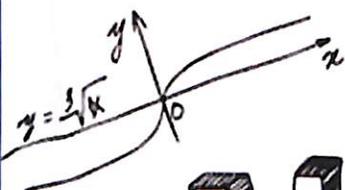


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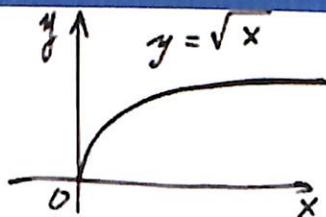
ALGEBRA VA SONLAR NAZARIYASI  
MODUL TEXNOLOGIYASI ASOSIDA  
TAYYORLANGAN MISOL  
VA MASHQLAR TO'PLAMI

2

$$\cos \alpha + \cos \beta = 2 \cos \frac{\alpha + \beta}{2} \cos \frac{\alpha - \beta}{2}$$

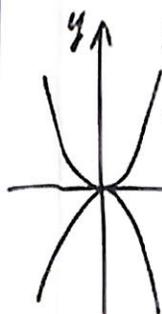


$$\cos^2 \alpha = \frac{1 + \cos 2\alpha}{2}$$



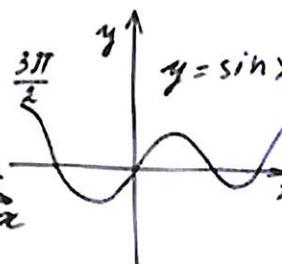
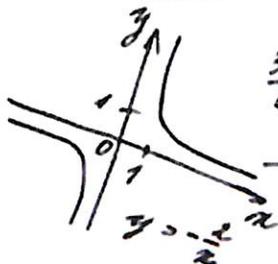
$$(\sqrt{a} = b) \Leftrightarrow \begin{cases} b \geq 0 \\ a = b^2 \end{cases}$$

# ALGEBRA



$$\sin \alpha - \sin \beta = 2 \cos \frac{\alpha + \beta}{2} \sin \frac{\alpha - \beta}{2}$$

$$\sqrt{\frac{a}{b}} = \frac{\sqrt{|a|}}{\sqrt{|b|}}$$



O'ZBEKISTON RESPUBLIKASI  
OLIV VA O'RTA MAXSUS TA'LIM VAZIRLIGI

D.YUNUSOVA, A.YUNUSOV

ALGEBRA VA SONLAR NAZARIYASI  
MODUL TEXNOLOGIYASI  
ASOSIDA TAYYORLANGAN  
MISOL VA MASHQLAR TO'PLAMI

*O'quv qo'llanma*

5140100-matematika va informatika

5140100-matematika

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Algebra va sonlar nazariyasi modul texnologiyasi asosida tayyorlangan  
misol va mashqlar to'plami. 2-Kitob. / O'quv qo'llanma/. - Toshkent:  
"Innovatsiya-Ziyo", 2022, 166 b.

Ushbu qo'llanma "Algebra va sonlar nazariyasi" fan davlat ta'lim standarti hamda  
matbuot ta'limining bo'lib ularda keltirilgan nazariy savollar va amaliy topshiriqlar  
kengitirishga qaratilgan bo'lib, talabalarni mustaqil yodiy izlanishga yo'naltiradi.  
Ushbu qo'llanma o'rta o'rta maxsus matematika faniga ushbu fanning keng ta'biqlarini o'quvchilari;  
o'qituvchilari, malakalarini oshirish va qayta tayyorlash kurslari tinglovchilari foydalanishlari  
mumkin.

Данное пособие соответствует учебной программе дисциплины «Алгебра и  
теория чисел» педагогических высших учебных заведений  
Примеры и задачи собраны в модуль, что создает некоторые удобства  
и позволяет использовать пособие в учебнике, а также использовать теоретические  
и практические задания в учебнике, а также использовать теоретические  
и практические задания в учебнике.

The given manual meets to the curriculum of discipline «Algebra and the theory of  
numbers» pedagogical higher educational institutions. Examples and tasks are collected in  
book and expand knowledge of students. Taking into account wide application of  
mathematics of schools and the licea, the given manual we recommend teachers of schools,  
licea students of courses of improvement of qualification and retrainings of the pedagogical  
staff.

Taqrizchilar: t.f.d., professor N.Sherboyev, f.-m.f.n., dotsent A.Dusumbetov  
O'zbekiston Respublikasi Oliy va o'rta maxsus ta'lim vazirligi tomonidan  
nashrga tavsiya etilgan.

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## IX MODUL. CHIZIQLI AKSLANTIRISHLAR

### 21-§. Chiziqli akslantirish. Chiziqli operator yadrosi va obrazi. Chizikli operator matrisasi.

Asosiy tushunchalar: chiziqli akslantirish, chiziqli operator, operator yadrosi, tasviri, defekti, rangi, matrisasi.

$\mathcal{F}$  sonlar maydoni ustida aniqlangan U vektor fazoning V vektor fazoga akslantiruvchi  $\varphi$  akslantirish uchun ushbu

$$1. \varphi(\bar{x}_1 + \bar{x}_2) = \varphi(\bar{x}_1) + \varphi(\bar{x}_2),$$

2.  $\varphi(\lambda \bar{x}) = \lambda \varphi(\bar{x})$  ( $\lambda \in \mathcal{F}$ ) shartlar bajarilsa, u holda U vektor fazo V vektor fazoga chiziqli akslanadi deyiladi.

U fazoni V fazoga chiziqli akslantirishlar to'plamini  $Hom(U, V)$  orqali belgilanadi.

U vektor fazoni o'zini o'ziga chiziqli akslantirish U fazoda aniqlangan chiziqli operator deyiladi.

U vektor fazoning ixtiyoriy  $\bar{x}_1$  va  $\bar{x}_2$  elementlari va U da aniqlangan  $\varphi$  operator uchun  $\varphi(\bar{x}_1 + \bar{x}_2) = \varphi(\bar{x}_1) + \varphi(\bar{x}_2)$  tenglik bajarilsa, u holda  $\varphi$  ga U da aniqlangan additiv operator deyiladi.

Agar  $\lambda$  ixtiyoriy son bo'lganda U fazoning ixtiyoriy  $\bar{x}$  elementi uchun  $\varphi(\lambda \bar{x}) = \lambda \varphi(\bar{x})$  tenglik o'rinli bo'lsa, u holda  $\varphi$  ga U da aniqlangan bir jinsli operator deyiladi.

Agar  $\forall \bar{x} \in U$  uchun  $\varphi(\bar{x}) = 0$  tenglik bajarilsa, u holda  $\varphi$  operatorga nol operator deyiladi.

Agar  $\forall \bar{x} \in U$  uchun  $e(\bar{x}) = \bar{x}$  tenglik bajarilsa, u holda e ga ayniy (birlik) operator deyiladi.

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Agar  $\forall \bar{x} \in U, \lambda \in P$  uchun  $\varphi(\bar{x}) = \lambda \bar{x}$  tenglik bajarilsa, u holda  $\varphi$  ga o'xshashlik operatori deyiladi.

Agar  $U_n$  fazoning ixtiyoriy  $\bar{x}$  vektori uchun  $f(\bar{x}) = \varphi(\bar{x}) + \Psi(\bar{x})$  tenglik bajarilsa u holda  $f$  ga  $\varphi$  va  $\Psi$  operatorlarning yig'indisi deyiladi va u  $\varphi + \Psi = f$  orqali yoziladi.

$\alpha \in F, \forall \bar{x} \in U_n$  uchun  $(\alpha\varphi)\bar{x} = \alpha\varphi(\bar{x})$  tenglik bajarilsa, u holda  $\alpha\varphi$  ga  $\alpha$  skalyarni  $\varphi$  operatorga ko'paytmasi deyiladi.

$U_n$  fazoning  $\varphi$  operator yordamida nolga akslanuvchi barcha elementlari to'plamiga  $\varphi$  operatorning yadrosi deyiladi va u  $\text{Ker } \varphi$  orqali belgilanadi.  $\varphi$  chiziqli operator yadrosining o'lchovi shu operatorning defekti deyiladi.

$\varphi U_n$  fazoostiga  $\varphi$  operatorning obrazi deyiladi.  $\varphi U_n$  obrazning o'lchoviga  $\varphi$  operatorning rangi deyiladi.

Agar  $\bar{x} = (x_1, x_2, \dots, x_n) \in U$  bo'lib,  $\varphi(\bar{x}) = \varphi(x_1, x_2, \dots, x_n) = (x_1, x_2, \dots, x_k)$  ( $1 \leq k < n$ ) bo'lsa, ya'ni  $\varphi$  operator  $n$  o'lchovli fazodagi vektorni  $k$  o'lchovli fazodagi vektorga o'tkazuvchi operator bo'lsa, u holda  $\varphi$  ga proektsiyalovchi operator deyiladi.

$\mathcal{F}$  maydon ustida  $V_n$  vektor fazo berilgan bo'lib,  $\bar{e}_1, \bar{e}_2, \dots, \bar{e}_n$  (1) uning bazisi bo'lsin. Agar  $\varphi$  operator  $V_n$  fazoda aniqlangan chiziqli operator bo'lsa, u holda  $\varphi(\bar{e}_1), \varphi(\bar{e}_2), \dots, \varphi(\bar{e}_n) \in V_n$  vektorlar (1) bazis orqali chiziqli ifodalanadi, ya'ni

$$\begin{cases} \varphi(\bar{e}_1) = \alpha_{11}\bar{e}_1 + \alpha_{21}\bar{e}_2 + \dots + \alpha_{n1}\bar{e}_n, \\ \varphi(\bar{e}_2) = \alpha_{12}\bar{e}_1 + \alpha_{22}\bar{e}_2 + \dots + \alpha_{n2}\bar{e}_n, \\ \dots \\ \varphi(\bar{e}_n) = \alpha_{1n}\bar{e}_1 + \alpha_{2n}\bar{e}_2 + \dots + \alpha_{nn}\bar{e}_n. \end{cases}$$

Ushbu

$$M(\varphi) = \begin{pmatrix} \alpha_{11} & \alpha_{12} & \dots & \alpha_{1n} \\ \alpha_{21} & \alpha_{22} & \dots & \alpha_{2n} \\ \dots & \dots & \dots & \dots \\ \alpha_{n1} & \alpha_{n2} & \dots & \alpha_{nn} \end{pmatrix}$$

matrisa  $\varphi$  chiziqli operatorning (1) bazisdagi matrisasi deyiladi.

$\bar{x}$  va  $\varphi(\bar{x})$  vektorlarning (1) bazis orqali  $x = \beta_1\bar{e}_1 + \dots + \beta_n\bar{e}_n$ ,  $\varphi(\bar{x}) = \gamma_1\bar{e}_1 + \dots + \gamma_n\bar{e}_n$  ko'rinishda ifodalansin.  $\bar{x}$  va  $\varphi(\bar{x})$  vektorlarning (1) bazisga nisbatan ustun koordinatalarini mos ravishda ushbu

$$M(\bar{x}) = \begin{bmatrix} \beta_1 \\ \beta_2 \\ \dots \\ \beta_n \end{bmatrix}, \quad M(\varphi(\bar{x})) = \begin{bmatrix} \gamma_1 \\ \gamma_2 \\ \dots \\ \gamma_n \end{bmatrix}$$

ko'rinishlarda belgilaymiz. U holda  $\forall \bar{x} \in V_n$  uchun  $M(\varphi(\bar{x})) = M(\varphi)M(\bar{x})$  tenglik bajariladi.

Agar  $\mathcal{F}$  maydon ustida  $A, B \in F^{m \times n}$  matrisalar uchun teskarilanuvchi  $T \in F^{m \times n}$  matrisa mavjud bo'lib, ular uchun  $B = T^{-1}AT$  tenglik o'rinli bo'lsa, u holda  $A$  va  $V$  matrisalar o'xshash matrisalar deyiladi.

$\mathcal{F}$  maydon ustida  $V_n$  vektor fazoning (1)dan boshqa  $\bar{e}'_1, \bar{e}'_2, \dots, \bar{e}'_n$  (2) bazisi berilgan bo'lsin. (2) bazisning vektorlarini (1) orqali chiziqli ifodalaymiz:

$$\begin{cases} \bar{e}'_1 = \beta_{11}\bar{e}_1 + \beta_{21}\bar{e}_2 + \dots + \beta_{n1}\bar{e}_n, \\ \bar{e}'_2 = \beta_{12}\bar{e}_1 + \beta_{22}\bar{e}_2 + \dots + \beta_{n2}\bar{e}_n, \\ \dots \\ \bar{e}'_n = \beta_{1n}\bar{e}_1 + \beta_{2n}\bar{e}_2 + \dots + \beta_{nn}\bar{e}_n. \end{cases}$$

U holda  $T = \begin{pmatrix} \beta_{11} & \beta_{12} & \dots & \beta_{1n} \\ \beta_{21} & \beta_{22} & \dots & \beta_{2n} \\ \dots & \dots & \dots & \dots \\ \beta_{n1} & \beta_{n2} & \dots & \beta_{nn} \end{pmatrix}$  matrisaga (2) bazisdan (1) bazisga o'tish

matrisasi deyiladi.

$\bar{x}$  vektorning birinchi va ikkinchi bazislardagi ustun koordinatalarini mos ravishda  $M(\bar{x})$  va  $M'(\bar{x})$  deb belgilasak, u holda  $\forall \bar{x} \in V_n$  vektor uchun  $M(\bar{x}) = TM'(\bar{x})$  va  $M'(\bar{x}) = T^{-1}M(\bar{x})$  tengliklar o'rinli bo'ladi.

$V_n$  fazoda aniqlangan  $\varphi$  chiziqli operator uchun  $M(\varphi)$  va  $M'(\varphi)$  lar  $\varphi$  chizikli operatorning birinchi va ikkinchi bazislarga nisbatan mos matrisalari bo'lsa, u holda  $M'(\varphi) = T^{-1}M(\varphi)T$  tenglik o'rinli bo'ladi.

**Misol.** Berilgan  $f(x) = (x_1 - x_2 + x_3; x_1; x_2)$  akslantirish chiziqli operator ekanligini isbotlang va uning rangi, defektini aniqlang.

**Yechish.** Chiziqli operator ta'rifiga ko'ra  $f$  akslantirish berilgan fazoni o'ziga akslantirishi va  $f(\bar{a} + \bar{b}) = f(\bar{a}) + f(\bar{b})$ ,  $f(\lambda\bar{a}) = \lambda f(\bar{a})$  shartlarga bo'ysunishi kerak.

$$1) \forall \bar{x}, \bar{y} \in R^3 \quad \bar{x} = (x_1, x_2, x_3), \quad \bar{y} = (y_1, y_2, y_3) \text{ lar uchun } f(\bar{x} + \bar{y}) = f((x_1 + y_1; x_2 + y_2; x_3 + y_3)) = (x_1 + y_1 - (x_2 + y_2) + x_3 + y_3; x_1 + y_1; x_2 + y_2) =$$

$$= (x_1 - x_2 + x_3; x_1; x_2) + (y_1 - y_2 + y_3; y_1; y_2) = f(\bar{x}) + f(\bar{y})$$

2)  $\forall \bar{x} \in R^3$  va  $\forall \alpha \in R$  lar uchun

$$f(\alpha\bar{x}) = f(\alpha(x_1, x_2, x_3)) = f((\alpha x_1, \alpha x_2, \alpha x_3)) = (\alpha x_1 - \alpha x_2 + \alpha x_3; \alpha x_1; \alpha x_2) = \alpha(x_1 - x_2 + x_3; x_1; x_2) = \alpha \cdot f(\bar{x})$$

Demak,  $f$  akslantirish chiziqli akslantirish va u  $R^3$  ni o'ziga akslantirganligi uchun  $f$  - chiziqli operator.

3) *defect*  $f$  ni topish uchun  $\text{Ker } f$  ni aniqlaymiz.  $\text{Ker } f = \{\bar{x} \mid f(\bar{x}) = \bar{0}\}$  ta'rifdan  $f(\bar{x}) = \bar{0}$  shartni qanoatlantiruvchi vektorlarni topamiz:

$$f(\bar{x}) = (x_1 - x_2 + x_3; x_1; x_2) = \bar{0}. \text{ Bundan}$$

$$\begin{cases} x_1 - x_2 + x_3 = 0 \\ x_1 = 0 \\ x_2 = 0 \end{cases} \Leftrightarrow \begin{cases} x_1 = 0 \\ x_2 = 0 \\ x_3 = 0 \end{cases}. \text{ Demak, } \text{Ker } f = \{\bar{0}\}.$$

Nol chiziqli fazoning o'lchovi 0 ga teng. Bundan *defect*  $f = 0$

$\dim V = \text{defect } f + \text{rang } f$  tenglikdan  $\text{rang } f = \dim V - \text{defect } f$  ni, bundan,  $\text{rang } f = 3 - 0 = 3$  ni hosil qilamiz.

Demak, 1)  $f$  - chiziqli operator;

$$2) \text{ defect } f = 0$$

$$3) \text{ rang } f = 3$$

**Misol.** Agar  $M - \bar{e}_1, \bar{e}_2, \bar{e}_3$  bazisdagi.  $M' - \bar{e}_1, \bar{e}_2, \bar{e}_3$  bazisdagi  $f(\bar{x}) = (x_1, x_2, x_3)$  operator matrisalari bo'lsa, u holda

$$a) M(f(\bar{x})) = M(f)M(\bar{x})$$

$$b) M'(\bar{x}) = T^{-1}M(\bar{x}) \wedge M(\bar{x}) = TM'(\bar{x})$$

$$v) M'(f) = T^{-1}M(f)T \wedge M(f) = TM'(f)T^{-1} \text{ shartlar bajarilishini tekshiring.}$$

$$\bar{x} = (1, 3, 1); \quad \bar{e}_1 = (1, 0, 2), \quad \bar{e}_2 = (2, 1, 1), \quad \bar{e}_3 = (1, 3, 0)$$

**Yechish.** 1) Berilgan  $f(\bar{x}) = (x_1, x_2, x_3)$  operatorning  $\bar{e}_1, \bar{e}_2, \bar{e}_3$  bazisdagi matrisasi  $M(f)$  ni topamiz. Buning uchun  $f(\bar{e}_1), f(\bar{e}_2), f(\bar{e}_3)$  vektorlarning  $\bar{e}_1, \bar{e}_2, \bar{e}_3$  bazisdagi chiziqli kombinasiyalarini aniqlaymiz:

$$f(\bar{e}_1) = f((1, 0, 0)) = (1, 0, 0) = 1 \cdot \bar{e}_1 + 0 \cdot \bar{e}_2 + 0 \cdot \bar{e}_3$$

$$f(\bar{e}_2) = f((0, 1, 0)) = (0, 0, 1) = 0 \cdot \bar{e}_1 + 0 \cdot \bar{e}_2 + 1 \cdot \bar{e}_3$$

$$f(\bar{e}_3) = f((0, 0, 1)) = (0, 1, 0) = 0 \cdot \bar{e}_1 + 1 \cdot \bar{e}_2 + 0 \cdot \bar{e}_3$$

Chiziqli kombinasiyalar koeffitsientlaridan matrisa hosil qilamiz. Bunda  $f(\bar{e}_i)$  ( $i = 1, \bar{3}$ ) ning chiziqli kombinasiyasida qatnashgan koeffitsientlar ustun qilib yoziladi:

$$M(f) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}.$$

2)  $M(f)$  matrisa yordamida  $f(\bar{x})$  vektorning ustun koordinatalarini topamiz:

$$M(f(\bar{x})) = M(f)M(\bar{x}) \text{ dan}$$

$$M(f(\bar{x})) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 3 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 3 \end{pmatrix} \text{ ni hosil qilamiz.}$$

Olingan natijani tekshirish uchun operator talabini  $\bar{x} = (1,3,1)$  vektorga qo'llaymiz  $f(\bar{x}) = f((1,3,1)) = (1,1,3)$  kelib chiqadi.

3) Berilgan birinchi bazisdan ikkinchi bazisga o'tish matrisasini topamiz:

$$f(\bar{e}_1) = \alpha_1 \bar{e}_1 + \alpha_2 \bar{e}_2 + \alpha_3 \bar{e}_3$$

$$f(\bar{e}_2) = \beta_1 \bar{e}_1 + \beta_2 \bar{e}_2 + \beta_3 \bar{e}_3$$

$$f(\bar{e}_3) = \gamma_1 \bar{e}_1 + \gamma_2 \bar{e}_2 + \gamma_3 \bar{e}_3$$

Bundan  $f((1,0,2)) = (1,2,0) = 1 \cdot \bar{e}_1 + 2 \cdot \bar{e}_2 + 0 \cdot \bar{e}_3$

$$f((2,1,1)) = (2,1,1) = 2 \cdot \bar{e}_1 + 1 \cdot \bar{e}_2 + 1 \cdot \bar{e}_3$$

$$f((1,3,0)) = (1,0,3) = 1 \cdot \bar{e}_1 + 0 \cdot \bar{e}_2 + 3 \cdot \bar{e}_3 \text{ kelib chiqadi.}$$

Birinchi bazisdan ikkinchi bazisga o'tish matrisasi  $T = \begin{pmatrix} 1 & 2 & 1 \\ 2 & 1 & 0 \\ 0 & 1 & 3 \end{pmatrix}$  dan iborat bo'ladi. Uning teskarisini topamiz:

$$\begin{pmatrix} 1 & 2 & 1 & | & 1 & 0 & 0 \\ 2 & 1 & 0 & | & 0 & 1 & 0 \\ 0 & 1 & 3 & | & 0 & 0 & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & 2 & 1 & | & 1 & 0 & 0 \\ 0 & -3 & -2 & | & -2 & 1 & 0 \\ 0 & 1 & 3 & | & 0 & 0 & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & 2 & 1 & | & 1 & 0 & 0 \\ 0 & 1 & 3 & | & 0 & 0 & 1 \\ 0 & 0 & 7 & | & -2 & 1 & 3 \end{pmatrix} \sim \begin{pmatrix} 1 & 2 & 0 & | & -\frac{1}{7} & \frac{1}{7} & \frac{1}{7} \\ 0 & 1 & 0 & | & \frac{2}{7} & -\frac{3}{7} & -\frac{2}{7} \\ 0 & 0 & 1 & | & -\frac{2}{7} & \frac{1}{7} & \frac{3}{7} \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & 0 & | & -\frac{1}{7} & \frac{1}{7} & \frac{1}{7} \\ 0 & 1 & 0 & | & \frac{2}{7} & -\frac{3}{7} & -\frac{2}{7} \\ 0 & 0 & 1 & | & -\frac{2}{7} & \frac{1}{7} & \frac{3}{7} \end{pmatrix}.$$

Demak,  $T^{-1} = \frac{1}{7} \begin{pmatrix} -3 & 5 & 1 \\ 6 & -3 & -2 \\ -2 & 1 & 3 \end{pmatrix}$ .

U holda  $M'(\bar{x}) = T^{-1}M(\bar{x}) = \frac{1}{7} \begin{pmatrix} -3 & 5 & 1 \\ 6 & -3 & -2 \\ -2 & 1 & 3 \end{pmatrix} \begin{pmatrix} 1 \\ 3 \\ 1 \end{pmatrix} = \frac{1}{7} \begin{pmatrix} 13 \\ -5 \\ 4 \end{pmatrix} = \begin{pmatrix} \frac{13}{7} \\ -\frac{5}{7} \\ \frac{4}{7} \end{pmatrix}$ .

Tekshirish maqsadida  $M(\bar{x}) = TM'(\bar{x})$  tenglikni tuzamiz:

$$\begin{pmatrix} 1 \\ 3 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 1 \\ 2 & 1 & 0 \\ 0 & 1 & 3 \end{pmatrix} \begin{pmatrix} \frac{13}{7} \\ -\frac{5}{7} \\ \frac{4}{7} \end{pmatrix} \Rightarrow \begin{pmatrix} 1 \\ 3 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \\ 1 \end{pmatrix}.$$

Demak,  $T, T^{-1}$  to'g'ri topilgan.

4)  $f$  operatorning 1- va 2-bazisdagi matrisalari orasidagi bog'lanishni o'rnatamiz:

$$M'(f) = T^{-1}M(f)T = \frac{1}{7} \begin{pmatrix} -3 & 5 & 1 \\ 6 & -3 & -2 \\ -2 & 1 & 3 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 2 & 1 \\ 2 & 1 & 0 \\ 0 & 1 & 3 \end{pmatrix} = \frac{1}{7} \begin{pmatrix} -3 & 5 & 1 \\ 6 & -2 & -3 \\ -2 & 1 & 3 \end{pmatrix} \begin{pmatrix} 1 & 2 & 1 \\ 2 & 1 & 0 \\ 0 & 1 & 3 \end{pmatrix} = \frac{1}{7} \begin{pmatrix} -1 & 0 & 12 \\ 2 & 7 & -3 \\ 4 & 0 & 1 \end{pmatrix};$$

Tekshirish uchun  $M(f) = TM'(f)T^{-1}$  tenglikka topilgan qiymatlarni qo'yamiz:

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 1 \\ 2 & 1 & 0 \\ 0 & 1 & 3 \end{pmatrix} \cdot \frac{1}{7} \begin{pmatrix} -1 & 0 & 12 \\ 2 & 7 & -3 \\ 4 & 0 & 1 \end{pmatrix} \cdot \frac{1}{7} \begin{pmatrix} -3 & 5 & 1 \\ 6 & -3 & -2 \\ -2 & 1 & 3 \end{pmatrix} \\ \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} = \frac{1}{49} \begin{pmatrix} 7 & 14 & 7 \\ 0 & 7 & 21 \\ 14 & 7 & 0 \end{pmatrix} \begin{pmatrix} -3 & 5 & 1 \\ 6 & -3 & -2 \\ -2 & 1 & 3 \end{pmatrix} = \frac{1}{7} \cdot \begin{pmatrix} 1 & 2 & 1 \\ 0 & 1 & 3 \\ 2 & 1 & 0 \end{pmatrix} \begin{pmatrix} -3 & 5 & 1 \\ 6 & -3 & -2 \\ -2 & 1 & 3 \end{pmatrix} = \frac{1}{7} \begin{pmatrix} 7 & 0 & 0 \\ 0 & 0 & 7 \\ 0 & 7 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}.$$

Demak, berilgan misol to'g'ri yechilgan.

### ⇒ Misol va mashqlar

1.  $\mathcal{F}$  sonlar maydoni ustida aniqlangan  $U$  vektor fazoda aniqlangan additiv operatorning quyidagi xossalari isbotlang:

1.1.  $\varphi(\vec{0}) = \vec{0}$ .

$$1.2. \varphi(\bar{x}) = -\varphi(\bar{x}) \quad (\forall \bar{x} \in U).$$

$$1.3. \varphi(r\bar{x}) = r\varphi\bar{x} \quad (\forall r \in Q).$$

$$1.4. \varphi(\bar{x}_1 - \bar{x}_2) = \varphi(\bar{x}_1) - \varphi(\bar{x}_2) \quad (\forall \bar{x}_1, \bar{x}_2 \in U).$$

2.  $\varphi$  operator chiziqli operator bo'lishi uchun  $U$  fazoning ixtiriy  $\bar{x}_1$  va  $\bar{x}_2$  elementlari va  $\lambda_1, \lambda_2 \in F$  berilganda  $\varphi(\lambda_1\bar{x}_1 + \lambda_2\bar{x}_2) = \lambda_1\varphi(\bar{x}_1) + \lambda_2\varphi(\bar{x}_2)$  tenglikning bajarilishi zarur va etarli ekanligini isbotlang.

3. Agar  $\varphi$  chiziqli operator bo'lsa, u holda  $\forall x_i \in U, \lambda_i \in P (i = \overline{1, n})$  uchun Ushbu  $\varphi(\lambda_1\bar{x}_1 + \lambda_2\bar{x}_2 + \dots + \lambda_n\bar{x}_n) = \lambda_1\varphi(\bar{x}_1) + \lambda_2\varphi(\bar{x}_2) + \dots + \lambda_n\varphi(\bar{x}_n)$  tenglik o'rinli bo'lishini isbotlang.

4. Nol operator ham chiziqli operator bo'lishini isbotlang.

5.  $\bar{a}_1, \bar{a}_2, \bar{a}_3$  vektorlarni  $\bar{b}_1, \bar{b}_2, \bar{b}_3$  vektorlarga o'tkazuvchi yagona chiziqli akslantirish mavjudligini isbotlang va uning matrisasini toping:

$$5.1. \bar{a}_1 = (2, 3, 5), \bar{a}_2 = (0, 1, 2), \bar{a}_3 = (1, 0, 0);$$

$$\bar{b}_1 = (1, 1, 1), \bar{b}_2 = (1, 1, -1), \bar{b}_3 = (2, 1, 2).$$

$$5.2. \bar{a}_1 = (2, 0, 3), \bar{a}_2 = (4, 1, 5), \bar{a}_3 = (3, 1, 2);$$

$$\bar{b}_1 = (1, 2, -1), \bar{b}_2 = (4, 5, -2), \bar{b}_3 = (1, -1, 1)$$

6. Berilgan akslantirishlar chiziqli operator bo'lishini tekshiring:

$$6.1. f(x) = (x_2 + x_3; 2x_1 + x_3; 3x_1 - x_2 + x_3).$$

$$6.2. f(x) = (x_1 + x_2; 4x_3; x_1 + x_3 + 3).$$

$$6.3. f(x) = (x_1 - x_2; x_2 + x_3; x_3).$$

$$6.4. f(x) = (x_1; x_2 + 2x_3; -x_3).$$

$$6.5. f(x) = (-3(x_1 + x_2); x_2 + x_3; x_1).$$

$$6.6. f(x) = (0; 3(x_2 + x_3); x_1).$$

$$6.7. f(x) = (x_1 - x_2; 3x_2 - x_3; 0).$$

$$6.8. f(x) = (x_2; x_3; 2).$$

$$6.9. f(x) = (x_2; x_3; x_1).$$

$$6.10. f(x) = (-x_2; x_2 + x_3; x_3).$$

$$6.11. f(x) = (x_2 + x_3; 0; x_1 - 2x_2 + x_3).$$

$$6.12. f(x) = (0; x_1; -2x_2 + x_3).$$

$$6.13. f(x) = (x_1 - x_2 + x_3; x_3; x_1).$$

$$6.14. f(x) = (1 - x_2 + x_3; x_3; x_1 - 2x_2).$$

7. 6-misoldagi chiziqli operatorlarning rangi defektini aniqlang.

8. Nol bo'lmagan chekli o'lchovli vektor fazodagi  $\varphi$  chiziqli operatorning rangi  $\varphi$  chiziqli operator matrisasining rangiga teng bo'lishini isbotlang.

9. Berilgan chiziqli operatorlarning rangi  $r$  va defekti  $d$  ni toping:

$$9.1. \begin{pmatrix} 1 & 2 \\ 1 & 2 \end{pmatrix}.$$

$$9.2. \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}.$$

$$9.3. \begin{pmatrix} 1 & 2 & 3 \\ -2 & -4 & -6 \\ 4 & 8 & 12 \end{pmatrix}.$$

$$9.4. \begin{pmatrix} 1 & 2 & 3 \\ -2 & -1 & -1 \\ 0 & 3 & -1 \end{pmatrix}.$$

$$9.5. \begin{pmatrix} 1 & 0 & 2 \\ 3 & 1 & 1 \\ -1 & 0 & 3 \end{pmatrix}.$$

$$9.6. \begin{pmatrix} 1 & 3 & 1 & 2 \\ -1 & -3 & -1 & -1 \\ 1 & 3 & 1 & 2 \\ 2 & 6 & 2 & 4 \end{pmatrix}.$$

$$9.7. \begin{pmatrix} 1 & -2 & 1 & 2 \\ 2 & 1 & 0 & 1 \\ 3 & -1 & 1 & 3 \\ 4 & -3 & 2 & 5 \end{pmatrix}.$$

$$9.8. \begin{pmatrix} 1 & 1 & 0 & 1 \\ -1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 3 \\ 1 & 2 & 1 & 3 \end{pmatrix}.$$

10.  $\varphi$  chiziqli operatorlar yadrosi shu operator qaralayotgan fazoning fazoosti bo'lishini isbotlang.

11. 6-topshiriqdagi chiziqli operatorlar uchun quyidagi shartlar bajarilishini tekshiring:

$$a) M(f(\bar{x})) = M(f) \cdot M(\bar{x});$$

$$b) M'(\bar{x}) = T^{-1} \cdot M(\bar{x}) \wedge M(x) = T \cdot M'(\bar{x});$$

$$c) M'(f) = T^{-1} \cdot M(f) \cdot T \wedge M(f) = T \cdot M'(f) \cdot T^{-1}$$

$$11.1. \bar{x} = (1, 2, 3); \bar{e}'_1(2, 1, 0), \bar{e}'_2(0, 3, 1), \bar{e}'_3(-1, 0, 2).$$

$$11.2. \bar{x} = (6, -5, 13); \bar{e}'_1(3, 1, -2), \bar{e}'_2(1, 3, 1), \bar{e}'_3(1, 5, 0).$$

- 11.3.  $\bar{x} = (1, 2, 3)$ ;  $\bar{e}_1(1, 0, 3)$ ,  $\bar{e}_2(1, 1, -2)$ ,  $\bar{e}_3(2, -1, 2)$ .  
 11.4.  $\bar{x} = (-8, 5, 2)$ ;  $\bar{e}_1(1, 1, 1)$ ,  $\bar{e}_2(1, 2, 3)$ ,  $\bar{e}_3(1, 3, 3)$ .  
 11.5.  $\bar{x} = (7, -2, -4, 3)$ ;  $\bar{e}_1(1, 2, 3, -1)$ ,  $\bar{e}_2(2, 3, 4, 1)$ ,  $\bar{e}_3(1, 4, 3, 3)$ .  
 11.6.  $\bar{x} = (3, -5, 7)$ ;  $\bar{e}_1(-1, 1, 1)$ ,  $\bar{e}_2(-1, 2, 3)$ ,  $\bar{e}_3(-1, 3, 3)$ .  
 11.7.  $\bar{x} = (3, 1, 9)$ ;  $\bar{e}_1(1, 2, 3)$ ,  $\bar{e}_2(2, 3, 4)$ ,  $\bar{e}_3(1, 4, 3)$ .  
 11.8.  $\bar{x} = (6, -4, 5)$ ;  $\bar{e}_1(2, 2, 3)$ ,  $\bar{e}_2(1, -1, 0)$ ,  $\bar{e}_3(-1, 2, 1)$ .  
 11.9.  $\bar{x} = (5, 0, 1)$ ;  $\bar{e}_1(1, 2, 3)$ ,  $\bar{e}_2(-1, 2, 0)$ ,  $\bar{e}_3(-1, 2, 1)$ .  
 11.10.  $\bar{x} = (-4, 5, -2)$ ;  $\bar{e}_1(2, -1, 0)$ ,  $\bar{e}_2(0, -3, -1)$ ,  $\bar{e}_3(1, 0, -2)$ .  
 11.11.  $\bar{x} = (3, 3, -2)$ ;  $\bar{e}_1(1, 1, 0)$ ,  $\bar{e}_2(0, 1, 1)$ ,  $\bar{e}_3(1, 0, 1)$ .  
 11.12.  $\bar{x} = (5, 6, 7)$ ;  $\bar{e}_1(2, 3, 0)$ ,  $\bar{e}_2(0, 2, 3)$ ,  $\bar{e}_3(2, 0, 3)$ .  
 11.13.  $\bar{x} = (-1, -2, -3)$ ;  $\bar{e}_1(4, 0, 4)$ ,  $\bar{e}_2(4, 4, 0)$ ,  $\bar{e}_3(0, 4, 4)$ .



