6}{x^2-11x+30}<0$.

14.69. $\frac{x-1}{x+1}<x$

14.70. $\frac{1}{x+2}<\frac{3}{x-3}$.

14.71. $\frac{14x}{x+1}-\frac{9x-30}{x-4}<0$.

14.72. $\frac{15-4x}{x^2-x-12}<4$.

14.73. $\frac{1}{x^2-5x+6}\ge\frac{1}{2}$

14.74. $\frac{(2-x^2)(x-3)^3}{(x+1)(x^2-3x-4)}\ge 0$.

14.75. $\frac{4}{1+x}+\frac{2}{1-x}<1$.

14.76. $2+\frac{3}{x+1}>\frac{2}{x}$.

14.77. $\frac{2(x-3)}{x(x-6)}\le\frac{1}{x-1}$.

14.78. $\frac{7}{(x-2)(x-3)}+\frac{9}{x+3}+1<0$.

14.79. $(x^2+3x+1)(x^2+3x-3)\ge 5$.

14.80. $(x^2-x-1)(x^2-x-7)<-5$.

15-§. MODUL QATNASHGAN TENGLAMALAR

1 - m i s o l. Tenglamani yeching:

a) $|x|=-2,5$; b) $|x|=2,5$; d) $|x^2-1|=0$.

Y e c h i s h. a) $|x|\ge 0$ bo'lgani uchun tenglama yechimga ega emas.

b) $|x|=2,5 \Leftrightarrow x=\pm 2,5$. Javob. $x=\pm 2,5$.

d) $|x^2-1|=0 \Leftrightarrow x^2-1=0$. Javob. $x=\pm 1$.

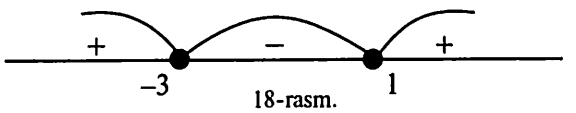
2 - m i s o l. $|x^2+2x-3|=2x+x^2-3$.

Y e c h i s h. $|f(x)|=f(x)$ ko'rinishdagi tenglamaga egamiz. Bu tenglama $f(x)\ge 0$ tengsizlikka teng kuchlidir:

$|x^2+2x-3|=2x+x^2-3$;

$2x+x^2-3\ge 0$;

$(x-(-3))(x-1)\ge 0$ (18-rasm).



18-rasm.

J a v o b. $(-\infty;-3]\cup[1;+\infty)$.

3 - m i s o l. $|x^2+2x-3|=3-x^2-2x$ tenglamani yeching.

Y e c h i s h. $|f(x)|=-f(x)$ ko'rinishidagi tenglamaga egamiz. Bu tenglama $f(x)\le 0$ tengsizlikka teng kuchli (19-rasm):

$|x^2+2x-3|=-(x^2+2x-3)$; $x^2+2x-3\le 0$; $(x-(-3))(x-1)\le 0$.



19-rasm.

J a v o b. $[-3;1]$.

4 - m i s o l. $x^2-5|x|+6=0$ tenglamani yeching.

Y e c h i s h. 1 - u s u l. $|x|^2=x^2$ ekanidan foydalanamiz:

$$|x|^2-5|x|+6=0;$$
$$|x|=t;$$
$$t^2-5t+6=0;$$
$$t_1=2;\ t_2=3;$$
$$|x|=2;\ |x|=3;$$
$$x_1=2,\ x_2=-2;\ x_3=3,\ x_4=-3.$$

J a v o b. ± 2; ± 3.

2- u s u l. Modulning ta'rifidan foydalanamiz:

$$\begin{cases} x^2-5x+6=0, \\ x\ge 0 \end{cases} \quad \text{yoki} \quad \begin{cases} x^2+5x+6=0, \\ x<0. \end{cases}$$

Bu sistemalarni yechib, ± 2; ± 3 ildizlarni topamiz.

5 - m i s o l. $|3x-8|+|3x-2|=-3$ tenglamani yeching..

Y e c h i s h. $|3x-8|\ge 0$, $|3x-2|\ge 0 \Rightarrow$ yechim yo'q. J a v o b. $\varnothing$.

6 - m i s o l. $|3x-8|+|3x-2|=3$ tenglamani yeching.

Y e c h i s h. $3x-8=0$, $3x-2=0$ tenglamani yechib, $x=2\frac{2}{3}$ va $x=\frac{2}{3}$ sonlarni topamiz. Ular son to'g'ri chizig'ini

uchta oraliqqa bo'ladi. Tenglamani shu oraliqlarning har birida yechamiz:

$x<\frac{2}{3}$	$\frac{2}{3}\le x<2\frac{2}{3}$	$x\ge 2\frac{2}{3}$
$\lvert 3x-8\rvert = 8-3x$ $\lvert 3x-2\rvert = 2-3x$ $(8-3x)+(2-3x)=3$ $x=\frac{7}{6}>\frac{2}{3}$	$\lvert 3x-8\rvert = 3x-8$ $\lvert 3x-2\rvert = 2-3x$ $(3x-8)+(2-3x)=3$ $0\cdot x=9$	$\lvert 3x-8\rvert = 3x-8$ $\lvert 3x-2\rvert = 3x-2$ $(3x-8)+(3x-2)=3$ $6x=13$ $x=2\frac{1}{3}<2\frac{2}{3}$
$\varnothing$	$\varnothing$	$\varnothing$

Javob. $\varnothing$.

7-misol. $|2+3x| = |4+2x|+|x-2|$ tenglamani yeching.

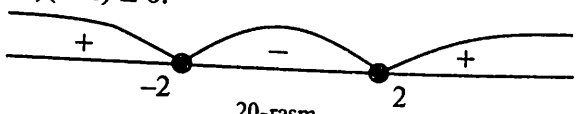
Yechish. $(4+2x)+(x-2) = 2+3x$ ekanini ko'rish qiyin emas.

$|f(x)+g(x)| = |f(x)|+|g(x)|$ tenglama $f(x)\cdot g(x)\ge 0$ tengsizlikka teng kuchli ekanligidan foydalanamiz:

$|(4+2x)+(x-2)| = |4+2x|+|x-2|$;

$(4+2x)(x-2) \ge 0$;

$2(x+2)(x-2) \ge 0$.



20-rasm.

Javob: $(-\infty;-2]\cup[2;+\infty)$.

$|f(x)| = a (a\in R)$ ko'rinishdagi tenglamani yeching:

15.1. $|x| = -2$. **15.7.** $|2x-5| = -1$. **15.13.** $|x^2-3x+1| = 1$.
15.2. $|x| = 2$. **15.8.** $|2x-5| = 1$. **15.14.** $|x^3-x| = 0$.
15.3. $|x| = 0$. **15.9.** $|2x-5| = 0$. **15.15.** $|x^4-x| = 0$.
15.4. $|x-1| = -2$. **15.10.** $|3-x| = -1$. **15.16.** $|x^2| = 9$.
15.5. $|x-1| = 2$. **15.11.** $|a+x| = -2$. **15.17.** $|x^2-1| = 0$.
15.6. $|x-1| = 0$. **15.12.** $|4-x| = 0$. **15.18.** $|x-|x|| = 0$.

$|f(x)| = f(x)$ ko'rinishdagi tenglamani yeching:

15.19. $|3x^2-7x+4| = 3x^2-7x+4$. **15.20.** $|x^2-14x-15| = x^2-14x-15$.
15.21. $|2-x-x^2| = 2-x-x^2$. **15.22.** $|3x^2-7x+6| = 3x^2-7x+6$.

$|f(x)| = -f(x)$ ko'rinishdagi tenglamani yeching:

15.23. $|3x^2-7x+6| = 7x-6-3x^2$. **15.24.** $|x^4-x^2| = x^2-x^4$.
15.25. $|-x^2-4x-4| = x^2+4x+4$.
15.26. $|(x-1)^2(x-2)(x-3)| = (x-1)^2(2-x)(x-3)$.

$f(|x|) = g(x)$ ko'rinishdagi tenglamani yeching:

15.27. $|x| = 3x-5$. **15.28.** $|x^2+|x|-6| = 0$.
15.29. $|x| = x^2-3x+5$. **15.30.** $x+|x|+5 = x^2$.

$|f(x)| = g(x)$ ko'rinishdagi tenglamani yeching:

15.31. $|x+2| = 2(3-x)$. **15.32.** $|3x-2| = 11-x$.
15.33. $2|x^2+2x-5| = x-1$. **15.34.** $|3x+1| = 5+6x$.

Tenglamani oraliqlar usuli bilan yeching:

15.35. $|3x-8|-|3x-2| = 6$. **15.36.** $|x-1|+|x-3| = 2$.
15.37. $|x-1|+|x-3| = 3$. **15.38.** $|x|-|x-2| = 2$.
15.39. a) $|x-3|+|x+2|-|x-4| = 3$. b) $|x-4|-|2x-3|+|3x-2| = 2$.

$|f(x)+g(x)| = |f(x)|+|g(x)|$ ko'rinishdagi tenglamani yeching:

15.40. $|7-2x| = |5-3x|+|x+2|$. **15.41.** $\left|\frac{x^2}{x-1}\right| = \left|\frac{x}{x-1}\right| + |x|$.
15.42. $|5x-4| = |x|+4|x-1|$. **15.43.** $|6x+13|+|7-6x| = 20$.
15.44. $|6x|-|6x-5| = 5$. **15.45.** $13-|-x+13| = |x|$.

Ichma-ich modullar qatnashgan tenglamani yeching:

15.46. $|2-|1-|x||| = 1$. **15.47.** $||x|-3| = 3-|x|$.
15.48. $||6x|-|6x-3|| = 3$. **15.49.** $|x-|4-x||-2x = 4$.

$|f(x)| = |g(x)|$ ko'rinishdagi tenglamani yeching:

15.50. $|3x-5| = |5-2x|$. **15.51.** $|x+1| = |x-1|$.
15.52. $|1-|2-x|| = |3+x|$. **15.53.** $||3-2x|-1| = |x-1|$.

Tenglamani yeching:

15.54. $(x^2-5x+6)^2-5|x^2-5x+6|+6 = 0$. **15.55.** $|x^2-4x+3| = -(4+2\sqrt{3})x$.
15.56. $|x^2-4x+3|+|x^2-5x+6| = 1$. **15.57.** $\frac{|x^2-4x|+3}{x^2+|x-5|} = 1$.
15.58. $x^2-6x+|x-4|+8 = 0$.
15.59. $(x^2-2|x|)\cdot(2|x|-2)-9\cdot\frac{2|x|-2}{x^2-2|x|} = 0$.
15.60. $12x-3x^2-\frac{x}{\sqrt{x-1}}-\frac{|4-x|}{\sqrt{x-1}}+|4-x| = 3x|4-x|-\frac{4}{\sqrt{x-1}}+4$.
15.61. $|2-3x|-|5-2x| = 0$. **15.62.** $|9-2x| = |4-3x|+|x+5|$.
15.63. $|x^2-3x+2| = |x|-x^2+4$. **15.64.** $\frac{|x^2-1|}{x-2} = x$.

Parametr qatnashgan tenglamani yeching:

15.65. $2|x+a|-|x-2a| = 3a$. **15.66.** $a-\frac{2a^2}{x+a} = a$.
15.67. $|x^2-a^2| = (x+3a)^2$. **15.68.** $x = 2|x-a|-2|x-2a|$.

Sistemani yeching:

15.69. $\begin{cases} 2u + v = 7, \\ |u - v| = 2. \end{cases}$ **15.71.** $\begin{cases} y + x - 1 = 0, \\ |y| - x - 1 = 1. \end{cases}$

15.70. $\begin{cases} 3u - v = 1, \\ |u - 2v| = 2. \end{cases}$ **15.72.** $\begin{cases} |x - 1| + y = 0, \\ 2x - y = 1. \end{cases}$

15.73. $\begin{cases} |x + 2|y| = 3, \\ 5y + 7x = 2. \end{cases}$ **15.75.** $\begin{cases} y - 2|x| + 3 = 0, \\ |y| + x - 3 = 0. \end{cases}$

15.74. $\begin{cases} |x-1| + |y-2| = 1, \\ y = 3 - |x - 1|. \end{cases}$ **15.76.** $\begin{cases} |xy - 4| = 8 - y^2, \\ xy = 2 + x^2. \end{cases}$

15.77. $|x^2+x-1| = 2x-1$ tengjamaning $x < \frac{\sqrt{3}}{3}$ shartni qanoatlantiruvchi barcha yechimlarini toping.

15.78. a ning $\begin{cases} 2x + 2(a-1)y = a - 4, \\ 2|x + 1| = ay + 2 \end{cases}$ sistema yagona echimga ega bo'ladigan barcha qiymatlarini toping. Sistemaning yechimini toping.

15.79. a ning $\begin{cases} ax + (a-1)y = 2 + 4a, \\ 3|x| + 2y = a - 5 \end{cases}$ sistema yagona echimga ega bo'ladigan barcha qiymatlarini toping. Sistemaning yechimini toping.

15.80. Tenglamani grafik usulda yeching:

a) $|x| = x+1$; d) $|3x-1| = 3-x$;

b) $|x+1| = x+3$; e) $|3x+1| = 5+6x$.

16-§. MODUL QATNASHGAN TENGSIZLIKLAR

1 - m i s o l. $|x-3| > x+2$ tengsizlikni yeching.

Y e c h i s h. $|x-3| = \begin{cases} x-3, \text{ agar } x \ge 3 \text{ bo'lsa}, \\ 3-x, \text{ agar } x < 3 \text{ bo'lsa} \end{cases}$ bo'lgani uchun

$\begin{cases} x \ge 3, \\ x-3 > x+2 \end{cases}$ va $\begin{cases} x < 3, \\ 3-x > x+2 \end{cases}$ sistemalar hosil bo'ladi.

Birinchi sistema yechimga ega emas, ikkinchi sistemaning yechimi $x \in (-\infty ; \frac{1}{2})$ dan iborat.

J a v o b. $x \in (-\infty ; \frac{1}{2})$.

2 - m i s o l. $|x^2-5x+6| \le x^2-5x+6$ tengcizlikni yeching.

Y e c h i s h. $|f(x)| \le f(x)$ tengsizlik $|f(x)| = f(x)$ tenglamaga va demak, $f(x) \ge 0$ tengsizlikka teng kuchli ekanligidan foydalanamiz:

$x^2-5x+6 \ge 0$.

Bu tengsizlikning yechimlarini topamiz: $(-\infty; 2] \cup [3; +\infty)$.

J a v o b: $(-\infty; 2] \cup [3; +\infty)$.

3 - m i s o l. $|x-5x+6| \ge x^2-5x+6$.

Y e c h i s h. $|f(x)| \ge f(x)$ tengsizlik $f(x)$ ifoda o'z ma'nosini yo'qotmaydigan barcha x lar uchun o'rinli ekanligidan foydalanamiz:

x^2-5x+6 ifoda barcha $x \in R$ da ma'noga ega.

J a v o b: $(-\infty ; +\infty)$.

4 - m i s o l. $|x^2-5x+6| > x^2-5x+6$ tengsizlikni yeching.

E c h i s h. $|f(x)| > f(x)$ tengsizlik $f(x) < 0$ tengsizlikka teng kuchli ekanligidan foydalanamiz: $|x^2-5x+6| > x^2-5x+6$; $x^2-5x+6 < 0$; $x \in (2;3)$.

J a v o b: $(2 ; 3)$.

Tengsizlikni yeching:

16.1. $|a| < 1$. **16.7.** $|a| < -3$. **16.14.** $3|x-4| \le 0$.

16.2. $|a| \le 1$. **16.8.** $|a| > -1$. **16.15.** $3|x-4| \ge 0$.

16.3. $|a| > 1$. **16.9.** $|a| \ge -1$. **16.16.** $13|x-4| > 0$.

16.4. $|a| \ge 1$. **16.10.** $|a| \le -3$. **16.17.** $|x^2-1| \le 0$.

16.5. $|a| < 0$. **16.11.** $|x-1| \le 0$. **16.18.** $|x^2-1| > 0$.

16.6. $|a| \le 0$. **16.12.** $|2x-3| \le 0$. **16.19.** $|x^3-8| > 0$.

16.13. $-3|x-4| < 0$. **16.20.** $\sqrt{x^2} \le 1$.

Tengsizlikni modulning ta'rifidan foydalanib yeching:

16.21. $2|x+10| > x+4$. **16.22.** $3|x-1| \le x+3$.

16.23. $x^2-7x+12 < |x-4|$. **16.24.** $x^2-5x+9 > |x-6|$.

16.25. $|x^2+3x| \ge 2-x^2$. **16.26.** $|x^2-6x+8| < 5x-x^2$.

Tengsizlikni yeching:

16.27. $|x-2| < 2x-10$. **16.28.** $|x^2-x-3| < 9$.

16.29. $\left|\frac{x^2-3x-1}{x^2+x+1}\right| < 3$. **16.30.** $\left|\frac{x^2-1}{x}\right| + 12 < 3x-1$.

16.31. $|2x-7| \le 5$. **16.32.** $|2x-1| \le |x-1|$.

16.33. $\left|\frac{x+4}{x+2}\right| < 1$. **16.34.** $|13-2x| \ge |4x-9|$.

16.35. $|x+1| + 4 \ge 2|x|$. **16.36.** $|2x+3| > |x| - 4x - 1$.

16.37. $|x-2| + |3-x| > 2+x$. **16.38.** $|x-1| > |x+2| - 3$.

16.39. $|5-x| < |x-2| + |7-2x|$. **16.40.** $|x-6| \le |x^2-5x+9|$.

16.41. $|x^3-1| > 1-x$. **16.42.** $\frac{2x-5}{|x-3|} > -1$. **16.43.** $\frac{|4-x|}{x+6} < 3$.

16.44. $\frac{|x-2|}{x^2-5x+6} \ge 3$. **16.45.** $\left|\frac{x^2-5x+4}{x^2-4}\right| \le 1$.

16.46. $\frac{|x^2-3x+2|}{|x^2+3x+2|} \ge 1$. **16.47.** $\frac{x^2-|x|-6}{x-2} \ge 2$.

16.48. $\frac{4x-1}{|x-1|} > |x+1|$. **16.49.** $\frac{2x}{|x-3|} < |x|$.

16.50. $x^2 \le \left|1-\frac{2}{x^2}\right|$. **16.51.** $\frac{|x^2-4x|+3}{x^2+|x-5|} \ge 1$.

16.52. $\frac{|x^2-2x|+4}{x^2+|x+2|} \ge 1$.

16.53. $|x-1|-|x-2|+|x+1| > |x+2|+|x|-3$.

16.54. $|x-1|-|x-2|+|x-3| \le 3+|x-4|-|x-5|$.

16.55. $|x+2|-|x+1|+|x| \ge \frac{5}{2}+|x-1|-|x-2|$.

17-§. IRRATSIONAL TENGLAMALAR VA IRRATSIONAL TENGLAMALAR SISTEMALARI

1 - m i s o l. $\sqrt{x+3}+\sqrt{2x-7}=-5$ tenglamani yeching.