### Takrorlash uchun savollar

1. Chiziqli akslantirish, chiziqchli operator deb nimaga aytiladi?
2. Additiv, bir jinsli operatorlarga misol keltiring.
3. Nol, birlik operatorlar deb nimaga aytiladi?
4. O'xshashlik, proektsiyalovchi operatorlar ta'rifini ayting.
5. Chiziqli operatorning yadrosi haqida tushuncha bering.
6. Chiziqli operatorning obrazini misol yordamida tushuntiring.
7. Chiziqli operator matrisasi qanday topiladi?
8. Chiziqli operator rangi deb nimaga aytiladi?
9. Chiziqli operatorning turli bazislarga nisbatan matrisalari orasidagi bog'lanish formulasini tushuntiring.
10. O'xshash matrisalar deb nimaga aytiladi?
11.  $\bar{x}$  va  $\varphi(\bar{x})$  vektorlar ustun koordinatalari orasidagi bog'lanish qanday o'rnatiladi?



### 22-§. Chiziqli operatorlar ustida amallar. Chiziqli algebra. Teskari operator. Xos vektorlar va xos qiymatlar.

**Asosiy tushunchalar:** chiziqli algebra, rangi, chiziqli algebra ustida amallar, operatorlar chiziqli algebra, chiziqli operator xos qiymati, xos vektorlari.

$\mathcal{F}$  maydon ustidagi  $V$  chiziqli fazo elementlari uchun quyidagi shartlar bajarilsa,

1.  $\overline{xy} \in V (\forall \bar{x}, \bar{y} \in V)$ ;
2.  $\overline{x(\bar{y}\bar{z})} = (\overline{xy})\bar{z} (\forall \bar{x}, \bar{y}, \bar{z} \in V)$ ;
3.  $\overline{x(\bar{y} + \bar{z})} = \overline{xy} + \overline{xz}$  ba  $(\bar{y} + \bar{z})\bar{x} = \bar{y}\bar{x} + \bar{z}\bar{x} (\forall \bar{x}, \bar{y}, \bar{z} \in V)$
4.  $\lambda(\overline{xy}) = (\lambda\bar{x})\bar{y} = \bar{x}(\lambda\bar{y}) (\lambda \in F, \forall \bar{x}, \bar{y} \in V)$

u holda  $V$  fazoni  $\mathcal{F}$  maydon ustidagi chiziqli algebra deyiladi.

Agar  $V$  chiziqli algebra  $\bar{x} \bullet \bar{y} = \bar{y} \bullet \bar{x} (\forall \bar{x}, \bar{y} \in V)$  shart bajarilsa,  $V$  kommutativ chiziqli algebra deyiladi.

$V$  chiziqli algebra rangi deb  $V$  fazoning o'lchoviga aytiladi.

$V$  fazo  $\mathcal{F}$  maydon ustidagi vektor fazo bo'lib,  $\varphi, \psi$  lar shu vektor fazoning chiziqli operatorlari bo'lsin, u holda :

- 1)  $(\varphi + \psi)(\bar{x}) = \varphi(\bar{x}) + \psi(\bar{x})$ .
- 2)  $(\lambda\varphi)(\bar{x}) = \lambda\varphi(\bar{x})$ .
- 3)  $(\varphi\psi)(\bar{x}) = \varphi(\psi(\bar{x}))$ .

$\text{Hom}(V, V)$  to'plam  $\mathcal{F}$  maydon ustida vektor fazo tashkil qiladi.

$\langle \text{Hom}(V, V), +, \{\omega_i | \lambda \in F\}, \bullet \rangle$  algebra  $V$  vektor fazoning chiziqli operatorlar algebra deyiladi va quyidagicha belgilanadi:

$$\text{End } V = \langle \text{Hom}(V, V), +, \{\omega_i | \lambda \in F\}, \bullet \rangle.$$

$U$  va  $U'$  algebra  $\mathcal{F}$  maydon ustidagi chiziqli algebra va  $\varphi: U \rightarrow U'$  akslantirish biekktiv akslantirish bo'lib, quyidagi shartlar bajarilsa:

$$\varphi(\bar{a} + \bar{b}) = \varphi(\bar{a}) + \varphi(\bar{b});$$

$$\varphi(\lambda \bar{a}) = \lambda \varphi(\bar{a});$$

$$\varphi(\bar{a} \cdot \bar{b}) = \varphi(\bar{a}) \cdot \varphi(\bar{b}), \forall \bar{a}, \bar{b} \in V \wedge \forall \lambda \in F$$

u holda  $\varphi$  akslantirishga U va U' chizikli algebra izomorfizmi deyiladi.

Kompleks sonlar maydoni ustida qurilgan  $V_n$  vektor fazo va  $\varphi: V_n \rightarrow V_n$  chizikli operator berilgan bo'lsin. Ushbu  $\varphi(\bar{x}) = \lambda \bar{x} (\forall \bar{x} \in V_n, x \neq \bar{0}, \lambda \in F)$  tenglikni qanoatlantiruvchi  $\alpha$  songa  $\varphi$  chizikli operatorning xos qiymati,  $\bar{x}$  vektor esa  $\lambda$  xos qiymatga mos keluvchi xos vektor deyiladi.

$V_n$  vektor fazoning  $\bar{e}_1, \bar{e}_2, \dots, \bar{e}_n$  bazisida  $\varphi$  chizikli operator

$$A = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{pmatrix} \text{ matrisa yordamida berilgan bo'lsa, } |A - \lambda E| = 0 \text{ tenglamaga } \varphi$$

chizikli operatorning xarakteristik tenglamasi deyiladi.

**Misol.** Berilgan  $A = \begin{pmatrix} 1 & 2 & 1 & 0 \\ 2 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 \end{pmatrix}$  operatorga teskari operatorni toping.

**Yechish:** Berilgan A operatorga teskari operatorni topish uchun A matrisaga teskari matrisa topiladi.

$$\begin{pmatrix} 1 & 2 & 1 & 0 & 1 & 0 & 0 & 0 \\ 2 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & 2 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & -4 & -1 & 0 & -2 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & -1 & -1 & 0 & -1 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 2 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & -4 & -1 & 0 & -2 & 1 & 0 & 0 \\ 0 & -1 & -1 & 0 & -1 & 0 & 0 & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & 2 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 4 & -2 & 1 & 4 & 0 \\ 0 & 0 & -1 & 1 & -1 & 0 & 1 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 2 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & -4 & 2 & -1 & -4 & 0 \\ 0 & 0 & 0 & -3 & 1 & -1 & -3 & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & 2 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & -4 & 2 & -1 & -4 & 0 \\ 0 & 0 & 0 & 1 & -\frac{1}{3} & \frac{1}{3} & 1 & -\frac{1}{3} \end{pmatrix}$$

$$\sim \begin{pmatrix} 1 & 2 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & \frac{2}{3} & \frac{1}{3} & 0 & -\frac{4}{3} \\ 0 & 0 & 1 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & -\frac{4}{3} \\ 0 & 0 & 0 & 1 & -\frac{1}{3} & \frac{1}{3} & 1 & -\frac{1}{3} \end{pmatrix} \sim \begin{pmatrix} 1 & 2 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & \frac{1}{3} & -\frac{1}{3} & 0 & \frac{1}{3} \\ 0 & 0 & 1 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & -\frac{4}{3} \\ 0 & 0 & 0 & 1 & -\frac{1}{3} & \frac{1}{3} & 1 & -\frac{1}{3} \end{pmatrix}$$

$$\sim \begin{pmatrix} 1 & 2 & 0 & 0 & \frac{1}{3} & -\frac{1}{3} & 0 & \frac{4}{3} \\ 0 & 1 & 0 & 0 & \frac{1}{3} & -\frac{1}{3} & 0 & \frac{1}{3} \\ 0 & 0 & 1 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & -\frac{4}{3} \\ 0 & 0 & 0 & 1 & -\frac{1}{3} & \frac{1}{3} & 1 & -\frac{1}{3} \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & 0 & 0 & -\frac{1}{3} & -\frac{1}{3} & 0 & \frac{2}{3} \\ 0 & 1 & 0 & 0 & \frac{1}{3} & -\frac{1}{3} & 0 & \frac{1}{3} \\ 0 & 0 & 1 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & -\frac{4}{3} \\ 0 & 0 & 0 & 1 & -\frac{1}{3} & \frac{1}{3} & 1 & -\frac{1}{3} \end{pmatrix}$$

$$A^{-1} = \frac{1}{3} \begin{pmatrix} -1 & 1 & 0 & 2 \\ 1 & -1 & 0 & 1 \\ 2 & 1 & 0 & -4 \\ -1 & 1 & 3 & -1 \end{pmatrix}$$

Tekshirish:  $\frac{1}{3} \begin{pmatrix} 1 & 2 & 1 & 0 \\ 2 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} -1 & 1 & 0 & 2 \\ 1 & -1 & 0 & 1 \\ 2 & 1 & 0 & -4 \\ -1 & 1 & 3 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

**Misol.** f chizikli operator (a) bazisda A matrisa orqali,  $\varphi$  chizikli operator (b) bazisda B matrisa orqali berilgan bo'lsa,  $4f + 2\varphi$  operatorning (e) bazisdagi matrisasini toping:

$$(a): \bar{a}_1 = (1,1), \bar{a}_2 = (2,1); A = \begin{pmatrix} 1 & 2 \\ 3 & 1 \end{pmatrix};$$

$$(b): \bar{b}_1 = (2,3), \bar{b}_2 = (3,2); B = \begin{pmatrix} 0 & 1 \\ 2 & 3 \end{pmatrix};$$

*Yechish:* (a) bazisdan (e) bazisga o'tish matricasini topamiz:

$$\bar{a}_1 = (1,1) = 1 \cdot \bar{e}_1 + 1 \cdot \bar{e}_2; \quad \bar{a}_2 = (2,1) = 2 \cdot \bar{e}_1 + 1 \cdot \bar{e}_2; \quad \text{va} \quad T = \begin{pmatrix} 1 & 2 \\ 1 & 1 \end{pmatrix}$$

O'tish matricasiga teskari matricani topamiz:

$$\begin{pmatrix} 1 & 2 & | & 1 & 0 \\ 1 & 1 & | & 0 & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & 2 & | & 1 & 0 \\ 0 & -1 & | & -1 & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & | & -1 & 2 \\ 0 & 1 & | & 1 & -1 \end{pmatrix} \text{ bundan } T^{-1} = \begin{pmatrix} -1 & 2 \\ 1 & -1 \end{pmatrix}.$$

3-misoldagi  $M'(f) = T^{-1}M(f)T$  bog'lanishdan foydalanamiz:

$$M'(f) = \begin{pmatrix} -1 & 2 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 5 & 0 \\ -2 & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 5 & 2 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} 1 & 10 \\ -1 & -3 \end{pmatrix}$$

Demak,  $f$  operatorning (e) bazisdagi matrisasi  $\begin{pmatrix} 5 & 10 \\ -1 & -3 \end{pmatrix}$ .

Endi (b) bazisdan (e) bazisga o'tish matricasini topamiz:

$$\bar{b}_1 = (2,3) = 2 \cdot \bar{e}_1 + 3 \cdot \bar{e}_2; \quad \bar{b}_2 = (3,2) = 3 \cdot \bar{e}_1 + 2 \cdot \bar{e}_2$$

O'tish matrisasi  $T = \begin{pmatrix} 2 & 3 \\ 3 & 2 \end{pmatrix}$ . Uning teskarisini topamiz:

$$\begin{pmatrix} 2 & 3 & | & 1 & 0 \\ 3 & 2 & | & 0 & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & 3/2 & | & 1/2 & 0 \\ 0 & -5/2 & | & -3/2 & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & | & -2/5 & 3/5 \\ 0 & 1 & | & 3/5 & -2/5 \end{pmatrix} \text{ Bundan,}$$

$$T^{-1} = \frac{1}{5} \begin{pmatrix} -2 & 3 \\ 3 & -2 \end{pmatrix}.$$

Endi  $\varphi$  operatorning (e) bazisdagi matricasini topamiz:

$$M'(\varphi) = \frac{1}{5} \begin{pmatrix} -2 & 3 \\ 3 & -2 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 2 & 3 \end{pmatrix} \begin{pmatrix} 2 & 3 \\ 3 & 2 \end{pmatrix} = \frac{1}{5} \begin{pmatrix} 6 & 7 \\ -2 & -3 \end{pmatrix} \begin{pmatrix} 2 & 3 \\ 3 & 2 \end{pmatrix} = \frac{1}{5} \begin{pmatrix} 33 & 32 \\ -13 & -12 \end{pmatrix}$$

Demak, (e) bazisda  $f$  operator matrisasi  $\begin{pmatrix} 5 & 10 \\ -1 & -3 \end{pmatrix}$ ;  $\varphi$  operator matrisasi  $\frac{1}{5} \begin{pmatrix} 33 & 32 \\ -13 & -12 \end{pmatrix}$  ga teng. U holda

$$4f + 2\varphi = 4 \begin{pmatrix} 5 & 10 \\ -1 & -3 \end{pmatrix} + \frac{2}{5} \begin{pmatrix} 33 & 32 \\ -13 & -12 \end{pmatrix} = \begin{pmatrix} 20 & 40 \\ -4 & -12 \end{pmatrix} + \begin{pmatrix} \frac{66}{5} & \frac{64}{5} \\ -\frac{26}{5} & -\frac{24}{5} \end{pmatrix} =$$

$$= \begin{pmatrix} \frac{166}{5} & \frac{264}{5} \\ -\frac{46}{5} & -\frac{84}{5} \end{pmatrix}.$$

*Misol.*  $\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 3 & 1 & 0 \end{pmatrix}$  operatorning xos vektorlari va xos qiymatlarini toping.

*Yechish.* Berilgan operatorning xos qiymatlarini topish uchun  $|\lambda E - A| = 0$  tenglikdan foydalanamiz.

$$\begin{vmatrix} \lambda - 1 & 0 & -1 \\ 0 & \lambda - 1 & -1 \\ -3 & -1 & \lambda \end{vmatrix} = \lambda(\lambda - 1)^2 - 3(\lambda - 1) =$$

$$(\lambda - 1)(\lambda(\lambda - 1) - 4) = (\lambda - 1)(\lambda^2 - \lambda - 4) = 0$$

Bundan  $\lambda_1 = 1$ ,  $\lambda_{2/3} = \frac{1 \pm \sqrt{17}}{2}$  xos qiymatlarni topamiz.

Endi, berilgan operatorning xos vektorlarini topish uchun  $(A - \lambda E)X = 0$  tenglamadan foydalanamiz. Bu tengliklarning noldan farqli yechimlari berilgan operatorning xos vektorlari bo'ladi.

$\lambda_1 = 1$  uchun xos vektorlarni aniqlaymiz:

$$(A - E)\vec{0} = 0 \Leftrightarrow \left( \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 3 & 1 & 0 \end{pmatrix} - \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \right) \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\Leftrightarrow \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 3 & 1 & -1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \Leftrightarrow \begin{cases} x_3 = 0 \\ x_3 = 0 \\ 3x_1 + x_2 - x_3 = 0 \end{cases} \Leftrightarrow \begin{cases} x_1 \in \mathbb{R} \\ x_2 = -3x_1 \\ x_3 = 0 \end{cases}$$

Demak,  $\lambda_1 = 1$  xos qiymat uchun berilgan operatorning xos vektorlari

$\{(x_1, -3x_1, 0) \mid x_1 \in R \wedge x_1 \neq 0\}$  to'plamdan iborat.

$\lambda_2 = \frac{1-\sqrt{17}}{2}$  uchun xos vektorlarni topamiz:

$$(A - \lambda E)X = 0 \Leftrightarrow \left( A - \frac{1-\sqrt{17}}{2} E \right) X = 0 \Leftrightarrow$$

$$\left( \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 3 & 1 & 0 \end{pmatrix} - \begin{pmatrix} 1-\frac{\sqrt{17}}{2} & 0 & 0 \\ 0 & 1-\frac{\sqrt{17}}{2} & 0 \\ 0 & 0 & 1-\frac{\sqrt{17}}{2} \end{pmatrix} \right) \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \Leftrightarrow$$

$$\Leftrightarrow \begin{pmatrix} 1-\frac{1-\sqrt{17}}{2} & 0 & 1 \\ 0 & 1-\frac{1-\sqrt{17}}{2} & 1 \\ 3 & 1 & -\frac{1-\sqrt{17}}{2} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \Leftrightarrow$$

$$\begin{cases} \left(1-\frac{1-\sqrt{17}}{2}\right)x_1 + x_3 = 0 \\ \left(1-\frac{1-\sqrt{17}}{2}\right)x_2 + x_3 = 0 \\ 3x_1 + x_2 - \frac{1-\sqrt{17}}{2}x_3 = 0 \end{cases} \Leftrightarrow \begin{cases} \left(1-\frac{1-\sqrt{17}}{2}\right)x_1 + x_3 = 0 \\ \left(1-\frac{1-\sqrt{17}}{2}\right)x_2 + x_3 = 0 \\ \left(\frac{1-\sqrt{17}}{2}\right)x_2 + x_3 = 0 \end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} x_1 = -\frac{2}{1+\sqrt{17}}x_3 = 0 \\ x_2 = -\frac{2}{1+\sqrt{17}}x_3 = 0 \\ x_3 \in R \end{cases}$$

Demak,  $\lambda_2 = \frac{1-\sqrt{17}}{2}$  xos qiymat uchun operatorning xos vektorlari  $\left\{ \left( -\frac{2}{1+\sqrt{17}}x_3; -\frac{2}{1+\sqrt{17}}x_3; x_3 \right) \mid x_3 \in R \wedge x_3 \neq 0 \right\}$  to'plamdan iborat.

$\lambda_3 = \frac{1+\sqrt{17}}{2}$  uchun xos vektorlarni topamiz:

$$\left( A - \frac{1+\sqrt{17}}{2} E \right) X = 0 \Leftrightarrow \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 3 & 1 & 0 \end{pmatrix} - \begin{pmatrix} \frac{1+\sqrt{17}}{2} & 0 & 0 \\ 0 & \frac{1+\sqrt{17}}{2} & 0 \\ 0 & 0 & \frac{1+\sqrt{17}}{2} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \Leftrightarrow$$

$$\Leftrightarrow \begin{pmatrix} 1-\frac{1+\sqrt{17}}{2} & 0 & 1 \\ 0 & 1-\frac{1+\sqrt{17}}{2} & 1 \\ 3 & 1 & -\frac{1+\sqrt{17}}{2} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} \frac{1-\sqrt{17}}{2}x_1 + x_3 = 0 \\ \frac{1-\sqrt{17}}{2}x_2 + x_3 = 0 \\ 3x_1 + x_2 - \frac{1+\sqrt{17}}{2}x_3 = 0 \end{cases} \Leftrightarrow \begin{cases} \frac{1-\sqrt{17}}{2}x_1 + x_3 = 0 \\ \frac{1-\sqrt{17}}{2}x_2 + x_3 = 0 \\ \frac{1-\sqrt{17}}{2}x_2 - x_3 = 0 \end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} \frac{1-\sqrt{17}}{2}x_1 + x_3 = 0 \\ \frac{1-\sqrt{17}}{2}x_2 + x_3 = 0 \\ -2x_3 = 0 \end{cases} \Leftrightarrow \begin{cases} x_1 = 0 \\ x_2 = 0 \\ x_3 = 0 \end{cases}$$

Demak,  $\lambda = \frac{1-\sqrt{17}}{2}$  xos qiymat uchun xos vektorlar mavjud emas.

### ⇒ Misol va mashqlar

1.f chiziqli operator  $\vec{a}_1, \vec{a}_2$  bazisda A matrisa,  $\varphi$  chiziqli operator  $\vec{b}_1, \vec{b}_2$  bazisda V matrisa yordamida berilgan bo'lsa,  $f + \varphi$  operatorning matrisasini toping:

1.1.  $\vec{a}_1 = (1, -2), \vec{a}_2 = (3, -5), A = \begin{pmatrix} 37 & -13 \\ 108 & -38 \end{pmatrix},$

$$\bar{b}_1 = (1,2), \bar{b}_2(2,5), \quad B = \begin{pmatrix} 1 & 1 \\ 2 & 2 \end{pmatrix}.$$

$$1.2. \bar{a}_1 = (7,3), \bar{a}_2(2,1), \quad A = \begin{pmatrix} 3 & 5 \\ 1 & 2 \end{pmatrix}.$$

$$\bar{b}_1 = (6,1), \bar{b}_2(5,1), \quad B = \begin{pmatrix} 3 & -2 \\ 6 & -6 \end{pmatrix}.$$

2.  $f$  chiziqli operator  $\bar{a}_1 = (-3,-1), \bar{a}_2(7,2)$  bazisda  $A = \begin{pmatrix} 2 & -1 \\ -2 & 2 \end{pmatrix}$  matrisa,  $\varphi$  chiziqli operator  $\bar{b}_1 = (3,2), \bar{b}_2 = (4,3)$  bazisda  $B = \begin{pmatrix} -14 & 10 \\ -21 & 15 \end{pmatrix}$  matrisa yordamida berilgan bo'lsa,  $\varphi\varphi$  operatorning matrisasini toping.

3.  $f$  chiziqli operator  $(a)$  bazisda  $A$  matrisa orqali,  $\varphi$  chiziqli operator  $(b)$  bazisda  $V$  matrisa orqali berilgan bo'lsa,  $\lambda f + \mu \varphi$  operatorning  $(e)$  bazisdagi matrisasini toping:

$$3.1. (a): \bar{a}_1(1,2), \bar{a}_2(0,1); \quad A = \begin{pmatrix} 2 & 1 \\ 3 & -1 \end{pmatrix};$$

$$(b): \bar{b}_1(3,1), \bar{b}_2(2,1); \quad V = \begin{pmatrix} -1 & 1 \\ 4 & 2 \end{pmatrix}; \quad 3f + \varphi.$$

$$3.2. (a): \bar{a}_1(-1,3), \bar{a}_2(1,1); \quad A = \begin{pmatrix} 2 & 1 \\ 3 & -1 \end{pmatrix};$$

$$(b): \bar{b}_1(-2,1), \bar{b}_2(2,4); \quad V = \begin{pmatrix} -1 & 1 \\ 4 & 2 \end{pmatrix}; \quad -2f + 3\varphi.$$

$$3.3. (a): \bar{a}_1(8,9), \bar{a}_2(3,1); \quad A = \begin{pmatrix} 7 & -2 \\ 1 & 5 \end{pmatrix};$$

$$(b): \bar{b}_1(4,5), \bar{b}_2(1,-4); \quad V = \begin{pmatrix} 6 & -5 \\ 0 & -2 \end{pmatrix}; \quad f - 4\varphi.$$

$$3.4. (a): \bar{a}_1(1,2), \bar{a}_2(3,4); \quad A = \begin{pmatrix} -4 & 5 \\ -7 & 6 \end{pmatrix};$$

$$(b): \bar{b}_1(-1,-2), \bar{b}_2(-3,-4); \quad V = \begin{pmatrix} 1 & -8 \\ -9 & 10 \end{pmatrix}; \quad 5f - 2\varphi.$$

$$3.5. (a): \bar{a}_1(-7,6), \bar{a}_2(5,-4); \quad A = \begin{pmatrix} 0 & 2 \\ -3 & 4 \end{pmatrix};$$

$$(b): \bar{b}_1(2,-1), \bar{b}_2(-4,3); \quad V = \begin{pmatrix} 3 & -5 \\ -4 & 0 \end{pmatrix}; \quad -3f + \varphi.$$

$$3.6. (a): \bar{a}_1(1,-1), \bar{a}_2(-2,1); \quad A = \begin{pmatrix} 3 & 7 \\ 5 & 4 \end{pmatrix};$$

$$(b): \bar{b}_1(2,1), \bar{b}_2(1,3); \quad V = \begin{pmatrix} 1 & 5 \\ 3 & 2 \end{pmatrix}; \quad 2f + 5\varphi.$$

4.  $C = \{a + bi \mid \forall a, b \in \mathbb{R}, i^2 = -1\}$  to'plam  $\mathbb{R}$  maydon ustida rangi ikkiga teng bo'lgan chiziqli algebra tashkil etishini isbotlang.

5. Barcha  $n$ -tartibli kvadrat matrisalar to'plami  $F^{n \times n}$ ,  $\mathcal{F}$  maydon ustida rangi  $n^2$  bo'lgan chiziqli algebra tashkil etishini isbotlang.

6. Berilgan operatorlarga teskari operatorni toping:

$$6.1. \quad A = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}.$$

$$6.2. \quad A = \begin{pmatrix} 0 & -1 & -1 \\ 1 & 2 & 3 \\ -2 & 1 & -1 \end{pmatrix}.$$

$$6.2. \quad A = \begin{pmatrix} 1 & 0 & 1 & 2 \\ 0 & 1 & -1 & -1 \\ 3 & 1 & 2 & 4 \\ 4 & 2 & 3 & 1 \end{pmatrix}.$$

$$6.4. \quad A = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

$$6.3. \quad A = \begin{pmatrix} 5 & 4 & 62 & -79 \\ 0 & 0 & 2 & 3 \\ 6 & 5 & 183 & 201 \\ 0 & 0 & 3 & 4 \end{pmatrix}.$$

$$6.6. \quad A = \begin{pmatrix} 3 & 2 & 1 & 2 \\ 7 & 5 & 2 & 5 \\ 0 & 0 & 9 & 4 \\ 0 & 0 & 11 & 5 \end{pmatrix}.$$

$$6.4. \quad A = \begin{pmatrix} 2 & 5 & 4 & 1 \\ 1 & 3 & 2 & 1 \\ 2 & 10 & 9 & 7 \\ 3 & 8 & 9 & 2 \end{pmatrix}.$$

$$6.8. \quad A = \begin{pmatrix} 3 & 5 & -3 & 2 \\ 4 & -2 & 5 & 3 \\ 7 & 8 & -1 & 5 \\ 6 & 4 & 5 & 3 \end{pmatrix}.$$

$$6.5. A = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 1 & 2 \\ 1 & 1 & 1 & -1 \\ 1 & 0 & -2 & -6 \end{pmatrix}$$

$$6.10. A = \begin{pmatrix} 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 1 & 0 & 0 & 2 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & -2 & 4 & 1 & 5 \end{pmatrix}$$

7. Quyidagi chiziqli operatorlarning xos qiymatlari va xos vektorlarini toping:

$$7.1. \begin{pmatrix} 2 & 4 \\ 1 & 2 \end{pmatrix}$$

$$7.2. \begin{pmatrix} 2 & 0 \\ 1 & 2 \end{pmatrix}$$

$$7.3. \begin{pmatrix} 1 & -3 & 0 \\ -1 & -2 & 3 \\ -1 & -4 & 4 \end{pmatrix}$$

$$7.4. \begin{pmatrix} 2 & 0 & 0 \\ 0 & 3 & 2 \\ 0 & -2 & 3 \end{pmatrix}$$

$$7.5. \begin{pmatrix} 6 & 1 & -5 \\ 3 & 12 & -3 \\ 7 & 1 & -6 \end{pmatrix}$$

$$7.6. \begin{pmatrix} 0 & 1 & 0 \\ -4 & 4 & 0 \\ -2 & 1 & 2 \end{pmatrix}$$

$$7.7. \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$7.8. \begin{pmatrix} 3 & -1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 3 & 0 & 5 & -3 \\ 4 & -1 & 3 & -1 \end{pmatrix}$$

$$7.9. \begin{pmatrix} 1 & 1 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & 2 & 4 \\ 0 & 1 & 1 & 2 \end{pmatrix}$$

$$7.10. \begin{pmatrix} 3 & -1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 3 & 0 & 5 & -3 \\ 4 & -1 & 3 & -1 \end{pmatrix}$$



### Takrorlash uchun savollar

1. Chiziqli operatorlar ustida qanday amallar bajariladi?
2. Chiziqli algebra deb nimaga aytiladi?
3. Chiziqli operatorlar algebrasi deb nimaga aytiladi?
4. Matritsalar chiziqli algebrasini tushuntiring.
5. Chiziqli operatorning teskarisi qanday topiladi?
6. Chiziqli operatorning xos qiymatlar, xos vektorlari deb nimaga aytiladi?

## X MODUL. CHIZIQLI TENGSIZLIKLAR SISTEMASI



### 23-§. Chiziqli tengsizliklar sistemasi. Qavariq konus.

**Asosiy tushunchalar:** chiziqli tengsizliklar sistemasi, yechim, chiziqli kombinatsiya, qavariq konus, yo'ldosh sistema.

Ushbu  $a_1x_1 + a_2x_2 + \dots + a_nx_n + b \geq 0$  (1) tengsizlik R haqiqiy sonlar maydoni ustidagi n ta noma'lumli chiziqli tengsizlik deyiladi. (1) da  $x_1, x_2, \dots, x_n$  - noma'lumlar,  $a_i, b \in R$  ( $i = \overline{1, n}$ ) esa koeffitsientlar deyiladi.

Agar (1) da  $b=0$  bo'lsa (1) ni bir jinsli,  $b \neq 0$  bulsa, (1) ni bir jinsli bo'lmagan chiziqli tengsizlik deyiladi.

$a_{i_1}x_1 + a_{i_2}x_2 + \dots + a_{i_m}x_n + b_i \geq 0$   $i = \overline{1, m}$ , (2) sistemaning barcha tengsizliklarini qanoatlantiruvchi  $x_1 = \alpha_1, x_2 = \alpha_2, \dots, x_n = \alpha_n$  sonlar (2) sistemaning yechimi deyiladi.

Agar (2) tengsizlik bitta ham yechimga ega bo'lmasa, ya'ni  $0x_1 + 0x_2 + \dots + 0x_n + b \geq 0$  ( $b < 0$ ) bo'lsa, u ziddiyatli tengsizlik deyiladi.

(2) sistemaning tengsizliklarini mos ravishda  $k_1 \geq 0, k_2 \geq 0, \dots, k_m \geq 0$  sonlarga ko'paytirib, ularni hadlab qo'shsak hosil bo'lgan ushbu tengsizlik

$$\sum_{j=1}^m k_j a_{j1} x_1 + \sum_{j=1}^m k_j a_{j2} x_2 + \dots + \sum_{j=1}^m k_j a_{jn} x_n + \sum_{j=1}^m k_j b_j \geq 0 \text{ ga (2) sistemaning}$$

manfiymas chiziqli kombinatsiyasi deyiladi.

Bir xil  $x_1, x_2, \dots, x_n$  noma'lumli ikkita hamjoyli tengsizliklar sistemasidan birining istalgan yechimi ikkinchisi uchun xam yechim bo'lsa yoki ikkala sistema ham hamjoysiz sistema bo'lsa, ular teng kuchli sistemalar deyiladi.

Vektorlarni ko'shish va manfiymas haqiqiy songa ko'paytirish amallariga nisbatan yopiq bo'lgan V vektor fazoning vektorlaridan tuzilgan bo'sh bo'lmagan to'plamga V vektor fazoning qavariq konusi deyiladi.

Chiziqli tengsizliklar sistemasidan noma'lumlar sonini bittaga kamaytirib tuzilgan yangi sistemani berilgan sistemaga yo'ldosh sistema deyiladi.

(2) sistemadan

$$\begin{cases} P_1 \geq x_n, \\ P_2 \geq x_n, \\ \dots \\ P_p \geq x_n; \end{cases} \begin{cases} x_n \geq Q_1, \\ x_n \geq Q_2, \\ \dots \\ x_n \geq Q_q; \end{cases} \begin{cases} R_1 \geq 0, \\ R_2 \geq 0, \\ \dots \\ R_r \geq 0 \end{cases} \quad (3) \text{ sistemani hosil qilamiz.}$$

Bundan  $\begin{cases} P_\alpha \geq Q_\beta \ (\alpha = \overline{1, p}; \beta = \overline{1, q}). \\ R_\gamma \geq 0 \ (\gamma = \overline{1, r}) \end{cases}$  sistemani hosil qilamiz.

Agar (3) sistemada birinchi yoki ikkinchi blok tengsizliklari bo'lmasa, u holda yo'ldosh sistema faqat  $R_\gamma \geq 0$  tengsizliklardan iborat bo'ladi. Fgar (3) sistemada birinchi va uchinchi yoki ikkinchi va uchinchi bloklar bo'lmasa, u holda yo'ldosh sistema mavjud emas. Ya'ni, bu sistemani ayniy ( $0 \geq 0$ ) deb qarash va uning yechimlari sifatida ixtiyoriy n o'lchovli arifmetik vektorni olish mumkin.

**Misol.** Chiziqli tengsizliklar sistemasini algebraik va geometrik usullarda

yeching:  $\begin{cases} 2x_1 + x_2 \leq 1 \\ 3x_1 - x_2 \leq 2 \\ x_1 + 2x_2 \leq 4 \\ -x_1 - 3x_2 \leq 3 \end{cases}$

**Yechish:** 1) Algebraik usul

$$\begin{cases} 2x_1 + x_2 \leq 1 \\ 3x_1 - x_2 \leq 2 \\ x_1 + 2x_2 \leq 4 \\ -x_1 - 3x_2 \leq 3 \end{cases} \Leftrightarrow \begin{cases} 2x_1 \leq 1 - x_2 \\ 3x_1 \leq 2 + x_2 \\ x_1 \leq 4 - 2x_2 \\ -3x_2 \leq 3 + x_1 \end{cases} \Leftrightarrow \begin{cases} x_1 \leq \frac{1}{2} - \frac{1}{2}x_2 \\ x_1 \leq \frac{2}{3} + \frac{1}{3}x_2 \\ x_1 \leq 4 - 2x_2 \\ -3x_2 - 3 \leq x_1 \end{cases}$$

$$\Leftrightarrow \begin{cases} -3x_2 - 3 \leq \frac{1}{2} - \frac{1}{2}x_2 \\ -3x_2 - 3 \leq \frac{2}{3} + \frac{1}{3}x_2 \\ -3x_2 - 3 \leq 4 - 2x_2 \end{cases} \Leftrightarrow \begin{cases} -\frac{7}{2} \leq \frac{5}{2}x_2 \\ -\frac{11}{3} \leq \frac{10}{3}x_2 \\ -7 \leq x_2 \end{cases} \Leftrightarrow \begin{cases} x_2 \geq -\frac{7}{5} \\ x_2 \geq -\frac{11}{10} \\ x_2 \geq -7 \end{cases} \Leftrightarrow x_2 \geq -\frac{11}{10}$$

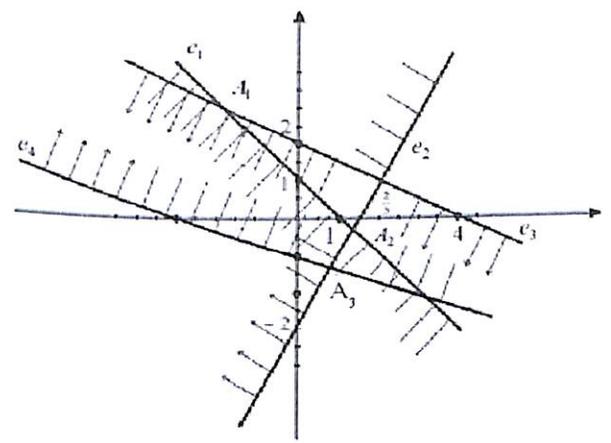
$x_2 = -1$  deb olamiz. U holda,

$$\begin{cases} x_1 \leq \frac{1}{2} + \frac{1}{2} \\ x_1 \leq \frac{2}{3} - \frac{1}{3} \\ 3 - 3 \leq x_1 \end{cases} \Leftrightarrow \begin{cases} x_1 \leq 1 \\ x_1 \leq \frac{1}{3} \\ x_1 \geq 0 \end{cases} \Leftrightarrow 0 \leq x_1 \leq \frac{1}{3}$$

$x_1 = 0$  deb olsak, u holda berilgan chiziqli tengsizliklar sistemasining xususiy yechimi sifatida  $(0; -1)$  vektorni olish mumkin.

2)  $\begin{cases} 2x_1 + x_2 \leq 1 \\ 3x_1 - x_2 \leq 2 \\ x_1 + 2x_2 \leq 4 \\ -x_1 - 3x_2 \leq 3 \end{cases}$  chiziqli tengsizliklar sistemasini tashkil etgan 4 ta

tengsizlikning har biri tekislikda yarim tekislikni bildiradi. Ularning umumiy qismi berilgan sistemaning yechimi bo'ladi.

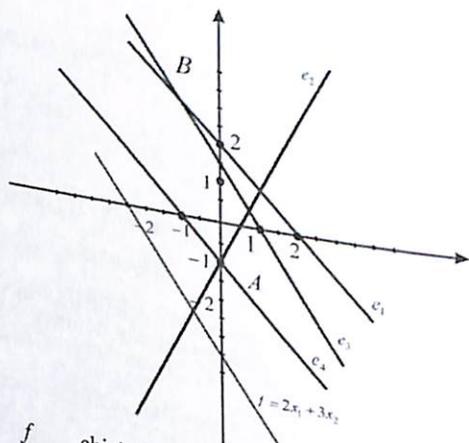


Chiziqli tengsizliklar sistemasining yechimi Dekart koordinatalar sistemasidagi  $A_1 A_2 A_3$  siniq chiziq bilan chegaralangan sohadan iborat.

*Misol.* 
$$\begin{cases} x_1 + x_2 \leq 2 \\ 2x_1 - x_2 \leq 1 \\ 3x_1 + 2x_2 \leq 3 \\ -x_1 - x_2 \leq 1 \end{cases}$$
 chiziqli tengsizliklar sistemasini geometrik usulda

yechib, manfiymas yechimlari orasidan berilgan  $f = 2x_1 + 3x_2$  chiziqli formani minimallashtiruvchi va maksimallashtiruvchi nuqtalarini aniqlang.

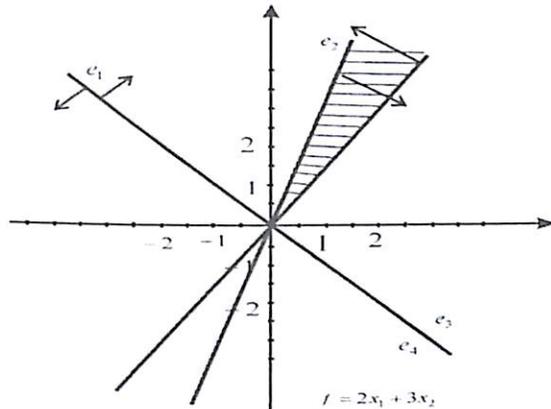
*Yechish.*



$f$  chiziqli formani minimallashtiruvchi nuqtalar  $A(-1; 0)$ , maksimallashtiruvchi nuqta  $B(-1; 3)$ .

*Misol.* 
$$\begin{cases} 2x_1 + x_2 \geq 0 \\ 3x_1 - x_2 \geq 0 \\ -x_1 + 2x_2 \geq 0 \end{cases}$$
 chiziqli tengsizliklar sistemasini yechimlarining qavariq konusini tekislikda tasvirlang.

*Yechish.* Bir jinsli chiziqli tengsizliklar sistemasini yechimlarining qavariq konusini tekislikda tasvirlang. SHuning uchun tekislikda berilgan bir jinsli chiziqli tengsizliklar sistemasining nolmas yechimlarini topamiz.



Demak, berilgan chiziqli tengsizliklar sistemasini yechimlar to'plami tashkil etgan qavariq konus chizmadagi shtrixlangan sohadan iborat.

*Misol.* 
$$\begin{cases} 2x_1 + 3x_2 - x_3 \geq 1 \\ -x_1 + 2x_2 + 3x_3 \leq 2 \\ -3x_1 - x_2 - 2x_3 \geq -2 \end{cases}$$
 chiziqli tengsizliklar sistemasini yeching.

*Yechish.* Berilgan chiziqli tengsizliklar sistemasidan  $x_2$  ni yo'qotamiz.

Buning uchun berilgan tengsizliklar sistemasiga yo'ldosh sistemani hosil qilamiz:

$$\begin{cases} 2x_1 + 3x_2 - x_3 \geq 1 \\ -x_1 + 2x_2 + 3x_3 \leq 2 \\ -3x_1 - x_2 - 2x_3 \geq -2 \end{cases} \Leftrightarrow \begin{cases} 3x_2 \geq -2x_1 + x_3 + 1 \\ 2x_2 \leq x_1 - 3x_3 + 2 \\ x_2 \leq -3x_1 - 2x_3 + 2 \end{cases} \Leftrightarrow \begin{cases} x_2 \geq -\frac{2}{3}x_1 + \frac{1}{3}x_3 + \frac{1}{3} \\ x_2 \geq \frac{1}{2}x_1 - \frac{3}{2}x_3 + 1 \\ x_2 \leq -3x_1 - 2x_3 + 2 \end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} -\frac{2}{3}x_1 + \frac{1}{3}x_3 + \frac{1}{3} \leq \frac{1}{2}x_1 - \frac{3}{2}x_3 + 1 \\ -\frac{2}{3}x_1 + \frac{1}{3}x_3 + \frac{1}{3} \leq -3x_1 - 2x_3 + 2 \end{cases}$$

$$\left(-\frac{2}{3} - \frac{1}{2}\right)x_1 + \left(\frac{1}{3} + \frac{3}{2}\right)x_3 + \left(\frac{1}{3} - 1\right) \leq 0$$

$$\left(-\frac{2}{3} + 3\right)x_1 + \left(\frac{1}{3} + 2\right)x_3 + \left(\frac{1}{3} - 2\right) \leq 0 \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} -\frac{7}{6}x_1 + \frac{11}{6}x_3 - \frac{2}{3} \geq 0 \\ \frac{7}{3}x_1 + \frac{7}{3}x_3 - \frac{5}{3} \leq 0 \end{cases} \Leftrightarrow \begin{cases} x_1 \leq -x_3 + \frac{5}{7} \\ x_1 \geq \frac{11}{7}x_3 - \frac{4}{7} \end{cases} \Leftrightarrow \frac{11}{7}x_3 - \frac{4}{7} \leq -x_3 + \frac{5}{7} \Leftrightarrow$$

$$\Leftrightarrow \left(\frac{11}{7} + 1\right)x_3 \leq \frac{5}{7} + \frac{4}{7} \Leftrightarrow \frac{18}{7}x_3 \leq \frac{9}{7} \Leftrightarrow 2x_3 \leq 1 \Leftrightarrow x_3 \leq \frac{1}{2}$$

$x_3$  ning topilgan sohasidan  $x_3 = 0$  qiymatni olsak  $\begin{cases} x_1 \leq \frac{5}{7} \\ x_1 \geq -\frac{4}{7} \end{cases}$ , ya'ni

$-\frac{4}{7} \leq x_1 \leq \frac{5}{7}$  hosil bo'ladi. Agar  $x_1 = 0$  deb olsak, u holda,

$$\begin{cases} x_2 \geq \frac{1}{3} \\ x_2 \leq 1, \text{ ya'ni } \frac{1}{3} \leq x_2 \leq 1 \\ x_2 \leq 2 \end{cases} \text{ hosil bo'ladi.}$$

Demak, berilgan chiziqli tengsizliklar sistemasining xususiy yechimlaridan biri  $(0,1,0)$  bo'ladi.



### Misol va mashqlar

1. Quyidagi tengsizliklar sistemalarining yechimlar sohasini aniqlang:

1.1.  $\begin{cases} x_1 + x_2 \geq -2, \\ 2x_1 - x_2 \geq -1. \end{cases}$

1.2.  $\begin{cases} x_1 - x_2 \geq -1, \\ 2x_1 - x_2 \geq 3. \end{cases}$

1.3.  $\begin{cases} 3x_1 - 2x_2 \geq 6, \\ x_1 \leq 3. \end{cases}$

1.4.  $\begin{cases} 4x_1 - 3x_2 \leq 1, \\ x_1 + x_2 \geq 0. \end{cases}$

1.5.  $\begin{cases} 2x_1 + x_2 \leq -3, \\ x_1 - 3x_2 \leq 1, \\ 4x_1 + 3x_2 \leq 2. \end{cases}$

1.6.  $\begin{cases} -x_1 + x_2 \geq 4, \\ x_1 - 3x_2 \leq -1, \\ 3x_1 - x_2 \leq -2. \end{cases}$

1.7.  $\begin{cases} 4x_1 + 5x_2 - 20 \leq 0, \\ -7x_1 + 3x_2 - 12 \leq 0, \\ -3x_1 + 8x_2 + 15 \geq 0. \end{cases}$

1.8.  $\begin{cases} 2x_1 - x_2 - 3 \leq 0, \\ x_1 + x_2 - 6 \leq 0, \\ x_1 - 2x_2 \leq 0. \end{cases}$

1.9.  $\begin{cases} 8x_1 + 8x_2 - 64 \leq 0, \\ 4x_1 - 4x_2 - 16 \leq 0, \\ -4x_1 + 4x_2 - 16 \leq 0, \\ 8x_1 + 8x_2 + 64 \geq 0. \end{cases}$

1.10.  $\begin{cases} x_1 - x_2 \leq 0, \\ -x_1 + x_2 - 4 \leq 0, \\ x_1 - x_2 + 6 \geq 0, \\ 2x_1 - 2x_2 + 8 \leq 0. \end{cases}$

2. Chiziqli tengsizliklar sistemaning har bir manfiymas chiziqli kombinatsiyasi shu sistemaning natijasi bo'lishini isbotlang.

3. Quyidagi tengsizliklar sistemalarining biri ikkinchisiga natija bo'ladimi? Teng kuchli sistemalarni aniqlang:

3.1. 1)  $x_1 - x_2 \leq 0,$  2)  $x_1 - x_2 + 6 \geq 0.$

3.2. 1)  $-x_1 + x_2 - 4 \leq 0,$  2)  $x_1 - x_2 + 6 \geq 0.$

3.3. 1)  $\begin{cases} 4x_1 + 5x_2 - 20 \leq 0, \\ -7x_1 + 3x_2 - 12 \leq 0, \\ -3x_1 + 8x_2 + 15 \geq 0. \end{cases}$  2)  $x_2 - 5 \leq 0.$

3.4. 1)  $\begin{cases} x_1 - 2x_2 \leq 0, \\ -2x_1 + x_2 + 3 \leq 0, \\ x_1 + x_2 - 10 \leq 0. \end{cases}$  2)  $\begin{cases} 2x_1 - x_2 - 3 \geq 0, \\ x_1 + x_2 - 6 \leq 0, \\ x_1 - 2x_2 \leq 0. \end{cases}$

3.5. 1)  $\begin{cases} x_1 - x_2 \geq 0, \\ x_2 - 3 \geq 0, \\ 4x_1 + 4x_2 + 16 \leq 0. \end{cases}$  2)  $\begin{cases} x_1 + x_2 - 3 \leq 0, \\ -x_1 - x_2 + 5 \leq 0. \end{cases}$

3.6. 1)  $\begin{cases} 8x_1 + 8x_2 - 64 \leq 0, \\ 4x_1 - 4x_2 - 16 \leq 0, \\ -4x_1 + 4x_2 - 16 \leq 0, \\ 8x_1 + 8x_2 + 64 \geq 0. \end{cases}$  2)  $5x_1 - 3x_2 - 30 \leq 0.$

4.  $\bar{a} \in R^n$  va  $\bar{a} \neq \bar{0}$  Lar uchun  $\{\lambda \bar{a} | \lambda \geq 0, \lambda \in R\}$  to'plam  $R^n$  fazoning kavarik konusi bo'lishini isbotlang. Bu kavarik konus  $\bar{a}$  vektor yaratgan tug'ri chiziq deyiladi.

5.  $\bar{a}_1, \dots, \bar{a}_m \in R^n$  vektorlar sistemasining barcha manfiymas chiziqli kombinatsiyalar to'plami  $R^n$  fazoning qavariq konusi bo'lishini isbotlang.

6. Bir jinsli chiziqli tengsizliklar sistemasining barcha yechimlar to'plami  $V=\mathbb{R}^n$  fazoning qavariq konusi bo'lishini isbotlang.

7. Chiziqli tengsizliklar sistemasi xamjoysiz bo'lishi uchun uning biror chiziqli kombinatsiyasi ziddiyatli tengsizlik bo'lishi zarur va etarli ekanligini isbotlang.

8. Bir jinsli chiziqli tengsizliklar sistemasining har bir natijasi bu sistemaning manfiy koeffitsientli chiziqli kombinatsiyasidan iboratligini isbotlang.

9. Yuldosh sistemaning har bir tengsizligi berilgan tengsizliklar sistemasining chiziqli kombinatsiyasi bo'lishini isbotlang.

10. Chiziqli tengsizliklar sistemasini algebraik va geometrik usullarda yeching:

$$10.1. \begin{cases} -2x_1 + 5x_2 \leq -1, \\ x_1 + 3x_2 \leq 13, \\ -3x_1 + 2x_2 \leq 5, \\ -x_1 + 3x_2 \leq 2. \end{cases}$$

$$10.3. \begin{cases} 8x_1 + 4x_2 \leq 1, \\ -8x_1 - 4x_2 \leq -1, \\ 2x_1 - 3x_2 \leq 5, \\ -2x_1 + 3x_2 \leq -5. \end{cases}$$

$$10.5. \begin{cases} 5x_1 - 9x_2 \leq 4, \\ -5x_1 + 9x_2 \leq -4, \\ -2x_1 + 3x_2 \leq 5, \\ 2x_1 - 3x_2 \geq -5, \\ x_1 + x_2 \leq -1. \end{cases}$$

$$10.7. \begin{cases} -2x_1 + x_2 \geq -1, \\ 4x_1 + 3x_2 \leq 3, \\ -x_1 + 7x_2 \geq 5, \\ -4x_1 + 3x_2 \leq -2. \end{cases}$$

$$10.2. \begin{cases} 5x_1 + 3x_2 \geq 3, \\ x_1 - x_2 \geq 6, \\ 2x_1 - x_2 \geq 1, \\ -x_1 + 8x_2 \geq 2. \end{cases}$$

$$10.4. \begin{cases} 11x_1 - 3x_2 \leq 30, \\ 2x_1 + 5x_2 \leq -6, \\ 12x_1 - 6x_2 \leq 4, \\ -5x_1 + 7x_2 \leq 12. \end{cases}$$

$$10.6. \begin{cases} 2x_1 + 5x_2 \leq -10, \\ x_1 + 3x_2 \leq 2, \\ 6x_1 + 7x_2 \geq 5, \\ 4x_1 + 3x_2 \leq 12. \end{cases}$$

$$10.8. \begin{cases} 5x_1 - x_2 \leq 4, \\ 3x_1 + 11x_2 \leq 2, \\ 6x_1 + 7x_2 \geq -5, \\ -x_1 + 3x_2 \leq 0. \end{cases}$$

11. Chiziqli tengsizliklar sistemasi yechimlarining qavariq konusini tekislikda tasvirlang:

$$11.1. \begin{cases} x_1 + x_2 \geq 0, \\ 2x_1 - x_2 \geq 0. \end{cases}$$

$$11.3. \begin{cases} -x_1 + x_2 \geq 0, \\ 2x_1 - 3x_2 \geq 0. \end{cases}$$

$$11.5. \begin{cases} 9x_1 + 8x_2 \geq 0, \\ 2x_1 + 3x_2 \geq -0, \\ -4x_1 + 2x_2 \geq 0, \\ -7x_1 + 3x_2 \geq 0. \end{cases}$$

$$11.7. \begin{cases} 2x_1 + 8x_2 \geq 0, \\ x_1 + 3x_2 \geq 0, \\ -x_1 + 2x_2 \geq 0, \\ -3x_1 + 3x_2 \geq 0. \end{cases}$$

$$11.9. \begin{cases} x_1 + x_2 - 8 \leq 0, \\ x_1 - x_2 - 4 \leq 0, \\ -x_1 + x_2 - 3 \leq 0, \\ 2x_1 + 2x_2 + 16 \geq 0. \end{cases}$$

$$11.2. \begin{cases} x_1 - x_2 \geq 0, \\ 2x_1 - x_2 \geq 0. \end{cases}$$

$$11.4. \begin{cases} 4x_1 - 3x_2 \leq 0, \\ x_1 + x_2 \geq 0. \end{cases}$$

$$11.6. \begin{cases} 3x_1 + -4x_2 \geq 0, \\ -2x_1 + 13x_2 \geq 0, \\ -4x_1 + 3x_2 \geq 0, \\ 7x_1 + 3x_2 \geq 0. \end{cases}$$

$$11.8. \begin{cases} 4x_1 + 8x_2 \geq 0, \\ 6x_1 - 3x_2 \geq 0, \\ 7x_1 + 2x_2 \geq 0, \\ -x_1 - 3x_2 \geq 0. \end{cases}$$

$$11.10. \begin{cases} -8x_1 + 8x_2 + 64 \leq 0, \\ 4x_1 - 4x_2 - 16 \leq 0, \\ -4x_1 + 4x_2 - 6 \leq 0, \\ -8x_1 + 8x_2 - 64 \geq 0. \end{cases}$$

12. Chiziqli tengsizliklar sistemasini yeching:

$$12.1. \begin{cases} 6x_1 - 5x_2 + 2x_3 \leq 11 \\ 2x_1 + 4x_2 - x_3 + 3x_4 \leq -1 \\ x_1 + 2x_2 + 2x_3 + 13x_4 \geq 10 \\ 2x_1 - 4x_2 + 7x_3 - 2x_4 \geq 0 \end{cases}$$

$$12.3. \begin{cases} -2x_1 + 3x_2 - 6x_3 - x_4 \leq 11 \\ 3x_1 + 6x_2 + 5x_3 - 12x_4 \leq 2 \\ x_1 - 7x_2 + x_3 + 4x_4 \geq 23 \\ x_2 + 23x_4 \leq 2 \\ -2x_1 + 7x_2 + 2x_3 + 2x_4 \leq 14 \end{cases}$$

$$12.5. \begin{cases} -4x_1 + 5x_2 + 3x_3 + 4x_4 \leq 1 \\ -2x_1 + 3x_2 - 9x_3 - x_4 \geq 2 \\ -9x_1 + 10x_2 - x_3 \geq 3 \end{cases}$$

$$12.2. \begin{cases} 6x_1 - 5x_2 + x_3 - 2x_4 \leq 11 \\ x_1 + 4x_2 - x_3 + 3x_4 \leq 1 \\ 2x_2 + 2x_3 + 13x_4 \leq 17 \\ 2x_1 - 7x_3 - 2x_4 \geq 0 \end{cases}$$

$$12.4. \begin{cases} -2x_1 + 3x_2 + 3x_3 + 2x_4 \leq 4 \\ 2x_1 + 4x_2 - 3x_3 + 3x_4 \leq -1 \\ 13x_1 - x_2 + 2x_3 + 11x_4 \geq 0 \\ 12x_1 + 4x_2 - 6x_3 + 2x_4 \leq 10 \end{cases}$$

$$12.6. \begin{cases} -9x_1 + 5x_2 + 3x_3 - 5x_4 \leq 1 \\ 3x_1 + 3x_2 - 9x_3 + x_4 \leq 2 \\ 7x_1 + 10x_2 - x_3 + x_4 \leq 3 \end{cases}$$

$$12.7. \begin{cases} 2x_1 + 5x_2 + 3x_3 - 3x_4 \geq 1 \\ x_1 + 3x_2 - 9x_3 + x_4 \leq 2 \\ 2x_1 + 10x_2 - x_3 + 3x_4 \leq 3 \end{cases}$$

$$12.8. \begin{cases} 2x_1 - 3x_2 - 9x_3 + 5x_4 \leq 11 \\ 3x_1 + x_2 + 8x_3 - 3x_4 \leq 0 \\ -2x_1 - 4x_2 - 2x_3 \geq 2 \end{cases}$$

$$12.9. \begin{cases} x_1 + 2x_2 + x_3 - x_4 + x_5 \geq 21 \\ 2x_1 - 3x_2 - 9x_3 - 3x_4 \geq 3 \\ 3x_1 + x_2 + 8x_3 + 2x_4 + x_5 \leq 3 \end{cases}$$

$$12.10. \begin{cases} x_1 - x_2 + x_3 + 5x_4 - 4x_5 \geq 7 \\ x_1 - x_2 + x_3 + 3x_4 - 5x_5 \leq 6 \\ x_1 - 3x_2 - x_3 + x_4 \geq 9 \end{cases}$$

$$12.11. \begin{cases} -2x_1 - x_2 + 3x_3 + 4x_4 \leq -11 \\ -3x_1 + 3x_2 - 9x_3 - x_4 \geq 2 \\ x_1 + 10x_2 - x_3 \geq 3 \end{cases}$$

$$12.12. \begin{cases} 2x_1 - x_2 + 5x_3 - 5x_4 \leq 3 \\ x_1 + 3x_2 - 4x_3 + x_4 \geq 2 \\ -x_1 + 10x_2 - x_3 + x_4 \leq 3 \end{cases}$$

$$12.13. \begin{cases} 5x_1 - 2x_2 + 3x_3 - 4x_4 \leq 1 \\ x_1 + 3x_2 - 9x_3 - x_4 \geq 2 \\ 5x_1 + 3x_2 - x_3 \geq 3 \end{cases}$$

$$12.14. \begin{cases} 3x_1 + x_2 - 2x_3 - x_4 \leq 4 \\ -2x_1 + 3x_2 - 4x_3 + x_4 \leq 2 \\ -3x_1 - x_2 - x_3 + x_4 \leq 3 \end{cases}$$

13.  $f$  chiziqli formani minimum qiymatini va uni minimum qiymatga keltiruvchi nuqtani toping:

$$13.1. f = -x_2 + x_1, \begin{cases} x_1 + 3x_2 \leq 12, \\ 3x_1 - x_2 \geq 6, \\ 3x_1 + 4x_2 \geq 0, \\ x_1 \geq 0, \\ 3x_2 \geq 0. \end{cases}$$

$$13.2. f = x_1 - 4x_2, \begin{cases} x_1 + 2x_2 \leq 4, \\ x_1 \leq 3, \\ x_1 - 2x_2 \geq -1, \\ x_1 \geq 0, \\ 3x_2 \geq 0. \end{cases}$$

$$13.3. f = 2x_1 - x_2, \begin{cases} 2x_1 - x_2 \leq 12, \\ x_1 + x_2 \leq 6, \\ x_1 + 3x_2 \geq 1, \\ x_1 \geq 0, \\ 3x_2 \geq 0. \end{cases}$$

$$13.4. f = x_1 + 2x_2 + 3, \begin{cases} 2x_1 + 4x_2 \leq 8, \\ 3x_1 \leq 6, \\ 5x_2 \leq 5, \\ x_1 \geq 0, \\ 3x_2 \geq 0. \end{cases}$$

14.  $f$  chiziqli formani maksimum qiymatini va uni maksimum qiymatga keltiruvchi nuqtani toping:

$$14.1. f = 2x_1 + 4x_2, \begin{cases} 4x_1 + 3x_2 \leq 40, \\ 12x_1 + 3x_2 \leq 24, \\ 2x_1 \leq 6, \\ x_2 \leq 3, \\ x_1 \geq 0, \\ x_2 \geq 0. \end{cases}$$

$$14.2. f = -x_1 + 4x_2, \begin{cases} 3x_1 + 2x_2 \leq 12, \\ 2x_1 - x_2 \leq 0, \\ -3x_1 + 2x_2 \leq 3, \\ x_1 + 2x_2 \leq 3, \\ x_1 \geq 0, \\ x_2 \geq 0. \end{cases}$$

$$14.3. f = 2x_1 + x_2, \begin{cases} 4x_1 - x_2 \geq -4, \\ 2x_1 + 3x_2 \leq 12, \\ 5x_1 - 3x_2 \leq 15, \\ x_2 \leq 7, \\ x_1 \geq 0, \\ x_2 \geq 0. \end{cases}$$

$$14.4. f = 3x_1 + 2x_2, \begin{cases} x_1 + x_2 \geq 1, \\ -5x_1 + x_2 \leq 0, \\ 5x_1 - x_2 \geq 0, \\ x_1 - x_2 \geq -1, \\ x_1 + x_2 \leq 6, \\ x_1 \geq 0, \\ x_2 \geq 0. \end{cases}$$

$$12.7. \begin{cases} 2x_1 + 5x_2 + 3x_3 - 3x_4 \geq 1 \\ x_1 + 3x_2 - 9x_3 + x_4 \leq 2 \\ 2x_1 + 10x_2 - x_3 + 3x_4 \leq 3 \end{cases}$$

$$12.8. \begin{cases} 2x_1 - 3x_2 - 9x_3 + 5x_4 \leq 11 \\ 3x_1 + x_2 + 8x_3 - 3x_4 \leq 0 \\ -2x_1 - 4x_2 - 2x_3 \geq 2 \end{cases}$$

$$12.9. \begin{cases} x_1 + 2x_2 + x_3 - x_4 + x_5 \geq 21 \\ 2x_1 - 3x_2 - 9x_3 - 3x_4 \geq 3 \\ 3x_1 + x_2 + 8x_3 + 2x_4 + x_5 \leq 3 \end{cases}$$

$$12.10. \begin{cases} x_1 - x_2 + x_3 + 5x_4 - 4x_5 \geq 7 \\ x_1 - x_2 + x_3 + 3x_4 - 5x_5 \leq 6 \\ x_1 - 3x_2 - x_3 + x_4 \geq 9 \end{cases}$$

$$12.11. \begin{cases} -2x_1 - x_2 + 3x_3 + 4x_4 \leq -11 \\ -3x_1 + 3x_2 - 9x_3 - x_4 \geq 2 \\ x_1 + 10x_2 - x_3 \geq 3 \end{cases}$$

$$12.12. \begin{cases} 2x_1 - x_2 + 5x_3 - 5x_4 \leq 3 \\ x_1 + 3x_2 - 4x_3 + x_4 \geq 2 \\ -x_1 + 10x_2 - x_3 + x_4 \leq 3 \end{cases}$$

$$12.13. \begin{cases} 5x_1 - 2x_2 + 3x_3 - 4x_4 \leq 1 \\ x_1 + 3x_2 - 9x_3 - x_4 \geq 2 \\ 5x_1 + 3x_2 - x_3 \geq 3 \end{cases}$$

$$12.14. \begin{cases} 3x_1 + x_2 - 2x_3 - x_4 \leq 4 \\ -2x_1 + 3x_2 - 4x_3 + x_4 \leq 2 \\ -3x_1 - x_2 - x_3 + x_4 \leq 3 \end{cases}$$

13.  $f$  chiziqli formani minimum qiymatini va uni minimum qiymatga keltiruvchi nuqtani toping:

$$13.1. f = -x_2 + x_1, \begin{cases} x_1 + 3x_2 \leq 12, \\ 3x_1 - x_2 \geq 6, \\ 3x_1 + 4x_2 \geq 0, \\ x_1 \geq 0, \\ 3x_2 \geq 0. \end{cases}$$

$$13.2. f = x_1 - 4x_2, \begin{cases} x_1 + 2x_2 \leq 4, \\ x_1 \leq 3, \\ x_1 - 2x_2 \geq -1, \\ x_1 \geq 0, \\ 3x_2 \geq 0. \end{cases}$$

$$13.3. f = 2x_1 - x_2, \begin{cases} 2x_1 - x_2 \leq 12, \\ x_1 + x_2 \leq 6, \\ x_1 + 3x_2 \geq 1, \\ x_1 \geq 0, \\ 3x_2 \geq 0. \end{cases}$$

$$13.4. f = x_1 + 2x_2 + 3, \begin{cases} 2x_1 + 4x_2 \leq 8, \\ 3x_1 \leq 6, \\ 5x_2 \leq 5, \\ x_1 \geq 0, \\ 3x_2 \geq 0. \end{cases}$$

14.  $f$  chiziqli formani maksimum qiymatini va uni maksimum qiymatga keltiruvchi nuqtani toping:

$$14.1. f = 2x_1 + 4x_2, \begin{cases} 4x_1 + 3x_2 \leq 40, \\ 12x_1 + 3x_2 \leq 24, \\ 2x_1 \leq 6, \\ x_2 \leq 3, \\ x_1 \geq 0, \\ x_2 \geq 0. \end{cases}$$

$$14.2. f = -x_1 + 4x_2, \begin{cases} 3x_1 + 2x_2 \leq 12, \\ 2x_1 - x_2 \leq 0, \\ -3x_1 + 2x_2 \leq 3, \\ x_1 + 2x_2 \leq 3, \\ x_1 \geq 0, \\ x_2 \geq 0. \end{cases}$$

$$14.3. f = 2x_1 + x_2, \begin{cases} 4x_1 - x_2 \geq -4, \\ 2x_1 + 3x_2 \leq 12, \\ 5x_1 - 3x_2 \leq 15, \\ x_2 \leq 7, \\ x_1 \geq 0, \\ x_2 \geq 0. \end{cases}$$

$$14.4. f = 3x_1 + 2x_2, \begin{cases} x_1 + x_2 \geq 1, \\ -5x_1 + x_2 \leq 0, \\ 5x_1 - x_2 \geq 0, \\ x_1 - x_2 \geq -1, \\ x_1 + x_2 \leq 6, \\ x_1 \geq 0, \\ x_2 \geq 0. \end{cases}$$



## Takrorlash uchun savollar

1. Chiziqli tengsizliklar sistemasining umumiy ko'rinishini yozing.
2. ChTS ning yechimi deb nimaga aytiladi?
3. Hamjoyli va hamjoysiz ChTS ta'riflarini ayting.
4. ChTSning natijasi deb nimaga aytiladi?
5. ChTSning manfiymas chiziqli kombinatsiyasini tuzing.
6. Bir jinsli ChTS deb nimaga aytiladi?
7. Qavariq konus ta'rifini ayting.
8. Ziddiyatli tengsizlik deb nimaga aytiladi?

## XI MODUL. BUTUN SONLAR HALQASIDA BO'LINISH MUNOSABATI

### 24-§. Tub va murakkab sonlar. EKUB. EKUK.

**Asosiy tushunchalar:** tub son, murakkab son, natural son natural bo'luvchilar soni va yig'indisi, EKUB, EKUK, Eylar funksiyasi.