Ye ch i sh. $\sqrt{x+3} \ge 0$, $\sqrt{2x-7} \ge 0$ bo'lgani uchun tenglama yechimga ega emas.

J a v o b. $\varnothing$.

2 - m i s o l. $\sqrt{15+\sqrt{x-\sqrt{7}}}=2{,}5$ tenglamani yeching.

Y e c h i s h. x ning joiz qiymatlarida $\sqrt{x-\sqrt{7}} \ge 0$ bo'lgani uchun $\sqrt{15+\sqrt{x-\sqrt{7}}} \ge \sqrt{15+0} > 2{,}5$.

J a v o b: $\varnothing$.

3 - m i s o l. $\sqrt{x-7}+\sqrt{7-x}=8$ tenglamani yeching.

E ch i sh. Tenglamaning aniqlanish sohasi: $\{7\}$, $x=7$ soni yechim bo'la olmaydi.

J a v o b. $\varnothing$.

Tenglamani mantiqiy mulohazalar yuritib yeching:

17.1. $\sqrt{x+2}+\sqrt{2x-1}=-3$. **17.2.** $4+\sqrt{2y-3}=1$.

17.3. $6-\sqrt{x+\sqrt{2}}=7$. **17.4.** $\sqrt{10+\sqrt{x-\sqrt{3}}}=3$.

17.5. $\sqrt{x-3}+\sqrt{2-x}=5$. **17.6.** $\sqrt{x-4}+\sqrt{4-x}=1$.

17.7. $\sqrt{x-4}+\sqrt{4-x}=-1$. **17.8.** $\sqrt{x+4}+\sqrt{-x-5}=0$.

Tenglamani aniqlanish sohasini topish bilan yeching:

17.9. $x+\sqrt{x-1}+2=\sqrt{x-1}$. **17.10.** $\sqrt{-x^2+x+6}=2x-7$.

17.11. $\sqrt{-x^2-3x-2}=x-1$. **17.12.** $\sqrt{x^2-4x+3}=\sqrt{5x-6-x^2}$.

17.13. $\sqrt{2x^2-7x+3}=\sqrt{5x-2-x^2}$. **17.14.** $\sqrt{y-3}-6\sqrt{2-y}=8$.

17.15. $(x^2-1)\sqrt{2x-1}=0$. **17.16.** $(x^2-4)\sqrt{x+1}=0$.

17.17. $(9-x^2)\sqrt{2-x}=0$. **17.18.** $(16-x^2)\sqrt{3-x}=0$.

17.19. $\sqrt{2x-3}-\sqrt{x+3}=0$.

Tenglamani $\sqrt[2n]{f(x)}=g(x)$ tenglama bilan

$\begin{cases} f(x) = (g(x))^{2n}, \\ g(x) \ge 0 \end{cases}$ sistemaning teng kuchliligidan foydalanib yeching:

17.20. $\sqrt{12-x}=x$. **17.21.** $\sqrt{7-x}=x-1$.

17.22. $x-\sqrt{x+1}=5$. **17.23.** $21+\sqrt{2x-7}=x$.

1724. $1-\sqrt{1+5x}=x$. **17.25.** $2\sqrt{x+5}=x+2$.

17.26. $4\sqrt{x+6}=x+1$. **17.27.** $\sqrt{4+2x-x^2}=x-2$.

17.28. $\sqrt{37-x^2}+5=x$. **17.29.** $\sqrt{6-4x-x^2}=x+4$.

17.30. $\sqrt{1+4x-x^2}=x-16$.

Tenglamani yangi o'zgaruvchi kiritib yeching:

17.31. $x^2-4x+6=\sqrt{2x^2-8x+12}$.

17.32. $2x^2+3x-5\sqrt{x^2+3x+9}+3=0$.

17.33. $x^2+\sqrt{x^2+2x+8}=12-2x$.

17.34. $2x^2+\sqrt{2x^2-4x+12}=4x+8$.

17.35. $3x^2+15x+2\sqrt{x^2+5x+1}=2$.

17.36. $\sqrt[3]{x}+2\sqrt[3]{x^2}=3$. **17.37.** $\sqrt[3]{x^2}-\sqrt[3]{x}-6=0$.

17.38. $\frac{4}{\sqrt[3]{x}+2}+\frac{\sqrt[3]{x}+3}{5}=2$. **17.39.** $\frac{8}{\sqrt{10-2x}}-\sqrt{10-2x}=2$.

17.40. $\sqrt{2-x}+\frac{4}{\sqrt{2-x}+3}=2$. **17.41.** $\sqrt{\frac{3-x}{2+x}}+3\sqrt{\frac{2+x}{3-x}}=4$.

17.42. $\sqrt{\frac{2x+1}{x-1}}-2\sqrt{\frac{x-1}{2x+1}}=1$. **17.43.** $\sqrt{\frac{x+1}{x-1}}\sqrt{\frac{x-1}{x+1}}=\frac{3}{2}$.

Tenglamani darajaga ko'tarish usuli bilan yeching:

17.44. $\sqrt{x+1}=8-\sqrt{3x+1}$.

17.45. $\sqrt{x+\sqrt{x+11}}+\sqrt{x-\sqrt{x+11}}=4$.

17.46. $\sqrt{x^2+1}+\sqrt{x^2-2x+3}=3$.

17.47. $\sqrt{x^2+x-5}+\sqrt{x^2+8x-4}=5$.

17.48. $\sqrt{4x-3}+\sqrt{5x+1}=\sqrt{15x+4}$.

17.49. $\sqrt{x+5}+\sqrt{x+3}=\sqrt{2x+7}$.

17.50. $\sqrt[3]{x+34}-\sqrt[3]{x-3}=1$.

17.51. $\sqrt[3]{x}+\sqrt[3]{x-16}=\sqrt[3]{x-8}$.

17.52. $\sqrt[3]{x+5}+\sqrt[3]{x+6}=\sqrt[3]{2x+11}$.

17.53. $\sqrt[3]{x+1}+\sqrt[3]{3x+1}=\sqrt[3]{x-1}$.

17.54. $\sqrt[3]{1+\sqrt{x}}+\sqrt[3]{1-\sqrt{x}}=2$.

17.55. $\sqrt[3]{5x+7} - \sqrt[3]{5x-12} = 1$.

17.56. $\sqrt[3]{9-\sqrt{x+1}} + \sqrt[3]{7+\sqrt{x+1}} = 4$.

17.57. $\sqrt[3]{24+\sqrt{x}} - \sqrt[3]{5+\sqrt{x}} = 1$.

17.58. $\sqrt[3]{x^2-2x} - \sqrt[3]{2x^2-7x+6} = 0$.

17.59. $\sqrt[3]{x+34} - \sqrt[3]{x-3} = 1$.

17.60. $\sqrt[3]{x+45} - \sqrt[3]{x-16} = 1$.

Tenglamani «qo'shmasiga ko'paytirish» usuli bilan yeching:

17.61. $\sqrt{3x^2+5x+8} - \sqrt{3x^2+5x+1} = 1$.

17.62. $\sqrt{3x^2-2x+15} + \sqrt{3x^2-2x+8} = 7$.

17.63. $\sqrt{x^2+9} - \sqrt{x^2-7} = 2$. **17.64.** $\sqrt{15-x} + \sqrt{3-x} = 6$.

Irratsional tenglamalarni yeching:

17.65. $\sqrt{x^2+3x-3} = 2x-3$.

17.66. $\sqrt{9x^2+2x-3} = 3x-2$.

17.67. $(x+2)(x-5) + \sqrt[3]{x(x+3)} = 0$.

17.68. $\sqrt{x+2\sqrt{x-1}} - \sqrt{x-2\sqrt{x-1}} = 2$.

17.69. $\sqrt{x-3-2\sqrt{x-4}} + \sqrt{x-4\sqrt{x-4}} = 1$.

17.70. $\sqrt{5x+7} - \sqrt{x+3} = \sqrt{3x+1}$.

17.71. $\sqrt{x+4} + 2\sqrt{x+1} = \sqrt{x+20}$.

17.72. $\sqrt[3]{x+1} + \sqrt[3]{x-1} = \sqrt[3]{5x}$.

17.73. $\sqrt[3]{x-2} + \sqrt[3]{x+3} = \sqrt{2x+1}$.

17.74. $\sqrt[3]{(3-x)^2} + \sqrt[3]{(6+x)^2} - \sqrt[3]{(3-x)(6+x)} = 3$.

17.75. $\sqrt[3]{x+1} - \sqrt[3]{x-1} = \sqrt[6]{x^2-1}$. **17.76.** $\sqrt{x+1} = a$.

17.77. $\sqrt{x+3} = \sqrt{a-x}$. **17.78.** $\sqrt{\frac{x+a}{x-a}} + 2\sqrt{\frac{x-a}{x+a}} = 3$.

17.79. $\sqrt{7-x} - \sqrt{x-3} = a$. **17.80.** $\sqrt{2x-1} - x + a = 0$.

17.81. $x + \frac{2x}{\sqrt{2+x^2}} = \sqrt{2}$ tenglamani yeching.

Irratsional tenglamalar sistemasini yeching:

17.82. $\begin{cases} x+y+\sqrt{x+y} = 20, \\ x^2+y^2 = 136. \end{cases}$ **17.83.** $\begin{cases} \sqrt{\frac{x}{y}} + \sqrt{\frac{y}{x}} = 2{,}5, \\ x+y = 5. \end{cases}$

17.84. $\begin{cases} \sqrt{\frac{2x-1}{y+2}} + \sqrt{\frac{y+2}{2x-1}} = 2, \\ x+y = 2. \end{cases}$ **17.85.** $\begin{cases} \sqrt{\frac{x}{y}} + \sqrt{\frac{y}{x}} = 2{,}5, \\ x^2+y^2 = 15. \end{cases}$

17.86. $\begin{cases} \sqrt{\frac{x}{y}} - \sqrt{\frac{y}{x}} = 1{,}5, \\ x+y+xy = 9. \end{cases}$ **17.87.** $\begin{cases} \sqrt{x^2+y} = 5, \\ y^2-x = 7. \end{cases}$

17.88. $\begin{cases} \sqrt[3]{x} - \sqrt[3]{y} = 2, \\ xy = 27. \end{cases}$ **17.89.** $\begin{cases} x+y = 9, \\ \sqrt[3]{x} + \sqrt[3]{y} = 3. \end{cases}$

17.90. $\begin{cases} x\sqrt{y} + y\sqrt{x} = 6, \\ x^2y + xy^2 = 20. \end{cases}$ **17.91.** $\begin{cases} \sqrt{\frac{20y}{x}} = \sqrt{x+y} - \sqrt{x-y}, \\ \sqrt{\frac{16x}{8}} = \sqrt{x+y} - \sqrt{x-y}, \end{cases}$

17.92. $\begin{cases} x\sqrt{(x+y)^2} = 3x, \\ x(\sqrt{(x-y)^2}-1)^2 = 0. \end{cases}$

17.93. $\begin{cases} x-y+\sqrt{x^2-4y^2} = 2, \\ x\sqrt[5]{x^2-4y^2} = 0. \end{cases}$

18-§. IRRATSIONAL TENGSIZLIKLAR

1 - m i s o l. $\sqrt{x+5} > -8$ tengsizlikni yeching.

Y e c h i s h. $\sqrt{x+5} > 0$ bo'lgani uchun $\sqrt{x+5} > -8$ tengsizlik o'zining aniqlanish sohasidagi barcha x lar uchun, ya'ni $x \geq -5$ da bajariladi.

J a v o b. $[-5;+\infty)$.

2 - m i s o l. $\sqrt{x^2-3x+1} < 0$ tengsizlikni yeching.

Y e c h i s h. $\sqrt{x^2-3x+1} \geq 0$ bo'lgani uchun berilgan tengsizlik yechimga ega emas.

J a v o b. $\varnothing$.

Tengsizlikni mantiqiy mulohazalar yuritib yeching:

18.1. $\sqrt{x+3} \geq -5$. **18.2.** $\sqrt{x^2+1} > -1$.

18.3. $\sqrt{x^2-2x+4} > -\frac{1}{2}$. **18.4.** $\sqrt{x^2-2x+4} < 0$.

18.5. $\sqrt{x^2-2x+4} \geq 0$. **18.6.** $\sqrt{x^2-2x+4} > 0$.

18.7. $\sqrt{x^2-6x+9} \geq 0$. **18.8.** $\sqrt{|x-2|+x^2+4} < 0$.

18.9. $\sqrt{x^2-2x+3} \geq -0{,}3$. **18.10.** $\sqrt{x^2} > 0$.

18.11. $\sqrt{x-4} + \sqrt{3-x} > 0$. **18.12.** $\sqrt{x-4} + \sqrt{3+x} < 0$.

18.13. $\sqrt{x+4} + \sqrt{x+3} > 0$. **18.14.** $\sqrt{x^3+1} > 0$.

18.15. $\sqrt{x^2-3x+2} \geq 0$. **18.16.** $\sqrt{4y^2+4y+1} > 0$.

18.17. $\sqrt{x^2+x+1} > 0$. **18.18.** $\sqrt{5x-6-x^2} > 0$.

18.19. $\sqrt{5x-6-x^2} \leq 0$. **18.20.** $\sqrt{-4x^2-12x-9} \geq 0$.

18.21. $\sqrt{x-1-x^2} > 0$. **18.22.** $\sqrt{5x-18-x^2} > 0$.

18.23. $(x-1)\sqrt{x^2-x-2} \geq 0$. **18. 24.** $(3-x)\sqrt{x^2+x-2} \leq 0$.

18.25. $(x+2)\sqrt{x^2-2x-3} \geq 0$. **18.26.** $(1-x)\sqrt{6+x-x^2} \leq 0$.

18.27. $\frac{\sqrt{2x^2+15x-17}}{10-x} < 0$. **18.28.** $\frac{x-7}{\sqrt{4x^2-19x+12}} \leq 0$.

18.29. $\frac{\sqrt{x^2-3x-6}}{\sqrt{x^2-4x+3}} > 0$. **18.30.** $\frac{\sqrt{x^2-x-2}}{x^2+2x-3} < 0$.

Eslatmalar:

1) $\sqrt[2n+1]{f(x)}$ V $g(x)$ ko'rinishdagi tengsizlikni (bu yerda V ni $<$, $>$, $\leq$, $\geq$ larning istalgan bittasi deb tushunamiz) yechish uchun $f(x)$ V $g^{2n+1}(x)$ tengsizlikni yechish kifoya;

2) $\sqrt[2n]{f(x)} < g(x)$ tengsizlikni yechish uchun $\begin{cases} f(x) \geq 0, \\ g(x) > 0, \\ f(x) < g^{2n}(x) \end{cases}$ sistemani yechish kerak;

3) $\sqrt[2n]{f(x)} \leq g(x)$ tengsizlikni yechish uchun $\begin{cases} f(x) \geq 0, \\ g(x) \geq 0, \\ f(x) \leq g^{2n}(x) \end{cases}$ sistemani yechish kerak;

4) $\sqrt[2n]{f(x)} > g(x)$ tengsizlikni yechish uchun $\begin{cases} g(x) < 0, \\ f(x) \geq 0 \end{cases}$ va $\begin{cases} g(x) \geq 0, \\ f(x) > g^{2n}(x) \end{cases}$ sistemalarni yechish kerak;

5) $\sqrt[2m]{f(x)} \geq g(x)$ tengsizlikni yechish uchun $\begin{cases} g(x) < 0, \\ f(x) \geq 0 \end{cases}$ va $\begin{cases} g(x) \geq 0, \\ f(x) \geq g^{2n}(x) \end{cases}$ sistemalarni yechish kerak. Bu sistemalar yechimlari to'plamlarining birlashmasi berilgan tengsizlikning yechimlari to'plami bo'ladi.

Quyidagi irratsional tengsizliklarni yeching:

18.31. $\sqrt{x+7} < x$.

18.32. $\sqrt{9x-20} < x$.

18.33. $\sqrt{x^2+4x+4} < x+6$.

18.34. $\sqrt{2x^2-3x-5} < x-1$.

18.35. $\sqrt{x+78} < x+6$.

18.36. $\sqrt{(x+2)(x-5)} < 8-x$.

18.37. $1-\sqrt{13+3x^2} > 2x$.

18.38. $\sqrt{x^2+x-12} < x$.

18.39. $\sqrt{2x+4} > x+3$.

18.40. $\sqrt{x^2+x-2} > x$.

18.41. $\sqrt{9-24x+16x^2} > 8$.

18.42. $\sqrt{(x+4)(x+3)} > 6-x$.

18.43. $\sqrt{x^2-5x-24} > x+2$.

18.44. $3\sqrt{6+x-x^2} > 4x-2$.

18.45. $\sqrt{x^2-x-2} > 2x+3$.

18.46. $\sqrt{x^2-4x} > x-4$.

18.47. $\sqrt{x^2-x-6} \geq x+5$.

18.48. $\sqrt{x^2-5x+6} \geq x+1$.

18.49. $\sqrt{x^2-7x+12} \geq 1-x$.

18.50. $\sqrt{3x^2+13x+4} \geq x-2$.

Tomonlarida nomanfiy ifodalar hosil qilib yechiladigan tengsizliklarni yeching:

18.51. $3\sqrt{x}-\sqrt{x+3} > 1$.

18.52. $\sqrt{x+3}+\sqrt{x+2}-\sqrt{2x+4} > 0$.

18.53. $\sqrt{x-6}-\sqrt{10-x} \geq 1$.

18.54. $\sqrt{x+3}-\sqrt{x-1} > \sqrt{2x-1}$.

18.55. $\sqrt{3x^2+5x+7}-\sqrt{3x^2+5x+2} > 1$.

18.56. $\sqrt{1-x} \leq \sqrt[4]{5-x}$.

18.57. $\sqrt[4]{5x-1} \leq \sqrt{\sqrt{6}}$.

18.58. $\sqrt{1-x^2+1} < \sqrt{3-x^2}$.

18.59. $\sqrt{x+3} < \sqrt{x+1}+\sqrt{x-2}$.

18.60. $\sqrt{x+2\sqrt{x-1}}+\sqrt{x-2\sqrt{x-1}} > -\frac{3}{2}$

Tengsizlikni yeching:

18.61. $\sqrt{x^2-x-12} < 7-x$.

18.62. $\sqrt{x^2-5x+6} < 2x-3$.

18.63. $\frac{\sqrt{x+2}}{x} < 1$.

18.64. $\sqrt{x+\sqrt{x}}-\sqrt{x-\sqrt{x}} > 1{,}5\sqrt{\frac{x}{x+\sqrt{x}}}$.

18.65. $\sqrt{x^2-5x+6}+\frac{1}{\sqrt{x^2-5x+6}} \geq 2$.

18.66. $\frac{1}{\sqrt{3x-2}}+\sqrt{3x+2} > 2$.

18.67. $\sqrt{x^2-x-2}+\frac{1}{\sqrt{x^2-x-2}} > 2$.

18.68. $\sqrt{3-4x}+\frac{1}{\sqrt{3-4x}} < 2$.