Faqat ikkita turli natural bo'luvchilarga ega bo'lgan natural son tub son deyiladi.

Natural bo'luvchilarining soni ikkita dan ortiq bo'lgan natural son murakkab son deyiladi.

$a > 1$  natural son bo'lsin.  $a = p_1^{\alpha_1} p_2^{\alpha_2} \dots p_n^{\alpha_n}$  tenglik  $a$  sonning kanonik yoyilmasi deyiladi.

Agar  $a$  va  $b \neq 0$  butun sonlar uchun  $a = bq$  munosabatni qanoatlantiruvchi  $q$  butun son mavjud bo'lsa, u holda  $a$  son  $b$  songa bo'linadi yoki  $b$  son  $a$  sonni bo'ladi deyiladi.

$a = p_1^{\alpha_1} p_2^{\alpha_2} \dots p_n^{\alpha_n}$  son uchun  $\tau(a) = (\alpha_1 + 1)(\alpha_2 + 1) \dots (\alpha_n + 1)$  va

$$\sigma(n) = \frac{p_1^{\alpha_1+1} - 1}{p_1 - 1} \cdot \frac{p_2^{\alpha_2+1} - 1}{p_2 - 1} \dots \frac{p_n^{\alpha_n+1} - 1}{p_n - 1} \text{ bo'ladi.}$$

$a$  va  $b$  butun sonlarning ikkisini ham bo'ladigan son shu sonlarning umumiy bo'luvchisi deyiladi.

$a$  va  $b$  natural sonlar umumiy buluvchilarining eng kattasiga shu sonlarning eng katta umumiy bo'luvchisi (EKUB) deyiladi va uni  $(a; b)$  ko'rinishda belgilanadi.

Agar  $(a; b) = 1$  bo'lsa, u holda  $a$  va  $b$  natural sonlar o'zaro tub sonlar deyiladi.

$a_1, a_2, \dots, a_n$  butun sonlarning barchasini bo'ladigan son shu sonlarning umumiy bo'luvchisi deyiladi.

~~150 151 152 153 154 155 156 157 158 159~~  
~~160 161 162 163 164 165 166 167 168 169~~  
~~170 171 172 173 174 175 176 177 178 179~~  
~~180 181 182 183 184 185 186 187 188 189~~  
 190 191 192 193 194 195 196 197 198 199

200

Demak, 150 bilan 200 orasidagi tub sonlarni topish uchun 2.3.5.7.11.13 ga bo'linadigan sonlar qatordan o'chirildi va berilgan oraliqdagi tub sonlar Eratosfen yordamida aniqlandi. Ular 151, 157, 163, 167, 173, 179, 181, 191, 193, 197, 199.

Misol. Berilgan 1321 sonning tub yoki murakkab ekanligini aniqlang.

Yechish. Berilgan  $a$  natural sonning tub yoki murakkab ekanligini aniqlash uchun  $\sqrt{a}$  songacha bo'lgan tub sonlarga berilgan sonning bo'linishi yoki bo'linmasligi aniqlanadi. Agar berilgan  $a$  son  $\sqrt{a}$  gacha bo'lgan birorta ham tub songa bo'linmasa, u holda  $a$  tub son bo'ladi.

Demak,  $\sqrt{1321} \approx 36$  ni topamiz. 36 gacha bo'lgan tub sonlar 2,3,5,7,11,13,17,19,23,29,31 ga berilgan 1321 sonni bo'linish-bo'linmasligini tekshiramiz.

2 ga bo'linmaydi, chunki 1321 toq son;

3 ga bo'linmaydi, chunki  $1+3+2+1=7/3$ ;

5 ga bo'linmaydi, chunki 1321 ning oxirgi raqami 1;

$$1321:7 \approx 188$$

$$1321:11 \approx 120$$

$$1321:13 \approx 101$$

$$1321:17 \approx 77$$

$$1321:19 \approx 69$$

$$1321:23 \approx 54$$

$$1321:29 \approx 45$$

$$1321:31 \approx 42$$

Demak, 1321 36 gacha bo'lgan tub sonlarga bo'linmaydi. U tub son.

Misol. Berilgan 123 va 321 sonlarning EKUB va EKUKlarini ikki usulda toping. EKUBni berilgan sonlar orqali chiziqli ifodalang.

Yechish. Berilgan natural sonlarning EKUB va EKUKlarini topish uchun ularni tub ko'paytiruvchilarga yoyilmasidan yoki Evklid algoritmidan foydalanish mumkin.

1-usul. Berilgan sonlarni tub ko'paytiruvchilarga kanonik yoyilmasini topamiz:

$$\begin{array}{l|l} 123 & 3 \\ 41 & 41 \\ 1 & 1 \end{array} \quad \begin{array}{l|l} 321 & 3 \\ 107 & 107 \\ 1 & 1 \end{array}$$

$$123 = 3 \cdot 41 = 3^1 \cdot 41^1 \cdot 107^0;$$

$$321 = 3 \cdot 107 = 3^1 \cdot 41^0 \cdot 107^1$$

$$n = p_1^{\alpha_1} \dots p_n^{\alpha_n} \text{ va } m = p_1^{\beta_1} \dots p_n^{\beta_n} \text{ sonlarning}$$

$$\text{EKUBi } (n, m) = P_1^{\min(\alpha_1, \beta_1)} \cdot P_2^{\min(\alpha_2, \beta_2)} \dots P_n^{\min(\alpha_n, \beta_n)}$$

$$\text{EKUKi } [n, m] = P_1^{\max(\alpha_1, \beta_1)} \cdot P_2^{\max(\alpha_2, \beta_2)} \dots P_n^{\max(\alpha_n, \beta_n)}$$

$$\text{Demak, } (123; 321) = 3 \text{ va } [123; 321] = 3 \cdot 41 \cdot 107 = 13161.$$

2-usul. Berilgan sonlar uchun qoldiqli bo'lish teoremasi yordamida Evklid algoritmini tuzamiz:

$$321 = 123 \cdot 2 + 75;$$

$$75 = 321 - 123 \cdot 2;$$

$$123 = 75 \cdot 1 + 48;$$

$$48 = 123 - 75 \cdot 1;$$

$$75 = 48 \cdot 1 + 27;$$

$$27 = 75 - 48 \cdot 1;$$

$$48 = 27 \cdot 1 + 21;$$

$$21 = 48 - 27 \cdot 1;$$

$$27 = 21 \cdot 1 + 6;$$

$$6 = 27 - 21 \cdot 1;$$

$$21 = 6 \cdot 3 + 3;$$

$$3 = 21 - 6 \cdot 3$$

$$6 = 3 \cdot 2 + 0$$

Evklid algoritmidagi oxirgi noldan farqli qoldiq EKUB ni beradi. Demak,

$$(321, 123) = 3. \text{ Bundan } [321, 123] = \frac{321 \cdot 123}{(321, 123)} = 13161.$$

Topilgan EKUB  $(321, 123) = 3$  ning 123 va 321 lar yordamidagi chiziqli

ifodasini topamiz. Tuzilgan Evklid algoritmidagi qoldiqlarni bo'linuvchi va bo'luvchilar yordamidagi ifodalarini topamiz:

$$\begin{aligned} 3 &= 21 - 6 \cdot 3 = (48 - 27 \cdot 1) - (27 - 21 \cdot 1) \cdot 3 = 48 - 27 \cdot 4 + 21 \cdot 3 = 123 - 75 \cdot 1 - \\ &- (75 - 48 \cdot 1) \cdot 4 + (48 - 27 \cdot 1) \cdot 3 = 123 - 75 \cdot 5 + 48 \cdot 7 - 27 \cdot 3 = \\ &= 123 - (321 - 123 \cdot 2) \cdot 5 + (123 - 75 \cdot 1) \cdot 7 - (75 - 48 \cdot 1) \cdot 3 = \\ &= 123 \cdot 18 - 321 \cdot 5 - 75 \cdot 10 + 48 \cdot 3 = 123 \cdot 18 - 321 \cdot 5 - \\ &- (321 - 123 \cdot 2) \cdot 10 + (123 - 75 \cdot 1) \cdot 3 = 123 \cdot 41 - 321 \cdot 15 - 75 \cdot 3 = \\ &= 123 \cdot 41 - 321 \cdot 15 - (321 - 123 \cdot 2) \cdot 3 = 123 \cdot 47 - 321 \cdot 18 = \\ &= 123 \cdot 47 + 321 \cdot (-18). \end{aligned}$$

Bundan,  $3 = 123 \cdot 47 + 321 \cdot (-18)$  kelib chiqadi.

**Misol.** Berilgan  $n = 126$  soning natural bo'linuvchilari soni va yig'indisini, undan katta bo'lmagan va  $u$  bilan o'zaro tub sonlar sonini toping.

**Yechish.** Berilgan  $n$  sonining natural bo'luvchilari soni  $\tau(n)$  va natural bo'luvchilari yig'indisini  $\sigma(n)$ ,  $n$  dan katta bo'lmagan  $u$  bilan o'zaro tub sonlar soni  $\varphi(n)$  larni aniqlash uchun  $n$  sonining tub ko'paytuvchilarga kanonik yoyilmasini topamiz. Agar  $n = p_1^{\alpha_1} \dots p_n^{\alpha_n}$  bo'lsa, u holda

$$\tau(n) = (\alpha_1 + 1)(\alpha_2 + 1) \dots (\alpha_n + 1);$$

$$\sigma(n) = \frac{p_1^{\alpha_1+1} - 1}{p_1 - 1} \cdot \frac{p_2^{\alpha_2+1} - 1}{p_2 - 1} \dots \frac{p_n^{\alpha_n+1} - 1}{p_n - 1};$$

$$\varphi(n) = n \left(1 - \frac{1}{p_1}\right) \left(1 - \frac{1}{p_2}\right) \dots \left(1 - \frac{1}{p_n}\right) \text{ bo'ladi.}$$

$n = 126$  ning tub bo'luvchilarga kanonik yoyilmasini topamiz:

$$\begin{array}{l|l} 126 & 2 \\ 63 & 3 \\ 21 & 3 \\ 7 & 7 \\ 1 & \end{array}$$

Bundan,  $126 = 2^1 \cdot 3^2 \cdot 7^1$  ekan. U holda

a)  $\tau(126) = (1+1)(2+1)(1+1) = 2 \cdot 3 \cdot 2 = 12$ . Demak, 126 ning bo'luvchilari 12 ta. Haqiqatdan ham ular: 1, 2, 3, 6, 7, 9, 14, 18, 21, 42, 63, 126

b)  $\sigma(126) = \frac{2^2 - 1}{2 - 1} \cdot \frac{3^3 - 1}{3 - 1} \cdot \frac{7^2 - 1}{7 - 1} = \frac{3}{1} \cdot \frac{26}{2} \cdot \frac{48}{6} = 26 \cdot 12 = 312$ .

Haqiqatdan ham,  $1 + 2 + 3 + 6 + 7 + 9 + 14 + 18 + 21 + 42 + 63 + 126 = 312$ .

c)  $\varphi(126) = 126 \cdot \left(1 - \frac{1}{2}\right) \left(1 - \frac{1}{3}\right) \left(1 - \frac{1}{7}\right) = 126 \cdot \frac{1}{2} \cdot \frac{2}{3} \cdot \frac{6}{7} = 36$ .

Demak, 126 dan katta bo'lmagan,  $u$  bilan o'zaro tub sonlar soni 36 ta.

**Misol.** 23! ni tub ko'paytuvchilarga kanonik yozilmasini toping.

**Yechish.** Berilgan  $n!$  sonning tub ko'paytuvchilarga yoyilmasini topish uchun,  $n$  dan katta bo'lmagan tub sonlar qanday daraja bilan kanonik yoyilmada qatnashishini topamiz.

23 dan katta bo'lmagan tub sonlar 2, 3, 5, 7, 11, 13, 17, 19, 23

2 ning 23! ning kanonik yoyilmasidagi darajasini topamiz. Buning uchun 23 ni 2 ga bo'lamiz. Bo'linma 2 dan kichik son bo'lguncha bu jarayonni davom ettiramiz:

$$23 = 2 \cdot 11 + 1$$

$$11 = 2 \cdot 5 + 1$$

$$5 = 2 \cdot 2 + 1$$

$$2 = 2 \cdot 1 + 0$$

Demak, 2 ning kanonik yoyilmadan darajasi  $11 + 5 + 2 + 1 = 19$ .

3 ning darajasini topamiz:

$$23 = 3 \cdot 7 + 2$$

$$7 = 3 \cdot 2 + 1$$

3 ning darajasi  $7 + 2 = 9$ .

5 ning darajasini topamiz:  $23 = 5 \cdot 4 + 3$

5 ning darajasi 4.

$$23 = 7 \cdot 3 + 2$$

7 ning darajasi 3.

$$23 = 11 \cdot 2 + 1$$

11 ning darajasi 2.

13 ning darajasi 1, chunki  $23 = 13 \cdot 1 + 10$ .

Huddi shunday 17, 19, 23 larning ham yoyilmadagi darajalari 1 ga teng.

Demak,  $23! = 2^{19} \cdot 3^9 \cdot 5^4 \cdot 7^3 \cdot 11^2 \cdot 13 \cdot 17 \cdot 19 \cdot 23$ .

Misol.  $\begin{cases} a \cdot b = 768 \\ (a, b) = 8 \end{cases}$  sistemani qanoatlantiruvchi  $a$  va  $b$  sonlarni toping.

**Yechish.** Berilgan  $a$  va  $b$  sonlarning eng katta umumiy bo'luvchisi 8 ekanligidan, bu sonlarni  $a = 8k$  va  $b = 8l$ ,  $k, l \in \mathbb{Z}$  ko'rinishda yozib olamiz. Bu erda  $(k, l) = 1$ . Bundan  $a \cdot b = 8k \cdot 8l = 64 \cdot k \cdot l = 768$  va  $k \cdot l = 12$  larni hosil qilamiz. Demak, 12 o'zaro tub  $k$  va  $l$  sonlarning ko'paytmasi ko'rinishida ifodalanadi. Quyidagi holatlar bo'lishi mumkin:

| $k$ | $l$ | $k \cdot l$ |
|-----|-----|-------------|
| 1   | 12  | 12          |
| 3   | 4   | 12          |
| 4   | 3   | 12          |
| 12  | 1   | 12          |

| $a$ | $b$ | $a \cdot b$ |
|-----|-----|-------------|
| 8   | 96  | 768         |
| 24  | 32  | 768         |
| 32  | 24  | 768         |
| 96  | 8   | 768         |

Bundan,

Demak,  $(a, b): (8, 96), (24, 32), (32, 24), (96, 8)$



### Misol va mashqlar

- Tub va murakkab sonlarning quyidagi xossalarini isbotlang:
  - $a > 1$  murakkab sonning 1 dan boshqa eng kichik natural bo'luvchisi  $r$  bo'lsa, u holda  $r$  son tub son bo'ladi.
  - Har qanday natural  $a$  va  $r$  tub sonlari yoki o'zaro tub, yoki  $a$  son  $r$  ga bo'linadi.
  - Agar  $ab$  ko'paytma biror  $r$  tub songa bo'linsa, u holda

ko'paytuvchilardan kamida bittasi  $r$  ga bo'linadi.

1.4. Agar ko'paytma  $r$  tub songa bo'linib, uning barcha ko'paytuvchilari tub sonlardan iborat bo'lsa, u holda bu ko'paytuvchilardan biri  $r$  ga teng bo'ladi.

1.5. 1 dan boshqa ixtiyoriy natural son yoki tub son yoki tub sonlar ko'paytmasi shaklida yoziladi, agar bu ko'paytmada ko'paytuvchilarning o'rnini e'tiborga olinmasa, u holda bu ko'paytma yagona bo'ladi.

2. Eratosfen g'alviri yordamida berilgan sonlar orasidagi barcha tub sonlarni aniqlang:

- |                    |                     |
|--------------------|---------------------|
| 2.1. 1050 va 1150. | 2.2. 2100 va 2200.  |
| 2.3. 1100 va 1200. | 2.4. 2550 va 2650.  |
| 2.5. 1880 va 2000. | 2.6. 4550 va 4670.  |
| 2.7. 5555 va 5750. | 2.8. 4660 va 4770.  |
| 2.9. 4422 va 4525. | 2.10. 1122 va 1222. |

3. Berilgan natural sonning tub yoki murakkab ekanligini aniqlang:

- |                   |                    |
|-------------------|--------------------|
| 3.1. $n = 1559$ . | 3.2. $n = 1627$ .  |
| 3.3. $n = 1783$ . | 3.4. $n = 3061$ .  |
| 3.5. $n = 3709$ . | 3.6. $n = 4057$ .  |
| 3.7. $n = 1987$ . | 3.8. $n = 2339$ .  |
| 3.9. $n = 2671$ . | 3.10. $n = 3343$ . |

4. Butun sonlar halqasida bo'linish munosabatining quyidagi xossalarini isbotlang:

- $(\forall a \in \mathbb{Z}, a \neq 0) 0 : a;$
- $(\forall a \in \mathbb{Z}, a \neq 0) a : a;$
- $(\forall a \in \mathbb{Z}) a : 1;$
- $(\forall a, b, s \in \mathbb{Z}, b \neq 0, s \neq 0) ((a : b) \wedge (b : c)) \Rightarrow (a : s);$
- $(\forall a, b \in \mathbb{Z}, a \neq 0, b \neq 0) ((a : b) \wedge (b : a)) \Rightarrow (b = \pm a);$
- $(\forall a, b, s \in \mathbb{Z}, s \neq 0) a : s \Rightarrow ab : c;$
- $(\forall a, b \in \mathbb{Z}, s \neq 0) ((a : s) \wedge (b : s)) \Rightarrow (a \pm b) : s;$
- $(\forall a, b_i \in \mathbb{Z}, a \neq 0, i = \overline{1, n}) ((b_1 : a) \wedge (b_2 : a) \wedge \dots \wedge (b_n : a)) \Rightarrow$

$$\Rightarrow (b_1 s_1 \pm b_2 s_2 \pm \dots \pm b_n c_n) : a \quad (s_i \in \mathbb{Z}, i = \overline{1, n}).$$

5. Ixtiyoriy  $a$  butun son,  $b$  natural sonlar uchun shunday yagona  $q$  butun son va yagona manfiy  $r$  butun son topiladiki, natijada ushbu  $a = bq + r$   $0 \leq r < b$  munosabatlar o'rinli bo'lishini isbotlang.

6.  $n = p_1^{\alpha_1} p_2^{\alpha_2} \dots p_k^{\alpha_k}$  sonning bo'luvchisi  $d$  bo'lishi uchun  $d$  sonning kanonik yoyilmasi  $d = p_1^{\beta_1} p_2^{\beta_2} \dots p_k^{\beta_k}$  bo'lib, bunda  $\beta_i \leq \alpha_i$  ( $i = \overline{1, k}$ ) bo'lishi zarur va etarli ekanligini isbotlang.

7. Berilgan  $n$  natural sonning natural bo'luvchilari soni va yig'indisini:

7.1.  $n = 60$ .

7.2.  $n = 100$ .

7.3.  $n = 360$ .

7.4.  $n = 375$ .

7.5.  $n = 720$ .

7.6.  $n = 957$ .

7.7.  $n = 988$ .

7.8.  $n = 990$ .

7.9.  $n = 1000$ .

7.10.  $n = 1200$ .

7.11.  $n = 1542$ .

7.12.  $n = 3500$ .

7.13.  $n = 680$ .

7.14.  $n = 865$ .

7.15.  $n = 779$ .

7.16.  $n = 410$ .

8.  $n!$  ni tub ko'paytuvchilarga kanonik yoyilmasini toping:

8.1.  $n = 55$ .

8.2.  $n = 92$ .

8.3.  $n = 87$ .

8.4.  $n = 63$ .

8.5.  $n = 34$ .

8.6.  $n = 99$ .

8.7.  $n = 53$ .

8.8.  $n = 45$ .

8.9.  $n = 50$ .

8.10.  $n = 38$ .

8.11.  $n = 90$ .

8.12.  $n = 100$ .

9. Quyidagi xossalarni isbotlang:

9.1.  $(a, b) = (a+b, a+2b)$ .

9.2.  $(a; b) = d$  bo'lsa,  $u$  holda shunday  $u$  va  $v$  butun sonlar topiladiki, ular uchun  $au + bv = d$  tenglik bajariladi.

9.3.  $((a; c) = 1 \wedge (b; c) = 1) \Rightarrow ((ab; c) = 1)$ .

9.4.  $((ab; c) \wedge (a; c) = 1) \Rightarrow (b; c) (c \neq 0)$ .

9.5.  $((a; b) = 1) \Rightarrow ((a^n; b^n) = 1) (\forall n \in \mathbb{N})$ .

9.6.  $((a; b) = d) \Rightarrow ((\frac{a}{d}; \frac{b}{d}) = 1)$ ;

9.7.  $((a; b) \wedge (a; c) \wedge ((b; c) = 1) \Rightarrow (a; bc) (b \neq 0, c \neq 0)$

9.8.  $a = bq + r \Rightarrow (a; b) = (b; r)$ .

9.9.  $d$  son  $a$  va  $b$  sonlarning EKUBi bo'lishi uchun  $d$  umumiy bo'luvchi  $a$  va  $b$  sonlarning har qanday umumiy bo'luvchisiga bo'linishi zarur va etarli.

9.10. Agar  $(a_1, a_2, \dots, a_n) = d$  bo'lib,  $(a_1, a_2) = d_2, (d_2, a_3) = d_3, \dots, (d_{n-1}, a_n) = d_n$  bo'lsa,  $u$  holda  $d_n = d$  bo'ladi.

10. Ikki usulda berilgan sonlarning EKUBini toping:

10.1. 1232, 1672.

10.2. 135, 8211.

10.3. 589, 343.

10.4. 29719, 76501.

10.5. 469459, 519203.

10.6. 179370199, 4345121.

10.7. 12606, 6494.

10.8. 162891, 32176.

10.9. 7650, 25245.

10.10. 35574, 192423.

10.11. 10140, 92274.

10.12. 46550, 37730.

11.  $x$  va  $y$  natural sonlarni toping:

11.1.  $\begin{cases} x + y = 150, \\ (x, y) = 30; \end{cases}$

11.6.  $\begin{cases} x \cdot y = 20, \\ [(x, y) = 10; \end{cases}$

11.2.  $\begin{cases} x \cdot y = 8400, \\ (x, y) = 20; \end{cases}$

11.7.  $\begin{cases} (x, y) = 24, \\ [(x, y) = 2496; \end{cases}$

11.3.  $\begin{cases} x + y = 667, \\ [(x, y) = 120 \cdot (a, b); \end{cases}$

11.8.  $\begin{cases} x \cdot y = 168, \\ (x, y) = 14; \end{cases}$

$$11.4. \begin{cases} \frac{x}{y} = \frac{11}{7}, \\ (x, y) = 45; \end{cases}$$

$$11.9. \begin{cases} \frac{x}{y} = \frac{5}{9}, \\ (x, y) = 28; \end{cases}$$

$$11.5. \begin{cases} \frac{x}{(x, y)} + \frac{y}{(x, y)} = 18, \\ [x, y] = 975; \end{cases}$$

$$11.10. \begin{cases} x + y = 100, \\ [x, y] = 495; \end{cases}$$



### Takrorlash uchun savollar

1. Arifmetikaning asosiy teoremasini bayon eting.
2. Tub va murakkab sonlarning qanday xossalariini bilasiz?.
3. Bo'linish munosabati xossalariini bayon eting?.
4. Qoldiqli bo'lish haqidagi teoremani bayon eting.
5. Sonli funktsiya deb nimaga aytiladi?
6.  $\tau(n)$  va  $\sigma(n)$  sonli funktsiyalar qanday hisoblanadi?
7. Ikkita sonning EKUBi deb nimaga aytiladi?
8.  $n$  ta sonning EKUBi qanday topiladi?
9. Ikkita sonning EKUKi deb nimaga aytiladi?
10.  $n$  ta sonning EKUKi qanday topiladi?
11. O'zaro tub sonlar deb nimaga aytiladi?
12. Evklid algoritimini tushuntiring.

### 25-§. Chekli zanjir kasrlar. Munosib kasrlar.

Asosiy tushunchalar: uzluksiz zanjir kasr, chekli zanjir kasr, munosib kasr.  
Ushbu

$$a_0 + \frac{b_1}{a_1 + \frac{b_2}{a_2 + \dots}}$$

$$\dots + \frac{b_k}{a_k}$$

$(a_i (i=0, \bar{k}), b_j (j=1, \bar{k}))$  butun sonlar ko'rinishdagi ifoda uzluksiz zanjir kasr deyiladi.

Agar (1) da  $b_1 = b_2 = \dots = b_k = 1$ ,  $a_0$ -butun son,  $a_1, a_2, \dots, a_k$ -natural sonlar bo'lib  $a_k > 1$

bo'lsa, u holda ushbu  $a_0 + \frac{1}{a_1 + \frac{1}{a_2 + \dots + \frac{1}{a_k}}}$  ifodani chekli zanjir kasr

deyiladi.

$$T = a_0 + \frac{1}{a_1 + \frac{1}{a_2 + \dots + \frac{1}{a_n}}}$$
 bo'lsin.

$A_0 = a_0$  deb olaylik. U holda buni nolinchii tartibli munosib kasr deyiladi.

$A_1 = a_0 + \frac{1}{a_1} = \frac{a_0 a_1 + 1}{a_1}$  - birinchi tartibli munosib kasr deyiladi.

$A_2 = a_0 + \frac{1}{a_1 + \frac{1}{a_2}}$  - ikkinchi tartibli munosib kasr deyiladi.

.....  
 $A_n = T$  esa  $n$ -tartibli munosib kasr deyiladi.

$A_0 = \frac{a_0}{1} = \frac{P_0}{Q_0}$  deb belgilaylik. U holda  $R_0 = a_0$ ,  $Q_0 = 1$  hosil bo'ladi;

$A_1 = a_0 + \frac{1}{a_1} = \frac{a_0 a_1 + 1}{a_1} = \frac{P_1}{Q_1}$  desak, u xolda  $R_1 = a_0 a_1 + 1$ ,  $Q_1 = a_1$  xosil buladi;

$A_2 = a_0 + \frac{1}{a_1 + \frac{1}{a_2}} = \frac{P_2}{Q_2}$  - ikkinchi tartibli munosib kasr;

.....  
 $A_n = T = \frac{P_n}{Q_n}$   $n$ - tartibli munosabat kasr.

Shu yo'l bilan  $R_0, R_1, R_2, \dots, Q_0, Q_1, Q_2, \dots$  ketma-ketliklarni hosil qilamiz.  
 Bu ketma-ketliklardan quyidagi rekurrent formulalarni hosil qilamiz:

$$R_k = P_{k-1} a_k + R_{k-2}, \quad Q_k = Q_{k-1} a_k + Q_{k-2}$$

$\frac{P_k}{Q_k} - k$  - tartibli munosib kasr deyiladi.

$R_2=0, R_1=1, Q_2=1, Q_1=0$  deb belgilaylik. Lekin ularning o'zi ma'noga ega emas. Yuqoridagi tushunchalardan quyidagi jadvalni tuzamiz:

|       |    |    |       |       |       |     |           |       |
|-------|----|----|-------|-------|-------|-----|-----------|-------|
| $k$   | -2 | -1 | 0     | 1     | 2     | ... | $n-1$     | $n$   |
| $A_k$ | -  | -  | $a_0$ | $a_1$ | $a_2$ | ... | $a_{n-1}$ | $a_n$ |
| $P_k$ | 0  | 1  | $P_0$ | $P_1$ | $P_2$ | ... | $P_{n-1}$ | $P_n$ |
| $Q_k$ | 1  | 0  | $Q_0$ | $Q_1$ | $Q_2$ | ... | $Q_{n-1}$ | $Q_n$ |

**Misol.** Berilgan  $\frac{104}{23}$  kasrni chekli zanjir kasr ko'rinishida ifodalang va uning munosib kasrlarini toping.

**Yechish.**  $\frac{104}{23}$  kasrni chekli zanjir kasr ko'rinishida ifodalash uchun 104 va 23 sonlari uchun Evklid algoritmini tuzamiz.

$$\begin{aligned} 104 &= 23 \cdot 4 + 12; \\ 23 &= 12 \cdot 1 + 11; \\ 12 &= 11 \cdot 1 + 1; \\ 11 &= 1 \cdot 11 + 0. \end{aligned}$$

Evklid algoritmidagi tengliklarning har ikkala tomonini bo'luvchilarga bo'lamiz:

$$\begin{aligned} \frac{104}{23} &= 4 + \frac{12}{23}; \\ \frac{23}{12} &= 1 + \frac{11}{12}; \\ \frac{12}{11} &= 1 + \frac{1}{11}; \end{aligned}$$

$$\frac{11}{1} = 11.$$

Hosil bo'lgan tengliklarning o'ng tomonidagi kasr sonni uning teskarisi bilan almashtirish natijasida

$$\frac{104}{23} = 4 + \frac{12}{23} = 4 + \frac{1}{\frac{23}{12}} = 4 + \frac{1}{1 + \frac{11}{12}} = 4 + \frac{1}{1 + \frac{1}{1 + \frac{1}{11}}}$$

chekli zanjiri hosil qilamiz. Uni qisqacha  $\frac{104}{23} = [4; 1, 1, 11]$  ko'rinishida ifodalaymiz. Agar berilgan kasr manfiy bo'lsa, birinchi qoldiqni musbat qilib olamiz. Masalan,  $-\frac{23}{13} = -2 + \frac{3}{13}$  va kasr qismi chekli zanjir ko'rinishida

$$\text{ifodalanadi: } -\frac{23}{13} = -2 + \frac{3}{13} = -2 + \frac{1}{\frac{13}{3}} = -2 + \frac{1}{4 + \frac{1}{3}} = [-2; 4, 3]$$

Berilgan  $\frac{104}{23} = [4; 1, 1, 11]$  ning munosib kasrlarini topish uchun quyidagi jadvalni tuzamiz:

|       |    |   |   |   |     |
|-------|----|---|---|---|-----|
| $k$   | -1 | 0 | 1 | 2 | 3   |
| $q_k$ | -  | 4 | 1 | 1 | 11  |
| $P_k$ | 1  | 4 | 5 | 9 | 104 |
| $Q_k$ | 0  | 1 | 1 | 2 | 23  |

Demak,  $\frac{P_0}{Q_0} = 4; \frac{P_1}{Q_1} = 5; \frac{P_2}{Q_2} = \frac{9}{2}; \frac{P_3}{Q_3} = \frac{104}{23}$ .

**Misol.** Berilgan  $\sqrt{14}$  sonni zanjir kasr ko'rinishida ifodalang.

**Yechish.**

$$\sqrt{14} = 3 + \frac{1}{\alpha_1};$$

$$\alpha_1 = \frac{1}{\sqrt{14} - 3} = \frac{\sqrt{14} + 3}{5} = 1 + \frac{1}{\alpha_2};$$

$$\alpha_2 = \frac{1}{\frac{\sqrt{14+3}}{5} - 1} = \frac{5}{\sqrt{14-1}} = \frac{\sqrt{14+2}}{2} = 2 + \frac{1}{\alpha_1};$$

$$\alpha_3 = \frac{1}{\frac{\sqrt{14+2}}{2} - 2} = \frac{2}{\sqrt{14-2}} = \frac{\sqrt{14+2}}{5} = 1 + \frac{1}{\alpha_2};$$

$$\alpha_4 = \frac{1}{\frac{\sqrt{14+2}}{5} - 1} = \frac{5}{\sqrt{14-3}} = \sqrt{14+3} = 6 + \frac{1}{\alpha_3};$$

$$\alpha_5 = \frac{1}{\sqrt{14+3} - 6} = \frac{1}{\sqrt{14-3}};$$

$\alpha_5 = \alpha_1$  bo'lganligi uchun, yana yuqoridagi jarayon hosil bo'ladi.

Demak,  $\sqrt{14} = [3; (1, 2, 1, 6)]$ .

**Misol.**  $-117x + 343y = 119$  tenglamani butun sonlar to'plamida yeching.

**Yechish.** Tenglamani  $117(-x) + 343y = 119$  ko'rinishida yozib olamiz va  $ax + by + c$  tenglama agar  $(a, b) = 1$  bo'lsa

$$x = (-1)^{n-1} \cdot c \cdot Q_{n-1} + bt$$

$$y = (-1)^n \cdot c \cdot P_{n-1} - at, \quad t \in \mathbb{Z}$$

formulalar orqali topiladigan butun yechimlarga ega. Buning uchun  $\frac{a}{b}$  kasrning munosib kasrlari topiladi.

$$\frac{a}{b} = \frac{117}{343}$$

uchun chekli zanjir kasrni topamiz:

$$117 = 0 \cdot 343 + 117;$$

$$343 = 117 \cdot 2 + 109;$$

$$117 = 109 \cdot 1 + 8;$$

$$109 = 8 \cdot 13 + 5;$$

$$8 = 5 \cdot 1 + 3;$$

$$5 = 3 \cdot 1 + 2;$$

$$3 = 2 \cdot 1 + 1;$$

$$2 = 1 \cdot 2 + 0.$$

Demak,  $\frac{117}{343} = [0; 2, 1, 13, 1, 1, 1, 2]$ . Munosib kasrlar jadvalini tuzamiz:

|       |    |   |   |   |    |    |    |     |     |
|-------|----|---|---|---|----|----|----|-----|-----|
| $k$   | -1 | 0 | 1 | 2 | 3  | 4  | 5  | 6   | 7   |
| $q_k$ | -  | 0 | 2 | 1 | 13 | 1  | 1  | 1   | 2   |
| $P_k$ | 1  | 0 | 1 | 1 | 14 | 15 | 29 | 44  | 117 |
| $Q_k$ | 0  | 1 | 2 | 3 | 41 | 44 | 85 | 129 | 343 |

$P_6 = 44$ ,  $Q_6 = 129$  lardan foydalanamiz.

$$\text{Xususiy yechim: } \begin{cases} -x_0 = (-1)^6 \cdot 119 \cdot 129 = 15351; \\ y_0 = (-1) \cdot 119 \cdot 44 = -5236 \end{cases}$$

Umumiy yechim:

$$\begin{cases} -x = 15351 + 343t \\ y = -5236 - 117t, \quad t \in \mathbb{Z} \end{cases} \quad \text{yoki} \quad \begin{cases} x = -15351 - 343t \\ y = -5236 - 117t, \quad t \in \mathbb{Z} \end{cases}$$

Berilgan misolni yechishda  $-\frac{117}{343}$  uchun zanjir kasrni tuzish ham mumkin.

U holda  $-\frac{117}{343} = [-1; 1, 1, 1, 13, 1, 1, 1, 2]$  bo'lib,  $k = 8$ ,  $a = -117$ ,  $b = 343$ ,

$c = 119$ ,  $P_{n-1} = P_7 = -44$ ,  $Q_{n-1} = Q_7 = 129$  bo'ladi.

Undan  $\begin{cases} x = -15351 + 343t \\ y = 5236 + 117t, \quad t \in \mathbb{Z} \end{cases}$  yechimlar hosil bo'ladi.



## Misol va mashqlar

1. Quyidagi tasdiqlarni isbotlang:

1.1. Har qanday ratsional con chekli zanjir kasrga yoyiladi va bu yoyilma yagona bo'ladi.

1.2.  $A_k = \frac{P_k}{Q_k} (k = \overline{0, n})$ .

1.3.  $R_k Q_{k-1} - P_{k-1} Q_k = (-1)^{k-1}$  tenglik  $k$  ning har qanday qiymatida to'g'ri bo'ladi.

1.4.  $A_i = \frac{P_i}{Q_i}$  munosib kasrning surati bilan maxraji o'zaro tub, ya'ni

$(R_k; Q_k) = 1$  bo'ladi.

2. Berilgan kasrni chekli zanjir kasr ko'rinishida ifodalang:

2.1.  $\frac{323}{17}$     2.2.  $\frac{135}{279}$     2.3.  $-\frac{187}{63}$     2.4.  $\frac{96}{67}$

2.5.  $\frac{30}{337}$     2.6.  $-\frac{12}{15}$     2.7.  $\frac{127}{52}$     2.8.  $\frac{24}{35}$

2.9. 1,23.    2.10.  $\frac{71}{41}$     2.11.  $\frac{157}{225}$     2.12.  $\frac{507}{1001}$

3. Berilgan irratsional sonlarni chekli zanjir kasr orqali ifodalang:

3.1.  $\sqrt{11}$ .    3.2.  $\sqrt{12}$ .    3.3.  $\sqrt{13}$ .

3.4.  $\sqrt{28}$ .    3.5.  $\sqrt{30}$ .    3.6.  $\sqrt{59}$ .

3.7.  $1 + \sqrt{2}$ .    3.8.  $\frac{1 + \sqrt{3}}{2}$ .    3.9.  $\frac{2 + \sqrt{5}}{3}$ .

3.10.  $\frac{3 + \sqrt{5}}{2}$ .    3.11.  $\frac{2 + \sqrt{7}}{2}$ .    3.12.  $\frac{3 + \sqrt{10}}{3}$ .

4. Quyidagi zanjir kasrlar orqali ifodalanuvchi qisqarmas kasrlarni toping:

4.1. [2;1,3,4,2].    4.7. [4;(3,2,1)].

4.2. [2;1,19,1,3].    4.8. [(2,1)].

4.3. [2;1,1,3,1,2].    4.9. [3;(3,6)].

4.4. [1;1,2,3,4].    4.10. [1;(1,2)].

4.5. [0;4,1,2,5,6].    4.11. [1;7,(1,6)].

4.6. [-2;1,3,1,1,5].    4.12. [3;(5,2,1,2)].

5. Berilgan tenglamalarni butun sonlar to'plamida yeching:

5.1.  $38x + 117y = 209$ ;    5.2.  $23x - 42y = 72$ ;

5.2.  $119x - 68y = 34$ ;    5.4.  $15x + 28y = 185$ ;

5.3.  $41x + 114y = 5$ ;    5.6.  $90x - 5y = 5$ ;

5.4.  $49x + 9y = 400$ ;    5.8.  $10x - 11y = 15$ ;

5.5.  $12x + 31y = 170$ ;    5.10.  $31x - 47y = 23$ ;

5.6.  $37x + 23y = 15$ ;    5.12.  $101x + 39y = 89$ ;

5.7.  $53x + 17y = 25$ ;    5.14.  $-26x + 174y = 2$ ;

5.8.  $64x - 39y = 15$ ;    5.16.  $-6x + 11y = 29$ ;

5.9.  $3827x + 3293y = 1869$ ;    5.18.  $-10x + 23y = 17$ ;

5.10.  $571x + 359y = -10$ ;    5.20.  $903x + 5y = 43$ .



## Takrorlash uchun savollar

1. Uzluksiz kasr deb nimaga aytiladi?
2. Chekli zanjir kasr deb nimaga aytiladi?
3. Ratsional sonni chekli zanjir kasrga yagona yul bilan yoyishni bayon eting.
4. Munosib kasrlar haqida tushuncha bering.
5. Munosib kasrlar haqidagi teoremlarni bayon eting.
6. Chekli zanjir kasrlar tatbiqiga misollar keltiring.



## 26-§. Sistematik sonlar va ular ustida amallar

**Asosiy tushunchalar:** sanoq sistemasi, asosi g ga teng bo'lgan sistematik son, sistematik sonlarni qo'shish, ayirish, ko'paytirish, bo'lish.

O'nlik sanoq sistemasidan boshqa 2, 5, 7, 12, 60, ... sanoq sistemalari ham mavjud. Bu sanoq sistemalarining barchasi bitta umumiy yo'nalish asosida quriladi.

$m > 1$  natural son bo'lib,  $M = \{0, 1, 2, \dots, m-1\}$  to'plam berilganda har qanday  $a$  natural son uchun ushbu  $a = a_0 + a_1m + a_2m^2 + \dots + a_nm^n = a_0m^0 + a_1m^1 + \dots + a_nm^n$  ( $a_i \in M, i = \overline{1, n}, a_n \neq 0$ ) yoyilma mavjud va yagonadir.  $a$  natural sonning bu ko'rinishiga  $a$  ni  $m$  ning darajalari bo'yicha yoyish deyiladi.

Ixtiyoriy  $g \geq 2$  natural son va har qanday  $m$  natural son uchun

$$m = a_n g^n + a_{n-1} g^{n-1} + \dots + a_1 g + a_0 \quad (0 \leq a_i \leq g-1, i = \overline{0, n-1}, 1 \leq a_n \leq g-1)$$

tenglikni yoza olamiz. Undagi  $a_0, a_1, \dots, a_n$  lar  $m$  sonning raqamlari deyiladi uni



$m = (a_{n-1} a_{n-2} \dots a_1 a_0)_2$  ko'rinishida qisqacha yozish mumkin. Bu ko'rinishidagi son asosli **qaratilgan** sistematik son deyiladi.

$g$  asosli ixtiyoriy  $a$  va  $b$  sonlarni qo'shish, ayirish, ko'paytirish va bo'lish ko'phadni ko'phadga ko'paytirish kabi bajariladi.

**Misol.** Hisoblang:

$$(202332_4 + 22201_4) + (220111_4 - 32303_4) - 23230301_4 : 113_4$$

**Yechish.** 4 lik sanoq sistemasida berilgan amallarni bajarish uchun qo'shish va ko'paytirish amallari jadvalarini tuzib olamiz:

|   |   |    |    |    |
|---|---|----|----|----|
| + | 0 | 1  | 2  | 3  |
| 0 | 0 | 1  | 2  | 3  |
| 1 | 1 | 2  | 3  | 10 |
| 2 | 2 | 3  | 10 | 11 |
| 3 | 3 | 10 | 11 | 12 |

|   |   |   |    |    |
|---|---|---|----|----|
| x | 0 | 1 | 2  | 3  |
| 0 | 0 | 0 | 0  | 0  |
| 1 | 0 | 1 | 2  | 3  |
| 2 | 0 | 2 | 10 | 12 |
| 3 | 0 | 3 | 12 | 21 |

Berilgan misoldagi amallarni bajaramiz

1)

$$\begin{array}{r} 202332_4 \\ + 22201_4 \\ \hline 231133_4 \end{array}$$

Tekshirish:

$$\begin{array}{r} - 231133_4 \\ + 202332_4 \\ \hline 22201_4 \end{array}$$

2)

$$\begin{array}{r} - 220111_4 \\ + 32303_4 \\ \hline 121202_4 \end{array}$$

Tekshirish:

$$\begin{array}{r} + 121202_4 \\ + 32303_4 \\ \hline 220111_4 \end{array}$$

3)

$$\begin{array}{r} + 231133_4 \\ + 121202_4 \\ \hline 1013001_4 \end{array}$$

Tekshirish:

$$\begin{array}{r} - 1013001_4 \\ + 231133_4 \\ \hline 121202_4 \end{array}$$

4)

$$\begin{array}{r} - 23230301_4 \quad | \quad 113_4 \\ \hline 232 \quad | \quad 200203_4 \\ \hline - 303 \\ \hline 232 \\ \hline - 1101 \\ \hline 1011 \\ \hline 30_4 \end{array}$$

Tekshirish:

$$\begin{array}{r} \times 200203_4 \\ \hline 113_4 \\ + 1201221_4 \\ \hline 2002031_4 \\ + 20020344_4 \\ \hline 23230211_4 \end{array}$$

$$23230211_4 + 30_4 = 2323301_4$$

5)

$$\begin{array}{r} - 1013001_4 \\ + 200203_4 \\ \hline 1213210_4 \end{array}$$

Demak, javob: 1213210<sub>4</sub>

**Misol.**  $n$  asosda berilgan  $a$  sonni  $m$  va  $k$  asoslarga o'tkazing:

$$a = 211, n = 3, m = 2, k = 4$$

**Yechish.** Berilgan  $a$  sonni 3 lik sanoq sistemasida uni 2 lik sanoq sistemasiga o'tkazish uchun berilgan sonni hosil bo'ladigan bo'linmalarni 2 ga bo'lamiz:

$$\begin{array}{l} 211_3 \quad | \quad 2_3 \\ - 2 \quad | \quad 102_3 \\ \hline 11 \end{array} \quad \begin{array}{l} 102_3 \quad | \quad 2_3 \\ - 2 \quad | \quad 12_3 \\ \hline 12 \end{array} \quad \begin{array}{l} 12 \quad | \quad 2_3 \\ - 11 \quad | \quad 2_3 \\ \hline 1 \end{array} \quad \begin{array}{l} 2_3 \quad | \quad 2_3 \\ - 2_3 \quad | \quad 1_3 \\ \hline 0 \end{array} \quad \begin{array}{l} 1_3 \quad | \quad 2_3 \\ - 0 \quad | \quad 0_3 \\ \hline 1 \end{array}$$

Bu jarayonni bo'linmada 0 hosil bo'lguncha davom ettiramiz. Oxirgi qoldiqdan boshlab barcha qoldiqlar yordamida berilgan sonning 2 lik sanoq sistemasidagi ifodasini topamiz:  $211_3 = 10110_2$

Tekshirish ikki usulda bajariladi:

1-usul.  $211_3$  va  $10110_2$  sonlarni o'nlik asosga o'tkazilib solishtiriladi.

2-usul.  $10110_2$  uchlik asosga o'tkaziladi.

$$211_3 = 2 \cdot 3^2 + 1 \cdot 3^1 + 1 \cdot 3^0 = 18 + 3 + 1 = 22_{10}$$

$$10110_2 = 1 \cdot 2^4 + 0 \cdot 2^3 + 1 \cdot 2^2 + 1 \cdot 2^1 + 0 \cdot 2^0 = 16 + 4 + 2 = 22_{10}$$

Demak,  $211_3$  ni ikkilik asosda to'g'ri ifodalangan.  $211_3$  ni to'rtlik asosdagi ifodasini topamiz. Buning uchun  $211_3$  ning o'nlik asosdagi ifodasini topib, hosil bo'lgan sonni to'rtlik asosga o'tkazamiz:

$$211_3 = 22_{10}$$

$$\begin{array}{r} - \\ - 22_{10} \\ \hline 20 \\ \hline 2 \end{array} \begin{array}{r} 4_{10} \\ 5_{10} \\ \hline 1 \end{array} \quad \begin{array}{r} - 5_{10} \\ - 4 \\ \hline 1 \end{array} \begin{array}{r} 4_{10} \\ 1_{10} \\ \hline 1 \end{array} \quad \begin{array}{r} - 1_{10} \\ - 0 \\ \hline 1 \end{array} \begin{array}{r} 4_{10} \\ 0_{10} \\ \hline 1 \end{array}$$

Demak,  $22_{10} = 112_4$  bundan  $211_3 = 112_4$ . Tekshirish yuqoridagi usullarda bajariladi.



### Misol va mashqlar

1. Hisoblang:

1.1.  $1101_2 + 1011_2$ .

1.2.  $1011_2 \cdot 1101_2$ .

1.3.  $1000110_2 - 11011_2$ .

1.4.  $100011_2 : 101_2$ .

1.5.  $3604_7 \cdot 423_7$ .

1.6.  $7(10)_{12} \cdot 5(11)73_{12}$ .

1.7.  $23054_7 + 4326_7$ .

1.8.  $(10)(11)792_{12} + 9534(10)_{12} + 70(10)0_{12}$ .

1.9.  $26153_7 : 326_7$ .

1.10.  $8(1005(11)_{12} : 9(10)_{12}$ .

1.11.  $101_8 : 32_8$ .

2. Hisoblang:

2.1.  $11011,101_2 + 101,011_2$ ;

2.2.  $11,001_2 \cdot 1,01_2$ ;

2.3.  $111,01_2 \cdot 101,101_2$ ;

2.4.  $0,25_8 \cdot 0,43_8$ ;

2.5.  $2,5_8 \cdot 3,4_8$ .

3. Amallarni bajaring:

3.1.  $7306_8 + 25645_8 - 6774_8 - 26156_8$ ;

3.2.  $(425_6 \cdot 54_6 - 531_6 \cdot 43_6) : 245_6$ ;

3.3.  $20671_8 : 131_8 - 140_8$ ;

3.4.  $23213_5 : 32_5 + 113_5 \cdot 3_5 - 1242_5$ ;

3.5.  $232011_5 : 104 + 1234_5 \cdot 322_5 - 1022131_5$ ;

3.6.  $(563_8 + 217_8) \cdot 15_8 + (2365_8 - 636_8) : 17_8 - 15122_8$ ;

3.7.  $120111_3 : 102_3 + 201_3 \cdot 12_3 - 11220_3$ ;

3.8.  $6325_7 - 456_7 - 150335_7 : 23_7 - 551_7$ ;

3.9.  $3215_7 \cdot 24_7 - 11461_7 : 25_7 + 1532_7 - 115044_7$ ;

3.10.  $(4123_8 - 4221_8) \cdot 11_8 + (1222_8 + 773_8) : 3_8$ ;

3.11.  $(3333_4 + 2222_4) \cdot 12_4 - (231020_4 + 3333333_4) : 23_4$ ;

3.12.  $[(215_8 + 532_8) \cdot 16_8 - (11031_8 - 527_8)32_8] : 14775_8$ ;

3.13.  $[(351_6 \cdot 14_6 - 1153_6 : 31_6 - 150_6) : 205_6] : 25_6$ .

4. Berilgan sonlarni o'nlik sanoq sistemasida ifodalang:

4.1.  $100111_2$ ;

4.2.  $11001101_2$ ;

4.3.  $345_8$ ;

4.4.  $5071_8$ ;

4.5.  $1300_8$ ;

4.6.  $33311_7$ ;

4.7.  $4602_7$ ;

4.8.  $(10)6(11)_{12}$ ;

4.9.  $26014_7$ ;

4.10.  $42125_6$ ;

4.11.  $530415_6$ .

5. Berilgan sonlarni o'nlik sanoq sistemasida ifodalang:

5.1.  $0,111_2$ ;

5.2.  $0,110_2$ ;

5.3.  $11001, 1111_2$ ;

5.4.  $437,321_8$ ;

5.5.  $0,027_8$ .

6. Bir sanoq sistemasidan ikkinchisiga o'ting:

6.1.  $33311_7 \rightarrow x_{12}$ ;

6.2.  $21000122122_3 \rightarrow x_9$ ;

6.3.  $4672510_9 \rightarrow x_3$ ;

6.4.  $11110111011100001_2 \rightarrow x_8$ ;

6.5.  $21066754_8 \rightarrow x_2$ ;

6.6.  $206315_7 \rightarrow x_5$ ;

6.7.  $32014 \rightarrow x_8$ .

7. O'nlik sanoq sistemasidan berilgan sanoq sistemalariga o'ting:

7.1.  $2042 \rightarrow x_2, y_3, z_5$ ;

7.2.  $2786 \rightarrow x_2, y_3, z_5$ ;

7.3.  $729 \rightarrow x_7$ ;

7.4.  $231632 \rightarrow x_7$ ;

7.5.  $23163 \rightarrow x_8$ ;

7.6.  $17527 \rightarrow x_8$ ;

8. x ni toping:

8.1.  $201_x = 41_8$ ;

8.2.  $203_x = 53_{10}$ ;

8.3.  $106_x = 153_7$ ;

8.4.  $236_x = 1240_5$ ;

8.5.  $324_x = 10022_3$ ;

8.6.  $541_x = 2014_6$ ;

8.7.  $364_x = 3001_4$ ;

8.8.  $401_x = 265_7$ ;

8.9.  $100_x = 34_7$ .

9. Quyidagi tengliklar o'rinli bo'lgan sanoq sistemasini toping:

a)  $12 + 13 = 30$ ;

b)  $15 + 16 = 33$ ;

v)  $35 + 40 = 115$ ;

g)  $236 - 145 = 61$ ;

d)  $263 - 214 = 46$ ;

e)  $216 \cdot 3 = 654$ ;

j)  $656 : 5 = 124$ ;

z)  $736 : 6 = 121$ ;

k)  $1520 : 12 = 123$ ;

l)  $10 \cdot 10 = 100$ .





## Takrorlash uchun savollar

1. Sanoq sistemalari haqida tushuncha bering.
2. Sistematik son deb nimaga aytiladi?
3. Sistematik sonlar ustida amallar qanday bajariladi?
4. Bir sanoq sistemasidan boshqa sanoq sistemasiga o'tishni tushuntiring.

## XII MODUL. TAQQOSLAMALAR



### 27-§. Butun sonlar halqasida taqqoslamalar. Eyler va Ferma teoremlari

**Asosiy tushunchalar:** "taqqoslanadi" munosabati, chegirmalar sinfi, chegirmalarning to'la sistemasi, chegirmalarning keltirilgan sistemasi, Eyler teoremasi, Ferma teoremasi.

$Z$ -butun sonlar halqasi bo'lib,  $m \geq 1$  natural son bo'lsin.

Agar  $Z$  halqaga tegishli  $a$  va  $b$  sonlarni  $m$  natural songa bo'lganda hosil bo'lgan qoldiqlar teng bo'lsa, yoki  $a-b$  ayirma  $m$  ga bo'linsa, ya'ni  $a=b+mq$  tenglik o'rinli bo'lsa,  $u$  holda  $a$  va  $b$  sonlar  $m$  modul bo'yicha taqqoslanadi deyiladi va uni  $a \equiv b \pmod{m}$  ko'rinishda belgilanadi.

$m$  ga bo'linganda  $r$  ga teng bir hil qoldiq beradigan butun sonlar to'plami  $m$  modul buyicha chegirmalar sinfi deyiladi va  $\bar{r}$  kabi belgilanadi.

$m$  modul bo'yicha tuzilgan har bir chegirmalar sinfidan ixtiyoriy bittadan element olib tuzilgan to'plamga  $m$  modul bo'yicha chegirmalarning to'la sistemasi deyiladi.

$m$  modul bilan o'zaro tub bo'lgan barcha chegirmalar sinfidan ixtiyoriy bittadan chegirma olib tuzilgan to'plam chegirmalarning  $m$  modul bo'yicha keltirilgan sistemasi deyiladi.

Eyler teoremasi. Agar  $(a;m)=1$  bo'lsa,  $u$  holda  $a^{\varphi(m)} \equiv 1 \pmod{m}$  taqqoslama o'rinli bo'ladi.

Ferma teoremasi. Agar  $(a;r)=1$ ,  $u$  holda  $a^{r-1} \equiv 1 \pmod{r}$  taqqoslama o'rinli bo'ladi.

**Misol.**  $a=2511$  sonini  $b=123$  ga bo'lgandagi qoldiqni toping.

**Yechish.** Qoldiqli bo'lishi xaqidagi teorimadan foydalanib  $a=bq+r$ ,

$0 \cdot r = b$  ifodani topamiz.  $2511 = 123 \cdot 20 + 51$

Demak,  $a = 2511$  ni  $b = 123$ ga bo'lganda  $r = 51$  qoldiq qoladi.

**Misol.**  $a = 25^{112}$  ni  $b = 16$  ga bo'lgandagi qoldiqni toping.

**Yechish.**  $a = 25^{112}$  sonini 16ga bo'lish uchun taqqoslamaning xossalariidan foydalanamiz.  $25 = 16 \cdot 1 + 9$  ekanligidan  $25 \equiv 9 \pmod{16}$  qilib chiqadi. Bundan  $25^{112} \equiv 9^{112} \equiv (9^2)^{56} \equiv 81^{56}$ ,  $81 = 16 \cdot 5 + 1$  ekanligini e'tiborga olsak, u xolda  $25^{112} \equiv 81^{56} \equiv 1^{56} \equiv 1 \pmod{16}$ .

Demak,  $25^{112}$  ni 16ga bo'lganda 1 qoldiq qoladi.

**Misol.** Agar  $100a + 100b + c \equiv 0 \pmod{21}$  bo'lsa, u xolda  $a - 2b + 4c \equiv 0 \pmod{21}$  ekanligini isbotlang.

**Isbot.** Taqqoslamaning ikkala tomonini modul bilan o'zaro tub 4 songa ko'paytiramiz:  $400a + 40b + 4c \equiv 0 \pmod{21}$ .

$400 = 21 \cdot 19 + 1$ ,  $40 = 21 \cdot 2 + (-2)$ ,  $4 = 21 \cdot 0 + 4$  lardan foydalanib quyidagi

taqqoslamalarni yozamiz:

$$400a \equiv a \pmod{21}, \text{ chunki } 400a - a = 399a \div 21;$$

$$40b \equiv -2b \pmod{21}, \text{ chunki } 40b - (-2b) = 42b \div 21;$$

$$4c \equiv 4c \pmod{21}, \text{ chunki } 4c - 4c = 0 \div 21;$$

Birilgan taqqoslamadan yuqoridagi taqqoslamalarni e'tiborga olib

$$400a + 40b + 4c \equiv a - 2b + 4c \pmod{21} \text{ taqqoslamani hosil qilamiz.}$$

Demak,  $400a + 40b + 4c \equiv 0 \pmod{21}$  shartdan  $a - 2b + 4c \equiv 0 \pmod{21}$  kelib chiqadi.



### Misol va mashqlar

1. Butun sonlar halqasida aniqlangan taqqoslama munosabatining quyidagi xossalari isbotlang:

1.1. Taqqoslama ekvivalent binar munosabat.

1.2. Bir hil modulli taqqoslamalarni hadma-had qo'shish (ayirish) mumkin.

1.3. Taqqoslamaning bir qismidagi sonni uning ikkinchi qismiga qarama-qarshi ishora bilan o'tkazish mumkin.

1.4. Taqqoslamaning ixtiyoriy qismiga modulga karrali sonni qo'shish mumkin.

1.5. Bir hil modulli taqqoslamalarni hadma-had ko'paytirish mumkin.

1.6. Taqqoslamaning ikki qismini (modulni o'zgartirmay) bir hil natural darajaga ko'tarish mumkin.

1.7. Modulni o'zgartirmagan holda taqqoslamaning ikki qismini bir hil butun songa ko'paytirish mumkin.

1.8. Agar  $x \equiv y \pmod{m}$  bo'lsa, u holda ixtiyoriy butun koeffitsientli  $f(x) = a_0x^n + a_1x^{n-1} + \dots + a_{n-1}x + a_n$ ,  $f(y) = a_0y^n + a_1y^{n-1} + \dots + a_{n-1}y + a_n$  ko'phadlar uchun  $f(x) \equiv f(y) \pmod{m}$  taqqoslama o'rinli bo'ladi.

1.9. Agar bir vaqtda  $a_i \equiv b_i \pmod{m}$  ( $i = \overline{1, n}$ ) va  $x \equiv y \pmod{m}$  taqqoslamalar o'rinli bo'lsa, u holda

$$a_0x^n + a_1x^{n-1} + \dots + a_{n-1}x + a_n \equiv b_0y^n + b_1y^{n-1} + \dots + b_{n-1}y + b_n \pmod{m}$$

taqqoslama o'rinli bo'ladi.

1.10. Taqqoslamada qatnashuvchi qo'shiluvchini o'zi bilan teng qoldiqli bo'lgan ikkinchi songa almashtirish mumkin.

1.11. Taqqoslamaning ikkala qismini modul bilan o'zaro tub bo'lgan ko'paytuvchiga qisqartirish mumkin.

1.12. Taqqoslamaning ikkala qismi va modulini bir xil musbat songa ko'paytirish mumkin.

1.13. Taqqoslamaning ikkala qismi va moduli umumiy ko'paytuvchiga ega bo'lsa, u holda bu taqqoslamaning ikkala qismi va modulini bu umumiy ko'paytuvchiga bo'lish mumkin.

1.14. Agar taqqoslama bir nechta modul bo'yicha o'rinli bo'lsa, u holda bu taqqoslama shu modullarning eng kichik umumiy bo'linuvchisi bo'yicha ham o'rinli bo'ladi.

1.15. Agar taqqoslama biror  $m$  modul bo'yicha o'rinli bo'lsa, u holda bu taqqoslama modulning ixtiyoriy bo'luvchisi bo'yicha ham o'rinli bo'ladi.

1.16. Taqqoslamaning bir qismi va modulining EKUBi bilan uning ikkinchi qismi va modulining EKUBi o'zaro teng bo'ladi.

1.17. Sinfning bitta chegirmasi  $m$  modul bilan o'zaro tub bo'lsa, u holda bu sinfning barcha elementlari ham  $m$  modul bilan o'zaro tub bo'ladi.

2. Bo'lish natijasida hosil bo'lgan qoldiqni toping:

2.1.  $15^{231}$  ni 14 ga;

2.2.  $15^{231} + 2$  ni 16 ga;

2.3.  $1532^5 - 1$  ni 9 ga;

2.4.  $12^{1231} + 14^{4324}$  ni 13 ga;

2.5.  $208^{208}$  ni 23 ga;

2.6.  $2^{15783} - 7$  ni 25 ga;

2.7.  $3^{79821} + 5$  ni 17 ga;

2.8.  $10^{2732} + 10$  ni 22 ga;

2.9.  $18^{2815} - 3$  ni 14 ga;

2.10.  $2^{100} + 5^{200}$  ni 29 ga;

2.11.  $13^{1054} - 23 \cdot 16^{285} + 22^{17}$  ni 15 ga;

2.12.  $29^{2929} - 34^{3434} + 29 \cdot 41 \cdot 6^{231} - 24 \cdot 17^{120}$  ni 31 ga;

3. Har qanday  $a, b$  lar uchun quyidagilarni isbotlang:

3.1.  $(11a+5)^{2n+1} + (11b+6)^{2n+1} \equiv 0 \pmod{11}$ ;

3.2.  $(13a+3)^{3n+2} + (13b-4)^{3n+2} + 1 \equiv 0 \pmod{13}$ ;

3.3.  $9^{3n+1} + 3^{3n+1} + 1 \equiv 0 \pmod{13}$ .

4. Berilgan sonlarning oxirgi ikkita raqamini toping:

4.1.  $2^{999}$ ;

4.3.  $2^{341}$ ;

4.5.  $203^{203203}$ ;

4.7.  $9^9$ ;

5. Isbotlang:

5.1. Agar  $(a+b-c) \div 2$  bo'lsa, u holda  $(a-b-c) \div 2$ .

5.2. Agar  $(11a+2b) \div 19$  bo'lsa, u holda  $(18a+5b) \div 19$ .

5.3. Agar  $(a-5b) \div 17$  bo'lsa, u holda  $(2a+7b) \div 17$ .

5.4. Agar  $(12a-7b) \div 16$  bo'lsa, u holda  $(4a+23b) \div 16$ .

5.5. Agar  $(a-5b) \div 19$  bo'lsa, u holda  $(10a+7b) \div 19$ .

5.6. Agar  $(16a-11b+c) \div 21$  bo'lsa, u holda  $(11a-b+2c) \div 21$ .

5.7. Agar  $(6a-11b) \div 31$  bo'lsa, u holda  $(a-7b) \div 31$ .

5.8. Agar  $(50a+8b+c) \div 21$  bo'lsa, u holda  $(a+b+8c) \div 21$ .

5.9. Agar  $(15a+3b) \div 17$  bo'lsa, u holda  $(5a+b) \div 17$ .

5.10. Agar  $(50a-b+60c) \div 388$  bo'lsa, u holda  $(a-4b+41c) \div 194$ .

6. Quyidagilarning qaysilari uchun Eyler teoremasi o'rinli ekanligini aniqlang:

6.1.  $a=2, m=9$ ;

6.2.  $a=2, m=15$ ;

6.3.  $a=3, m=4$ ;

6.4.  $a=3, m=9$ ;

6.5.  $a=3, m=16$ ;

6.6.  $a=4, m=9$ ;

6.7.  $a=5, m=24$ ;

6.8.  $a=2, m=33$ ;

6.9.  $a=3, m=24$ .

7. Quyidagilarning qaysilari uchun Ferma teoremasi o'rinli ekanligini aniqlang:

7.1.  $a=2, p=3$ ;

7.2.  $a=2, p=5$ ;

7.3.  $a=3, p=2$ ;

7.4.  $a=10, p=5$ ;

7.5.  $a=5, p=2$ ;

7.6.  $a=5, p=3$ ;

7.7.  $a=5, p=7$ ;

7.8.  $a=4, p=3$ ;

7.9 a 4, p 5.

7.10 a 14, p 7

8. Eylar teoremasi yordamida bo'lishdan hosil bo'lgan qoldiqni toping:

8.1.  $7^{67}$  ni 12 ga;

8.2.  $109^{145}$  ni 14 ga;

8.3.  $197^{157}$  ni 35 ga;

8.4.  $356^{273}$  ni 39 ga;

8.5.  $383^{175}$  ni 45 ga;

8.6.  $293^{275}$  ni 48 ga;

8.7.  $439^{291}$  ni 60 ga;

8.8.  $527^{144}$  ni 65 ga;

8.9.  $353^{160}$  ni 75 ga;

8.10.  $485^{84}$  ni 129 ga.

9. Ferma teoremasi yordamida bo'lishdan hosil bo'lgan qoldiqni toping:

9.1.  $93^{253}$  ni 7 ga;

9.2.  $5008^{10000}$  ni 5, 7, 11, 13 ga;

9.3.  $42^{50}$  ni 17 ga;

9.4.  $20^{59}$  ni 17 ga;

9.5.  $2598^{33}$  ni 17 ga;

9.6.  $230^{347}$  ni 37 ga;

9.7.  $71^{50}$  ni 67 ga;

9.8.  $512^{402}$  ni 101 ga.

10. Bo'lish natijasida hosil bo'lgan qoldiqni toping:

10.1.  $45^{83}$  ni 24 ga;

10.2.  $6^{76}$  ni 26 ga;

10.3.  $96^{113}$  ni 92 ga;

10.4.  $204^{41}$  ni 111 ga;

10.5.  $460^{150}$  ni 425 ga.

10.6.  $763^{17}$  ni 29 ga;

10.7.  $342^{256}$  ni 29 ga;

10.8.  $581^{3792}$  ni 37 ga;

10.9.  $10^{10}$  ni 67 ga;

10.10.  $244^{408}$  ni 73 ga;

10.11.  $749^{193}$  ni 79 ga;

10.12.  $341^{245}$  ni 89 ga;

10.13.  $175^{411}$  ni 629 ga;

10.14.  $272^{1141}$  ni 135 ga;

10.15.  $35^{100}$  ni 1242 ga;

10.16.  $20^{6n-5}$  ni 9 ga,  $n \in N$ .

11. Bo'lish natijasida hosil bo'lgan qoldiqni toping:

11.1.  $7^{100} + 8^{100}$  ni 5 ga;

11.2.  $10^{100} + 40^{100}$  ni 7 ga;

11.3.  $3^{100} + 4^{100}$  ni 7 ga;

11.4.  $5^{50} + 25^{70}$  ni 9 ga;

11.5.  $25^{80} + 40^{80}$  ni 11 ga;

11.6.  $15^{60} + 20^{30}$  ni 13 ga;

11.7.  $5^{70} + 7^{50}$  ni 12 ga;

11.8.  $3^{500} + 7^{500}$  ni 101 ga;

11.9.  $(12371^{56} + 145)^{28}$  ni 111 ga;

11.10.  $3 \cdot 5^{75} + 4 \cdot 7^{100}$  ni 132 ga.

11.11.  $53^{29} \cdot 43^{17}$  ni 37 ga;

11.12.  $378^{561} \cdot 427^{921}$  ni 41 ga;

11.13.  $37^{20} \cdot 23^{12}$  ni 61 ga;

11.14.  $3^{19 \cdot 37 - 1}$  ni  $19 \cdot 37$  ga;

11.15.  $(5622 + 179 - 346) \cdot 923$  ni 23 ga;

11.16.  $631^{57} + 250^{28}$  ni 23 ga;

11.17.  $7^{161} - 3^{80}$  ni 100 ga;

11.18.  $(12371^{56} + 34)^{28}$  ni 111 ga.