18.69. $\sqrt{x^2+4x+4} < x+6$.

18.70. $\sqrt{16x^2-24x+9} < \sqrt{4x^2+12x+9}$.

18.71. $\sqrt{x^2+2x+1}+\sqrt{x^2-6x+9} < 8$.

18.72. $\sqrt{x^4+2x^2+1}+\sqrt{4x^4-4x^2+1} \leq 2x-1$.

18.73. $\sqrt{x}-3 \leq \frac{2}{\sqrt{x-2}}$.

18.74. $5\sqrt{x} > x+6$.

18.75. $\frac{x-1}{\sqrt{x}+1} > 4+\frac{\sqrt{x-1}}{2}$.

18.76. $\frac{1}{\sqrt{2-x}} > \frac{1}{x-1}$.

18.77. $\frac{1}{\sqrt{1+x}} > \frac{1}{2-x}$.

18.78. $\frac{\sqrt{3x^2+4}}{x-1} \geq 4$.

18.79. $\frac{1-3\sqrt{16-x^2}}{x} \leq 1$.

18.80. $\frac{1-\sqrt{1-4x^2}}{x} > 1{,}5$.

18.81. $a\sqrt{x+1} < 1$.

18.82. $x+\sqrt{a-x} > 0$ $(a \geq 0)$.

18.83. $\sqrt{a+x}+\sqrt{a-x} > a$.

VII b o b. FUNKSIYALAR VA GRAFIKLAR

1-§. FUNKSIYANING ASOSIY XOSSALARI

1 - m i s o l. $y = \begin{cases} -1, & x < 1 \text{ da}, \\ 0, & x = 1 \text{ da}, \\ 2, & x > 1 \text{ da} \end{cases}$ funksiyaning qiymatlar sohasini toping.

Y e c h i s h. x har qanday qiymat qabul qilganda ham y o'zgaruvchi faqat -1, 0, 2 qiymatlardan birortasiga teng bo'ladi. Shuning uchun $E(y)=\{-1;0;2\}$ ($E(y)$ bilan $y(x)$ funksiya qiymatlar sohasi belgilangan).

2 - m i s o l. $y=\sqrt{x+5}$ funksiyaning qiymatlar sohasini toping.

Ye ch i sh. a ning $\sqrt{x+5}=a$ tenglama kamida bitta ildizga ega bo'ladigan qiymatlarini topamiz.

$a<0$ bo'lsa, tenglama yechimga ega emas. $a \geq 0$ bo'lsin. $\sqrt{x+5}=a$ ni kvadratga ko'tarib, $x+5=a^2$ ga yoki $x=a^2-5$ ga ega bo'lamiz.

Demak, $\sqrt{x+5}$ tenglama $a \geq 0$ bo'lganda yechimga ega.

J a v o b. $E(y)=[0;+\infty)$.

3 - m i s o l. $f\left(\frac{x-2}{5x+1}\right)=\frac{x+2}{x-3}$ bo'lsa, $f(x)$ ni toping.

Y e c h i s h. $\frac{x-2}{5x+1}=t$ deb, x ni topamiz: $x=\frac{t+2}{1-5t}$.

Berilgan tenglikka ko'ra: $f(t)=\frac{\frac{t+2}{1-5t}+2}{\frac{t+2}{1-5t}-3}$. Bundan $f(x)=\frac{4-9x}{16x-1}$ ekani kelib chiqadi.

4 - m i s o l. $f(x)+3f\left(\frac{1}{x}\right)=2x$ bo'lsa, $f(x)$ ni toping.

Y e c h i s h. Berilgan tenglikda, x ga $x=t$ va $x=\frac{1}{t}$ qiymatlarni beramiz:

$f(t)+3f\left(\frac{1}{t}\right)=2t$; $f\left(\frac{1}{t}\right)+3f(t)=\frac{2}{t}$.

Hosil bo'lgan tenglamalarning ikkinchisini 3 ga ko'paytirib, undan birinchisini hadma-had ayiramiz va $8f(t)=\frac{6}{t}-2t=\frac{6-2t^2}{t}$ ga ega bo'lamiz. Bu tenglik yordamida, $f(x)=\frac{3-x^2}{4x}$ ni topamiz.

5 - m i s o l. $f(x)=|x-1|x^3-1$ funksiyani juft va toq funksiyalarning yig'indisi shaklida tasvirlang.

Y e c h i s h. f(x) funksiya juft funksiya ham emas, toq funksiya ham emas. Uning aniqlanish sohasi koordinatalar boshiga nisbatan simmetrik, ya'ni $\forall\ x \in D(f)$ uchun $-x \in D(f)$. Shuning uchun, $f(x)$ funksiyani juft va toq funksiyalarning yig'indisi shaklida tasvirlash mumkin.

$$\varphi(x)=\frac{f(x)+f(-x)}{2}=\frac{|x-1|x^3-|x+1|x^3-2}{2} \text{ va}$$

$$\varphi(x)=\frac{|x-1|x^3+|x+1|x^3}{2}$$

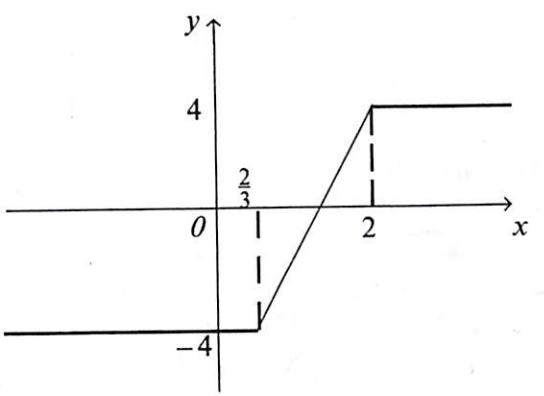
funksiyalarni qaraymiz. $\varphi(x)$ funksiyaning juft funksiya, $\psi(x)$ ning esa toq funksiya va $f(x)=\varphi(x)+\psi(x)$ ekanligini ko'rish qiyin emas.

J a v o b. $f(x)=\frac{|x-1|x^3+|x+1|x^3-2}{2}+\frac{|x-1|x^3+|x+1|x^3}{2}$.

6 - m i s o l. $f(x)=|3x-2|-3|x-2|$ funksiyaning grafigini yasang.

Y e c h i s h. Funksiya ifodasini modul belgisisiz yozib olib, so'ngra grafigini yasaymiz (21-rasm):

$$f(x)=\begin{cases} -4, \text{ agar } x<\frac{2}{3} \text{ bo'lsa}, \\ 6x-8, \text{ agar } \frac{3}{2} \leq x < 2 \text{ bo'lsa}, \\ 4, \text{ agar } x \geq 2 \text{ bo'lsa}. \end{cases}$$



21-rasm.

7 - m i s o l. $y=9-|x-3|$ funksiyaning eng katta qiymatini va argumentining unga mos qiymatlarini toping.

Y e c h i s h. $D(y)=R$ va $\forall\ x \in R$ uchun $y=9-|x-3| \leq 9+0=9$

ga egamiz. $y(x)=9$ tenglik bajariladigan x lar mavjud yoki mavjud emasligini aniqlaymiz:

$$9-|x-3|=9;\quad |x-3|=0 \Rightarrow x_1=0,\ x_2=3.$$

Demak, $y(x)=9$ tenglik o'rinli bo'ladigan x lar mavjud. Shunday qilib, barcha $x\in R$ lar uchun $y(x)\le 9$ bo'lib, $y(x)=9$ tenglik o'rinli bo'ladigan x lar mavjud ekan. Bu hol, $\max\limits_{x\in R} y(x) = y(0) = y(3) = 9$ deyish uchun asos bo'la oladi.

J a v o b. $\max\limits_{x\in R} y(x) = 9,\quad x_{max} = 0,\quad x_{max} = 3.$

Quyidagi funksiyalarning aniqlanish sohasini toping:

1.1. $f(x)=\frac{3}{x-2}$. **1.2.** $f(x)=\frac{3x}{x-3,4}$. **1.3.** $f(x)=\frac{4x-1}{3x-2}$.

1.4. $f(x)=\frac{4x+13}{7x+14}$. **1.5.** $f(x)=\frac{4x}{(x-1)(x-2)}$.

1.6. $f(x)=\frac{3x-1}{(x-1)(x-2)(x-3)}$. **1.7.** $f(x)=\frac{4x^2-1}{x^2-7x+12}$.

1.8. $f(x)=\frac{4x+1}{x^2-8x+15}$. **1.9.** $f(x)=\frac{1}{x^2+3}$.

1.10. $f(x)=\frac{1}{x^2-x+1}$. **1.11.** $f(x)=\frac{x}{x^2+x+1}$.

1.12. $f(x)=\frac{x-2}{x^2+x+1}$. **1.13.** $f(x)=x+x^2+\frac{1}{x-3}$.

1.14. $f(x)=x^2+x-3$. **1.15.** $f(x)=x+\frac{1}{x}+\frac{1}{x^2-1}$.

1.16. $f(x)=\left(x+\frac{1}{2}\right)^2+\frac{1}{x^2-1}$. **1.17.** $f(x)=x+x^{-1}+x^{-2}$.

1.18. $f(x)=x^1+\frac{2}{x}$. **1.19.** $f(x)=ax^2+bx+c$.

1.20. $f(x)=ax+b$. **1.21.** $f(x)=\frac{1}{x-2}+\frac{1}{(x^2-3x)^2}$.

1.22. $f(x)=\frac{1}{x^2+3}$. **1.23.** $y=\sqrt{3-5x}$.

1.24. $f(x)=\frac{1+x}{\sqrt{3-2(7-5x)}}$. **1.25.** $y=\sqrt{2(3x-1)-7x+2}$.

1.26. $y=\frac{3+4x}{\sqrt{3-2x-4(1-5x)}}$. **1.27.** $y=\sqrt{-\sqrt{2}(2-3x)}$.

1.28. $y=\frac{1}{\sqrt{(3x-1)\sqrt{2}-3x+2}}$. **1.29.** $y=\sqrt{2-\sqrt{3}(x+\sqrt{3})-2x}$.

1.30. $y=\frac{2}{\sqrt{(x-\sqrt{3})\sqrt{3}-2x+1}}$. **1.31.** $y=\sqrt{60x-25x^2-36}$.

1.32. $y=\frac{1}{\sqrt{112x+64+49x^2}}$. **1.33.** $y=\sqrt{5x^2+6x+1}+\frac{1}{3x+5}$

1.34. $y=\sqrt{3x+4}\frac{1}{\sqrt{-2x^2-5x-2}}$. **1.35.** $y=\sqrt{4-x|x|}$.

1.36. $y=\sqrt{|x|(x-1)}$. **1.37.** $y=\sqrt{(x-2)\sqrt{x}}$.

1.38. $y=\sqrt{(1-x)\sqrt{x-2}}$. **1.39.** $y=\sqrt{\frac{3-2x-x^2}{x^2+7x+12}}$.

1.40. $y=\sqrt{\frac{-x^2+6x-8}{x^2+5x+6}}$. **1.41.** $y=\frac{2}{\sqrt{x^2+x-20}}+\sqrt{x^2+5x-14}$.

1.42. $y=\sqrt{20-x-x^2}-\frac{3}{\sqrt{14-5x-x^2}}$.

1.43. $y=\sqrt{\frac{\sqrt{17-15x-2x^2}}{x+3}}$. **1.44.** $y=\sqrt{\frac{7-x}{\sqrt{4x^2-19x+12}}}$.

1.45. $y=\sqrt{\frac{-4x^2+4x+3}{\sqrt{2x^2-7x+3}}}$. **1.47.** $y=\sqrt{12x^2-4x^3-9x-\sqrt{2-|x|}}$.

1.48. $y=\sqrt{|x-1|(3x-6)}+\frac{3}{x^2+4x-21}$.

1.49. $y=\frac{\sqrt{(x^2-4x-21)|x+2|}}{x^2+x-72}$.

1.50. $y=\sqrt{5-\sqrt{4x^2-20x+25}-\sqrt{|x|(2x-10)}}$.

Quyidagi funksiyalarning qiymatlar sohasini toping:

1.51. $y=1$. **1.52.** $y=x$. **1.53.** $y=x^2$. **1.54.** $y=-x^2$.

1.55. $y=\begin{cases}0, & x\le 0 \text{ da},\\ 1, & x>1 \text{ da},\end{cases}$ **1.56.** $y=\begin{cases}-1, & x^2<1 \text{ da},\\ 0, & x^2=1 \text{ da},\\ 1, & x^2>1 \text{ da}.\end{cases}$

1.57. $y=x^2+2$. **1.58.** $y=3-4x^2$. **1.59.** $y=3x-x^2$.

1.60. $y=3x^2-6x+1$. **1.61.** $y=\frac{5}{x-2}$. **1.62.** $y=\frac{x}{x+1}$.

1.63. $y=\frac{2}{x^2+2}$. **1.64.** $y=\frac{x^2+1}{x}$. **1.65.** $y=\sqrt{x-2}+3$.

1.66. $y=|x-4|-2$. **1.67.** $y=5-\sqrt{2x+1}$.

1.68. $y=3-|2x+3|$. **1. 69.** $y=\sqrt{x^2+4}$.

1.70. $y=4-2\sqrt{x^2+9}$. **1.71.** $y=\sqrt{3x^2-6x+4}$.

1.72. $y=\sqrt{8x-2x^2-7}$. **1.73.** $y=1-\frac{5}{\sqrt{x-1}+1}$.

1.74. $y=2-\frac{3}{2x^2-8x+9}$. **1.75.** $y=1-\sqrt{9-\sqrt{2x^2+6\sqrt{2}x+9}}$.

1.76. $y=3-\sqrt{16-\sqrt{4x^2-4\sqrt{3}x+3}}$.

1.77. $y=\frac{x^3+8}{x+2}$. **1.78.** $y=\frac{(x^3+8)(x-4)}{x^2-2x-8}$.

1.79. $y=\frac{x^3-27}{x-3}$. **1.80.** $y=\frac{x^4+6x^3-27x-162}{x^2+3x-18}$.

1.81. $f(x)=\frac{x-1}{x+1}$ bo'lsa, $f\left(\frac{1}{x^2}\right)$ ni toping.

1.82. $f(x)=\sqrt{x^3-1}$ bo'lsa, $f(\sqrt[3]{x^2+1})$ ni toping.

1.83. $f(x)=\frac{x^2}{\sqrt{1+x^2}}$ bo'lsa, $f(tgx)$ ni toping.

1.84. $f\left(\frac{3x-1}{x+2}\right)=\frac{x+1}{x-1}$ bo'lsa, $f(x)$ ni toping.

1.85. $f(x)+2f\left(\frac{1}{x}\right)=x$ bo'lsa, $f(x)$ ni toping.

1.86. $(x-1)f(x)+f\left(\frac{1}{x}\right)=\frac{1}{x-1}$ bo'lsa, $f(x)$ ni toping.

1.87. $f(x)+xf\left(\frac{x}{2x-1}\right)=2$ bo'lsa, $f(x)$ ni toping.

1.88. $2f\left(\frac{x}{x-1}\right)-3f\left(\frac{3x-2}{2x+1}\right)=\frac{13x-4}{2x-3x^2}$ bo'lsa, $f(x)$ ni toping.

Quyidagi funksiyalarni juftlikka tekshiring (1.89. — 1.93):

1.89. a) $f(x)=19$; b) $\varphi(x)=0$; d) $g(x)=(2-3x)^3+(2+3x)^3$;
e) $h(x)=(5x-2)^4+(5x+2)^4$.

1.90. a) $f(x)=(x+3)|x-1|+(x-3)|x+1|$
b) $\varphi(x)=(x+5)|x-3|-(x-5)|x+3|$;
d) $g(x)=\frac{|x-7|}{x+1}+\frac{|x+7|}{x-1}$; e) $h(x)=\frac{|x-4|}{x+2}-\frac{|x+4|}{x-2}$.

1.91. a) $f(x)=(x+2)(x+3)(x+4)-(x-2)(x-3)(x-4)$;
b) $\varphi(x)=(x-5)^8(x+7)^{11}+(x+5)^8(x-7)^{11}$;
d) $g(x)=(x-6)^9(x+3)^5+(x+6)^9(x-3)^5$;
e) $h(x)=(x^2-3x+5)(x^3-8x^2+2x-1)-(x^2+3x+5)\cdot(x^3+8x^2+2x+1)$.

1.92. a) $f(x)=\frac{x^3-2x^2}{x+1}-\frac{x^3+2x^2}{x-1}$;
b) $\varphi(x)=\frac{x^5-2x^2+3}{x-4}+\frac{x^5+2x^2+3}{x+4}$;
d) $g(x)=\frac{(x-1)^5}{(3x+4)^3}+\frac{(x+1)^5}{(3x-4)^3}$;
e) $h(x)=\frac{(x-2)^3(x+1)^5(x-5)^7}{2x+1}+\frac{(x+2)^3(x-1)^5(x+5)^7}{2x-1}$.

1.93. a) $f(x)=8^{x^2}$; b) $f(x)=4,3^x$; d) $f(x)=x^3+3x^2-5$;
e) $f(x)=5x^4-4x^3+3x^2+1$.

Quyidagi funksiyalarni juft va toq funksiyalarning yig'indisi shaklida tasvirlang (1.94 — 1.95):

1.94. a) $f(x)=|x+1|\cdot x^2-1$; b) $f(x)=|2x-3|+x^2-1$;
d) $\varphi(x)=(x+3)|x-1|+|x+1|x$;
e) $g(x)=|x-1||x+1||x+2|x+3|x||(x-1)$.

1.95. a) $f(x)=\frac{(x-2)^2(x+3)^3}{2x+1}-\frac{(x+2)^2}{x-1}$;
b) $f(x)=2(x-2)|x+3|+\frac{5|x|+4x^2}{x-1}$;
d) $\varphi(x)=3x-2(x-1)+\frac{x^2-2x+1}{x+1}$;
e) $g(x)=3|x^2-4x+1|+|x^2-x|+8x^2$.

1.96. Quyidagi funksiyalarning chegaralanganligini isbot qiling:

a) $y=\frac{1}{1+x^2}$; b) $y=\frac{2}{4+x^2}$.

1.97. Quyidagi funksiyalarning chegaralanmaganligini isbot qiling:

a) $y=\frac{1}{1-x^2}$; b) $y=\frac{1}{(x-1)^2}$.

1.98. a) $y=\frac{5}{2x+1}$ funksiya $(-\infty;-0,5)$ da kamayishini;
b) $y=\frac{4}{2-x}$ funksiya $(2;+\infty)$ da o'sishini;
d) $y=\frac{21x-9}{3x-1}$ funksiya$(-\infty;1/3)$ da o'sishini;
e) $y=\frac{4x+31}{x+7}$ funksiya$(-7;\infty)$ da kamayishini isbotlang.

1.99. a) $y=3x^2-4x+7$ funksiya$(-\infty;2/3]$ da kamayishini;
b) $y=-5x^2+6x+19$ funksiya$(-\infty;0,6]$ da o'sishini;
d) $y=3\sqrt{4x+1}-1$ funksiya $[-0,25;+\infty)$ da kamayishini;
e) $y=2+\sqrt{3-5x}$ funksiya$(-\infty;0,6]$ da kamayishini isbotlang.

1.100. a) $y=x^3-3x$ funksiya $[1;+\infty)$ da o'sishini;
b) $y=12x-x^3$ funksiya $[2;+\infty)$ da kamayishini;
d) $y=0,5x^2-2\sqrt{x}$ funksiya $[1;\infty)$ da o'sishini va $[0;1]$ da kamayishini;
e) $y=\sqrt{x}-2x^2$ funksiya $[0;0,25]$ da o'sishini va $[0,25;+\infty)$ da kamayishini isbotlang.

1.101. $f(x)=x^2$ funksiya berilgan. Argumentning har qanday x_1 va x_2 qiymatlarida $f\left(\frac{x_1+x_2}{2}\right) \le \frac{f(x_1)+f(x_2)}{2}$ bo'lishini isbotlang.

1.102. $f(x)=\sqrt{x}$ funksiya berilgan. Argumentning har qanday x_1 va x_2 qiymatlarida $f\left(\frac{x_1+x_2}{2}\right) \ge \frac{f(x_1)+f(x_2)}{2}$ bo'lishini isbotlang.

1.103. $f(x)=x^2-4x+4$ va $g(x)=\frac{a^2+1}{x+3}$ funksiyalar berilgan.

a) f(x) funksiya $[2;+\infty)$ da o'sishini isbotlang;
b) $g(x)$ funksiya$[2;+\infty)$ da kamayishini isbotlang;
d) a ning $f(3)=g(3)$ bo'ladigan barcha qiymatlarini toping;
e) $(x-2)^2=\frac{6}{x+3}$ tenglamani $[2;+\infty)$ oraliqda yeching.

1.104. $f(x)=(x-3)^2$ va $g(x)=\frac{a^2+1}{4-x}$ funksiyalar berilgan.

a) $f(x)$ funksiya$(-\infty;3]$ da kamayishini isbotlang;
b) $g(x)$ funksiya$(-\infty;3]$ da 'o'sishini isbotlang;
d) a ning $f(2)=g(2)$ bo'ladigan barcha qiymatlarini toping;
e) $x^2-6x+9=\frac{2}{4-x}$ tenglamani $(-\infty;3]$ oraliqda yeching.

1.105. Agar f(x) funksiya X to'plamda o'suvchi (kamayuvchi), $g(x)$ funksiya esa X to'plamda kamayuvchi (o'suvchi) bo'lsa, $f(x)=g(x)$ tenglama X to'plamda ko'pi bilan bitta ildizga ega bo'lishini isbotlang.

1.106. Tenglamalarni yeching:

a) $(x+1)^3=41-3x-x^3$; b) $3x^3+2x=4+(2-x)^3$;
d) $(x-1)^5+x^5=45-x^3=2x$; e) $4x^5+2x^3+71=(3-x)^3+1$;
f) $x^{1991}+1=\sqrt{5-x}$; g) $\sqrt{10+x}+5=-2x^{13}-6x$;
h) $2+\sqrt{x-2}=\frac{9}{x}-1$; i) $\sqrt{3-x}=1-\frac{5}{x}$.

1.107. Quyidagi funksiyalarning nollarini toping:

a) $f(x)=3x^2-4$; f) $f(x)=|x-1|\cdot\left|\frac{x+1}{x^2-1}\right|$;
b) $f(x)=2x^2-5x+6$; g) $f(x)=x^3+8x-x$;
d) $f(x)=\sqrt{x-1}+\sqrt{2-x}$; h) $f(x)=\frac{x-1}{x^2-7x+12}$;
e) $f(x)=\frac{x}{x-1}-\frac{2x}{x+1}$; i) $f(x)=\frac{x^2-4}{x^2-11x+30}$.

Quyidagi funksiyalarning o'sish va kamayish oraliqlarini toping:

1.108. $y=1-2x$. **1.109.** $y=x^3$.
1.110. $y=3-2x-x^2$. **1.111.** $y=\frac{1}{x+1}$.

Quyidagi funksiyalarni davriylikka tekshiring:

1.112. $y=x$. **1.116.** $y=\{x\}+1$. **1.120.** $y=5$.
1.113. $y=x^2$. **1.117.** $y=[x]-1$. **1.121.** $y=5+x$.
1.114. $y=\{x\}$. **1.118.** $y=x^2+\{x\}$. **1.222.** $y=\{5+x\}$.
1.115. $y=[x]$. **1.119.** $y=[x]+x$. **1.123.** $y=[5+x]$.

1.124. $f(x)=\begin{cases}-x, & \text{agar } 0\le x<1 \text{ bo'lsa,}\\ \frac{1}{2}, & \text{agar } 1\le x<2 \text{ bo'lsa}\end{cases}$

funksiya berilgan. Shu funksiya yordamida davriy funksiya quring.

1.125. Davri faqat ratsional sonlarda bo'lgan funksiya quring.

Quyidagi funksiyalarga teskari funksiyalarni toping:

1.126. $f(x)=2x+3$; **1.127.** $f(x)=\frac{2x-1}{x+2}$;
1.128. $f(x)=x^2$, $x\in(0;+\infty]$; **1.129.** $f(x)=x^2$, $(-\infty;0]$;
1.130. $f(x)=-x^2$, $x\in(-\infty;0]$;
1.131. $f(x)=\begin{cases}x, & \text{agar } x\in[0;1) \text{ 1 bo'lsa,}\\ 3-x, & \text{agar } x\in[1;2] \text{ bo'lsa.}\end{cases}$

Quyidagi funksiyalar teskarilanuvchimi:

1.132. $f(x)=3x^2+1$; **1.133.** $f(x)=3x+4$;
1.134. $f(x)=4x-5$; **1.135.** $f(x)=\frac{3x+1}{4x-2}$;
1.136. $f(x)=\frac{7x-4}{3x+5}$; **1.137.** $f(x)=\frac{dx+b}{cx+d}$;
1.138. $f(x)=\begin{cases}x^2, & \text{agar } x\in[0;1) \text{ bo'lsa,}\\ x-1, & \text{agar } x\in[1;2] \text{ bo'lsa.}\end{cases}$
1.139. $f(x)=\begin{cases}3x+1, & \text{agar } x\in[0;1) \text{ bo'lsa,}\\ -3x+1, & \text{agar } x\in[1;2] \text{ bo'lsa.}\end{cases}$
1.140. $f(x)=\begin{cases}x^3, & x\le 0 \text{ bo'lsa,}\\ x, & x>0 \text{ bo'lsa.}\end{cases}$?

Quyidagi funksiyalarning eng katta qiymatlarini va argumentning unga mos qiymatlarini ko'rsating:

1.141. $y=5-|x+8|$. **1.142.** $y=2-\sqrt{x-2}$.
1.143. $y=x^2-2x+3$, $x\in[1;5]$.
1.144. $y=-x^2-4x+1$, $x\in[1;2]$.

1.145. $y=\frac{2}{5+|3x-2|}$. **1.146.** $y=\frac{2}{x^2-2x+2}$.

1.147. $y=\frac{2x}{x^2+1}$. **1.148.** $y=\frac{4x}{x^2+4}$.

1.149. $y=\frac{x}{4x^2+9}$.

Quyidagi funksiyalarning eng kichik qiymatlarini va argumentning funksiyalar bu qiymatlarga erishadigan qiymatlarini toping:

1.150. $y=\sqrt{4x^2-12x+9}-2$. **1.151.** $y=3+\sqrt{x^2-3x+2}$.

1.152. $y=x^2+6x+11$, $x\in[-4;2]$. **1.153.** $y=-x^2+2x+2$, $x\in[1;2]$.

1.154. $y=-\frac{3}{|x+1|+1}$. **1.155.** $y=\frac{2}{x^2+1}$.

1.156. $y=-\frac{x}{12x^3+3}$. **1.157.** $y=\frac{x^2+4x+4}{x^2+4x+5}$.

2-§. FUNKSIYA GRAFIGINI YASASHGA DOIR MISOLLAR

Quyidagi funksiyalarning grafiklarini yasang:

2.1. $y=\frac{|x|}{x}(x^2+6x)$. **2.2.** $y=\frac{x}{|x|}(4x-x^2-3)$.

2.3. $y=\frac{|x-2|}{2-x}(x^2-2x)$. **2.4.** $y=\frac{x+2}{|x+2|}(x^2+4x+3)$.

2.5. $y=|||x|-2|-1|$. **2.6.** $y=|2-|1-|x|||$.

2.7. $y=|x^2-5|x|+6|$. **2.8.** $y=\sqrt{4x^2-4x^2|x|+x^4}$.

2.9. $y=||1-x^2|-3|$. **2.10.** $y=||x^2-2x|-3|$.

2.11. $y=2-\sqrt{|x-3|}$. **2.12.** $y=2-\sqrt{3-|x|}$.

2.13. $y=|2-\sqrt{|x-3|}|$. **2.14.** $y=|2-\sqrt{3-|x|}|$.

2.15. $y=\frac{|x|}{x-1}$. **2.16.** $y=\frac{|x|}{|x|-1}$.

2.17. $y=\left|\frac{x}{x-1}\right|$. **2.18.** $y=\frac{x}{|x-1|}$.

2.19. $y=\frac{x^2-5x-6}{8x-x^2-12}$. **2.20.** $y=\frac{2x^2-17x+21}{7+6x-x^2}$.

2.21. $y=\frac{\frac{1}{x}+\frac{1}{x+1}}{\frac{1}{x}-\frac{1}{x+1}}$. **2.22.** $y=\frac{\frac{x-2}{x}+\frac{x-2}{x+1}}{\frac{x-2}{x}-\frac{x-2}{x+1}}$.

2.23. $y=\frac{2x}{\sqrt{1-\left(\frac{1-x^2}{1+x}\right)}}$. **2.24.** $\frac{\sqrt{\frac{9+x^2}{3x}+2}+\sqrt{\frac{9+x^2}{3x}-2}}{\sqrt{\frac{9+x^2}{3x}+2}-\sqrt{\frac{9+x^2}{3x}-2}}$.

2.25. $y=\frac{|x^3-3x+2|}{x-1}$.

2.26. $y=\begin{cases}3, \text{ agar } x\le -4 \text{ bo'lsa},\\ |x^2-4|x|+3|, \text{ agar } -4<x\le 4 \text{ bo'lsa}.\\ 3-(x-4)^2, \text{ agar } x>4 \text{ bo'lsa}.\end{cases}$

2.27. $y=\begin{cases}8-(x+6)^2, \text{ agar } x<-6 \text{ bo'lsa},\\ x^2-6|x|+8|, \text{ agar } -6\le x<5 \text{ bo'lsa},\\ 3, \text{ agar } x\ge 5 \text{ bo'lsa}.\end{cases}$

2.28. $f(x)$ juft funksiya va $x\ge 0$ da $f(x)=\sqrt{x}$ bo'lsa, $f(x)$ funksiya grafigini yasang.

2.29. $f(x)$ juft funksiya va $x\ge 0$ da $f(x)=x^2-3x$ bo'lsa, $f(x)$ funksiya grafigini yasang.

2.30. $f(x)$ toq funksiya va $x\ge 0$ da $f(x)=x^2$ bo'lsa, $f(x)$ funksiya grafigini yasang.

2.31. $f(x)$ toq funksiya va $x\le 0$ da $f(x)=x^2-2x$ bo'lsa, $f(x)$ funksiya grafigini yasang.

3-§. ARALASH MASALALAR

Quyidagi funksiyalarning aniqlanish sohasi va qiymatlar sohasini toping:

3.1. $y=\sqrt{x-1}$. **3.2.** $y=\frac{x^2-4}{x^2-9}$. **3.3.** $y=\frac{1}{\sqrt{x^2-x}}$.

3.4. $y=\sqrt[3]{1+x}$. **3.5.** $y=\frac{\sqrt{x(x+1)}}{x+4}$. **3.6.** $y=\sqrt{x^2-1}$.

3.7. $y=x$ va $y=\frac{x^2}{x}$ funksiyalarning aniqlanish sohalari ustma-ust tushadimi? Agar ustma-ust tushmasa, aniqlanish sohalarining umumiy qismini toping.

3.8. Jumlaning ma'nosini tushuntiring:

a) Funksiya yuqoridan (quyidan) chegaralangan;

b) Funksiya yuqoridan (quyidan) chegaralanmagan;

d) Funksiya chegaralangan;

e) Funksiya chegaralangan emas.

3.9. Isbotlang:

a) $y=\frac{1}{x}$ funksiya yuqoridan chegaralangan emas;

b) $y=\frac{1}{x}$ funksiya quyidan chegaralangan emas;

d) $y=x^2$ funksiya yuqoridan chegaralangan emas;
e) $y=x^2$ funksiya chegaralangan emas.

3.10. Shunday funksiya quringki, bu funksiya juft ham bo'lmasin va toq ham bo'lmasin.

3.11. Har qanday funksiyani ham juft va toq funksiyalarning yig'indisi shaklida yozish mumkinmi? $y=\sqrt{x}$ funksiyani misol sifatida qarang.

3.12. Funksiyaning monotonligini isbotlang:

a) $y=\sqrt{x}$; b) $y=x^3$.

3.13. Funksiya monoton funksiya bo'la oladimi (agar bo'la olmasa, monotonlik oraliqlarini toping):

a) $y=\frac{1}{|x|}$; b) $y=x-[x]$; d) $y=\sqrt[3]{x^2}$; e) $y=\sqrt{5-4x}$;

f) $y=\begin{cases}-1, & \text{agar } x<0 \text{ bo'lsa},\\ 1, & \text{agar } x>0 \text{ bo'lsa}\end{cases}$; g) $y=\frac{x+1}{x-2}$;

h) $y=|x^2-3x+2|$; i) $y=\sqrt{1-x^2}$.

3.14. Ikkita monoton funksiyaning yig'indisi monoton bo'lmasligi mumkinmi?

3.15. Monoton o'suvchi funksiyalarning ko'paytmasi hamma vaqt ham monoton o'suvchi funksiya bo'ladimi?

3.16. [0;2] oraliqda berilgan funksiyani ikkita monoton o'suvchi funksiyalarning ayirmasi shaklida tasvirlang:

$$y=\begin{cases}x^2, & \text{agar } 0\le x<1 \text{ bo'lsa},\\ 5, & \text{agar } x=1 \text{ bo'lsa},\\ x+3, & \text{agar } 1<x\le 2 \text{ bo'lsa}.\end{cases}$$

3.17. Monoton bo'lmagan funksiyani ikkita monoton funksiyalarning ayirmasi shaklida tasvirlash mumkinmi?

3.18. $y=\{x\}$ funksiya davriy funksiya ekanligini isbotlang. Uning davrini toping va grafigini yasang.

3.19. Davri 2π bo'lgan $f(x)$ davriy funksiya $[-\pi;\pi]$ oraliqda $y=\begin{cases}0, & \text{agar } -\pi\le x\le 0 \text{ bo'lsa},\\ x, & \text{agar } 0<x\le\pi \text{ bo'lsa}\end{cases}$ funksiya bilan ustma-ust tushadi. $f(x)$ funksiya grafigini yasang.

3.20. Davri T=2 bo'lgan $f(x)$ davriy funksiya $[-1;1]$ oraliqda $y=\begin{cases}x+1, & \text{agar } -1\le x\le 0,\\ x, & \text{agar } 0<x\le 1\end{cases}$ funksiya bilan ustma-ust tushadi. $f(x)$ funksiya grafigini yasang.

3.21. Davri T=3 bo'lgan f funksiya $(0;3]$ oraliqda $y=2-x$ funksiya bilan ustma-ust tushadi. $f(x)$ funksiya grafigini yasang.

3.22. Funksiyalarning grafiklarini ayni bir koordinatalar sistemasida yasang:

a) $y=x$, $y=x^2$, $y=x^3$, $y=x^4$, $y=x^5$;

b) $y=x$, $y=\sqrt{x}$, $y=\sqrt[3]{x}$, $y=\sqrt[4]{x}$, $y=\sqrt[5]{x}$.

Quyidagi funksiyalarning grafiklarini yasang:

3.23. $y=\sqrt{\frac{1}{x}}$. **3.24.** $y=\left[\frac{1}{x}\right]$. **3.25.** $y=[x^2]$.

3.26. $y=[\sqrt{x}]$. **3.27.** $y=\begin{cases}x^3, & \text{agar } x\le -1 \text{ bo'lsa},\\ \frac{1}{x}, & \text{agar } -1<x<0 \text{ bo'lsa},\\ x^2, & \text{agar } x\ge 0 \text{ bo'lsa}.\end{cases}$

3.28. $y=\begin{cases}x^2, & \text{agar } x\le -1 \text{ bo'lsa},\\ 2x-1, & \text{agar } -1<x\le 1 \text{ bo'lsa},\\ \sqrt{x}, & \text{agar } x>1 \text{ bo'lsa}.\end{cases}$

3.29. $y=\begin{cases}x^2, & \text{agar } x\le -2 \text{ bo'lsa},\\ \frac{1}{x^2}, & \text{agar } -2<x<0 \text{ bo'lsa},\\ \sqrt{x}, & \text{agar } 0\le x\le 4 \text{ bo'lsa},\\ x^2, & \text{agar } x\ge 4 \text{ bo'lsa}.\end{cases}$

3.30. $y=\begin{cases}x^3, & \text{agar } x\ge -2 \text{ bo'lsa},\\ \frac{1}{x}, & \text{agar } -2<x<-1 \text{ bo'lsa},\\ x^2, & \text{agar } -1\le x<2 \text{ bo'lsa},\\ \sqrt{x}, & \text{agar } x\ge 2 \text{ bo'lsa}.\end{cases}$

3.31. $y=x^2+5|x-1|+1$. **3.32.** $y=|-3x+2|-|2x-3|$.

3.33. $y=|x^2-3x+2|-|2x-3|$. **3.34.** $y=(x+1)(|x|-2)$.

3.35. $y=\frac{2x+1}{2-x}$. **3.36.** $y=1-\frac{1}{|x|}$

3.37. $y=\frac{2x-6}{|3-x|}$. **3.38.** $y=\sin^2x+\cos^2x$.

3.39. $y=\frac{|x-1|}{1-x^2}$. **3.40.** $y=(\sin^2x+\cos^2x)^4-x^2+5$.

Quyidagi funksiyalarga teskari funksiyalarni toping va teskari funksiyalarning grafigini yasang:

3.41. $y=3x-2$.

3.42. $y=-(x+2)^2-2$, $x\in(-\infty;-1)$.

3.43. $y=\frac{x+1}{x-1}$, $x\in(1;+\infty;)$.

3.44. $y=\sqrt{x^2-4}$, $x\in[2;+\infty)$.

Berilgan funksiyalarga teskari funksiyalarni toping. Teskari funksiyalarning va unga teskari funksiyalarning grafiklarini ayni bir koordinatalar sistemasida yasang:

3.45. a) $y=2x$; b) $y=-3x$; d) $y=5x-1$; e) $y=3x-4$.