12. Quyidagi sonlarning oxirgi ikkita raqamini toping:

12.1.  $3^{199}$ ;

12.3.  $11^{243}$ ;

12.5.  $17^{999}$ ;

12.7.  $903^{1293}$ ;

12.9.  $2^{199}$ ;

12.11.  $102^{14}$ ;

13. Isbotlang:

13.1.  $2^{11 \cdot 31} \equiv 2 \pmod{11 \cdot 31}$ ;

13.2.  $2^{19(73-1)} \equiv 1 \pmod{19 \cdot 73}$ ;

13.3.  $2^{17 \cdot 19} \equiv 23 \pmod{17 \cdot 19}$ ;

13.4.  $2^{1093 \cdot 1092} \equiv 1 \pmod{1093^2}$ ;

13.5.  $2^{73 \cdot 37 - 1} \equiv 1 \pmod{73 \cdot 37}$ ;

14. Isbotlang:

14.1.  $a^7 - a : 42$ ;

14.2.  $a^{11} - a : 66$ ;

14.3.  $a^{21} - a^3 : 27$ ;

14.4.  $a^{42} - a^2 : 100$ ;

14.5.  $a^{103} - a^3 : 125$ ;

14.6.  $a^{12} - b^{12} : 65, (a, 65) = (b, 65) = 1$ ;

14.7.  $a^{13} - a : 2730$ ;

14.8.  $a^{560} - 1 : 561, (a, 561) = 1$ ;

14.9.  $a^{561} - a : 11$ ;

14.10.  $a^{10} - a^6 - a^4 + 1 : 35, (a, 35) = 1$ ;

14.11.  $14^{120} - 1 : 45$ ;

14.12.  $13^{176} - 1 : 89$ ;

14.13.  $372654^{500} + 72 \cdot 10^7 : 18$ ;

14.14.  $2^{1093} - 2 : 1093^2$ ;

14.15.  $43^{23} + 23^{43} : 66$ ;

12.2.  $3^{219}$ ;

12.4.  $13^{219}$ ;

12.6.  $19^{882}$ ;

12.8.  $573^{1911}$ ;

12.10.  $2^{143}$ ;

14.16.  $222^{555} + 555^{222} : 7$ ;

14.17.  $220^{119^9} + 69^{220^{119}} + 119^{69 \cdot 220} : 102$ ;

15. Ixtiyoriy  $m, n$  natural sonlar uchun quyidagilarni isbotlang:

15.1.  $n^7 + 6n : 7$ ;

15.2.  $10^n(9n-1)+1 : 9$ ;

15.3.  $3 \cdot 5^{2n+1} + 2^{3n+1} : 17$ ;

15.4.  $6^{2n+1} + 5^{n+2} : 31$ ;

15.5. Agar  $m = 2n$  bo'lsa,  $20^m + 16m - 3m - 1 : 323$ ;

15.6.  $mn(m^{60} - n^{60}) : 56786730$ ;

15.7. Agar  $(m, 12) = (n, 12) = 1$  bo'lsa,  $m^{96} - n^{96} : 144$ ;

16. Isbotlang:

16.1. Agar  $a_1 + a_2 + \dots + a_n \equiv 0 \pmod{30}$ ,  $a_1, a_2, \dots, a_n \in \mathbb{Z}$  bo'lsa, u holda

$$a_1^5 + a_2^5 + \dots + a_n^5 \equiv 0 \pmod{30}.$$

16.2. Agar  $n \in \mathbb{N}$ ,  $(a, 10) = 1$  bo'lsa, u holda  $a^{100n+1} \equiv a \pmod{1000}$ ;

16.3. Agar  $(n, 6) = 1$  bo'lsa, u holda  $n^2 \equiv 1 \pmod{24}$ ;

16.4.  $a^{6m} + a^{6n} \equiv 0 \pmod{7} \Leftrightarrow a : 7, m, n \in \mathbb{N}$ ;

16.5. Agar  $a$  butun sonning kubi bo'lsa, u holda

$$(a-1)a(a+2) \equiv 0 \pmod{504}.$$



### Takrorlash uchun savollar

1. Taqqoslama deb nimaga aytiladi?
2. Taqqoslamaning sodda xossalarini bayon eting.
3. Modul bo'yicha chegirmalarning to'la sistemasi deb nimaga aytiladi?
4. Modul bo'yicha chegirmalarning keltirilgan sistemasi deb nimaga aytiladi?
5. Eylar va Ferma teoremlarini bayon eting.



## 28-§. Birinchi darajali va tub modul bo'yicha yuqori darajali taqqoslamalar

**Asosiy tushunchalar:** bir noma'lumli  $n$ -darajali taqqoslama, taqqoslamaning yechimi, teng kuchli taqqoslamalar, bir noma'lumli birinchi darajali taqqoslama tub modulli taqqoslama

Koeffitsientlari butun sonlardan iborat  $f(x) = a_0x^n + a_1x^{n-1} + \dots + a_{n-1}x + a_n$  ko'rinishdagi berilgan bo'lsin.

Ushbu  $f(x) \equiv 0 \pmod{m}$  ( $a_0$  son  $m$  ga bo'linmaydi,  $a_i \in \mathbb{Z}$ ,  $m \geq 1$ ) ko'rinishdagi taqqoslamaning bir noma'lumli  $n$ -darajali taqqoslama deyiladi.

Agar  $x=c$  bo'lganda  $f(c) \equiv 0 \pmod{m}$  taqqoslama to'g'ri bo'lsa, u holda  $c$  son  $f(x)$  taqqoslamaning qanoatlantiradi deyiladi.

Agar  $c$  son  $f(x)$  taqqoslamaning qanoatlantirsa, u holda  $\bar{c}$  chegirmalar sinfi  $f(x)$  taqqoslamaning yechimi deyiladi.

Yechimlari to'plami ustma-ust tushgan taqqoslamalarni teng kuchli taqqoslamalar deyiladi.

Ushbu  $ax \equiv b \pmod{m}$  ( $a, b \in \mathbb{Z}$ ,  $\forall m \in \mathbb{N}$ ) ko'rinishdagi taqqoslama bir noma'lumli birinchi darajali taqqoslama deyiladi.

Agar  $f(x) = a_0x^p + a_1x^{p-1} + \dots + a_{n-1}x + a_n$ ,  $a_i \in \mathbb{Z}$ ,  $r$ -tub son,  $(a_0, r) = 1$  bo'lsa, u holda  $f(x) \equiv 0 \pmod{p}$  taqqoslama tub modulli  $n$ -darajali bir noma'lumli taqqoslama deyiladi.

**Misol.**  $7 \cdot x \equiv 10 \pmod{4}$  taqqoslamaning yechimlarini taqqoslama xossalariidan foydalanib toping.

**Yechish.**  $(7, 4) = 1$  ekanligidan taqqoslama yagona yechimga ega ekanligi kelib chiqadi. 7 va 11 sonlari 4 dan katta bo'lganligi uchun  $7 \cdot x \equiv 3x \pmod{4}$  va  $10 \equiv 2 \pmod{4}$  lardan foydalanib  $3x \equiv 2 \pmod{4}$  ni hosil qilamiz. Bundan  $3x \equiv -x \pmod{4}$  ni e'tiborga olib  $-x \equiv 2 \pmod{4}$  ni, va nihoyat  $x \equiv -2 \pmod{4}$  ni hosil qilamiz.

Agar  $-2 \equiv 2 \pmod{4}$  ni qo'llasak, u holda  $x \equiv 2 \pmod{4}$  kelib chiqadi.

Tekshirish :

$$7 \cdot 2 \equiv 10 \pmod{4}$$

$$14 \equiv 10 \pmod{4} \Rightarrow (14 - 10) = 4 : 4$$

kelib chiqadi

**Misol.**  $27x \equiv 47 \pmod{38}$  taqqoslamaning taqqoslama xossalariidan foydalanib yechimlarini toping.

**Yechish.**  $47 \equiv 9 \pmod{38}$  dan  $27x \equiv 9 \pmod{38}$  hosil bo'ladi.  $(27, 38) = 1$  bo'lgani uchun taqqoslama yagona yechimga ega.  $(9, 38) = 1$  bo'lgani uchun taqqoslamaning ikkala tomonini 9 ga bo'lamiz:  $3x \equiv 1 \pmod{38}$ .

Taqqoslamaning o'ng tomoniga 38 ni qo'shamiz:  $3x = 39 \pmod{38}$ . Hosil bo'lgan taqqoslamaning ikkala tomonini  $(3, 38) = 1$  bo'lgani uchun 3 ga bo'lamiz:  $x \equiv 13 \pmod{38}$ .

Tekshirish:  $27 \cdot 13 - 47 = 304 = (38 \cdot 8) : 38$

**Misol.** Berilgan  $7x \equiv 10 \pmod{4}$  taqqoslamaning tanlash usuli bilan yeching.

**Yechish.**  $ax \equiv b \pmod{m}$  taqqoslamaning tanlash usuli bilan yechimlarini topish uchun avval yechimlar sonini aniqlaymiz. So'ngra  $m$  modul bo'yicha chegirmalar to'la sistemasidagi har bir sinfning yechim bo'lish bo'lmasligini tekshiramiz.

$$7 \cdot x \equiv 10 \pmod{4} \text{ taqqoslamada } (7, 4) = 1.$$

Demak, yagona yechim mavjud. 4 modul bo'yicha chegirmalar to'la sistemasi 0, 1, 2, 3.  $x$  noma'lum o'rniga birma-bir qo'yib tekshiriladi. Qaysidir chegirmalar sinfi yechim bo'lishi ma'lum bo'lsa tekshirish jarayonini to'xtatamiz:

$$x=0 \text{ da } 7 \cdot 0 \equiv 0 \pmod{4} \text{ o'rinli emas, chunki } (0 - 10) \not\equiv 4;$$

$$x=1 \text{ da } 7 \cdot 1 \equiv 7 \pmod{4} \text{ o'rinli emas, chunki } 7 - 10 = 3 \not\equiv 4;$$

$$x=2 \text{ da } 7 \cdot 2 \equiv 14 \pmod{4} \text{ o'rinli, chunki } 14 - 10 = 4 : 4.$$

$x \equiv 2 \pmod{4}$  yechim bo'ladi. Qolgan sinflar berilgan taqqoslamaning birgina yechimi mavjud bo'lganligi sababli, tekshirilmaydi.

Tekshirish:  $7 \cdot 2 - 10 = 14 - 10 = 4 : 4$ .

**Misol.**  $2x \equiv 5 \pmod{9}$  taqqoslamaning tanlash usuli yordamida yechimlarini toping.

**Yechish.** 9 modul bo'yicha  $0, \pm 1, \pm 2, \pm 3, \pm 4$  chegirmalar sinflaridan  $(2; 9) = 1$  bo'lganligi uchun berilgan taqqoslamaning yagona yechimini topamiz.

$2 \cdot 0 = 0 \not\equiv 5 \pmod{9};$

$2 \cdot 1 = 2 \not\equiv 5 \pmod{9};$

$2 \cdot (-1) = -2 \not\equiv 5 \pmod{9};$

$2 \cdot 2 = 4 \not\equiv 5 \pmod{9};$

$2 \cdot (-2) = -4 \equiv 5 \pmod{9}.$

Demak,  $x \equiv -2 \pmod{9}$ , ya'ni  $x \equiv 7 \pmod{9}$  berilgan taqqoslamaning yechimi.

Tekshirish:  $2 \cdot 7 - 5 = 14 - 5 = 9 : 9$

**Misol.**  $7 \cdot x \equiv 10 \pmod{4}$  taqqoslamaning Eyler teoremasi yordamida yeching.

**Yechish.** Agar  $a \cdot x \equiv b \pmod{m}$  taqqoslama  $(a, m) = 1$  bo'lsa, u holda uning yechimi  $x \equiv b \cdot a^{\varphi(m)-1} \pmod{m}$  formula yordamida topiladi. Haqiqatdan ham Eyler teoremasiga ko'ra  $a^{\varphi(m)} \equiv 1 \pmod{m}$ . Bundan  $a^{\varphi(m)} b \equiv b \pmod{m}$  va  $a \cdot a^{\varphi(m)-1} b \equiv b \pmod{m}$  larni hosil qilsak  $x \equiv b a^{\varphi(m)-1} \pmod{m}$  kelib chiqadi.

$7 \cdot x \equiv 10 \pmod{4}$  dan  $a=7, b=10, m=4$  yechim  $x \equiv 10 \cdot 7^{\varphi(4)-1} \pmod{4}$  ni topish uchun  $\varphi(4)$  ni aniqlaymiz.  $4 = 2^2$  ekanligidan  $\varphi(4) = 4 \cdot (1 - \frac{1}{2}) = 2$  kelib chiqadi.

Demak,  $x \equiv 10 \cdot 7^{2-1} \pmod{4}$ . Agar  $10 \equiv 2 \pmod{4}, 7 \equiv 3 \pmod{4}$  va  $6 \equiv 2 \pmod{4}$  taqqoslamalardan foydalansak,  $x \equiv 10 \cdot 7^{2-1} \equiv 2 \cdot 3 \equiv 6 \equiv 2 \pmod{4}$ , ya'ni  $x \equiv 2 \pmod{4}$  yechimni hosil qilamiz.

Tekshirish:  $2 \cdot 10 - 10 = 14 - 10 = 4 : 4$ .

**Misol.**  $27x \equiv 24 \pmod{102}$  taqqoslamaning Eyler metodidan foydalanib yechimlarini toping.

**Yechish.**  $(27, 102) = 3$  va  $24 = 3 \cdot 8$ . Demak, taqqoslama 3 ta yechimga ega. Berilgan taqqoslamaning ikkala qismi va modulni 3 ga bo'lamiz:  $9x \equiv 8 \pmod{34}$ .

Bunda  $a=9, m=34, b=8$  bo'lgani uchun  $x \equiv b \cdot a^{\varphi(m)-1} \pmod{m}$  dan  $x \equiv 8 \cdot 9^{\varphi(34)-1} \pmod{34}$  ga ega bo'lamiz.  $\varphi(34) = 2 \cdot 17 \left(1 - \frac{1}{2}\right) \left(1 - \frac{1}{17}\right) = 16$

ekanligini e'tiborga olamiz:

$x \equiv 8 \cdot 9^{15} \equiv 8 \cdot 9 \cdot 9^{14} \equiv 4 \cdot (9^2)^7 \equiv 4 \cdot 13^7 \equiv 4 \cdot 13^7 \equiv 4 \cdot 13 \cdot (13^2)^3 \equiv$   
 $\equiv 18 \cdot 33^3 \equiv 18 \cdot 33 \cdot (33)^2 \equiv 16 \cdot 1^2 \equiv 16 \pmod{34}$

Bundan  $x \equiv 16 \pmod{34}$  ga ega bo'lamiz.

Tekshirish:  $9 \cdot 16 - 8 = 136 : 34$ . U holda  $27x \equiv 24 \pmod{102}$  taqqoslama

$x \equiv 16 \pmod{102}$

$x \equiv 16 + 34 \pmod{102}$

$x \equiv 16 + 34 \cdot 2 \pmod{102}$  yechimlarga ya'ni,

$x \equiv 16 \pmod{102}$

$x \equiv 50 \pmod{102}$

$x \equiv 84 \pmod{102}$  yechimlarga ega.

Tekshirish:

$27 \cdot 16 - 24 = 408 : 102;$

$27 \cdot 50 - 24 = 3126 : 102;$

$27 \cdot 84 - 24 = 2244 : 102.$

**Misol.**  $7x \equiv 10 \pmod{4}$  taqqoslamaning munosib kasrlar yordamida yeching.

**Yechish.** Agar  $ax \equiv b \pmod{m}$  taqqoslamada  $(a, m) = 1$  va  $P_{n-1}$  son  $\frac{m}{a}$  ning oxiridan oldingi munosib kasr surati bo'lsa, u holda  $x \equiv b \cdot (-1)^{n-1} P_{n-1} \pmod{m}$  berilgan taqqoslamaning yechimi bo'ladi.

Berilgan taqqoslamada  $m = 4$ ,  $a = 7$  bo'lganidan  $\frac{4}{7}$  ning munosib kasrlarini topamiz:

$$4 = 7 \cdot 0 + 4;$$

$$7 = 4 \cdot 1 + 3;$$

$$4 = 3 \cdot 1 + 1;$$

$$3 = 1 \cdot 3 + 0.$$

Bundan  $\frac{4}{7} = [0; 1, 1, 3]$  ko'rinishda bo'ladi.

Munosib kasrlar jadvalini tuzamiz:

|       |    |   |   |   |   |
|-------|----|---|---|---|---|
| $k$   | -1 | 0 | 1 | 2 | 3 |
| $q_k$ | -  | 0 | 1 | 1 | 3 |
| $p_k$ | 1  | 0 | 1 | 1 | 4 |
| $Q_k$ | 0  | 1 | 1 | 2 | 7 |

Demak,  $P_{n-1} = P_2 = 1$  va  $x \equiv b \cdot (-1)^{n-1} P_{n-1} \equiv 10 \cdot (-1)^{3-1} \cdot 1 \equiv 10 \equiv 2 \pmod{4}$ . Berilgan taqqoslamaning  $x \equiv 2 \pmod{4}$  yechimi mavjud ekan.

Tekshirish.  $7 \cdot 2 - 10 = 14 - 10 = 4 : 4$ .

**Misol.**  $220x \equiv 28 \pmod{348}$  taqqoslamani munosib kasrlar yordamida yechimlarini toping.

**Yechish.**  $(220, 348) = 4$  va  $28 : 4$  dan berilgan taqqoslama 4 ta yechimga ega ekanligi kelib chiqadi. Taqqoslamani ikkala tomoni va modulni 4 ga bo'lamiz:  $55x \equiv 7 \pmod{87}$ .  $\frac{87}{55}$  kasmi chekli zanjir kasr ko'rinishiga keltirib, munosib kasrlar jadvalini tuzamiz:

$$\frac{87}{55} = [1; 1, 1, 1, 2, 1, 1, 4].$$

Bundan,

|       |   |   |   |   |   |    |    |    |
|-------|---|---|---|---|---|----|----|----|
| $k$   | 1 | 0 | 1 | 2 | 3 | 4  | 5  | 6  |
| $q_k$ | - | 1 | 1 | 1 | 2 | 1  | 1  | 4  |
| $p_k$ | 1 | 1 | 2 | 3 | 8 | 11 | 19 | 87 |

va  $n = 6$ ,  $P_{n-1} = P_5 = 19$ ,  $b = 7$ ,  $m = 87$  larni  $x \equiv (-1)^n P_{n-1} b \pmod{m}$  formulaga qo'ysak,  $x \equiv (-1)^6 \cdot 19 \cdot 7 \equiv 133 \equiv 46 \pmod{87}$  kelib chiqadi.

Demak,  $55x \equiv 7 \pmod{87}$  ning yechimi  $x \equiv 46 \pmod{87}$  va  $220x \equiv 28 \pmod{348}$  ning yechimlari  $x \equiv 46; 133; 220; 307 \pmod{348}$ .

Tekshirish:

$$220 \cdot 46 - 28 = 10092 : 348;$$

$$220 \cdot 133 - 28 = 29232 : 348;$$

$$220 \cdot 220 - 28 = 48372 : 348;$$

$$220 \cdot 307 - 28 = 67512 : 348.$$

**Misol.**  $7x \equiv 10 \pmod{4}$  taqqoslamani 7 ga 4 modul bo'yicha teskari sinfi orqali yeching.

**Yechish.**  $ax \equiv b \pmod{m}$  taqqoslamada  $(a, m) = 1$  bo'lsa, u holda 1 ning  $a$  va  $m$  sonlarga chiziqli yoyilmasini topamiz:  $1 = au + mv$  yoyilmadagi  $u$  soni  $a$  soniga  $m$  modul bo'yicha teskari son bo'ladi.

Evklid algoritmi yordamida berilgan  $\frac{7}{4}$  sonlarning eng katta umumiy bo'luvchisining chiziqli ifodasini topamiz:

$$7 = 4 \cdot 1 + 3;$$

$$3 = 7 - 4 \cdot 1;$$

$$4 = 3 \cdot 1 + 1;$$

$$1 = 4 - 3 \cdot 1$$

$$3 = 1 \cdot 3 + 0.$$

Bundan  $1 = 4 - 3 \cdot 1 = 4 - (7 - 4 \cdot 1) = 4 \cdot 2 - 7 = 4 \cdot 2 + 7(-1)$ . Demak,

$1 = 4 \cdot 2 + 7(-1)$ . 7 soniga 4 modul bo'yicha teskari son  $-1$  yoki  $-1 \equiv 3 \pmod{4}$  ekanligidan 3 soni bo'ladi.

$7x \equiv 10 \pmod{4}$  taqqoslamani ikkala tomonini 7 ga 4 modul bo'yicha teskari son 3ga ko'paytiramiz  $((3, 4) = 1)$ :

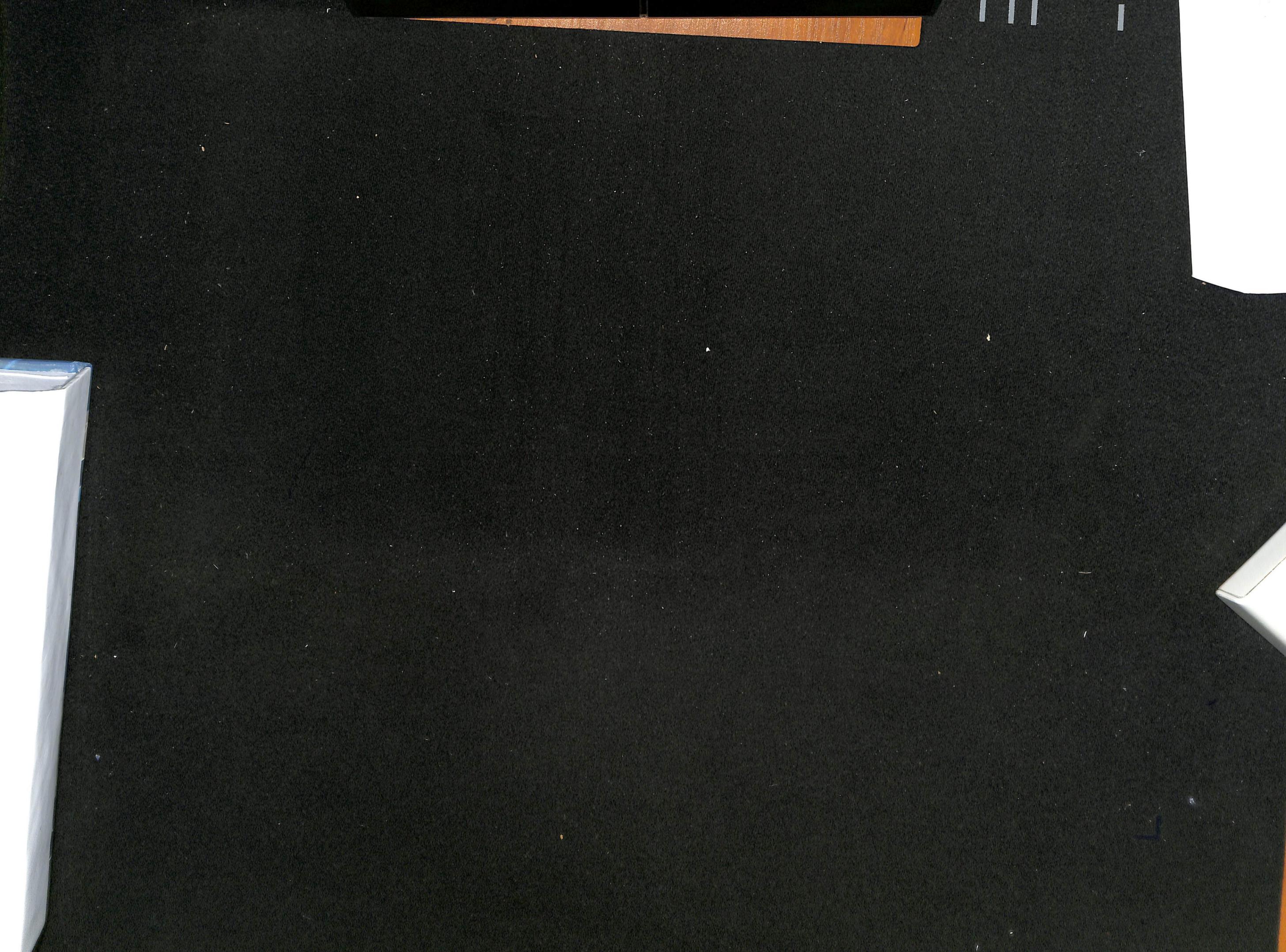
$$7 \cdot 3x \equiv 10 \cdot 3 \pmod{4}$$

$$21x \equiv 30 \pmod{4}$$

$$21x \equiv x \pmod{4}$$

$$30x \equiv 2 \pmod{4}$$

lardan  $x \equiv 2 \pmod{4}$  yechimni topamiz.



Tekshirish.  $7 \cdot 2 - 10 = 14 - 10 = 4 : 4$ .

**Misol.**  $37x \equiv 25 \pmod{107}$  taqqoslamani teskari sinf yordamida yeching.

**Yechish.**  $(37, 107) = 1$  dan berilgan taqqoslamani yagona yechimi mavjudligi kelib chiqadi. 107 modulda 37 ga teskari sonni topamiz:

$$107 = 37 \cdot 2 + 33;$$

$$37 = 33 \cdot 1 + 4;$$

$$33 = 4 \cdot 8 + 1;$$

$$4 = 1 \cdot 4 + 0.$$

$$1 = 33 - 4 \cdot 8 = 33 - (37 - 33 \cdot 1) \cdot 8 = 33 \cdot 9 + 37(-8) = (107 - 37 \cdot 2) \cdot 9 + 37(-8) = 107 \cdot 9 + 37(-26).$$

Bundan  $1 = 107 \cdot 9 + 37(-26)$ , ya'ni 107 modulda 37 ga teskari sinf  $-26$  musbat son bilan almashtiramiz:  $-26 + 107 = 81$ . Hosil bo'lgan 81 ga berilgan taqqoslamani ikkala qismini ko'paytiramiz va  $37 \cdot 81x \equiv 28 \cdot 81 \pmod{107}$  dan  $x \equiv 2025 \pmod{107}$  ya'ni  $x \equiv 99 \pmod{107}$  yechimni topamiz.

Tekshirish:  $37 \cdot 99 - 25 = 3638 : 107$ .

**Misol.**  $27x + 38y = 47$  tenglamani taqqoslamalar yordamida yeching.

**Yechish.** Tenglamani butun yechimlarini taqqoslamalardan foydalanib topish uchun  $27x \equiv 47 \pmod{38}$  bir o'zgaruvchili taqqoslamani tuzib olamiz.  $(27, 38) = 1$  ekanligidan taqqoslamani bitta yechimi mavjud.  $47 \equiv 9 \pmod{38}$  dan  $27x \equiv 9 \pmod{38}$  ni hosil qilamiz. Bundan  $3x \equiv 1 \pmod{38}$  va  $x \equiv 13 \pmod{38}$  kelib chiqadi.

$x \equiv 13 \pmod{38}$  berilgan  $27x \equiv 9 \pmod{38}$  taqqoslamani yechimi. U holda  $\left\{13, \frac{47 - 27 \cdot 13}{38}\right\} = \{13, -8\}$  berilgan tenglamaning yechimlaridan biri bo'ladi.

$ax + by = c$  tenglamaning barcha yechimlari  $x' = x_0 + \frac{m}{d}t$ ,  $y' = y_0 + \frac{a}{d}t$  ko'rinishda bo'lib, bu erda  $x_0 = 13$ ,  $y_0 = -8$ ,  $m = 18$ ,  $a = 27$ ,  $d = 1$ . Demak,

$$\begin{cases} x' = 13 + 38t, \\ y' = -8 - 27t, \quad t \in \mathbb{Z} \end{cases}$$

Tekshirish:  $27(13 + 38t) + 38(-8 - 27t) = 47$ ;

$$351 + 1026t - 304 - 1026t = 47$$

$$47 = 47$$

**Misol.**  $\begin{cases} 3x \equiv 11 \pmod{17} \\ 15x \equiv 35 \pmod{13} \\ 21x \equiv 33 \pmod{30} \end{cases}$  taqqoslamalar sistemasini yeching.

**Yechish.** Berilgan taqqoslamalar sistemasidagi har bir taqqoslama yechimlari yuqoridagi misollarda keltirilgan usullardan biri yordamida topiladi.

$$\begin{cases} x \equiv 15 \pmod{17} \\ x \equiv 11 \pmod{13} \\ x \equiv 3 \pmod{10} \end{cases}$$

Hosil qilingan taqqoslamalar sistemasidagi taqqoslamalar modullari o'zaro tub bo'lganligi uchun ularning eng kichik umumiy karralisi  $M$  bo'yicha quyidagi qiymatlarni topamiz:

$$M = 17 \cdot 13 \cdot 10 = 2210;$$

$$M_1 = \frac{2210}{17} = 130;$$

$$M_2 = \frac{2210}{13} = 170;$$

$$M_3 = \frac{2210}{10} = 221.$$

Quyidagi taqqoslamalarni tuzib yechimini topamiz:

$$1) \quad \begin{cases} 130y_1 \equiv 1 \pmod{17} \\ y_1 = 14; \end{cases}$$

$$2) \quad \begin{cases} 170y_2 \equiv 1 \pmod{13} \\ y_2 = 1; \end{cases}$$

$$3) \quad \begin{cases} 221y_3 \equiv 1 \pmod{10} \\ y_3 = 1. \end{cases}$$

Bundan berilgan taqqoslamalar sistemasining yechimi  $x = x_0 = 130 \cdot 14 \cdot 15 + 170 \cdot 1 + 11 + 211 \cdot 1 \cdot 3 = 29833 \equiv 1103 \pmod{2210}$  ya'ni,  $x \equiv 1103 \pmod{2210}$  kelib chiqadi.

Agar berilgan taqqoslamalar sistemasidagi uchinchi taqqoslamaning 3 ta yechimi borligini e'tiborga olsak, u holda taqqoslamalar sistemasining 3 ta yechimini topish mumkin:

$$\begin{cases} x \equiv 15 \pmod{17} \\ x \equiv 11 \pmod{13} \\ x \equiv 3 \pmod{30} \end{cases} \quad \begin{cases} x \equiv 15 \pmod{17} \\ x \equiv 11 \pmod{13} \\ x \equiv 13 \pmod{30} \end{cases} \quad \begin{cases} x \equiv 15 \pmod{17} \\ x \equiv 11 \pmod{13} \\ x \equiv 23 \pmod{30} \end{cases}$$

$$x \equiv 5523 \pmod{6630}; \quad x \equiv 3313 \pmod{6630} \quad x \equiv 1103 \pmod{6630}$$

yechimlar hosil qilinadi.

*Misol.*  $\begin{cases} x \equiv 2 \pmod{15} \\ x \equiv 7 \pmod{20} \\ x \equiv 12 \pmod{35} \end{cases}$  taqqoslamalar sistemasini yeching.

*Yechish.* Taqqoslama ta'rifiga ko'ra birinchi taqqoslamadan  $x = 2 + 15t$ ,  $t \in \mathbb{Z}$  ifodani hosil qilamiz. Bu qiymatni ikkinchi taqqoslamaga qo'yamiz:

$$2 + 15t \equiv 7 \pmod{20}. \text{ Bundan, } 15t \equiv 5 \pmod{20} \text{ yoki } t \equiv 3 \pmod{4} \text{ ni olamiz.}$$

Yana taqqoslama ta'rifini qo'llab  $z = 3 + 4k$ ,  $k \in \mathbb{Z}$  ifodani olamiz. Bu ifodadan  $x = 2 + 15t = 2 + 15(3 + 4k) = 47 + 60k$  kelib chiqadi. Hosil qilingan  $x$  ning ifodasini uchinchi taqqoslamaga qo'yamiz:  $47 + 60k \equiv 12 \pmod{35}$  taqqoslamani yechib  $k \equiv 0 \pmod{7}$  yechimni topamiz.

Bundan  $k = 7l$ ,  $l \in \mathbb{Z}$  kelib chiqadi. Hosil bo'lgan ifodani  $x$  ning ifodasiga qo'llaymiz:  $x = 47 + 60k = 47 + 60 \cdot 7l = 47 + 420l$

Demak,  $x \equiv 47 \pmod{420}$  berilgan taqqoslamalar sistemasining yechimi.

Tekshirish:  $\begin{cases} 47 - 2 = 45:15; \\ 47 - 7 = 40:20; \\ 47 - 12 = 35:35. \end{cases}$

*Misol.* soddalashtiring.

$$251x^{54} + 63x^{25} - 7x^{11} + 4x^3 + 2 \equiv 0 \pmod{5}$$

taqqoslamani

*Yechish.* Berilgan taqqoslamani soddalashtirish uchun taqqoslamalar xossalari va Eyler teoremasidan foydalanamiz:

1.6. Darajasi  $n$  ( $n-r$ ) bo'lgan  $r$  tub modulli taqqoslama darajasi  $r-1$  dan katta bo'lmagan taqqoslamaga teng kuchli bo'ladi.

1.7. Tub modulli  $n$ -darajali taqqoslama yechimlari soni  $n$  tadan ortiq emas.

2. Quyidagi taqqoslamalarni tanlash usulida yeching:

2.1.  $2x \equiv 1 \pmod{3}$ ;

2.2.  $8x \equiv 3 \pmod{4}$ ;

2.3.  $6x \equiv 7 \pmod{5}$ ;

2.4.  $3x \equiv 22 \pmod{7}$ ;

2.5.  $4x \equiv 6 \pmod{10}$ ;

2.6.  $12x \equiv 1 \pmod{7}$ ;

2.7.  $5x \equiv 7 \pmod{11}$ ;

2.8.  $8x \equiv 1 \pmod{16}$ .