3.46. a) $y=\frac{3}{x-1}$; b) $y=\frac{2}{2-x}$; d) $y=\frac{3x}{2x-1}$; e) $y=\frac{1-x}{x+2}$.

3.47. a) $y=(x+3)^2$, $x\le -3$; b) $y=(x-4)^2$, $x\ge 4$;
d) $y=x^2+8x-4$, $x\ge 4$; e) $y=(x-4)^2$, $x\le 1$.

3.48. a) $y=\sqrt{x-2}$; b) $y=\sqrt{3-x}$;
d) $y=4-\sqrt{x-1}$; e) $y=5+\sqrt{4-x}$.

3.49. Agar $A(1;2)$ nuqta $y=x^2+px+q$ parabolaning uchi bo'lsa, p va q larni toping.

3.50. Agar $M(-1;-7)$ nuqta ordinatalar o'qini $N(0;-4)$ nuqtada kesuvchi $y=ax^2+bx+c$ parabolaning uchi bo'lsa, a, b, c larni toping.

3.51. Agar $y=ax^2+bx+c$ funksiyaning grafigi $A(1;4)$, $B(-1;10)$, $C(2;7)$ nuqtalar orqali o'tsa, $y=ax^2+bx+c$ funksiyani toping.

3.52. Uchi $A(1;1)$ nuqta bo'lgan $y=ax^2+bx+c$ parabola $B(-1;5)$ nuqta orqali o'tadi. Bu parabolaning abssissasi 5 ga teng bo'lgan nuqtasining ordinatasini toping.

3.53. $x=2$ to'g'ri chiziq $y=ax^2-(a+6)x+9$ kvadrat uchhad grafigini yasang.

3.54. $y=x^2-6x+a$ funksiyaning eng kichik qiymati 1 ga teng. Funksiya grafigini yasang.

3.55. $y=-x^2+4x+a$ funksiyaning eng katta qiymati 2 ga teng. Funksiya grafigini yasang.

3.56. $y=2x^2+(a+2)x+a$ funksiyaning x_1 va x_2 nollari uchun $\frac{1}{x_1}+\frac{1}{x_2}=3$ munosabat o'rinli bo'lsa, uning grafigini yasang.

3.57. a ning qanday qiymatlarida $y=-x^2+4x+a$ funksiyaning qiymatlari to'plami $y=\sqrt{2x-a}$ funksiyaning aniqlanish sohasi bilan ustma-ust tushadi?

3.58. b ning qanday qiymatlarida $y=2bx^2+2x+1$ va $y=5x^2+2bx-2$ funksiyalarning grafiklari bitta nuqtada kesishadi?

3.59. $y=x^2+6x-3$ va $y=(x+3)^2-25$ funksiyalarning grafiklari $x=a$ to'g'ri chiziq bilan kesishgan. Kesishish nuqtalari orasidagi masofani toping.

3.60. c ning qanday qiymatlarida $y=cx^2-x+c$ va $y=cx+1-c$ funksiyalarning grafiklari umumiy nuqtaga ega bo'lmaydi?

3.61. b ning $x^2-2bx+b+6=0$ tenglama
a) manfiy ildizlarga;
b) musbat ildizlarga;
d) har xil ishorali ildizlarga ega bo'ladigan barcha qiymatlarini toping.

3.62. a ning qanday qiymatlarida quyidagi tengsizlik barcha $x\in(-\infty;+\infty)$ lar uchun o'rinli bo'ladi:
a) $x^2-(a+2)x+8a+1>0$; d) $ax^2+4x+a+3<0$;
b) $\frac{1}{24}x^2+ax-a+1>0$; e) $ax^2-4ax-3\le 0$?

3.63. Tengsizlik b ning qanday qiymatlarida yechimga ega emas:
a) $x^2+2bx+1<0$; d) $bx^2+(2b+3)x+b-1\ge 0$;
b) $bx^2+4bx+5\le 0$; e) $(4-b^2)x^2+2(b+2)x-1>0$?

3.64. Quyidagi funksiyalarning grafiklarini yasang va ularning yordamida funksiyalarning nollari, ishorasi saqlanadigan oraliqlarini, funksiyalarning eng katta va eng kichik qiymatlarini, qiymatlari sohalarini ko'rsating:

a) $x=\begin{cases}3, \text{ agar } x\le -4 \text{ bo'lsa},\\ |x^2-4|x|+3|, \text{ agar } -4<x\le 4 \text{ bo'lsa},\\ 3-(x-4)^2 \text{ agar } x>4 \text{ bo'lsa};\end{cases}$

b) $x=\begin{cases}8-(x+6)^2, \text{ agar } x<-6 \text{ bo'lsa},\\ |x^2-6|x|+8|, \text{ agar } -6\le x<5 \text{ bo'lsa},\\ 3, \text{ agar } x\ge 5 \text{ бўлса};\end{cases}$

d) $x=\begin{cases}|||x|-1|-1|, \text{ agar } |x|<2 \text{ bo'lsa},\\ \sqrt{|x|-2}, \text{ agar } |x|\ge 2 \text{ bo'lsa};\end{cases}$

e) $x=\begin{cases}2-\sqrt{4-|x|}, \text{ agar } |x|\le 4 \text{ bo'lsa},\\ \frac{8}{|x|}, \text{ agar } x>4 \text{ bo'lsa}.\end{cases}$

3.65. $f(x)=x^2-6x$ funksiya berilgan. Quyidagi funksiyalarning grafiklarini yasang:

a) $y=f(x)-2$; b) $y=f(x-2)$; d) $y=2f(x)$;
e) $y=f(2x)$; f) $y=-f(x)$; g) $y=f(-x)$;
h) $y=f(|x|)$; i) $y=|f(x)|$; j) $y=|f(|x|)|$.

3.66. Quyidagi funksiyalarning eng katta qiymatini toping:

a) $y = \frac{x}{1+x^2}$; b) $y = \frac{x}{1+x+x^2}$.

3.67. $y = \frac{x^2+3}{1+x}$ $(x>-1)$ funksiyaning eng kichik qiymatini toping.

3.68. $f(x) = \sqrt{x}$, $g(t) = \frac{t^2}{t-1}$ bo'lsa, $f(g(t))$ ni toping.

3.69. $f(x) = \frac{\sqrt{x-1}}{x}$, $g(t) = \frac{2t^2-2t+1}{(t-1)^2}$ bo'lsa, $f(g(t))$ ni toping.

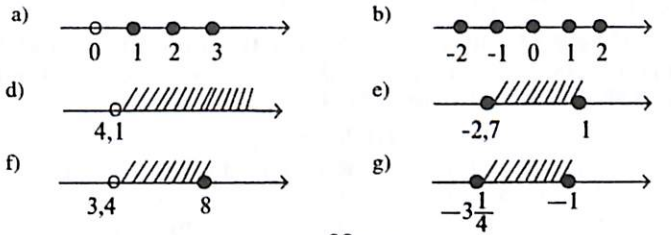
3.70. $f(x) = \frac{x^2}{\sqrt{x+1}}$, $g(t) = \frac{t^2-\sqrt{t}}{t}$ bo'lsa, $f(g(t))$ ni toping.

JAVOBLAR

I bob

1.1. {Toshkent, Andijon, Buxoro, Jizzax, Qashqadaryo, Navoiy, Namangan, Samarqand, Surxondaryo, Sirdaryo, Farg'ona, Xorazm, Qoraqalpog'iston}.

1.2. {S, E, R, Q, U, Y, O, SH, H, O', L, K, A, M, G, B, X, T, N, J, Z, I, D, F, NG, P, V, G', CH, '}. **1.3.** {1, 9, 2}. **1.4.** {10∈V, 136∈V}. **1.5.** S={–3; –2; –1; 4}, S_1={3; 2; 1; –4}. **1.6.** {B, O', SH, V, A, Q, T, D, N, U, M, L, I, F, O, Y}. **1.7.** a) {1, 2, 3, 4}; b) $\{-\frac{7}{4}\}$; {0; 12}; d) $\{-\sqrt{2}; \sqrt{2}\}$; e) { 1; 2}; f {1; 2; 3}. **1.8.**



22-rasm.

1.9. a) {111, 113, 131, 133, 311, 313, 331, 333}; b) {135, 153, 315, 351, 513, 531}; d) {104, 140, 203, 302, 320, 401, 410, 500}; e) 1, 11, 21, 31, 41, 51, 61, 71, 81, 91}. **1.10.** a), b). **1.12.** f). **2.1.** a) V⊂A, b) D⊄A, **2.3.** {3}, {6}, {9}, {12}, {3; 6}, {3; 9}, {3; 12;}, {6; 9}, {6; 12}, {9; 12}, {3; 6; 9}, {3; 6; 12}, {3; 9; 12}, {6; 9; 12}, ∅, A. **2.4.** a) A⊂B; b) C⊂D; d) E⊂F; e) K⊄M, M⊄K. **2.6.** a) B⊂A; b) A⊂B; d) A⊂B; B⊂A; e) A⊂B; f) A⊂B; g) B⊂A; h) B⊂A. **2.7.** a) to'g'ri; b) noto'g'ri; d) noto'g'ri; e) to'g'ri. **2.8.** a) A=B; b) A≠B; d) A≠B; e) A=B. **3.5.** [3; 5]. **3.6.** R∪E = {a, b, d, e, f, g, h, i, j}. **3.8.** a) A∪B = {x| x = 4k, k∈Z}. **3.11.** A\B= {x| x∈[–5; 3)∪(3; 4)∪(4; 5)∪(5; 6)∪(6; 7)∪(7; 8)∪(8; 9)∪(9; 10)}. **3.15.** A ={ x| x = 2k, k∈Z}. **3.16.** A= {x| x = 3k + 1, x = 3k + 2, k∈Z}. **4.1.** 20 kishi. **4.2.** 13 kishi. **4.4.** 68 kishi. **4.5.** 4 ta.

II bob

1.1. 1875. **1.2.** 51215. **1.3.** 89. **1.4.** 475385. **1.5.** 73450. **1.6.** 13174. **1.7.** 68654. **1.8.** 933333. **1.9.** 249480. **1.10.** 27396. **1. 12.** {7, 14, 21}. **1.13.** {117342, 1897524}. **1.15.** Hammasiga. **1.16.** k = 2431 bo'lishi mumkin, k∈{15; 18}. **1.17.** k = 1,3, 5, 7, 15, 21, 35, 105. **1.25.** a) 1, 11, 19, 209; b) 1, 11, 13, 143; d) 1, 11, 13, 17, 143, 187, 221, 2431; e) 1, 11, 13, 19, 143, 209, 247, 2717. **1.26.** a) 1; 11; b) 1; 11; d) 1; 11; 13; 143; e) 1; 11; 13; 143. **1.33.** a) 1; 13; 17; 221; b) 1; 17; 19; 23; 323; 391;.437; 7429. **1.34.** 8 ta. **1.36.** a) 2; b) 5555; d) 20; e) 1; f) 1; g) 28; h) 600. **1.40.** 1. **2.6.** a) 70=23·3+1; b) 180=20·9; d) 200=11·17+13; e) 76=8·9+4. **2.7.** a) 5=0·9+5; b) 9=0·18+9. **2.9.** $q_1 = -q-1$; $r_1 = b-r$. **2.13.** a) n = 3,

n = 5; b) n = 3; d) hech bir qiymatida; e) $n = 3$, $n = 9$; f) $n = 3$, $n = 5$, $n = 9$; g) hech bir qiymatida; h) $n = 3$, n = 9; i) n = 3, n = 5. **2.15.** 4. **2.17.** h) 1; i) 1; m) 5. **3.32.** a) $-\frac{5}{6}$; b) 1; d) 9. **4.9.** d) 3. **4.12.** A>B. **4.13.** Ko'rsatma: a va b sonlari orasida S_1 ratsional son topilishini isbotlang. Agar S soni $\frac{a}{\sqrt{2}}$ va $\frac{b}{\sqrt{2}}$ sonlari orasidagi ratsional son bo'lsa, $\sqrt{2}S$ sonini izlangan s son sifatida olish mumkin. **4.15.** a); e); f); i). **5.3.** b) -3; e) 0,8; f) 15. **5.5.** a) $a=b$ yoki $a=-b$; b) $b\in(-\infty; 0]$. **5.11.** $|a|+|b|+|c|+|d|\neq 0$. **5.12.** $|a-b| + |b-c| + |a-c| \neq 0$. **5.13.** $|a-b| + |b-c| + |a-c| \leq 0$. **5.15.** a) $x\in(-1; 5)$; g) $x\in[-1.25; -0.25]$. **5.16.** i) 3; j) -4; k) 3; m) 14. **5.18.** a) $x\in\left[7; 8\frac{1}{3}\right)$; b) $x\in[-4,5; -4)$. **5.19.** a) $\{-2; -1\}$. Ko'rsatma: $0\leq\frac{x-1}{2}-x<1$ ning butun yechimlarini toping. e) $\varnothing$. **5.25.** a) 1; b) $5\frac{1}{3}$; d) **5.28.** a) 25%; b) 60%; d) 250%. **5.31.** 1,75 kg. **5.33.** 240 ta. **5. 36.** 960 ta. **5.41.** 9 m va 10,8 m. **5.42.** 8,8 m va 11 m. **5.43.** 21%. **5.45.** 19%. **5.48.** 2 yildan keyin. **5.49.** 25 kundan keyin. **5.50.** 20 km; $5\frac{1}{4}$ soat. **5.51.** 4.

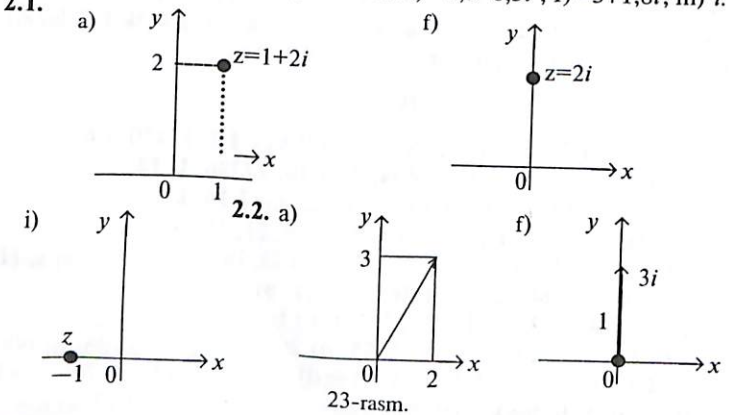
III bob

1.1. a) Re(z) = –5, Im(z) = 8; j) Re(z) = 0, Im(z) = 8; k) Re(z) =4, Im(z) = 0. **1.2.** a) $-4+8i$; b) 1,2. **1.5.** a) $\bar{z}=-3-5i$; d) $\bar{z}=-3+5i$; e) $\bar{z}=3-5i$; f) $\bar{z}=3i$; g) $\bar{z}=4{,}2$. **1.6.** a) $1+i$; b) 8; d) 0; i) $6-9i$; j) $4+2i$. **1.7.** a) $1+\frac{2}{3}i$; b) $1+i$; d) $1+3\frac{1}{9}i$; e) $4+13i$. **1.8.** a) $-13+11i$; f) $3\frac{5}{9}i$; g) $\frac{-1-\sqrt{2}}{2}+\frac{-1-\sqrt{2}}{2}i$; h) $\frac{67+5\sqrt{2}}{15}+i$. **1.9.** a) $-9+19i$; h) 13. **1.10.** b) $0{,}4-2{,}2i$; j) $12+3i$; k) $\frac{4}{51}-\frac{1}{51}i$. **1.11.** a) $a^2+4b^2 = (a-2bi)(a+2bi)$; l) $a^{2n}+b^{2k}=(a^n-ib^k)(a^n+ib^k)$.

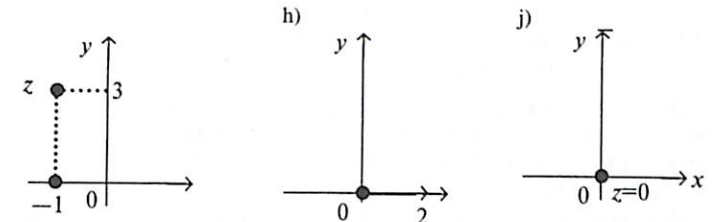
1.12. $i^n=\begin{cases} i, \text{ agar } n=4k+1,\ k=0,1,2,\ldots \text{ bo'lsa,}\\ -1, \text{ agar } n=4k+2,\ k=0,1,2,\ldots \text{ bo'lsa,}\\ -i, \text{ agar } n=4k+3,\ k=0,1,2,\ldots \text{ bo'lsa,}\\ 1, \text{ agar } n=4k,\ k=0,1,2,\ldots \text{ bo'lsa.}\end{cases}$

1.13. a) $13+21i$; d) $12i$; e) $8i$. **1.14.** a) $-6{,}5-6{,}5i$; f) $-3+1{,}8i$; m) i.

2.1.



23-rasm.



23-rasm (davomi).

2.3. a) $|z| = 5$; g) $|z| = 3\sqrt{2}$; i) $|z| = \sqrt{2}$; e) $|z| = 1$; f) $|z| = 4$; k) $|z|=|b|$; p) $|z| = 1$. **2.4.** a) $\frac{\pi}{4}$; b) $\frac{\pi}{3}$; d) $\frac{\pi}{2}$; g) 0; e) $\frac{\pi}{6}$; f) $\frac{3\pi}{2}$; g) $\frac{3\pi}{4}$; h) $\frac{3\pi}{6}$; i) 0; i) $\frac{\pi}{2}$; k) p. l) $\frac{3\pi}{2}$; m) $\frac{3\pi}{2}$. **2.5.** a) $\sqrt{2}\left(\cos\frac{5\pi}{4}+i\sin\frac{5\pi}{4}\right)$; b) $\sqrt{2}\cos\frac{7\pi}{4}+i\sin\frac{7\pi}{4}$; d) $2\left(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}\right)$; e) $2\left(\cos\frac{2\pi}{3}+i\sin\frac{2\pi}{3}\right)$; f) $2(\cos\pi+i\sin\pi)$; g) $\cos\frac{\pi}{2}+i\sin\frac{\pi}{2}$; h) $\cos0+i\sin0$; i) $\cos\frac{3\pi}{2}+i\sin\frac{3\pi}{2}$; j) $\sqrt{2}\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}$); k) $\cos\frac{2\pi}{3}+i\sin\frac{2\pi}{3}$; l) $\sqrt{11}\left(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}\right)$; m) $\cos\frac{5\pi}{6}+i\sin\frac{5\pi}{6}$; n) $2\left(\cos\frac{\pi}{2}+i\sin\frac{\pi}{2}\right)$. **2.6.** $-3-4i=5\left(\cos\left(\pi+a\operatorname{rctg}\frac{4}{3}\right)+i\sin\left(\pi+a\operatorname{rctg}\frac{4}{3}\right)\right)$. **2.7.** $z=2\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)$. **2.8.** $z=\cos\frac{16\pi}{17}+i\sin\frac{16\pi}{17}$. **3.1.** a) $\frac{\sqrt{2}}{2}\left(\cos\frac{3\pi}{8}+i\sin\frac{3\pi}{8}\right)$; d) $3\sqrt{3}\left(\cos\frac{\pi}{8}+i\sin\frac{\pi}{8}\right)$. **3.2.** a) $\frac{\sqrt{3}}{2}\cos\left(\frac{2\pi}{399}+i\sin\frac{2\pi}{399}\right)$; e) $\cos\frac{2\pi}{3}+i\sin\frac{2\pi}{3}$. **3.3.** a) 1; d) $\frac{1}{2}-\frac{\sqrt{3}}{2}i$; e) $\cos\frac{3\pi}{7}+i\sin\frac{3\pi}{7}$; f) $64+64\sqrt{3}i$. **3.4.** a) $z_0=\frac{\sqrt[4]{8}}{2}\left(\cos\frac{\pi}{8}+i\sin\frac{\pi}{8}\right)$; $z_1=\frac{\sqrt[4]{8}}{2}\left(-\cos-\frac{\pi}{8}i\sin\frac{\pi}{8}\right)$; d) $z_0=\frac{\sqrt{3}}{2}+\frac{1}{2}i$; $z_1=-\frac{\sqrt{3}}{2}-\frac{1}{2}i$. **4.2.** a) $x\in R$; b) $x\in\varnothing$; d) 1; e) 2. **4.3.** $x=-\frac{2}{3}$, $y=-\frac{28}{9}$. **4.4.** a) -2^{10}; b) $-2^{10}(1-i)$.

4.9. a) $x_{1,2}=\frac{-b\pm i\sqrt{4ac-b^2}}{2a}$; b) $z_1=\frac{1}{2}+\frac{\sqrt{3}}{2}i$, $z_2=-\frac{1}{2}-\frac{\sqrt{3}}{2}i$, $z_3=\frac{1}{2}-\frac{\sqrt{3}}{2}i$, $z_4=\frac{1}{2}-+\frac{\sqrt{3}}{2}i$.

IV bob

1.2. b) $59\frac{3}{7}$; d) $\frac{7}{416}$. **1.3.** l) 45; m) 222. **1.5.** a) $27x^6y^3z^3$; f) $243x^5z^{20}$. **1.6.** e) $\frac{1}{27}$; h) $\frac{11}{13}$; i) $1\frac{3}{14}$. **1.8.** a) a^3+x^2; d) $5a-12x$; i) $2x^3y+32xz^2$. **1.12.** b) $\frac{1}{2}$; e) 3; h) 192. **1.15.** $a+b+c=0 \Rightarrow c=-a-b \Rightarrow a^3+b^3+c^3=a^3+b^3-(a+b)^3=-3a^2b-3ab^2=3ab(-a-b)=3abc$.

1.17. $a+b=\begin{cases}|a|, & a\neq 0,\ b=0 \text{ da}\\ |b|, & a=0,\ b\neq 0 \text{ da.}\\ 0, & a=b=0 \text{ da.}\end{cases}$ **2.2.** a) $3^{1999}+147$. e) 17.

2.3. b) $a=-7$. **2.4.** a) 16; e) 84. **2.5.** b) $a=1, \forall b\in R$; e) $a=0, b=4$. **2.6.** $a=3;\ b=-7;\ c=4$. **2.8.** a) 12; d) 14. **2.10.** a) $f(x)g(x)=20x^5+16x^3+4x^2+8x$; e) $f(x)g(x)=26x^5+73x^4+100x^3+33x^2+12$.

2.11. a) $P(x)=D(x)(x+1)+2$; b) $P(x)=D(x)(x^2+4x+1)+2$;
d) $P(x)=D(x)(x^2+2x+2)+3x+4$; e) $P(x)=D(x)(x^2+3x+1)+3x+4$;
f) $P(x)=D(x)(3x^2+5x-8)-5x^2+14x+2$; k) $P(x)=D(x)\cdot(x^2+3x+5)$;
l) $P(x)=D(x)\cdot(x^3+4x)$. **2.12.** a) $x+1$; b) x^2+1; d) x^3+1; e) x^2-2x+2; f) x^3-x+1; g) $x+3$; h) x^2+x+1; i) 1.

V b o b.

1.1. a) $\{2\}$; b) $\varnothing$; d) $\{1;2\}$; e) $\{-3;3\}$; f) $\{-\frac{3}{2}\}$; g) $\varnothing$; h) $\{4,5\}$; i) $\{13\}$; n) $\varnothing$; o) $\{0;3\}$; p) $\{-5;9\}$; q) $\{1\}$. **1.2.** a) $\{x\mid x\neq -2\}$; b) R; k) $\{x\mid x\in R, x\neq 1, x\neq 7\}$; m) R; o) R. **1.3.** a) $\{(x;y)\mid x\in R, y\in R, x\neq 0, x\neq y\}$; d) $\{(x;y)\mid x\in R, y\in R, y\neq x^2\}$; f) $\{(x;y)\mid x\in R, y\in R, x\neq 2, x\neq 3, y\neq 0\}$; h) $\{(x;y)\mid x\in R, y\in R, x\neq \pm y\}$; k) $\{(x;y)\mid x\in R, y\in R\}$. **1.4.** a) $-\frac{a}{2}$; d) $\frac{x-2m}{x+2m}$; e) $\frac{2a+5}{a+2b}$; k) $\frac{1}{x+5}$; l) 1. **1.6.** a) $\frac{a-6}{6}$; b) $\frac{5x-3a}{4}$; d) $\frac{41a-5}{12}$; e) $\frac{a^2+x^2}{2a}$; f) $-\frac{x^2+c^2}{2x}$. **1.7.** a) $\frac{a+x}{x}$; b) $\frac{2y-x}{y}$; d) $-\frac{2a+x}{ax}$; f) $\frac{x-5}{5x}$. **1.8.** d) $\frac{2x}{(1-3a)(x+2)}$; e) $\frac{7x^2}{(2x-1)(2y+3)}$. **1.9.** a) $\frac{y(x-y)}{x^2}$; b) $\frac{a(a+b)}{3b}$; g) $-\frac{a}{xyb}$; k) $\frac{1}{axy}$. **1.10.** d) $\frac{b}{4a}$; f) $\frac{x+2}{6}$; h) $\frac{a-b}{a+b}$; i) $9c^2-b^2$. **1.12.** b) $-2x$; d) q^2-pq; e) $-\frac{1}{4x}$; f) $\frac{30x^2+6y^2-16xy}{x(x^2+y^2)}$; g) 2; i) $2x(x+y)$; j) $a-2$. **1.13.** a) $\frac{1}{x^2+x+1}$; b) $\frac{1}{x^7+1}$; d) $\frac{a}{xy-a^2}$; e) $\frac{x^{11}-1}{x^{11}}$. **1.14.** 1 va 9. **2.2.** a) $x\leq 0$; b) $x\in R$; d) $x\in R$; e) $x\in R$; f) $x\in R$; g) $x\in R$; h) $x\in R$; i) $x\in R$; j) $x\in\varnothing$; k) $x\in R$; l) $x\in R$; m) $x=3$. **2.3.** a) $x\leq 2$; b) $x\geq -3$; d) $x\geq 3$; e) $x\leq 4$; f) $x=3$; g) $x=3$; h) $x\in\varnothing$; i) $x=1$; j) $x=-8$; k) $x=8$; l) $x\in\{2;4\}$; m) $x=3$. **2.4.** a) 44; b) — 15; d) 6; e) 6; f) 630; g) 120; h) 60; i) 0,015.

2.5. a) $\frac{6}{7}$; b) $-\frac{4}{3}$; d) $\frac{2}{3}$; e) $\frac{3}{2}$; f) $\frac{5}{8}$; g) $\frac{4}{5}$; i) $\frac{3}{5}$; j) $\frac{1}{3}$.

2.7. a) 225; b) 225; d) –25; e) $\frac{1}{9}$; f) $-x$; g) x^2; i) x^2+1; j) x^3.

2.8. a) $\sqrt[9]{16}$; b) $\sqrt[12]{76}$; d) $\sqrt[15]{4}$; e) $\sqrt[21]{25}$; f) $\sqrt[21]{x^2}$; g) $\sqrt[6]{x}$; i) $\sqrt[12]{x}$; j) $\sqrt[9]{x}$.

2.9. a) $\sqrt[4]{8}$; b) 4; d) $\sqrt[3]{-32}$; e) 2; f) $\sqrt[4]{x^3}$; g) x^3; i) $\sqrt[4]{(x+2)^5}$; j) x^8.

2.10. a) $\sqrt[6]{27}$ va $\sqrt[6]{16}$; d) $\sqrt[4]{25}$ va $\sqrt[4]{6}$; j) $\sqrt[20]{(x-y)^4}$ va $\sqrt[20]{y^5}$. **2.11.** a) Ha; b) Yo‘q; d) Ha; e) Yo‘q; f) Yo‘q; g) Ha; h) Yo‘q; i) Ha.

2.12. a) $x\in Q$; b) $\{x\mid x=2k, k\in Z\}$; d) $x\geq -3$; e) $x\in R$; f) $x>0$; g) $x\in R$; h) $x\in[-1;1]$; i) $x\neq\pm 1$.

2.14. g) 1989; h) $\frac{1}{8}$. **2.15.** a) $c^{\frac{2}{3}}$; b) $\sqrt{b}$; d) $\frac{1}{m}$; e) y^3.

2.16. f) $7\sqrt{2}$; i) $2\sqrt[4]{3}$; k) $|x^2-2|\sqrt[4]{y}$; l) $(x-1)\sqrt[7]{z^2}$; m) $(y+1)^2\sqrt[5]{5x^2}$.

2.17. a) $\sqrt{80}$; b) $\sqrt[3]{-54}$; d) $-\sqrt{162}$; e) $\sqrt[5]{96}$; f) $-\sqrt{x^2y^3}$; g) $\sqrt[5]{x^5y^3}$; i) $\sqrt[4]{x^8y^3}$; j) $-\sqrt[4]{x^{12}y^3}$; k) $\sqrt[4]{(x-1)^8(y-2)}$; l) $-\sqrt[4]{(x-1)^{12}(y-2)}$; m) $-\sqrt[4]{x^4y}$; n) $-\sqrt{(7-4\sqrt{3})xy^3}$.

2.18. a) $\sqrt{2}$; b) $6\sqrt[5]{3}$; d) $2\sqrt[4]{8}$. **2.19.** a) $2\sqrt{2}$; d) $\sqrt[30]{3^{13}}$; g) $\sqrt{\sqrt{2}+1}$; i) $\sqrt[9]{32}$.

2.20. a) $2\sqrt{3}<3\sqrt{2}$; e) $3\sqrt[3]{4}>3\sqrt[3]{2}$; f) $\sqrt{2}<\sqrt[3]{3}$; i) $\sqrt{8}<\sqrt[3]{19}$.

2.21. a) 20; b) $2\sqrt[12]{2}$; d) 6; j) $\sqrt[4]{12}$. **2.22.** a) $\sqrt[6]{2}$; b) $\sqrt[3]{4}$; d) $\sqrt{6}$; e) $\sqrt[12]{\frac{16}{27}}$; f) $\frac{1}{\sqrt[12]{a}}$; g) $\sqrt[18]{a}$. **2.23.** a) $x\sqrt[3]{16x}$; b) $24x^2$; d) $36x^2-9$; $\left(|x|\geq\frac{1}{2}\right)$; e) x^{16}; g) $\sqrt[3]{(2+xy^2)^2}$; h) $(xy+z)\sqrt{xy+z}$. **2.24.** a) $\frac{2\sqrt{3}}{3}$; b) $\frac{5}{6}\sqrt[3]{18}$; d) $5+2\sqrt{6}$; e) $2-\sqrt{2}+\sqrt{6}$; f) $4+\sqrt[3]{75}+\sqrt[3]{45}$; k) $\frac{2(\sqrt{a}-\sqrt{x})}{a-x}$; m) $\frac{(x-y)\sqrt{x+y}}{x+y}$; n) $(1+\sqrt{a})\sqrt{1-\sqrt{a}}$. **2.25.** a) $\sqrt{37}-\sqrt{2}$; b) $\sqrt{23}-\sqrt{6}$; d) 2; e) $2\sqrt{5}$. **2.26.** a) to‘g‘ri; b) noto‘g‘ri; d) to‘g‘ri; e) to‘g‘ri. **2.27.** $\frac{1}{8}$. **2.28.** a) $x^{-\frac{2}{3}}y^{-\frac{2}{3}}z^{-\frac{1}{3}}$; b) $x^{\frac{5}{7}}y^{\frac{1}{2}}$; d) 0; e) $\frac{b^4}{a^3}\sqrt{\frac{3\sqrt{2b}\sqrt[3]{b}}{\sqrt[3]{a^7}}}$; f) $\frac{a}{b}$; g) $\left(\frac{b}{a}\right)^{\frac{1}{9}}$. **2.30.** e) $\sqrt[4]{18}+\sqrt[4]{2}$. **2.31.** 2. **2.32.** a) $a\sqrt[4]{b}$ $(\sqrt{a}+\sqrt[4]{b})$; b) 27; d) – 1, agar $0<a\leq 1$ va $-\left(\frac{\sqrt{1-a^2}+1}{a}\right)^2$, agar $-1<a<0$; e) 3; f) $\sqrt[6]{a}$; g) 9a; h) $\frac{x^2}{2x-1}$; i) $\sqrt[2]{(a-b)2}$. **2.33.** 1. **2.34.** 4.

VI b o b.

1.1. a) $\frac{a-1}{3}$; b) $a=1$ da yechim yo‘q, $a\neq 1$ da $\frac{5}{a-1}$; e) $a=\pm 1$ da x ixtiyoriy son, $a\neq\pm 1$ da $x=0$. **1.3.** Yo‘q. **1.4.** Yo‘q. **1.5.** 15 yildan keyin. **1.7.** a) – 4,5; b) istalgan son; d) – 1; e) ildizi yo‘q. **1.9.** a) $a\neq 1$ da $x=a-1$, $a=1$ da x — istalgan son; b) $a\neq\pm 1$ da $x=0$, $a=\pm 1$ da x — istalgan son; g) $a\neq 1$ da $x=\frac{b+1}{a-1}$; $a=1, b=-1$ da x — istalgan son; $a=1$, $b\neq -1$ da ildiz yo‘q. **2.1.** f) $(x-3)^2-1$; g) $a(x-2a)^2+3$; i) $\left(x+\frac{a+b^2}{2}\right)-\frac{(a-b)^2}{4}$. **2.7.** Yo‘q. **2.10.** Ko‘rsatma: $a^2+b^2=(a+b)^2-2ab$, $a^3+b^3=(a+b)^3-3ab(a+b)$. **2.17.** i) $a(x^2-(\alpha+\beta)x+\alpha\beta)=0, a\in R, a\neq 0$. **2.18.** $14x^2-3x-5=0$. **2.19.** $-\frac{1}{2}x^2-\frac{1}{2}x-3=0$. **2.20.** a) Ko‘rsatma: $(x-2-\sqrt{3})(x-\alpha)=0$. Tenglamaning chap tomonini kvadrat uchhad ko‘rinishida tasvirlang, bu yerda $\alpha\in R$. **3.1.** – 4, 5. **3.2.** 1. **3.3.** 15. **3.4.** $x=5$.

3.5. $x\in R, x\neq\frac{2}{3}$. **3.6.** $x\in R, x\neq -2$. **3.7.** $a\neq -c, c\neq 0$ da $x=\frac{a-c}{a+c}$; $a=-c$, $c=0$ da $\varnothing$. **3.8.** $a\neq 1$, $a\neq 2{,}25$, $a\neq -0{,}4$ da $x=\frac{31-2a}{4a-9}$; $a=2{,}25$,

a=−0,4 da ∅; a=1 da ma'noga ega emas. **3.9.** $-\frac{11}{7}$ va 2. **3.10.** ∅. **3.11.** −4 va 9. **3.12.** 0 va 1. **3.13.** 1. **4.1.** Ko'rsatma: $a^3+b^3=(a+b)^3-3(a+b)ab$. **4.2.** −1; 1; 8. **4.11.** − 1; 2. **4.12.** − 2; 1. **4.13.** $y=x^2+6x+1$ ga nisbatan kvadrat uchhad sifatida qarang. **4.14.** $y=(x^2-x+1)^2$ ga nisbatan kvadrat uchhad sifatida qarang. **4.15.** Ko'rsatma: $2\cdot\frac{x^2+36}{x^2-36}=\frac{x+6}{x-6}+\frac{x-6}{x+6}$. **4.16.** −4; −2; −1. **4.17.** $1;\frac{-1\pm\sqrt{17}}{2}$. **4.18.** Ko'rsatma: 40 = 8 + 32. **5.1.** −1 va 6. **5.7.** 0,2, $1\pm\sqrt{2}$. **5.10.** Ko'rsatma: $x^2-5x+6=t$ deb oling. **5.11.** Ko'rsatma: $x^2+5x=t$ deb oling. **5.15.** ∅. **5.16.** 5,5 va 6. **5.17.** −5; 1; $-1\pm\sqrt{6}$. **5.18.** $\pm2;\pm\frac{\sqrt[4]{24}}{2}$. **5.19.** Ko'rsatma: $x^2+2x=t$ deb oling. **5.20.** − 4; 2. **5.21.** $x_{1,2}=10\pm\sqrt{85}$, $x_{3,4}=5\pm\sqrt{10}$. **5.22.** $x_1=\frac{1}{2}$, $x_2=\frac{7}{2}$. **5.23.** $x=1$. **5.24.** $x_1=3, x_2=5, x_{3,4}=9\pm\sqrt{66}$. **5.25.** $x_{1,2}=\frac{-11\pm\sqrt{97}}{6}$. **5.26.** $a=b$ da $x\in R, x\neq a; a\neq b$ da ∅. **6.5.** $m=\pm\sqrt{15}$. **6.6.** $m\neq2$ bo'lsa. **6.7.** $a=b=-3$. **6.8.** m = 1, n = −30. **6.9.** 2x + 1. **6.10.** $\frac{r_1-r_2}{a-b}x+\frac{r_1b-r_2a}{b-a}$. **6.11.** b) P(x) = D(x) · (x^2-x+3); d) P(x)=D(x) $(2x^3-2x^2-x-4)+6$; h) P(x)=D(x) · $(x^3-3x^2+8x-21)$; m) P(x) = D(x) $(x^4+x^3-3x^2-x-1)-4$. **6.12.** a) 2; b) 0; d) 3. **6.13.** $a^3+b^3+c^3-3abc=(a+b+c)(a^2+b^2+c^2-ab-bc-ac)$. Ko'rsatma: $a^3+b^3+c^3-3abc$ ni a ga nisbatan ko'phad deb qarang va a= −b − c soni shu ko'phadning ildizi ekanini tekshirib ko'ring. **7.1.** b) $x=2\pm i$; e) $x=-2\pm3i$; g) $x=4\pm5i$; i) $x=-0,5\pm i$; k) $x=1\pm\frac{1}{4}i$; m) $x=3\pm\sqrt{2}$ i. **7.2.** a) $(x+1-2i)(x+1+2i)$; b) $(x-3i)(x-1+3i)$; e) $(5z+5-i)\cdot(5z+5+i)$. **7.3.** a) $\pm3i;\pm2$; h) $z_{1,2}=\pm\frac{1+i\sqrt{3}}{2}$, $z_{3,4}=\pm\frac{1-i\sqrt{3}}{2}$. **7.4.** $ax^2-4ax+13a=0, a\neq0$, $a\in R$. **7.5.** $ax^4-8ax^3-34ax^2-72ax+65=0, a\neq0$, $a\in R$. **7.6.** $a(x-2)(x^4-8x^3-34x^2-72x+65=0), a\neq0, a\in R$. **7.7.** 3 karrali. **7.8.** a) $(x^2+3)(x^2-3x+3)(x^2+3x+3)$; b) $x^2(x-4i)(x+4i)$. **8.1.** a) 2; b) −5; 3; 6; d) ratsional ildizi yo'q; e) $\frac{1}{2}$; g) $\frac{1}{2}$; $-\frac{3}{2}$; h) − 1; i) −3; 2. **8.2.** a) −2; 1; b) −4; − 2; −1; d) butun yechimlari yo'q. **8.3.** a) $1\frac{2}{3}$; b) $\pm\frac{1}{4}\sqrt{2(\sqrt{73}-5)}$; d) $\pm\sqrt{\frac{\sqrt{13}-2}{3}}$; e) $\pm\frac{1}{2}\sqrt{14}$; f) ±2; g) ±2; −3. **8.4.** a) $-2;\pm\sqrt{3};\frac{1}{2}$; b) $\frac{1\pm\sqrt{5}}{2};-\frac{1}{2}$; 2; d) 1; e) ±1; $\frac{3\pm\sqrt{73}}{8}$; f) ± 1; $-\frac{2}{3}$; 2; g) ± 1; $\frac{7\pm\sqrt{73}}{4}$. **8.5.** a) $-\frac{1}{2};\frac{3}{4};\frac{1\pm\sqrt{5}}{2}$; b) −1; d) $\frac{1}{2}$ 2,5; e) $-\frac{2}{3};-\frac{1}{2};3$; f) 1; $a\pm\sqrt{a}$; g) −5; 2; 3; 4. **8.6.** a) 4; 5; b) −1; 5; $2\pm2\sqrt{2}$; f) − 4; 3; $\frac{1\pm\sqrt{145}}{2}$; g) − 3; 4; $\frac{-1\pm\sqrt{145}}{2}$.