3. Quyidagi taqqoslamalarni taqqoslama xossalari yordamida yeching:

3.1.  $7x \equiv 8 \pmod{13}$ ;

3.2.  $6x \equiv 11 \pmod{14}$ ;

3.3.  $8x \equiv 10 \pmod{14}$ ;

3.4.  $11x \equiv -32 \pmod{27}$ ;

3.5.  $16x \equiv 50 \pmod{23}$ ;

3.6.  $25x \equiv 1 \pmod{37}$ ;

3.7.  $17x \equiv 23 \pmod{41}$ ;

3.8.  $32x \equiv 43 \pmod{51}$ .

4. Berilgan taqqoslamalarni Eylerni teoremasi yordamida yeching:

4.1.  $5x \equiv 7 \pmod{13}$ ;

4.2.  $29x \equiv 3 \pmod{12}$ ;

4.3.  $5x \equiv 26 \pmod{12}$ ;

4.4.  $8x \equiv 17 \pmod{19}$ ;

4.5.  $27x \equiv 11 \pmod{34}$ ;

4.6.  $24x \equiv 1 \pmod{15}$ ;

4.7.  $15x \equiv 23 \pmod{22}$ ;

4.8.  $12x \equiv 51 \pmod{39}$ .

5. Berilgan taqqoslamalarni chekli zanjir kasrlar yordamida yeching:

5.1.  $15x \equiv 37 \pmod{98}$ ;

5.2.  $32x \equiv 182 \pmod{119}$ ;

5.3.  $105x \equiv 72 \pmod{147}$ ;

5.4.  $97x \equiv 53 \pmod{169}$ ;

5.5.  $-50x \equiv 67 \pmod{177}$ ;

5.6.  $69x \equiv 393 \pmod{201}$ ;

5.7.  $192x \equiv 9 \pmod{327}$ ;

5.8.  $365x \equiv 50 \pmod{395}$ ;

5.9.  $-639x \equiv 177 \pmod{924}$ ;

5.10.  $1296x \equiv 1105 \pmod{2413}$ ;

5.11.  $1215x \equiv 550 \pmod{2755}$ ;

5.12.  $1919x \equiv 1717 \pmod{4009}$ .

6. Berilgan  $ax \equiv b \pmod{m}$  taqqoslamalarni  $a$  ga teskari sinf orqali yeching:

6.1.  $21x \equiv 17 \pmod{23}$ ;

6.2.  $5x \equiv 7 \pmod{24}$ ;

6.3.  $17x \equiv 19 \pmod{24}$ ;

6.4.  $13x \equiv -1 \pmod{30}$ ;

6.5.  $28x \equiv 33 \pmod{35}$ ;

6.6.  $12x \equiv 24 \pmod{30}$ ;

6.7.  $9x \equiv 18 \pmod{41}$ ;

6.8.  $11x \equiv 31 \pmod{50}$ .

7. Quyidagi taqqoslamalarni yeching:

7.1.  $(a+b)x \equiv a^2 + b^2 \pmod{ab}$ ,  $(a,b)=1$ ;

$$7.2. (a^2 + b^2)x \equiv a - b \pmod{ab}, (a, b) = 1;$$

$$7.3. (a + b)^2 x \equiv a^2 - b^2 \pmod{ab}, (a, b) = 1;$$

$$7.4. (a - b)x \equiv a^2 + b^2 \pmod{ab}, (a, b) = 1;$$

$$7.5. 2x \equiv 1 + p \pmod{p}, \text{ bu erda } p \text{ tub toh son.}$$

$$7.6. (m-1)x \equiv 1 \pmod{m},$$

$$7.7. (m+1)^2 x \equiv a \pmod{m};$$

$$7.8. ax \equiv 1 \pmod{p} \text{ bu erda } p \text{ tub son va } (a, p) = 1.$$

8. Berilgan tenglamalarni taqqoslamalar yordamida yeching:

$$8.1. 2x + 3y = 4;$$

$$8.2. 4x - 3y = 2;$$

$$8.3. 3x + 4y = 13;$$

$$8.4. 5x + 4y = 3;$$

$$8.5. 3x + 8y = 5;$$

$$8.6. 17x + 13y = 1;$$

$$8.7. 23x + 15y = 19;$$

$$8.8. 17x - 16y = 31;$$

$$8.9. 91x - 28y = 35;$$

$$8.10. 17x - 39y = 26;$$

$$8.11. 50x - 42y = 34;$$

$$8.12. 47x - 105y = 4;$$

$$8.13. 47x - 111y = 89.$$

9. Taqqoslamalar sistemasini yeching:

$$9.1. \begin{cases} 3x \equiv 5 \pmod{7}, \\ 2x \equiv 1 \pmod{5}; \end{cases}$$

$$9.2. \begin{cases} 3x \equiv 1 \pmod{20}, \\ 2x \equiv 3 \pmod{15}; \end{cases}$$

$$9.3. \begin{cases} 3x \equiv 1 \pmod{5}, \\ 5x \equiv 4 \pmod{7}; \end{cases}$$

$$9.4. \begin{cases} 14x \equiv 12 \pmod{18}, \\ x \equiv 5 \pmod{25}; \end{cases}$$

$$9.5. \begin{cases} x \equiv b_1 \pmod{13}, \\ x \equiv b_2 \pmod{17}; \end{cases}$$

$$9.6. \begin{cases} 3x + 4y \equiv 29 \pmod{143}, \\ 2x - 9y \equiv 59 \pmod{143}; \end{cases}$$

$$9.7. \begin{cases} x + 2y \equiv 0 \pmod{5}, \\ 3x + 2y \equiv 2 \pmod{5}; \end{cases}$$

$$9.8. \begin{cases} 5x - y \equiv 3 \pmod{6}, \\ 2x + 2y \equiv 5 \pmod{6}. \end{cases}$$

10. Taqqoslamalar sistemasini yeching:

$$10.1. \begin{cases} x \equiv 3 \pmod{8}, \\ x \equiv 11 \pmod{20}, \\ x \equiv 1 \pmod{15}; \end{cases}$$

$$10.2. \begin{cases} x \equiv 2 \pmod{3}, \\ x \equiv 3 \pmod{4}, \\ x \equiv 4 \pmod{5}; \end{cases}$$

$$10.3. \begin{cases} x \equiv 1 \pmod{2}, \\ x \equiv 3 \pmod{5}, \\ x \equiv 6 \pmod{9}; \end{cases}$$

$$10.4. \begin{cases} x \equiv 2 \pmod{7}, \\ x \equiv 5 \pmod{9}, \\ x \equiv 11 \pmod{15}; \end{cases}$$

$$10.5. \begin{cases} x \equiv 4 \pmod{7}, \\ x \equiv 9 \pmod{13}, \\ x \equiv 1 \pmod{17}; \end{cases}$$

$$10.6. \begin{cases} x \equiv 5 \pmod{12}, \\ x \equiv 2 \pmod{8}, \\ x \equiv 2 \pmod{11}; \end{cases}$$

$$10.7. \begin{cases} x \equiv 2 \pmod{15}, \\ x \equiv 7 \pmod{20}, \\ x \equiv 12 \pmod{35}; \end{cases}$$

$$10.8. \begin{cases} x \equiv 4 \pmod{5}, \\ x \equiv 1 \pmod{12}, \\ x \equiv 7 \pmod{14}; \end{cases}$$

$$10.9. \begin{cases} x \equiv 5 \pmod{8}, \\ x \equiv 4 \pmod{11}, \\ x \equiv 6 \pmod{17}; \end{cases}$$

$$10.10. \begin{cases} x \equiv b_1 \pmod{25}, \\ x \equiv b_2 \pmod{27}, \\ x \equiv b_3 \pmod{59}; \end{cases}$$

$$10.11. \begin{cases} x \equiv 1 \pmod{3}, \\ x \equiv 4 \pmod{5}, \\ x \equiv 2 \pmod{7}, \\ x \equiv 9 \pmod{11}, \\ x \equiv 3 \pmod{13}. \end{cases}$$

11. Taqqoslamalar sistemasini yeching:

$$11.1. \begin{cases} 3x \equiv 1 \pmod{10}, \\ 4x \equiv 3 \pmod{5}, \\ 2x \equiv 7 \pmod{9}; \end{cases}$$

$$11.2. \begin{cases} 2x \equiv 3 \pmod{5}, \\ 3x \equiv 5 \pmod{7}, \\ 3x \equiv 3 \pmod{9}; \end{cases}$$

$$11.3. \begin{cases} 4x \equiv 1 \pmod{9}, \\ 5x \equiv 3 \pmod{7}, \\ 4x \equiv 5 \pmod{12}; \end{cases}$$

$$11.4. \begin{cases} 7x \equiv 3 \pmod{11}, \\ 3x \equiv 2 \pmod{5}, \\ 15x \equiv 5 \pmod{35}; \end{cases}$$

$$11.5. \begin{cases} 3x \equiv 7 \pmod{10}, \\ 2x \equiv 5 \pmod{15}, \\ 7x \equiv 5 \pmod{12}; \end{cases}$$

$$11.6. \begin{cases} 5x \equiv 3 \pmod{9}, \\ 4x \equiv 7 \pmod{13}, \\ 8x \equiv 4 \pmod{14}, \\ x \equiv 2 \pmod{17}; \end{cases}$$

$$11.7. \begin{cases} 2x \equiv 7 \pmod{13}, \\ 5x \equiv 8 \pmod{17}, \\ 14x \equiv 35 \pmod{19}, \\ 3x \equiv 7 \pmod{31}. \end{cases}$$

12.  $a$  ning qanday qiymatlarida taqqoslamalar sistemasi yechimga ega?

$$12.1. \begin{cases} x \equiv a \pmod{6}, \\ x \equiv 1 \pmod{10}, \\ x \equiv 2 \pmod{21}, \\ x \equiv 3 \pmod{11}; \end{cases}$$

$$12.2. \begin{cases} 2x \equiv a \pmod{4}, \\ 3x \equiv 4 \pmod{10}; \end{cases}$$

$$12.3. \begin{cases} x \equiv 5 \pmod{18}, \\ x \equiv 8 \pmod{21}, \\ x \equiv a \pmod{35}; \end{cases}$$

$$12.4. \begin{cases} x \equiv a \pmod{6}, \\ x \equiv 1 \pmod{10}, \\ x \equiv 2 \pmod{21}, \\ x \equiv 3 \pmod{11}. \end{cases}$$

13. Darajasi berilgan taqqoslama darajasiga, bosh koeffitsienti 1ga teng bo'lgan tengkuchli taqqoslamani toping:

$$13.1. 3x^3 - 5x^2 - 2 \equiv 0 \pmod{11};$$

$$13.2. 27x^3 + 14x^2 - 10x + 13 \equiv 0 \pmod{59};$$

$$13.3. 70x^6 + 78x^5 + 25x^4 + 68x^3 + 52x^2 + 4x + 3 \equiv 0 \pmod{101};$$

$$13.4. a_0x^n + a_1x^{n-1} + \dots + a_{n-1}x + a_n \equiv 0 \pmod{m}, (a_0, m) = 1.$$

14. Darajasi moduldan kichik, berilgan taqqoslamaga tengkuchli taqqoslamani toping:

$$14.1. x^8 + 2x^7 + x^5 - x^4 - x + 3 \equiv 0 \pmod{5};$$

$$14.2. 3x^{14} + 4x^{13} + 3x^{12} + 2x^{11} + x^9 + 2x^8 + 4x^7 + x^6 + 3x^4 + x^3 + 4x^2 + 2x \equiv 0 \pmod{5};$$

$$14.3. x^{16} + 3x^8 - 5x^7 - x^4 + 6x - 2 \equiv 0 \pmod{7};$$

$$14.4. 2x^{17} + 6x^{16} + x^{14} + 5x^{12} + 3x^{11} + 2x^{10} + x^9 + 5x^8 + 2x^7 + 3x^5 + 4x^4 + 6x^3 + 4x^2 + x + 4 \equiv 0 \pmod{7}$$

$$14.5. 6x^{18} + 18x^{15} + 3x^4 - 8x^3 + x^2 + 3 \equiv 0 \pmod{11}.$$

15. Berilgan taqqoslamani soddalashtiring (darajasini pasaytiring, koeffitsientlarni moduldan kichik sonlar bilan almashtiring, bosh koeffitsienti 1ga teng bo'lsin) va tanlash usulida yeching:

$$15.1. x^5 + x^3 + x^2 + 4 \equiv 0 \pmod{3};$$

$$15.2. 6x^4 + 17x^2 - 16 \equiv 0 \pmod{3};$$

$$15.3. 28x^9 + 29x^8 - 26x^7 + 20x^4 - 17x + 23 \equiv 0 \pmod{3};$$

$$15.4. x^5 + 2x^4 - 2x^3 - 2x^2 + 2x - 1 \equiv 0 \pmod{3};$$

$$15.5. x^5 + x^4 - x^2 - 5x + 1 \equiv 0 \pmod{3};$$

$$15.6. x^7 + 2x^6 + x^5 + 4x^3 - 2x^2 - 4x + 2 \equiv 0 \pmod{5};$$

$$15.7. x^7 + 3x^6 + x^5 - x^3 - 3x^2 - 4x + 4 \equiv 0 \pmod{5};$$

$$15.8. x^7 + 5x^5 - x^3 - 9x + 3 \equiv 0 \pmod{5};$$

$$15.9. 34x^{10} - 29x^7 + 43x^4 - 19x + 37 \equiv 0 \pmod{5};$$

$$15.10. 6x^{10} - 12x + 1 \equiv 0 \pmod{5};$$

$$15.11. x^7 - 3x^6 + x^5 - 15x^4 - x^3 + 4x^2 - 4x + 2 \equiv 0 \pmod{5}.$$

16. Berilgan taqqoslamalarni soddalashtiring va tanlash usulida yeching:

$$16.1. 5x^{24} + 4x^{23} + 4x^{22} + 2x^{21} + x^{20} + 6x^{19} + 4x^{18} + 3x^{17} + 4x^{16} + 6x^{15} + 5x^{14} + 2x^{13} + x^{12} + 2x^{11} + x^{10} + 3x^9 + 4x^8 + 2x^7 + 5x^6 + 6x^5 + 5x^4 + 3x^3 + 4x^2 + 4x + 2 \equiv 0 \pmod{7};$$

$$16.2. x^{13} - x^{11} + x^9 - x^7 + x^5 + x^3 + x + 1 \equiv 0 \pmod{7};$$

$$16.3. 10x^{42} - 5x^{30} + 10x^{18} + 9x^{12} + 4 \pmod{7};$$

$$16.4. 75x^{13} - 62x^{12} - 53x^{11} - 24x^6 + 13x - 27 \equiv 0 \pmod{7};$$

$$16.5. 6x^{13} - 3x^{12} - 2x^{11} - 6x^3 + 3x^2 + 7x + 2 \equiv 0 \pmod{11};$$

$$16.6. 13x^{23} - 30x^{22} - 2x^{13} + 1 \equiv 0 \pmod{11};$$

$$16.7. 120x^{91} + 14x^{15} + x^{11} - 3x^5 + 9x^2 - x + 6 \equiv 0 \pmod{11};$$

$$16.8. x^{14} - x^{13} + 12x^2 + 2x + 1 \equiv 0 \pmod{13};$$

$$16.9. 300x^{90} + 259x^{67} - 95x^{23} - 1 \equiv 0 \pmod{23}.$$

17. Taqqoslamalarni berilgan modul bo'yicha chizihli ko'paytuvchilarga ajrating:

$$17.1. x^3 + 4x^2 - 3 \equiv 0 \pmod{5};$$

$$17.2. x^3 - 2x + 1 \equiv 0 \pmod{5};$$

$$17.3. x^4 - 20x^3 + 90x^2 - 135x + 54 \equiv 0 \pmod{5};$$

$$17.4. 3x^3 + 2x^2 - 2x - 3 \equiv 0 \pmod{5};$$

$$17.5. x^4 - 12x^3 + 46x^2 - 53x - 12 \equiv 0 \pmod{7};$$

$$17.6. 5x^3 + 4x^2 - 8x - 1 \equiv 0 \pmod{7};$$

$$17.7. 6x^3 + 5x^2 - 2x - 9 \equiv 0 \pmod{11};$$

$$17.8. x^3 + 3x^2 - 3 \equiv 0 \pmod{17};$$

$$17.9. x^3 + 11x^2 + 8x + 3 \equiv 0 \pmod{23};$$

$$17.10. x^4 + 15x^3 + 4x^2 + 4x - 15 \equiv 0 \pmod{29};$$

$$17.11. x^3 - 13x^2 - 3x + 11 \equiv 0 \pmod{31}.$$



## Takrorlash uchun savollar

1. Bir noma'lumli  $n$ -darajali taqqoslama deb nimaga aytiladi?
2. Taqqoslamaning yechimi deb nimaga aytiladi?
3. Teng kuchli takkoslamalarni tushuntiring.
4. Bir noma'lumli birinchi darajali taqqoslama qachon yechimga ega?
5. Bir noma'lumli birinchi darajali taqqoslamalarni qanday yechish usullarini bilasiz?
6. Tub modulli  $n$ -darajali taqqoslama yechimlari soni nimaga bog'liq bo'ladi?

### 29-§. Tub modul bo'yicha boshlang'ich ildizlar va indekslar

**Asosiy tushunchalar:** sonning ko'rsatkichi, boshlang'ich ildiz,  $g$  asosga nisbatan indeks, ikkinchi darajali taqqoslama, ikki hadli taqqoslama, kvadratik chegirma.

Agar  $(a; m)=1$  bo'lganda  $a^\delta=1 \pmod{m}$  taqqoslama o'rinli bo'lsa, u holda  $\delta$  son  $a$  sonning  $m$  modulga ko'ra ko'rsatkichi yoki  $m$  modul bo'yicha  $a$  soniga tegishli ko'rsatkich deyiladi.

Agar  $(a, m)=1$  bo'lib,  $\delta=\varphi(m)$  bo'lsa, u holda  $a$  son  $m$  modul bo'yicha boshlang'ich ildiz deyiladi.

Boshlang'ich ildizlar faqatgina  $m=2, 4, r^\alpha, 2p^\alpha$  ( $r$ -toq tub son,  $\alpha \geq 1$  natural son) sonlar uchun mavjud bo'ladi. Boshlang'ich ildizlar bevosita hisoblash usulida topiladi.

$r$  tub son bo'lib,  $\delta$  son  $r-1$  sonning bo'luvchisi bo'lsin, u holda  $r$  modul bo'yicha chegirmalarning keltirilgan sinflar sistemasida  $\delta$  ko'rsatkichga tegishli sinflar soni  $\varphi(\delta)$  ta bo'ladi.

Agar  $g$  son  $r$  tub modul bo'yicha boshlang'ich ildiz bo'lib,  $(a; r)=1$  bo'lganda  $g^\gamma=a \pmod{r}$  taqqoslama to'g'ri bo'lsa, u holda  $\gamma \geq 0$  butun son  $a$  sonning  $r$  modul

bo'yicha  $g$  asosga nisbatan indeks deyiladi va u  $\gamma = \text{ind}_g a$  kabi belgilanadi.

Agar  $a$  son  $m$  songa bo'linmasa, u holda ushbu  $ax^2+bx+c=0 \pmod{m}$  ko'rinishdagi taqqoslama ikkinchi darajali (kvadratik) taqqoslama deyiladi.

Agar  $a$  son  $r$  tub songa bo'linmasa, u holda ushbu  $ax^n=b \pmod{r}$  ( $\forall n \in \mathbb{N}$ ) ko'rinishdagi taqqoslamani  $n$ -darajali ikki hadli taqqoslama deyiladi. Agar  $(a; m)=1$  bo'lganda taqqoslama yechimga ega bo'lsa, u holda  $a$  son  $m$  modul buyicha  $n$ -darajali chegirma, aks xolda  $a$  son  $n$ -darajali chegirmamas deyiladi.

Ushbu  $x^2 \equiv a \pmod{m}$  ko'rinishdagi taqqoslamani ikki hadli kvadratik taqqoslama deyiladi. Agar  $(a; m)=1$  bo'lganda taqqoslama yechimga ega bo'lsa, u holda  $a$  son  $m$  modul bo'yicha kvadratik chegirma, aks holda  $a$  son  $m$  modul bo'yicha kvadratik chegirmamas deyiladi.

Ushbu  $x^2 \equiv a \pmod{p}$  ( $(a; r)=1, (2; r)=1$ ) ko'rinishdagi taqqoslamani toq tub modulli kvadratik taqqoslama deyiladi. Agar  $(a; r)=1$  bo'lib,  $a^{\frac{p-1}{2}} \equiv 1 \pmod{p}$  bo'lsa, u holda taqqoslama ikkita yechimga ega bo'ladi,  $a^{\frac{p-1}{2}} \equiv -1 \pmod{p}$  bo'lsa, u holda taqqoslama yechimga ega bo'lmaydi.

**Misol.**  $\frac{219}{383}$  ning Lejandr simvolini toping.

**Yechish:** Lejandr simvoli deb  $\frac{a}{p}$  kasr songa 1, -1 ni quyidagicha mos qo'yish tushuniladi:

$$\left(\frac{a}{p}\right) = \begin{cases} 1 & \text{agar } a \text{ soni } p \text{ modul bo'yicha kvadrat chegirma bo'lsa;} \\ -1 & \text{agar } a \text{ soni } p \text{ modul bo'yicha kvadrat chegirma bo'lmasa;} \end{cases}$$

Berilgan kasr sonning maxraji tub son bo'lsa, uning Lejandr simvoli topiladi.

Buning uchun quyidagi xossalardan foydalanamiz:

$$1. \text{ Agar } a \equiv b \pmod{p} \text{ bo'lsa, u xolda } \left(\frac{a}{p}\right) = \left(\frac{b}{p}\right);$$

$$2. \left(\frac{a^2}{p}\right) = 1;$$

$$3. \left(\frac{1}{p}\right) = 1;$$

$$4. \left(\frac{-1}{p}\right) = (-1)^{\frac{p-1}{2}};$$

$$5. \left(\frac{ab \dots c}{p}\right) = \left(\frac{a}{p}\right) \left(\frac{b}{p}\right) \dots \left(\frac{c}{p}\right);$$

$$6. \left(\frac{ab^2}{p}\right) = \left(\frac{a}{p}\right);$$

$$7. \left(\frac{a^n}{p}\right) = \left(\frac{a}{p}\right)^n;$$

$$8. \left(\frac{2}{p}\right) = (-1)^{\frac{p-1}{8}};$$

9. Agar  $(p; q) = 1$ , u xolda  $\left(\frac{q}{p}\right) = \left(\frac{p}{q}\right) \cdot (-1)^{\frac{p-1}{2} \cdot \frac{q-1}{2}}$ .

Berilgan  $\frac{219}{383}$  kasrning Lejandr simvolini topamiz:

**1-usul.**

$$\begin{aligned} \left(\frac{219}{383}\right) &= \left(\frac{3 \cdot 73}{383}\right) = |5 - \text{xossaga ko'ra}| = \left(\frac{3}{383}\right) \cdot \left(\frac{73}{383}\right) = |9 - \text{xossaga ko'ra}| = \\ &= \left(\frac{383}{3}\right) (-1)^{\frac{383-1}{2} \cdot \frac{3-1}{2}} \cdot \left(\frac{383}{73}\right) (-1)^{\frac{383-1}{2} \cdot \frac{73-1}{2}} = \left(\frac{2}{3}\right) (-1)^{191 \cdot 1} \left(\frac{18}{73}\right) (-1)^{191 \cdot 36} = -\left(\frac{2}{3}\right) \cdot \left(\frac{2 \cdot 3^2}{73}\right) = \\ &= |5,6 - \text{xossaga ko'ra}| = -\left(\frac{2}{3}\right) \cdot \left(\frac{2}{73}\right) = |8 - \text{xossaga ko'ra}| = -(-1)^{\frac{3^2-1}{8}} \cdot (-1)^{\frac{73^2-1}{8}} = \\ &= -(-1)(-1)^{666} = 1. \end{aligned}$$

Demak,  $\left(\frac{219}{383}\right) = 1$ . Bundan  $x^2 \equiv 219 \pmod{383}$  taqqoslama uchun 219 kvadrat chegirma bo'ladi, ya'ni hosil qilingan taqqoslama kamida bitta yechimga ega.

**2-usul.**  $\left(\frac{219}{383}\right) = \left(\frac{3}{383}\right) \cdot \left(\frac{73}{383}\right)$  tenglikdan foydalanib, ko'paytuvchilarni

alohida-alohida topish mumkin:

$$a) \left(\frac{3}{383}\right) = \left(\frac{383}{3}\right) \cdot (-1)^{\frac{383-1}{2} \cdot \frac{3-1}{2}} = -\left(\frac{383}{3}\right) = -\left(\frac{2}{3}\right) = -(-1)^{\frac{3^2-1}{8}} = -(-1) = 1.$$

$$\begin{aligned} v) \left(\frac{73}{383}\right) &= \left(\frac{383}{73}\right) \cdot (-1)^{\frac{383-1}{2} \cdot \frac{73-1}{2}} = -\left(\frac{383}{73}\right) = -\left(\frac{18}{73}\right) = \left(\frac{2 \cdot 3^2}{73}\right) = \\ &= \left(\frac{2}{73}\right) = -(-1)^{\frac{73^2-1}{8}} = 1. \end{aligned}$$

Bunda,  $\left(\frac{219}{383}\right) = \left(\frac{3}{383}\right) \cdot \left(\frac{73}{383}\right) = 1 \cdot 1 = 1$  kelib chiqadi.

**3-usul.** Berilgan  $\frac{219}{383}$  kasrning maxraji suratidan katta bo'lgani uchun 9-xossani qo'llash mumkin:

$$\begin{aligned} \left(\frac{219}{383}\right) &= \left(\frac{383}{219}\right) \cdot (-1)^{\frac{383-1}{2} \cdot \frac{219-1}{2}} = -\left(\frac{383}{219}\right) = \left(\frac{164}{219}\right) = \left(\frac{41 \cdot 2^2}{219}\right) = \\ &= -\left(\frac{41}{219}\right) = -\left(\frac{219}{41}\right) \cdot (-1)^{\frac{219-1}{2} \cdot \frac{41-1}{2}} = -\left(\frac{219}{41}\right) = \left(\frac{14}{41}\right) = -\left(\frac{2}{41}\right) \left(\frac{7}{41}\right) = (-1)^{\frac{41^2-1}{8}} \left(\frac{7}{41}\right) = \\ &= -\left(\frac{7}{41}\right) = -\left(\frac{41}{7}\right) \cdot (-1)^{\frac{41-1}{2} \cdot \frac{7-1}{2}} = -\left(\frac{41}{7}\right) = -\left(\frac{-1}{7}\right) = (-1)^{\frac{7-1}{2}} = 1 \end{aligned}$$

Demak,  $\left(\frac{219}{383}\right) = 1$ .

**Misol.**  $\frac{383}{219}$  ning Yakobi simvolini aniqlang.

**Yechish:** Yakobi simvolining Lejandr simvolidan farqi Yakobi simvoli o'zaro tub bo'lgan  $a$  va  $m$  ( $m > 1$ ) sonlardan tuzilgan  $\frac{a}{m}$  uchun aniqlanadi.  $\left(\frac{a}{m}\right)$  belgilash " $a$  ning  $m$  modul bo'yicha Yakobi simvoli" deb o'qiladi. Yuqoridagi 12-misoldagi Lejandr simvolining xossalari va  $\left(\frac{a}{m}\right) = \left(\frac{a}{p_1 \dots p_n}\right) = \left(\frac{a}{p_1}\right) \dots \left(\frac{a}{p_n}\right)$

xossadan:

$$\left(\frac{383}{219}\right) = \left(\frac{383}{3 \cdot 73}\right) = \left(\frac{383}{3}\right) \left(\frac{383}{73}\right) = \left(\frac{2}{3}\right) \left(\frac{18}{73}\right) = \left(\frac{2}{3}\right) \left(\frac{23^2}{73}\right) =$$

$$\left(\frac{2}{3}\right) \left(\frac{2}{73}\right) = (-1)^{\frac{2^2-1}{2}} = (-1)^{\frac{3^2-1}{2}} = (-1) \cdot 1 = -1$$

Demak,  $\left(\frac{383}{219}\right) = -1$ , ya'ni  $x^2 \equiv 383 \pmod{219}$  taqqoslama uchun 383 kvadrat chegirma emas.

**Misol.**  $p = 17$  modul bo'yicha  $g = 6$  boshlang'ich ildizning indekslar jadvalini tuzing.

**Yechish.**  $p$  tub modul bo'yicha boshlang'ich ildiz bu shunday  $g$  chegirmalar sinfini, uning uchun  $g^{p-1} \equiv 1 \pmod{p}$  bo'lib,  $p-1$  dan kichik natural darajalarda  $p$  modulda 1 bilan taqqoslanmaydi.

$g = 6$  ning mod 17 da boshlang'ich ildiz bo'lishini tekshiramiz. Buning uchun  $p-1$  ning  $n$  bo'luvchilarida  $6^n \equiv 1 \pmod{p}$  shartni tekshiramiz:

$p = 17$ ,  $p-1 = 16$ , 16 ning natural bo'luvchilari  $n = 1, 2, 4, 8, 16$ . Bundan:

$$6^1 \equiv 6 \pmod{17}$$

$$6^2 \equiv 2 \pmod{17}$$

$$6^4 \equiv 4 \pmod{17}$$

$$6^8 \equiv 16 \pmod{17}$$

$$6^{16} \equiv 1 \pmod{17}$$

Demak, 17 modulda 6 boshlang'ich ildiz bo'ladi.  $6^0, 6^1, 6^2, \dots, 6^{15}$  lardan 17 modul bo'yicha taqqoslamalar tuzamiz:

$$6^0 \equiv 1 \pmod{17};$$

$$6^1 \equiv 6 \pmod{17};$$

$$6^2 \equiv 2 \pmod{17};$$

$$6^3 \equiv 12 \pmod{17};$$

$$6^4 \equiv 4 \pmod{17};$$

$$6^5 \equiv 7 \pmod{17};$$

$$6^6 \equiv 8 \pmod{17};$$

$$6^7 \equiv 14 \pmod{17};$$

$$6^8 \equiv 16 \pmod{17};$$

$$6^9 \equiv 11 \pmod{17};$$

$$6^{10} \equiv 15 \pmod{17};$$

$$6^{11} \equiv 5 \pmod{17};$$

$$6^{12} \equiv 13 \pmod{17};$$

$$6^{13} \equiv 10 \pmod{17};$$

$$6^{14} \equiv 9 \pmod{17};$$

$$6^{15} \equiv 3 \pmod{17}.$$

Tuzilgan taqqoslamalar yordamida quyidagi jadvallarni tuzamiz:

1-jadval

| N | 0  | 1 | 2 | 3  | 4 | 5  | 6 | 7 | 8 | 9  |
|---|----|---|---|----|---|----|---|---|---|----|
| 0 |    | 0 | 2 | 15 | 4 | 11 | 1 | 5 | 6 | 14 |
| 1 | 13 | 9 | 3 | 12 | 7 | 10 | 8 |   |   |    |

1-jadval uchun taqqoslamalarning ikkinchi tomonidagi songa mos daraja topiladi.

2-jadval

| 1 | 0  | 1 | 2  | 3  | 4 | 5 | 6 | 7  | 8  | 9  |
|---|----|---|----|----|---|---|---|----|----|----|
| 0 | 1  | 6 | 2  | 12 | 4 | 7 | 8 | 14 | 16 | 11 |
| 1 | 15 | 5 | 13 | 10 | 9 | 3 |   |    |    |    |

2-jadval uchun taqqoslamalarning birinchi tomonidagi darajaga mos qoldiq topiladi.

**Misol.**  $15x^{19} \equiv 28 \pmod{17}$  taqqoslamani yeching.

**Yechish.**  $15x^{19} \equiv 28 \pmod{17}$  taqqoslamani taqqoslama xossalari yordamida soddalashtiramiz:  $15x^3 \equiv 11 \pmod{17}$ . Hosil bo'lgan taqqoslamani indekslar xossalariga ko'ra:  $ind15 + 3indx \equiv ind11 \pmod{16}$  taqqoslamani hosil qilamiz.

Yuqorida tuzilgan jadvaldan  $ind15 = 10$ ,  $ind11 = 9$  larni topamiz:

$$10 + 3indx \equiv 9 \pmod{16}$$

$$3indx \equiv -1 \pmod{16}$$

$$(3, 16) = 1$$

$$3indx \equiv 15 \pmod{16};$$

$$indx \equiv 5 \pmod{16}$$

xossalardan  $x \equiv 7 \pmod{17}$  yechimni hosil qilamiz.

Tekshirish:

$$15 \cdot 7^{19} - 28 = -2(7^2)^9 \cdot 7 - 11 \equiv -2(49)^9 \cdot 7 - 11 \equiv -2(-2)^9 \cdot 7 - 11 \equiv$$

$$-2(-2)^5 \cdot (-2)^4 \cdot 7 - 11 \equiv -2(-32) \cdot 16 \cdot 7 - 11 \equiv -2 \cdot 2 \cdot (-1) \cdot 7 - 11 \equiv$$

$$\equiv 28 - 11 \equiv 17 \equiv 0 \pmod{17}$$

Demak,  $15 \cdot 7^{19} \equiv 28 \pmod{17}$ .