8.7. a) 1; b) $\frac{1\pm\sqrt{5}}{2}$; $1+\sqrt{2}$. **8.8.** a) $\frac{-5\pm\sqrt{21}}{6}$; b) ∅; d) $\frac{-5\pm\sqrt{57}}{4}$; −1; 2. **8.9.** a) 1; b) 2; d) 1; −2; e) 5; −1; f) $\pm\sqrt{21}$; ± 3; g) ∅; i) ∅; j) ∅. **8.10.** a) 2; 3; b) $\frac{-1\pm\sqrt{21}}{2},\frac{-3\pm\sqrt{17}}{2}$; d) −1; 3; $\pm\sqrt{3}$. **9.1.** a) −15; b) 3; f) 0; g) 0; h) 1; i) $x^2(x^2-1)$. **9.2.** a) $a=6$; b) $a=-2$; d) 0; e) a — ixtiyoriy son. **9.3.** a) ± 2; b) 0; d) ∅. **9.5.** d) $x\in R$; e) $x\in R$. **9.6.** a) $8\frac{172}{495}$; b) 3,(13). **9.7.** a) − 80; b) 6; d) − 72; e) 0; f) 36; g) − 90. **9.8.** a) $-\frac{2}{3}$; b) 0 va 6; e) ∅. **10.1.** a) (4; − 1); b) $\left(\frac{9}{11};\frac{7}{11}\right)$; d) (t; 5 − t), $t\in R$; e) (4; − 3); f) (6; 9); g) ∅; h) $t;\left(\frac{21t-40}{7}\right)$, $t\in R$. **10.2.** a) (1; 2); b) ∅; d) $t;\left(\frac{7-2t}{3}\right)$, $t\in R$; e) $\left(\frac{1}{4};\frac{1}{2}\right)$; f) $\left(\frac{7}{11};\frac{3}{13}\right)$. **10.3.** a) −23; b) 6; d) 2a−5; e) − 4a+13b. **10.4.** a) $\Delta_x=7;\Delta_y=-1$; b) $\Delta_x=-3,5;\Delta_y=30$. **10.5.** a) (−5; 2); b) (2; 1); d) (6; 5); e) (5; − 2); f) ∅; g) ∅; h) ∅; i) ∅; j) (t; t−1), $t\in R$; k) $\left(t;\frac{3t}{5}\right)$, $t\in R$. **10.7.** a) Agar $a\neq\pm4$ bo'lsa, $\left(-\frac{12}{4-a};\frac{6}{4-a}\right)$. Agar $a=4$ bo'lsa, ∅. Agar $a=-4$ bo'lsa, $\left(t;\frac{t+3}{2}\right)$, $t\in R$; b) $a\neq\pm3$ da (2; 1), $a=\pm3$ da $\left(t;\frac{a-6+3t}{a}\right)$, $t\in R$; d) $a\neq\frac{1}{2}$ da $\left(\frac{a(2a-3)}{2(1-2a)};\frac{2a^2-a+1}{2(1-2a)}\right)$, $a=\frac{1}{2}$ da ∅. **10.8.** $a=1$, b = −1. **10.9.** (1; −1), (1; −2), (−1; −1), (−1; 2). **10.10.** $a=4$. **10.11.** $a=3$. **10.12.** a) (1; 1; −1); b) (1; −1; 1); d) (−1; 1; 1); e) (1; 1; 1); f) (1; −1; −1); g) (−1; −1; 1). **10.14.** a) ∅; e) $\left(3-2y;y;\frac{3y+1}{2}\right)$, $y\in R$. **11.1.** a) (1; 0); (0; −1); b) (5/4; − 1/8); (−1; 1); g) (−4; −5); (6; −5). **11.2.** a) (2; 3), (3; 2); e) (2; −3), (3; −2). **11.3.** a) ∅; b) ∅; d) (1 − t; t), $t\in R$. **11.4.** a) (− 2; − 4), (− 4; − 2), (2; 4), (4; 2); b) (2; 8), (8; 2), (−2; −8), (−8; −2); d) $\left(-\frac{9}{5};-\frac{16}{5}\right)$, $\left(\frac{9}{5};\frac{16}{5}\right)$; e) (−3; −2), (3; 2); f) (−7; −3), (7; 3); g) Ko'rsatma. Bir jinsli tenglama hosil qiling; i) (−3; −2), (3; 2). **11.5.** a) (1; 2), (2; 1); b) $(-3;-5),\left(-\frac{5}{3};-\frac{13}{3}\right),\left(\frac{5}{3};\frac{13}{3}\right)$, (3; 5); d) (− 4; −5), $(-3\sqrt{3};-\sqrt{3})$, $(3\sqrt{3};\sqrt{3})$, (4; 5); e) (1; −1), (3; −3), $\left(\sqrt{157}-13;\frac{\sqrt{157}-13}{2}\right),\left(-13-\sqrt{157};-\frac{13+\sqrt{157}}{2}\right)$; f) (2; −3), (t; 1), $t\in R$; g) (−1; 3), (t; 2), $t\in R$; h) (2; −1), (−1; t), $t\in R$; i) (−1; −2), $(-\sqrt{2};-\sqrt{2})$, (1; 2), $(\sqrt{2};\sqrt{2})$. **11.6.** a) (5; 1), (1; 5), (3; 2), (2; 3); b) (2; 1), (− 1; − 2), $(1-\sqrt{2};1+\sqrt{2})$, $(1+\sqrt{2};1-\sqrt{2})$; d) (−2; − 4), $\left(\frac{5}{3};\frac{10}{3}\right)$; e) (1; 4), (−5; −4), (5; −4), (−1; −4). **11.7.** a) (2; 3), (3; 2); b) (−1; −2), (2; 1); d) (−1; 2), (2; −1). Ko'rsatma. Ikkinchi tenglamani 3 ga ko'paytirib, birinchi tenglamaga qo'shing; e) (4; 8;), (8; 4); f) (−3; −1), (−1; −3), (1; 3), (3; 1); g) (2; −1), (−1; 2); h) (−3; −2), (−2; −3), (2; 3), (3; 2). Ko'rsatma. Birinchi tenglamadan $x^2+y^2=\frac{78}{xy}$ ni topamiz. Bu tenglamani kvadratga

ko'taring. **11.8.** a) (1; 3; 9), (9; 3; 1); b) $\left(\frac{12}{7}; \frac{12}{5}; -12\right)$; d) $\left(\frac{1}{\sqrt[3]{3}}; \frac{1}{\sqrt[3]{3}}; \frac{1}{\sqrt[3]{3}}\right)$; e) (2; 1; 3), (–2; –1; –3). **12.1.** 28 m. **12.2.** 2.5. t. **12.3.** 8 kunda. **12.4.** 21 qator. **12.5.** 20 km/soat. **12.6.** 20 km/soat. **12.7.** 7 km/soat. **12.9.** 5 soat, 7 soat. **12.10.** 30 kunda, 20 kunda. **12.12.** 18 km/soat, 24 km/soat. **12.14.** 11 ta. **12.15.** 22 kishi. **12.16.** 30 o'quvchi (Eslatma; 12.13. masalada 42 ta vektor hosil bo'ladi). **12.17.** 7 ta. **12.18.** sakkiz burchak. **12.19.** 40 km/soat. **12.20.** 30 km/soat. **12.21.** 10 sm va 4 sm. **12.22.** 15 sm; 8 sm. **12.24.** 12 sm; 16 sm; 20 sm. **12.25.** 36.4. **12.26.** 40 km/soat. 30 km/soat. **12.27.** 36 km/soat; 24 km/soat. **12.28.** 36 km/soat; 30 km/soat. **12.29.** 10 soat; 6 soat. **12.30.** 60 soat; 84 soat. **12.31.** 18 va 12. **12.32.** 15 yoki 95. **12.33.** 32. **13.1.** $(-\infty; -1)$.

13.2. $(-4, 6; +\infty)$. **13.3.** $(2\frac{13}{15}; +\infty)$. **13.4.** $(-\infty; 2\frac{28}{29})$. **13.5.** $(-\infty; -1,5)$.

13.6. $(-\infty; 3)$. **13.7.** $[1; +\infty)$. **13.8.** $(-\infty; -\frac{2}{3}]$. **13.9.** $(3; +\infty)$. **13.10.** $(-\infty; -2)$.

13.11. $(-\infty; -1\frac{2}{3})$. **13.23.** $y > \frac{3}{a^2+1}$.

13.26.
$$\begin{cases} a > 0 \text{ да } x < \frac{b}{a}; \\ a = 0,\ b \le 0 \text{ да } \varnothing; \\ a = 0,\ a > 0 \text{ да } x \in R; \\ a < 0 \text{ да } x > \frac{b}{a}. \end{cases}$$

13.35. a) $y<3$ da; b) $y>7$ da; d) $y>\frac{3}{17}$ da; e) $y<0.1$ da. **13.36.** $(-\infty; 1)\cup(3; +\infty)$.

13.37. $[1; 5]$. **13.38.** $\left[-\frac{2}{5}; 1\right]$. **13.39.** $(0; 1)$. **13.40.** $\varnothing$. 13.41. $(-\infty; +\infty)$.

13.42. $\varnothing$. **13.44.** $a \in (5/3; \infty)$.

13.47.
$$\begin{cases} k > 0 \text{ да},\ x \in \left(-\infty; \frac{1-\sqrt{1+4k}}{2k}\right) \cup \left(\frac{1+\sqrt{1+4k}}{2k}\right); \\ k = 1 \text{ да}, x \in (-\infty; -1); \\ -\frac{1}{4} < k < 0 \text{ да},\ x \in \left(\frac{1-\sqrt{1+4k}}{2k}\right) \cup \left(\frac{1+\sqrt{1+4k}}{2k}\right); \\ k \le -\frac{1}{4} \text{ да}, \varnothing. \end{cases}$$

13.49.
$$\begin{cases} |k| > 2\sqrt{6} \text{ да},\ x \in \left(\frac{-k-2\sqrt{6}}{4}; \frac{-k+2\sqrt{6}}{4}\right); \\ |k| \le 2\sqrt{6} \text{ да}, \varnothing. \end{cases}$$

13.50.
$$\begin{cases} k < 1 \text{ да},\ x \in (-\infty; 1-\sqrt{1-k})\cup(1+\sqrt{1-k}; +\infty); \\ k = 1 \text{ да},\ x \in (-\infty; 1) \cup (1; +\infty); \\ k > 1 \text{ да},\ x \in (-\infty; +\infty). \end{cases}$$

13.57. $x \in \left[1; \frac{4}{3}\right]$. **13.58.** $\varnothing$. **13.59.** $\left(-\frac{2}{3}; 3\right)$. **13.60.** $x \in R$. **13.61.** $x \in (-\infty; -1)\cup(15; +\infty)$. **13.62.** $[-2; 1]$. **14.2.** $x \in (-\infty; +\infty)$. **14.4.** $x \in (-\infty; 1)\cup(3; +\infty)$. **14.5.** $x \in (-\infty; -\frac{1}{3})\cup(2; +\infty)$. **14.8.** $x \in (-\infty; +\infty)$. **14.9.** $x \in (2; 5)\cup(12; +\infty)$. **14.10.** $x \in (-\infty; -7)\cup(-1; 4)$. **14.11.** $x \in (-\infty; -5)\cup(-1; 0)\cup(8; +\infty)$. **14.12.** $x \in (-48; 37)\cup(42; +\infty)$. **14.13.** $x \in (-\infty; -0.7)\cup(2.8; 9.2)$. **14.14.** $x \in (-17; -4)\cup(4; +\infty)$. **14.15.** $x \in (-\infty; -11)\cup(-1; 11)$. **14.16.** $x \in (-\infty; -5)\cup(0; 5)$. **14.17.** $x \in (-0.1; 0)\cup(0.1; +\infty)$. **14.18.** $x \in (-\infty; -3)\cup(-1; 1)\cup(3; +\infty)$.

14.19. $x \in (-6; 0)\cup(6; 15)$. **14.20.** $x \in (-2; 6)$. **14.21.** $x \in (-\infty; 0)\cup(4; +\infty)$. **14.22.** $x \in (-\infty; 1)\cup(1; 24)$. **14.23.** $x \in (-\infty; -7)\cup(21; +\infty)$. **14.24.** $x \in (-\infty; -4)\cup(8; +\infty)$. **14.25.** $x \in (-16; 11)$. **14.26.** $x \in [-1; 3)$. **14.27.** $x \in (-\infty; -4)\cup[6; +\infty)$ **14.28.** $x \in (-\infty; 1)\cup(1; 2)\cup(4; +\infty)$. **14.29.** $x \in (-\infty; -1]\cup\{1; 2\}\cup[4; +\infty)$. **14.30.** $x \in \{-2\}\cup[1; 2]$. **14.34.** $x \in (-\infty; 1)$. **14.35.** $(-\infty; -2)\cup(-2; 1)\cup(4; +\infty)$. **14.36.** $x \in (-\infty; -5]\cup\{1\}\cup[2; 7)\cup(7; +\infty)$. **14.49.** $(-\infty; +\infty)$. **14.50.** $(2; 3)$. **14.51.** $(-3; 1)$. **14.52.** $(-\infty; -2)\cup(-2; -\frac{1}{2})\cup(1; +\infty)$. **14.53.** $(-2; -1)\cup(1; 2)$. **14.54.** $[-3; 3]$. **14.55.** $(-\infty; 2)\cup(5; +\infty)$. **14.56.** $(-\infty; 1)\cup(1.5; +\infty)$. **14.57.** $(-\infty; 2.5)\cup(\frac{33}{8}; +\infty)$. **14.58.** $(-6; 3)$. **14.59.** $(-\infty; 1)\cup(4; +\infty)$. **14.60.** $(-3; 1)$. 14.61. $(-\infty; 0)\cup(4; +\infty)$. **14.63.** $(-\infty; +\infty)$. **14.64.** $(-\frac{1}{2}; 2)$. **14.65.** $[1; 3]\cup(5; +\infty)$. **14.66.** $(-\infty; -\frac{1}{\sqrt{2}})\cup(0; +\infty)$. **14.67.** $(-\infty; 6)\cup[-2; 0)\cup(3; +\infty)$. **14.68.** $(2; 3)\cup(5; 6)$. **14.69.** $(1; +\infty)$. **14.70.** $(-\frac{9}{2}; -2)\cup(3; +\infty)$. **14.71.** $(-1; 1)\cup(4; 6)$. **14.72.** $(-\infty; -3)\cup(-\frac{\sqrt{7}}{2}; \frac{\sqrt{7}}{2};)\cup(4; +\infty)$. **14.73.** $[1; 2)\cup(3; 4]$. **14.75.** $(-\infty; -1)\cup(1; +\infty)$. **14.76.** $(-\infty; -2)\cup(-1; 0)\cup(\frac{1}{2}; +\infty)$. **14.77.** $(-5; 1)\cup(2; 3)$. **14.78.** $(-\infty; 0)\cup(1; 6)$. **14.79.** $(-\infty; 4]\cup[-2; -1]$. **14.80.** $(-2; -1)\cup(2; 3)$ **15.19.** $[1; \frac{4}{3}]$. **15.20.** $(-\infty; -1]\cup[15; +\infty)$. **15.21.** $[-2; 1]$. **15.22.** R. **15.23.** $\varnothing$. **15.24.** $[-1; 1]$. **15.25.** R. **15.26.** $\{1\}\cup[2; 3]$. **15.27.** $x = -5/2$. **15.28.** $x = \pm 2$. **15.31.** $x = 4/3$. **15.32.** $x = -4,5$; $x = 3,25$. **15.33.** $x = \frac{\sqrt{113}-5}{4}$. **15.34.** $x = -\frac{2}{3}$. **15.35.** $(-\infty; 2/3]$. **15.36.** $[1; 3]$. **15.37.** $x = 0,5$, $x = 3,5$. **15.38.** $[2; +\infty)$. **15.39.** a) $x = 2$; $x = -6$. **15.40.** $[-2; 1\frac{2}{3}]$ **15.41.** $\{0\}\cup(1; +\infty)$. **15.42.** $(-\infty; 0]\cup[1; +\infty)$. **15.43.** $[-2\frac{1}{6}; 1\frac{1}{6}]$. **15.44.** $[\frac{5}{6}; +\infty)$. **15.45.** $[0; 13]$. **15.46.** $\{-4; -2; 0; 2; 4\}$ **15.47.** $[-3; 3]$. **15.48.** $(-\infty; 0]\cup[\frac{1}{2}; +\infty)$. Ko'rsatma. $|a-b| = |a| - |b| \Leftrightarrow (a-b)\,b \ge 0$. **15.49.** $\{0\}$. **15.50.** $\{0; 2\}$. **15.51.** $\{0\}$. **15.52.** $\{-1\}$. **15.54.** $\{1; 4; \frac{1}{2}(5 \pm \sqrt{13})\}$. **15.55.** $\{-\sqrt{3}\}$. **15.56.** $\{2; \frac{5}{2}; \frac{9+\sqrt{17}}{4}\}$. **15.57.** $\{-\frac{2}{3}; \frac{1}{2}; 2\}$. **15.58.** $\{3; 4\}$. **15.59.** $\{\pm 1; \pm 3\}$. **15.60.** $(1; 4]$. **15.61.** $\{1\frac{2}{3}; -3\}$. **15.62.** $\{-5; 1\frac{1}{3}]$. **15.63.** $\{\frac{1-\sqrt{5}}{2}; 1+\sqrt{2}\}$. **15.64.** $\{\frac{1-\sqrt{3}}{2}\}$. **15.65.** $a \le 0$ da $x = -a$; $a > 0$ da $x = -7a$, $x = a$. **15.66.** $a > 0$ da $\{-3a; a\}$; $a = 0$ da $x \ne 0$; $a < 0$ da $\varnothing$. **15.67.** $a \ne 0$ da $\{-\frac{5a}{3}\}$; $a = 0$ da $(-\infty; +\infty)$. **15.68.** $a \le 0$ da $x = \frac{6a}{5}$; $a > 0$ da $x = \pm 2a$. **15.69.** $(3; 1)$, $(\frac{5}{3}; \frac{11}{3})$. **15.70.** $(0; -1)$, $(\frac{4}{5}; \frac{7}{5})$. **15.71.** $(0; 1)$. **15.72.** $(0; -1)$. **15.73.** $(-\frac{11}{19}; \frac{23}{19})$, $(1; -1)$. **15.74.** (c; 4–c), bu yerda $c \in [0; 1]$ **15.75.** $(2; 1)$, $(0; -3)$, $(-6; 9)$. **15.76.** $(\sqrt{2}; 2\sqrt{2})$, $(-\sqrt{2}; -2\sqrt{2})$. **15.77.** $\{-\frac{3+\sqrt{17}}{2}\}$. **15.78.** $a \in [\frac{2}{3}; 3-\sqrt{5}]$ da $\left(\frac{4a-a^2}{2a-4}; \frac{a-4}{a-2}\right)$, $a \in ((3-\sqrt{5}; 2]$ da $\left(\frac{a^2-12a+8}{6a-4}; \frac{a}{3a-2}\right)$. **15.79.** $a = 7-4\sqrt{3}$ da $(0; 1-2\sqrt{3})$, $a = 7+4\sqrt{3}$ da $(0; 1+2\sqrt{3})$, $a = 1$ da $(6; -1)$. **16.1.** $(-1; 1)$. **16.2.** $[-1; 1]$. **16.3.** $(-\infty; -1)\cup(1; +\infty)$. **16.4.** $(-\infty; -1]\cup[; +\infty)$. **16.5.** $\varnothing$. **16.6.** $\{0\}$. **16.7.** $\varnothing$. **16.8.** $(-\infty; +\infty)$. **16.9.** $(-\infty; +\infty)$. **16.10.** $\varnothing$. **16.11.** $\{1\}$. **16.12.** $\{\frac{3}{2}\}$. **16.13.** $(-\infty; +\infty)$. **16.14.** $x = 4$. **16.15.** $(-\infty; +\infty)$. **16.16.** $(-\infty; 4\cup(4; +\infty)$.

16.17. $\{\pm1\}$. **16.18.** $(-\infty;-1)\cup(-1;1)\cup(1;+\infty)$. **16.19.** $R\setminus\{2\}$. **16.20.** $[-1;1]$. **16.21.** $(-\infty;+\infty)$. **16.22.** $[0;3]$. **16.23.** $(22;4)$. **16.24.** $(-\infty;1)\cup(3;+\infty)$. **16.25.** $(-\infty;-\frac{2}{3}]\cup[\frac{1}{2};+\infty)$. **16.26.** $\left(\frac{11-\sqrt{57}}{4};\frac{11+\sqrt{57}}{4}\right)$. **16.27.** $(8;+\infty)$. **16.28.** $(-3;4)$. **16.29.** $(-\infty;-2)\cup(-1;+\infty)$. **16.31.** $[1;6]$. **16.32.** $\varnothing$. **16.33.** $(-\infty;-3]$. **16.34.** $[-2;3\frac{2}{3}]$. **16.35.** $[-3;5]$. **16.36.** $(-\frac{4}{7};+\infty)$. **16.37.** $(-\infty;1)\cup(7;+\infty)$. **16.38.** $(-\infty;1)$. **16.39.** $(-\infty;2)\cup(3,5;+\infty)$. **16.40.** $(-;1]\cup[3;+\infty)$. **16.41.** $(-\infty;-1)\cup(0;1)\cup(1;+\infty)$. **16.42.** $(2;3)\cup(3;+\infty)$. **16.43.** $(-\infty;-6)\cup(-3,5;+\infty)$. **16.44.** $(3;3\frac{1}{3})$ **16.45.** $[0;1\frac{3}{5}]\cup[2,5;+\infty)$. **16.46.** $(-\infty;-2)\cup(-2;-1)\cup(-1;0]$. **16.47.** $(-\infty;2)$. **16.48.** $[\sqrt{6}-2;1)\cup(1;4]$. **16.49.** $(-\infty;1]\cup[5;+\infty)$. **16.50.** $[-1,0)\cup(0;1]$. **16.51.** $(-\infty;-\frac{2}{3}]\cup[\frac{1}{2};2]$. **16.52.** $(-\infty;\frac{1+\sqrt{17}}{4}]$. **16.53.** $(-3;3]$. **16.54.** $(1-\sqrt{3};2-\sqrt{2})$. **16.55.** $(-\infty;\frac{4-\sqrt{19}}{3})\cup(\frac{4+\sqrt{19}}{3};+\infty)$. **17.1.** $\varnothing$. **17.2.** $\varnothing$. **17.3.** $\varnothing$. **17.4.** $\varnothing$. **17.5.** $\varnothing$. **17.6.** $\varnothing$. **17.7.** $\varnothing$. **17.8.** $\varnothing$. **17.9.** $\varnothing$. **17.10.** $\varnothing$. **17.11.** $\varnothing$. **17.12.** x=3. **17.13.** x=0,5. **17.14.** $\varnothing$. **17.15.** $\{\frac{1}{2};1\}$. **17.16.** $\{-1;2\}$. **17.17.** $\{-3;2\}$. **17.18.** $\{-4;3\}$. **17.19.** x=6. **17.20.** x=3. **17.21.** x=3. **17.22.** x=8. **17.23.** x=28. **17.24.** x=0. **17.25.** x=4. **17.26.** x=19. **17.27.** x=3. **17.28.** x=6. **17.29.** x=–1. **17.30.** x=3. **17.31.** x=2. **17.33.** $x=-1\pm2\sqrt{17}$. **17.34.** $\varnothing$. **17.35.** $x=-5$, $x=0$. **17.36.** $-3\frac{3}{8}$; 1. **17.37.** –8;27. **17.38.** 8; 27. **17.39.** x=3. **17.40.** x=1. **17.41.** $\{-\frac{3}{2};\frac{1}{2}\}$. **17.42.** x=2,5. **17.43.** $x=1\frac{2}{3}$. **17.44.** x=8. **17.45.** x=5. **17.46.** $x=\frac{7\pm\sqrt{153}}{16}$. **17.47.** x=2. **17.48.** x=3. **17.49.** $\varnothing$. **17.50.** $x=-61$, x=30. **17.51.** x=8, $x=8\pm4\sqrt{3}$. **17.52.** $x=-6$, $x=-5$, $x=-\frac{11}{2}$. **17.53.** $x=-1$. **17.54.** x=0. **17.55.** x=3; x=4. **17.56.** x=0. **17.57.** x=9. **17.58.** x=2; x=3. **17.59.** $x=-61$; x=30. **17.60.** $x=+-109$; x=80. **17.61.** $x=-2\frac{2}{3}$; x=1. **17.62.** $x=-\frac{1}{3}$, x=1. **17.63.** $x=\pm4$. **17.64.** $x=-1$. **17.65.** x=4. **17.66.** $\varnothing$. **17.67.** x=–1; x=40. **17.68.** $[2;+\infty)$. **17.69.** $[5;8]$. **17.70.** $x=-\frac{1}{11}$. **17.71.** $x=\frac{5}{11}$. **17.72.** $x=\frac{\sqrt{5}}{2}$. **17.73.** x=2. **17.74.** $x=-5$; x=2. **17.75.** $\frac{\sqrt{5}}{2}$. **17.76.** $a<0$ da $\varnothing$, $a\geq0$ da $x=a^2-1$. **17.77.** $a<-3$ da $\varnothing$, $a\geq-3$ da $x=\frac{a-3}{2}$. **17.78.** $a\neq0$ da $x=\frac{5a}{3}$; a=0 da $(-\infty;0)\cup(0;+\infty)$. **17.79.** $a\in(-\infty;2)\cup(2\sqrt{2};+\infty)$ da $\varnothing$; $a\in[2;2\sqrt{2}]$ da $x=5\pm\frac{a\sqrt{8-a^2}}{2}$. **17.80.** $a<0$ da $\varnothing$, $0\leq a\leq\frac{1}{2}$ da $x=a+1\pm\sqrt{2a}$; $a>\frac{1}{2}$ da $x=a+1+\sqrt{2a}$. **17.81.** $x=\frac{\sqrt{2}(1-\sqrt{2\sqrt{3}-3}}{\sqrt{3}-1}$. **17.82.** (6;10), (10;6). **17.83.** (1;4), (4;1). **17.84.** $(\frac{5}{4};\frac{1}{3})$. **17.85.** $\left(-\sqrt{\frac{15}{17}};-4\sqrt{\frac{15}{17}}\right)$, $\left(-4\sqrt{\frac{15}{17}};-\sqrt{\frac{15}{17}}\right)$. **17.86.** $(-9;-\frac{9}{4})$, (4;1). **17.87.** (–6;–1), (–3;2), (9;–4), (2;3). **17.88.** (–1;–27), (27;1). **17.89.** (1;8) (8;1). **17.90.** (1;4), (4;1). **17.91.** (5;4). Ko'rsatma. Tenglamalarni ko'paytiring. **17.92.** (–2;–1), (–1;–2)

(1;2), (2;1), (0;c), $c\in R$. **17.93.** (4;2), (4/3;—2/3). **18.1.** $[-3;+\infty)$. **18.2.** $(-\infty;+\infty)$. **18.3.** $(-\infty;+\infty)$. **18.4.** $\varnothing$. **18.5.** x=2. **18.6.** $x\neq2$. **18.7.** $(-\infty;+\infty)$. **18.8.** $\varnothing$. **18.9.** $(-\infty;+\infty)$. **18.10.** $x\neq0$. **18.11.** $\varnothing$. **18.12.** $\varnothing$. **18.13.** $[-3;+\infty)$. **18.14.** $(-1;+\infty)$. **18.15.** $(-\infty;1]\cup[2;+\infty)$. **18.16.** $y\neq1/2$. **18.17.** $(-\infty;+\infty)$. **18.18.** (2;3). **18.19.** x=2; x=3. **18.20.** x==–1,5. **18.21.** $\varnothing$. **18.22.** $\varnothing$. **18.23.** $\{-1\}\cup[2;+\infty)$. **18.24.** $\{-2;1\}\cup[3;+\infty)$. **18.25.** $[-2;-1]\cup[3;+\infty)$. **18.26.** $\{-2\}\cup[1;3]$. **18.28.** $(-\infty;-8,5]\cup[1;10)$.

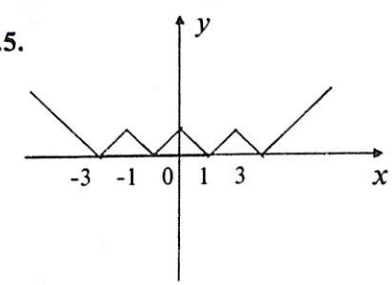
VII b o b.

1.1. $x\neq2$. **1.2.** $x\neq3.4$. **1.4.** $x\neq-2$. **1.6.** $x\neq1$, $x\neq2$, $x\neq3$. **1.7.** $x\neq3$. $x\neq4$. **1.10.** R. **1.12.** $x\neq2$. **1.13.** $x\neq3$. **1.14.** R. **1.16.** $x\neq0$, $x\neq\pm1$. **1.18.** $x\neq0$. **1.19.** R. **1.21.** $x\neq0$, $x\neq2$, $x\neq3$. **1.28.** $\left(-\frac{\sqrt{2}}{3};+\infty\right)$. **1.29.** $(-\infty;\sqrt{3}-2]$ **1.30.** $(-\infty;-2(\sqrt{3}+2))$. **1.31.** $\{1;2\}$. **1.32.** $x\neq-8/7$. **1.35.** $(-\infty;2]$. **1.36.** $\{0\}\cup[1;+\infty)$. **1.37.** $\{0\}\cup[2;+\infty)$. **1.38.** $\{2\}$. **1.45.** $[-0,5;0,5]$. **1.46.** $\left[-\frac{2}{3}\right]$; 2 $\cup\{3\}$. **1.47.** $[-2;0]\cup\{1,5\}$. **1.48.** $\{1\}\cup[2;3\}\cup(3;+\infty)$. **1.49.** $(-\infty;-9)\cup(-9;-3]\cup\{-2\}\cup[7;8)\cup(8;+\infty)$. **1.50.** $\{0,5\}$. **1.58.** $(-\infty;3]$. **1.59.** $(-\infty;2,25]$. **1.61.** $(-\infty;0)\cup(0;+\infty)$. **1.62.** $(-\infty;1)\cup(1;+\infty)$. **1.63.** $(0;1]$. **1.64.** $(-\infty;-2]\cup[2;+\infty)$. **1.66.** $[-2;+\infty)$. **1.67.** $(-\infty;5]$. **1.69.** $[2;-\infty]$. **1.70.** $(-8;-2]$. **1.71.** $[1;+\infty)$. **1.72.** $[0;1]$. **1.73.** $[-4;1)$. **1.74.** $[-1;2)$. **1.75.** $[-2;1]$. **1.76.** $[-1;3]$. **1.77.** $[-3;\infty)$. **1.78.** $[3;12)\cup(123;+\infty)$. **1.79.** $[6,75;+\infty)$. **1.80.** $[6,75;27)\cup(27;+\infty)$. **1.84.** Ko'rsatma: $\frac{3x-1}{x+2}=t$ deb oling va $f(t)$ni toping. **1.89.** a) juft; b) juft; d) juft; e) juft. **1.90.** d) toq; e) juft. **1.91.** a) juft; b) toq; d) juft; e) toq. **1.92.** a) toq; b) juft; d) juft; e) juft. **1.106.** a) 2; b) 1; d) 2; e) –1; f) 1; g) –1; –h) 3; i) –1. **1.107.** a) $\pm\frac{2\sqrt{3}}{3}$; d) $\varnothing$; f) $\varnothing$; h) 1. **1.108.** $(-\infty;+\infty)$ da $\downarrow$. **1.109.** $(-\infty;+\infty)$ da $\uparrow$. **1.126.** $g(x)=\frac{x-3}{2}$. **1.127.** $g(x)=\frac{2x+1}{2-x}$. **1.128.** $g(x)=\sqrt{x}$. **1.129.** $g(x)=-\sqrt{x}$. **1.130.** $g(x)=-\sqrt{-x}$.

1.131. $g(x)=\begin{cases}x, & \text{agar } x\in[0;1] \text{ bo'lsa,}\\ 3-x, & \text{agar } x\in[1;2] \text{ bo'lsa.}\end{cases}$ **1.147.** $y_{max}=1$, $x_{max}=1$.

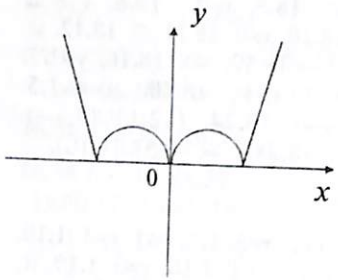
1.149. $y_{max}=\frac{1}{12}$, $x_{max}=1,5$. **1.157.** $y_{max}=0$, $x_{max}=-2$.

2.5.



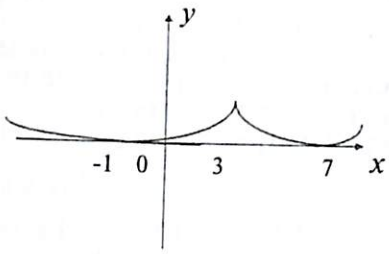
24-rasm.

2.8.



25-rasm.

2.13.



26-rasm.

3.49. $p=-2$, $q=-1$. **3.50.** $a=3$, $b=6$, $c=-4$. **3.51.** $y=2x^2-3x+5$. **3.52.** 17. **3.57.** $a=2$. **3.58.** $b=4$. **3.59.** $r=13$. **3.60.** $\forall c\in(-\infty; -\frac{1}{2})\cup(1;+\infty)$. **3.61.** a) $-6 < b \le 2$; b) $b \ge 3$; d) $b < -6$. **3.62.** a) $0 < a < 28$; b) $-\frac{1}{2} < a < \frac{1}{3}$; d) $a < -4$; e) $-\frac{3}{4} \le a \le 0$. **3.63.** a) $-1 \le b \le 1$; b) $0 \le b < 1{,}25$; d) $b < -\frac{9}{16}$; e) $b \le -2$.

MUNDARIJA

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ALGEBRA VA MATEMATIK ANALIZ ASOSLARIDAN MASALALAR TO'PLAMI

I q i s m

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Texnik muharrir *D. Gabdraxmanova*
Musahhih *Y. Bizaatova*

Diapozitivdan bosishga ruxsat etildi: 08.09.04. Bichimi $84x108^{1}/_{32}$. Tayms garniturasi. Ofset bosma. Shartli bosma tobog'i 7,98. Nashriyot-hisob tobog'i 8,2. Adadi 3814 nusxa. Buyurtma № 710. Bahosi shartnoma asosida.

**«Sharq» nashriyot-matbaa
aksiyadorlik kompaniyasi bosmaxonasi,
700083, Toshkent shahri, «Buyuk Turon» ko'chasi, 41**

A 45

Algebra va matematik analiz asoslaridan masalalar to'plami. I qism: Akademik litseylar va kasb-hunar kollejlari uchun o'quv qo'llanma /A. Abduhamidov, H. Nasimov, U. Nosirov, J. Husanov. — «Sharq». 2003. — 152b.

Sarl. oldida: O'zbekiston Respublikasi Oliy va o'rta maxsus ta'lim vazirligi; O'rta maxsus, kasb-hunar ta'limi markazi; O'rta maxsus kasb-hunar ta'limini rivojlantirish instituti.

I. Abduhamidov A. va boshq.

22.14ya722+22.16ya722