## Misol va mashqlar

1. Legendr simvolini toping

1.1.  $\left(\frac{13}{7}\right)$

1.2.  $\left(\frac{22}{13}\right)$

1.3.  $\left(\frac{19}{67}\right)$

1.4.  $\left(\frac{37}{67}\right)$

1.5.  $\left(\frac{56}{73}\right)$

1.6.  $\left(\frac{47}{73}\right)$

1.7.  $\left(\frac{54}{83}\right)$

1.8.  $\left(\frac{68}{113}\right)$

1.9.  $\left(\frac{63}{131}\right)$

2. Yakobi simvolini toping:

2.1.  $\left(\frac{283}{563}\right)$

2.2.  $\left(\frac{251}{577}\right)$

2.3.  $\left(\frac{241}{593}\right)$

2.4.  $\left(\frac{323}{607}\right)$

2.5.  $\left(\frac{346}{643}\right)$

2.6.  $\left(\frac{3153}{1201}\right)$

2.7.  $\left(\frac{20470}{1847}\right)$

2.8.  $\left(\frac{2108}{2003}\right)$

2.9.  $\left(\frac{3149}{5987}\right)$

3. Quyidagi taqqoslamalarning yechimlar sonini aniqlang:

3.1.  $x^2 \equiv 3 \pmod{31}$ ;

3.2.  $x^2 \equiv 2 \pmod{31}$ ;

3.3.  $x^2 \equiv 5 \pmod{73}$ ;

3.4.  $x^2 \equiv 3 \pmod{101}$ ;

3.5.  $x^2 \equiv 226 \pmod{563}$ ;

3.6.  $x^2 \equiv 429 \pmod{563}$ ,

3.7.  $x^2 \equiv 579 \pmod{821}$ ;

3.8.  $x^2 \equiv 728 \pmod{919}$ ;

3.9.  $x^2 \equiv 847 \pmod{1087}$ ;

3.10.  $x^2 \equiv 3766 \pmod{5987}$ .

4. Quyidagi tasdiqlarni isbotlang:

4.1. Biror  $m$  modul bo'yicha tuzilgan bitta sinfning chegirmalari shu modul bo'yicha bir xil ko'rsatkichga tegishli bo'ladi.

4.2. Agar  $(a; m) = 1$  bo'lganda  $a^{\delta} \equiv 1 \pmod{m}$  bo'lsa, u holda  $a^0, a^1, \dots, a^{\delta-1}$  sonlar sistemasi  $m$  modul bo'yicha o'zaro taqqoslanmaydi.

4.3. Agar  $\delta = \varphi(m)$  bo'lsa, u holda  $a^0, a^1, \dots, a^{\delta-1}$  sistema  $m$  modul bo'yicha chegirmalarning keltirilgan sistemasini tashkil qiladi.

4.4.  $a$  son  $m$  modul bo'yicha  $\delta$  ko'rsatkichga tegishli bo'lsa, u holda  $a^r = a^s \pmod{m}$  taqqoslama o'rinli bo'lishi uchun  $r \equiv s \pmod{\delta}$  taqqoslamaning o'rinli bo'lishi zarur va etarli.

4.5.  $r \equiv 0 \pmod{\delta}$  bo'lganda va faqat shu holdagina  $a^r \equiv 1 \pmod{m}$  taqqoslama o'rinli bo'ladi.

4.6.  $a$  sonning  $m$  modul bo'yicha  $\delta$  ko'rsatkichi  $\varphi(m)$ ning bo'luvchisi bo'ladi.

4.7. Agar  $a$  son  $m$  modul bo'yicha  $\delta$  ko'rsatkichga tegishli bo'lsa, u holda  $a^k$  soni shu modul bo'yicha  $\frac{\delta}{(\delta; k)}$  ko'rsatkichga tegishli bo'ladi.

4.8. Agar  $(\delta; k) = 1$  bo'lsa, u holda  $a$  son  $\delta$  ko'rsatkichga tegishli bo'ladi.

4.9.  $r$  tub modul bo'yicha tuzilgan  $r-1$  sonning har bir  $\delta$  buluvchisi  $\varphi(\delta)$  ta sinfning ko'rsatkichi bo'ladi. Xususi holda  $\varphi(r-1)$  ta boshlang'ich ildizlar sinfi mavjud.

5.  $a$  sonining  $m$  modul bo'yicha tartibini aniqlang:

5.1.  $a = 2, m = 5$ ;

5.2.  $a = 4, m = 5$ ;

5.3.  $a = 5, m = 8,$

5.4.  $a = 10, m = 13,$

5.5.  $a = 4, m = 15,$

5.6.  $a = 2, m = 15,$

5.7.  $a = 2, m = 17;$

5.8.  $a = 7, m = 20;$

5.9.  $a = 7, m = 22;$

5.10.  $a = 7, m = 43;$

5.11.  $a = 5, m = 108;$

5.12.  $a = 2, m = 133.$

6.  $a, b, c, d$  sonlarning  $m$  modul bo'yicha tartibini aniqlang:

6.1.  $a = 7, b = 9, c = 12; m = 13;$

6.2.  $a = 5, b = 8, c = 13; m = 17;$

6.3.  $a = 5, b = 8, c = 10; d = 16; m = 13;$

6.4.  $a = 10, b = 25, c = 50; m = 39;$

6.5.  $a = 5, b = 15, c = 21; d = 35; m = 44.$

7. Berilgan modul bo'yicha barcha boshlang'ich ildizlarni toping:

7.1. 11; 7.2. 13;

7.3. 15; 7.4. 19;

7.5. 49; 7.6. 81.

8. Berilgan modul bo'yicha boshlang'ich ildizlar sonini va ularning eng kichigini toping:

8.1. 10; 8.2. 18;

8.3. 19; 8.4. 31;

8.5. 37.

9. Berilgan modul bo'yicha boshlang'ich ildizlarning eng kichigini toping:

9.1. 7; 9.2. 17;

9.3. 23; 9.4. 41;

9.5. 53; 9.6. 50;

9.7. 54; 9.8. 71;

9.9. 242; 9.10. 289;

9.11. 578; 9.12. 625.

10.  $r$  modul bo'yicha  $g$  asosga ko'ra indekslar jadvalini tuzing:

10.1.  $p = 3, g = 2;$

10.2.  $p = 5, g = 2;$

10.3.  $p = 5, g = 3;$

10.4.  $p = 7, g = 3;$

10.5.  $p = 7, g = 5;$

10.6.  $p = 11, g = 2;$

10.7.  $p = 13, g = 2;$

10.8.  $p = 29, g = 2.$

11. Indeksning quyidagi xossalari isbotlang:

11.1.  $a \equiv b \pmod{r} \Leftrightarrow \text{ind } a = \text{ind } b.$

11.2. Agar  $(a;r)=1, (b;r)=1$  bo'lsa, u holda  $\text{ind}(ab) = \text{ind } a + \text{ind } b \pmod{p-1}$  bo'ladi.

11.3. Agar  $(a;r)=1$  va  $\forall n \in \mathbb{N}$  bo'lsa, u holda  $\text{ind}(a^n) \equiv n \cdot \text{ind } a \pmod{p-1}$  taqqoslama o'rinli bo'ladi.

11.4.  $\text{ind}\left(\frac{a}{b}\right) \equiv \text{ind } a - \text{ind } b \pmod{p-1}$  taqqoslama o'rinli.

11.5.  $\text{ind } 1 = 0, \text{ind } g = 1.$

12. Quyidagi taqqoslamalarni yeching:

12.1.  $7x \equiv 23 \pmod{17};$

12.2.  $5x \equiv 13 \pmod{27};$

12.3.  $8x \equiv -11 \pmod{37};$

12.4.  $47x \equiv 23 \pmod{73};$

12.5.  $53x \equiv 37 \pmod{79};$

12.6.  $125x \equiv 7 \pmod{79};$

$$12.7. 65x \equiv 38 \pmod{83};$$

$$12.8. 23x \equiv 9 \pmod{97};$$

$$12.9. 37x \equiv 5 \pmod{221};$$

13. Quyidagi ikkinchi darajali taqqoslamalarni yeching:

$$13.1. x^2 \equiv 15 \pmod{17};$$

$$13.2. x^2 \equiv 10 \pmod{27};$$

$$13.3. x^2 \equiv 47 \pmod{53};$$

$$13.4. x^2 \equiv 58 \pmod{61};$$

$$13.5. x^2 \equiv 59 \pmod{67};$$

$$13.6. x^2 \equiv -28 \pmod{67};$$

$$13.7. x^2 \equiv 54 \pmod{71};$$

$$13.8. x^2 \equiv 40 \pmod{83};$$

$$13.9. 3x^2 - 5x - 2 \equiv 0 \pmod{11};$$

$$13.10. 2x^2 - 7x + 28 \equiv 0 \pmod{43};$$

$$13.11. 3x^2 - 8x + 44 \equiv 0 \pmod{47};$$

$$13.12. x^2 \equiv 29 \pmod{59^2};$$

$$13.13. x^2 \equiv 61 \pmod{73^2};$$

14. Quyidagi taqqoslamalarning yechimlar sonini aniqlang:

$$14.1. x^{15} \equiv 6 \pmod{37};$$

$$14.2. x^{16} \equiv 10 \pmod{37};$$

$$14.3. 3x^3 \equiv 2 \pmod{37};$$

$$14.4. 7x^7 \equiv 11 \pmod{41};$$

$$14.5. 3x^{12} \equiv 31 \pmod{41};$$

$$14.6. 5x^{30} \equiv 37 \pmod{41};$$

$$14.7. x^5 \equiv 3 \pmod{71};$$

$$14.8. x^{21} \equiv 5 \pmod{71};$$

$$14.9. x^{15} \equiv 46 \pmod{97};$$

$$14.10. x^{55} \equiv 17 \pmod{97};$$

$$14.11. x^{60} \equiv 79 \pmod{97};$$

15. Quyidagi ikkihadli taqqoslamalarni yeching:

$$15.1. x^{10} \equiv 33 \pmod{37};$$

$$15.2. x^3 \equiv 34 \pmod{41};$$

$$15.3. x^8 \equiv 31 \pmod{41};$$

$$15.4. x^{12} \equiv 37 \pmod{41};$$

$$15.5. x^5 \equiv 37 \pmod{43};$$

$$15.6. x^{27} \equiv 39 \pmod{43};$$

$$15.7. x^{35} \equiv 17 \pmod{67};$$

$$15.8. x^{30} \equiv 14 \pmod{67};$$

$$15.9. x^{12} \equiv 27 \pmod{83};$$

$$15.10. x^{48} \equiv 2 \pmod{97};$$

16. Quyidagi ikkihadli taqqoslamalarni yeching:

$$16.1. 3x^3 \equiv 4 \pmod{7};$$

$$16.2. 2x^8 \equiv 5 \pmod{13};$$

$$16.3. 15x^4 \equiv 17 \pmod{23};$$

$$16.4. 27x^5 \equiv 25 \pmod{31};$$

$$16.5. 13x^3 \equiv 24 \pmod{37};$$

$$16.6. 37x^8 \equiv 59 \pmod{61};$$

$$16.7. 23x^5 \equiv 15 \pmod{73};$$

$$16.8. 37x^6 \equiv 69 \pmod{73};$$

$$16.9 \quad 37x^{17} = 62 \pmod{73},$$

$$16.10 \quad 44x^{21} = 53 \pmod{73},$$

$$16.11 \quad 27x^{19} = 41 \pmod{79}$$



### Takrorlash uchun savollar

1. Sonning modulga ko'ra ko'rsatkichi deb nimaga aytiladi?
2. Sonning modul bo'yicha boshlang'ich ildizi deb nimaga aytiladi?
3. Sonning modul bo'yicha indeksini tushuntiring.
4. Indeksning qanday xossalari bilasiz?
5.  $n$ -darajali ikki hadli taqqoslama deb nimaga aytiladi?
6.  $n$ -darajali ikki hadli taqqolama yechimlar soni nechta bo'ladi?
7. Tub modulli ikki hadli kvadratik taqqoslama yechimlari nechta?

## XIV MODUL. KO'PHADLAR

### 30-§. Bir o'zgaruvchili ko'phadlar

**Asosiy tushunchalar:**  $n$ -darajali ko'phad, ko'phadning ildizi, Bezu teoremasi, algebraik teng ko'phadlar, funktsional teng ko'phadlar.

Agar  $a_n \neq 0$  bo'lsa,  $u$  holda ushbu

$$a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0 = \sum_{i=0}^n a_i x^i \quad (a_i \in K, \quad i = \overline{0, n}, \forall n \in \mathbb{N})$$
 ifodani  $K$  maydon

ustidagi  $n$ -darajali ko'phad deyiladi.

Agar  $K$  butunlik sohasining biror  $c$  elementi uchun  $f(c) = 0$  tenglik o'rinli bo'lsa,  $u$  holda  $c$  element  $f(x)$  ko'phadning yoki  $f(x) = 0$  tenglamaning ildizi deyiladi.

Bezu teoremasi.  $f(x)$  ko'phadni  $x-s$  ikki hadga bo'lishdan xosil bo'lgan qoldiq  $f(c)$  ga teng.

$x=s$  element  $f(x)$  ko'phadning ildizi bo'lishi uchun  $f(x)$  ning  $x-s$  ikki hadga bo'linishi zarur va etarli.

Agar  $s_1, s_2, \dots, s_k$  lar  $f(x)$  ko'phadning turli ildizlari bo'lsa,  $u$  holda  $f(x)$  ko'phad  $(x-s_1)(x-s_2)\dots(x-s_k)$  ko'paytmaga bo'linadi.

Noldan farqli  $n$ -darajali ko'phad ( $n \geq 1$ )  $K$  butunlik sohasida  $n$  tadan ortiq ildizga ega emas.

Agar  $f(x) \in K[x]$  va  $0 \neq \varphi(x) \in K[x]$  ko'phadlar berilgan bo'lib, shunday  $g(x) \in K[x]$  ko'phad topilsaki, natijada  $f(x) = \varphi(x)g(x)$  tenglik o'rinli bo'lsa,  $u$  holda  $f(x)$  ko'phad  $\varphi(x)$  ko'phadga bo'linadi deyiladi va uni  $f(x) : \in \varphi(x)$  yoki  $f(x)/\varphi(x)$  ko'rinishlarda belgilanadi.

O'zgaruvchining bir xil darajalari oldidagi koeffitsientlari teng bo'lgan ko'phadlar o'zaro algebraik ma'nodagi teng ko'phadlar deyiladi.

Agar o'zgaruvchining biror cheksiz sohadan olingan xar qanday qiymatlariga mos keluvchi ko'phadlarning qiymatlari ustma-ust tushsa, u holda bunday ko'phadlarni o'zaro funksional ma'nodagi teng ko'phadlar deyiladi.

Berilgan  $f(x) = a_n x^n + \dots + a_1 x + a_0$  ko'phadni  $g(x) = b_m x^m + \dots + b_1 x + b_0$  ko'phadga bo'lishni quyidagi jadval asosida bajarish mumkin:

|           |                   |                                  |                                  |         |                                  |  |         |                              |                                      |       |
|-----------|-------------------|----------------------------------|----------------------------------|---------|----------------------------------|--|---------|------------------------------|--------------------------------------|-------|
|           | $a_n$             | $a_{n-1}$                        | $a_{n-2}$                        |         | $a_m$                            | $a_{m-1}$                                |         | $\dots$                      | $a_1$                                | $a_0$ |
| $b_m$     | $a_n$             | $\frac{a_{n-1}}{b_m}$            | $\frac{a_{n-2}}{b_m}$            |         |                                  |  |         |                              |                                      |       |
| $b_{m-1}$ |                   | $a_{n-1} - \sigma_1$             | $\frac{a_{n-2} - \sigma_2}{b_m}$ |         |                                  |  |         |                              |                                      |       |
|           |                   |                                  |                                  |         |                                  |  |         |                              |                                      |       |
|           |                   |                                  |                                  |         |                                  |  |         |                              |                                      |       |
| $b_1$     |                   |                                  |                                  |         |                                  |  |         |                              |                                      |       |
| $b_0$     |                   |                                  |                                  |         |                                  |  |         |                              |                                      |       |
|           |                   |                                  |                                  |         | $a_m - \sigma_{n-m}$             | $\frac{a_{m-1} - \sigma_{n-m}}{b_m}$     |         |                              | $b_0 \frac{a_n - \sigma_{n-m}}{b_m}$ |       |
|           | $\frac{a_n}{b_m}$ | $\frac{a_{n-1} - \sigma_1}{b_m}$ | $\frac{a_{n-2} - \sigma_2}{b_m}$ | $\dots$ | $\frac{a_m - \sigma_{n-m}}{b_m}$ | $\frac{a_{m-1} - \delta_{m-1}}{d_{m-1}}$ | $\dots$ | $\frac{a_0 - \delta_0}{d_0}$ |                                      |       |
|           | $c_{n-m}$         | $c_{n-m-1}$                      | $c_{n-m-2}$                      |         | $c_0$                            | $d_{m-1}$                                |         |                              |                                      |       |

**Misol.**  $g(x) \in Z[x]$ ,  $f(x) = x^4 + ax^3 + bx^2 - 8x + 4$  uchun  $f(x) = (g(x))^2$  shartni qanoatlantiruvchi barcha  $a$  va  $b$  butun sonlarni toping.

**Yechish.**  $f(x)$  ko'phadning darajasi 4 ga teng. Demak,  $g(x)$  ning darajasi 2 ga teng.  $g(x) = mx^2 + nx + p$ ,  $m \neq 0$  bo'lsin. Bundan  $(g(x))^2 = (mx^2 + nx + p)^2 = m^2 x^4 + 2mn(x^3 + 2mp + n^2)x^2 + 2np + p^2$   $f(x) = (g(x))^2$  dan quyidagi sistemani hosil qilamiz:

$$\begin{cases} m^2 = 1 \\ 2mn = a \\ 2mp + n^2 = b \\ 2np = -8 \\ p^2 = 4 \end{cases}$$

Sistemadan  $m = \pm 1$  va  $p = \pm 2$  larni hosil qilsak, u quyidagi 4 ta sistemalarga ajraladi:

$$1. \begin{cases} m = 1 \\ p = 2 \\ n = -2 \\ a = -4 \\ b = 8 \end{cases} \quad 2. \begin{cases} m = 1 \\ p = -2 \\ n = 2 \\ a = 4 \\ b = 0 \end{cases} \quad 3. \begin{cases} m = -1 \\ p = 2 \\ n = -2 \\ a = 4 \\ b = 0 \end{cases} \quad 4. \begin{cases} m = -1 \\ p = -2 \\ n = 2 \\ a = -4 \\ b = 8 \end{cases}$$

Demak, agar  $a = -4$  va  $b = 8$  bo'lsa,  $g_1(x) = -x^2 + 2x - 2$  va  $g_2(x) = x^2 - 2x + 2$ ;

Agar  $a = 4$  va  $b = 0$  bo'lsa,  $g_1(x) = x^2 + 2x - 2$  va  $g_2(x) = -x^2 - 2x + 2$  bo'ladi.

**Misol.** Ozod hadi 7 ga bo'linadigan barcha  $f(x) \in Z[x]$  lar to'plami  $K$  halqa tashkil etishini tekshiring.

**Yechish.**  $f(x) = a_n x^n + \dots + a_1 x + 7a_0$ ,  $g(x) = b_m x^m + \dots + b_1 x + 7b_0$  va  $m \geq n$  bo'lsin. U holda

$$\begin{aligned} f(x) + g(x) &= b_m x^m + \dots + (a_n + b_n)x^n + \dots + (a_1 + b_1)x + (7a_0 + 7b_0) = \\ &= b_m x^m + \dots + (a_n + b_n)x^n + \dots + (a_1 + b_1)x + 7(a_0 + b_0); \\ f(x) - g(x) &= (-b_m)x^m + \dots + (a_n - b_n)x^n + \dots + (a_1 - b_1)x + 7(a_0 - b_0); \\ f(x) \cdot g(x) &= a_n b_m x^{n+m} + \dots + 7(a_1 b_0 + a_0 b_1)x + 7 \cdot 7 a_0 b_0. \end{aligned}$$

Bundan  $f(x) + g(x)$ ,  $f(x) - g(x)$  va  $f(x) \cdot g(x)$  lar  $K$  to'plamning elementlari ekanligi kelib chiqadi. Demak,  $K$   $Z[x]$  ning qism halqasi.

**Misol**  $f(x) \in Z[x]$ ,  $\deg f(x) \leq 4$  uchun barcha  $\bar{a} \in Z_5$  da  $f(\bar{a}) = \bar{0}$  bo'lsa

$f(x)$

- nol ko'phad ekanligini isbotlang.

**Yechish**  $f(x) = ax^4 + bx^3 + cx^2 + dx + e$  bo'lsin.

U holda

$$f(0) = e = 0,$$

$$f(1) = \bar{a} + \bar{b} + \bar{c} + \bar{d} + \bar{e} = \bar{0},$$

$$f(2) = \bar{a} + 3\bar{b} + 4\bar{c} + 2\bar{d} + \bar{e} = \bar{0},$$

$$f(3) = \bar{a} + 2\bar{b} + 4\bar{c} + 3\bar{d} + \bar{e} = \bar{0},$$

$$f(4) = \bar{a} + 4\bar{b} + \bar{c} + 4\bar{d} + \bar{e} = \bar{0}.$$

$$\text{ya'ni, } \begin{cases} \bar{a} + \bar{b} + \bar{c} + \bar{d} = \bar{0}, \\ \bar{a} + 3\bar{b} + 4\bar{c} + 2\bar{d} = \bar{0}, \\ \bar{a} + 2\bar{b} + 4\bar{c} + 3\bar{d} = \bar{0}, \\ \bar{a} + 4\bar{b} + \bar{c} + 4\bar{d} = \bar{0}. \end{cases} \Leftrightarrow \begin{cases} \bar{a} + \bar{b} + \bar{c} + \bar{d} = \bar{0}, \\ 2\bar{b} + 3\bar{c} + \bar{d} = \bar{0}, \\ \bar{b} + 3\bar{c} + 2\bar{d} = \bar{0}, \\ 3\bar{b} + 3\bar{d} = \bar{0}. \end{cases} \Leftrightarrow \begin{cases} \bar{a} + \bar{b} + \bar{c} + \bar{d} = \bar{0}, \\ \bar{b} - \bar{d} = \bar{0}, \\ \bar{b} + 3\bar{c} + 2\bar{d} = \bar{0}, \\ \bar{b} + \bar{d} = \bar{0}. \end{cases}$$

$$\Leftrightarrow \bar{a} = \bar{b} = \bar{c} = \bar{d} = \bar{0}$$

Demak,  $f(x)$  - nol ko'phad.

**Misol.**  $f(x) = x^{19} + x^{17} + x^{13} + x^{11} + x^7 + x^5 - 6x^3$  ni  $g(x) = x^2 - 1$  ga  $Z[x]$  halqada bo'linishini isbotlang.

**Yechish.**

$$f(x) = (x^{19} - x^{17}) + (2x^{17} - 2x^{15}) + (2x^{15} - 2x^{13}) + (3x^{13} - 3x^{11}) + (4x^{11} - 4x^9) + (4x^9 - 4x^7) + (5x^7 - 5x^5) + (6x^5 - 6x^3) = x^{17}(x^2 - 1) + 2x^{15}(x^2 - 1) + 2x^{13}(x^2 - 1) + 3x^{11}(x^2 - 1) + 4x^9(x^2 - 1) + 4x^7(x^2 - 1) + 5x^5(x^2 - 1) + 6x^3(x^2 - 1).$$

$f(x)$  ko'phadlarning har bir hadi  $g(x)$  ga bo'linganligi uchun  $Z[x]$  halqada  $f(x):g(x)$ .

**Misol.**  $R[x]$  da  $f(x) = (x-2)^{100} + (x-1)^{50} + 1$  ko'phadni  $g(x) = x^2 - 3x + 2$  ko'phadga bo'lgandagi qoldiqni toping.

**Yechish.** Qoldiqli bo'lish haqidagi teoremaga asosan  $f(x) = g(x)h(x) + r(x)$  va  $\deg r(x) < 2$ , ya'ni  $r(x) = ax + b$ . Bundan

$$(x-2)^{100} + (x-1)^{50} + 1 = (x^2 - 3x + 2)h(x) + ax + b$$

$g(1) = g(2) = 0$  ekanligidan foydalanib  $x=1$  va  $x=2$  qiymatlarni tenglikka

qo'yamiz va quyidagi sistemani hosil qilamiz:

$$\begin{cases} a + b = 2 \\ 2a + b = 2 \end{cases} \Leftrightarrow \begin{cases} a = 0 \\ b = 2 \end{cases}$$

Demak,  $r(x) = 2$ .

**Misol.**  $f(x) = 4x^5 - 6x^3 + 2x^2 - 4$  ko'phadni  $g(x) = 2x^2 - 5x + 1$  ko'phadga  $Q[x]$  da bo'ling.

**Yechish.** Jadval tuzamiz:

|    |   |     |                |                 |                  |                 |
|----|---|-----|----------------|-----------------|------------------|-----------------|
|    | 4 | 0   | -6             | 2               | 0                | 4               |
| 2  | 4 | -10 | 2              |                 |                  |                 |
| -5 |   | 10  | -25            | 5               |                  |                 |
| 1  |   |     | 17             | $-\frac{85}{2}$ | $\frac{17}{2}$   |                 |
|    |   |     |                | $\frac{79}{2}$  | $-\frac{395}{4}$ | $\frac{79}{4}$  |
|    | 2 | 5   | $\frac{17}{2}$ | $\frac{79}{4}$  | $\frac{361}{4}$  | $-\frac{95}{4}$ |

Demak,  $f(x) = g(x) \left( 2x^3 + 5x^2 + \frac{17}{2}x + \frac{79}{4} \right) + \frac{361}{4}x - \frac{95}{4}$ .

$g(x) = 2x^2 - 5x + 1$  ikkihadga bo'ling.

**Misol.**  $R[x]$  da berilgan  $f(x) = 2x^7 + 4x^5 - x^4 - 6x^3 - x^2 + 3x - 2$  ko'phadni  $g(x) = 2x^3 - 1$  ikkihadga bo'ling.

**Yechish.**

|    |   |   |   |    |    |    |   |    |
|----|---|---|---|----|----|----|---|----|
|    |   |   | 4 | -1 | -6 | -1 | 3 | -2 |
| 2  | 2 | 0 |   | -1 | 0  | -2 | 0 | 3  |
| -1 |   |   | 2 | 0  | -3 | 1  | 3 | -5 |
|    | 1 | 0 |   |    |    |    |   |    |

Demak,  $f(x) = g(x)(x^4 + 2x^2 - 3) + (x^2 + 3x - 5)$

Misol.  $f(x) = 2x^3 + 4x^2 - x - 6x - x^2 + 3x - 2$  ko'phadni  $g(x) = x^2 + 1$  ikkihadning darajalari bo'yicha yoying

|   |   |   |    |    |    |    |    |    |
|---|---|---|----|----|----|----|----|----|
|   | 2 | 0 | 4  | -1 | -6 | -1 | 3  | -2 |
| 1 |   |   | 2  | 0  | 2  | -1 | -8 | 0  |
| 1 | 2 | 0 | 2  | -1 | -8 | 0  | 11 | -2 |
| 1 | 2 | 0 | 0  | -1 | -8 | 1  |    |    |
| 1 | 2 | 0 | -2 | -1 |    |    |    |    |
| 1 |   |   |    |    |    |    |    |    |
|   | 2 | 0 |    |    |    |    |    |    |

Demak,  $f(x) = 2x(x^2 + 1) + (-2x - 1)(x^2 + 1) + (-8x + 1)(x^2 + 1) + 11x - 2$

Misol.  $C[x]$ da berilgan.  $f(x) = (x + a + b)^{2005} - x^{2005} - a^{2005} - b^{2005}$  ning  $g_1(x) = x + a$  va  $g_2(x) = x + b$  ikkihadlarga bo'linishini isbotlang.

Isbot.  $f(x)$  ko'phadning  $x = -a$  va  $x = b$  dagi qiymatlarini topamiz:

$$f(-a) = b^{2005} + a^{2005} - a^{2005} - b^{2005} = 0$$

$$f(b) = a^{2005} + b^{2005} - a^{2005} - b^{2005} = 0$$

Bezu teoremasiga ko'ra  $f(x)$  ko'rhad  $g_1(x)$  va  $g_2(x)$  larga bo'linadi.

Misol.  $Q[x]$  halqada  $f(x) = 3x^2 - 6x^2 + 5x - 10$  va  $g(x) = 2x^3 - 4x^2 + 3x - 6$  ko'phadning EKUB, EKUKlarini toping.

Yechish.  $2f(x) - 3g(x) = x - 2$  va  $f(2) = g(2) = 0$  dan  $(f, g) = x - 2$  kelib chiqadi.

$$[f, g] = \frac{f(x)g(x)}{(f, g)}$$

ni e'tiborga olsak,  $[f, g] = (3x^3 - 6x^2 + 5x + 10)(2x^2 + 3)$  hosil bo'ladi.

Misol.  $Z_7[x]$  halqada  $f(x) = x^4 + 4x^3 + 4x^2 + 6x + 6$  va  $g(x) = x^3 - 1$  ko'phadlarning EKUBini chiziqli ifodasini toping.

Yechish:  $f(x)$  va  $g(x)$  larning EKUBini topamiz.

$$f(x) = g(x)h_1(x) + r_1(x) \text{ dagi } h_1(x) = x + 4, r_1(x) = 4x^2 + 3.$$

$$g(x) = r_1(x)h_2(x) + r_2(x) \text{ dagi } h_2(x) = 2x, r_2(x) = -6x - 1 = x + 6$$

$$r_1(x) = r_2(x)h_3(x) \text{ da } h_3(x) = 4x + 3.$$

Demak,  $(f, g) = x + 6 = r_3(x)$  ekan.

Evklid algoritmi yordamida

$$r_2(x) = g(x) - r_1(x)h_2(x) = g(x) - (f(x) - g(x)h_1(x))h_2(x) = f(x)(-h_2(x)) + g(x)(1 + h_1(x)h_2(x))$$

ifodani hosil qilamiz. Unda  $u(x) = -h_2(x) = -2x = 5x$  va

$$v(x) = 1 + h_1(x)h_2(x) = 1 + (x + 4)2x = 2x^2 + x + 1$$

belgilashlarni kiritsak

$$(f, g) = f(x)u(x) + g(x)v(x) \text{ hosil bo'ladi.}$$

Misol.  $Q[x]$  halqaning  $u(x) = (x^2 - 1) + v(x)(x^2 + 2x + 1) = x^3 + 1$  tenglamani qanoatlantiruvchi  $u(x)$  va  $v(x)$  ko'phadlarini toping.

Yechish. Berilgan tenglamada shakl almashtirish bajaramiz:

$$u(x)(x - 1)(x + 1) + v(x)(x + 1)^2 = (x + 1)(x^2 - x + 1) \text{ va}$$

$$u(x)(x - 1) + v(x)(x + 1) = x^2 - x + 1 \text{ ga ega bo'lamiz.}$$

$$g_1(x) = x - 1 \text{ va } g_2(x) = x + 1 \text{ ko'phadlar uchun } (g_1, g_2) = 1 \text{ bo'lganligi uchun}$$

$$\text{EKUBni } -\frac{1}{2}g_1(x) + \frac{1}{2}g_2(x) = 1 \text{ ko'rinishida ifodalash mumkin.}$$

Hosil bo'lgan ifodani ikkala tomonini  $(x^2 - x + 1)$  ga ko'paytirsak

$$-\frac{1}{2}(x^2 - x + 1)g_1(x) + \frac{1}{2}(x^2 - x + 1)g_2(x) = x^2 - x + 1 \text{ ga ega bo'lamiz. Bundan}$$

berilgan tenglamani qanoatlantiruvchi  $u(x)$  va  $v(x)$  larning xususiy qiymatlari

$$u_0(x) = -\frac{1}{2}(x^2 - x + 1)$$

$$v_0(x) = \frac{1}{2}(x^2 - x + 1)$$

kelib chiqadi.

Demak, berilgan tenglamani qanoatlantiruvchi  $u(x)$  va  $v(x)$  lar quyidagi ko'rinishda bo'ladi:

$$u(x) = \frac{1}{2}(x-x+1) \cdot (x+1)h(x)$$

$$v(x) = \frac{1}{2}(x-x+1) \cdot (x-1)h(x)$$

bu yerda  $h(x) \in Q[x]$ .



### Misol va mashqlar

1. Quyidagi ko'phadlarning qaysilari teng?

$$f_1(x) = 5^{\log_5 2} x^1 - \sqrt{4 - 2\sqrt{3}} x^2 + 2x - \operatorname{tg} \frac{\pi}{4};$$

$$f_2(x) = \left(1 - \sin \frac{\pi}{6}\right) x^2 + (i-1)^2 + 2 - i;$$

$$f_3(x) = \frac{1}{2} x^3 + (\sqrt{3} - 1) x^2 - 2i^2 x - i^4;$$

$$f_4(x) = \cos \frac{\pi}{3} x^3 + 2i^3 x^2 + i^7 + 2;$$

$$f_5(x) = \left(\frac{1}{6} + \operatorname{tg}^2 \frac{\pi}{6}\right) x^3 + \left(\operatorname{tg} \frac{\pi}{3} - 1\right) x^2 - 4 \cos^2 \frac{\pi}{3} x - 2.$$

2.  $a, b, c$  larning qanday qiymatlarida  $Z[x]$  ning quyidagi ko'phadlari teng:

$$2.1. f(x) = ax^2(x+1) + b(x^2+1)(x-6) + cx(x^2+1) \text{ va } g(x) = x^2 + 5x + 6$$

$$2.2. f(x) = ax(x^2+3) + bx(x-1) + c(x+1) \text{ va } g(x) = 2x^3 + 5x^2 + 8x + 7.$$

3. Berilgan halqada  $f(x) = (g(x))^2$  shartni qanoatlantiruvchi  $g(x)$  ko'phadni toping:

$$3.1. f(x) = x^4 + 6x^3 + 11x^2 + ax + 1, z[x];$$

$$3.2. f(x) = 4x^4 + \bar{a}x^2 - \bar{1}, z_s[x];$$

$$3.3. f(x) = 9x^4 - 12x^3 + 16x^2 - 8x + a, z[x].$$

4.  $Z[x]$  halqa uchun  $f(x) = (f(x))^2$  shart o'rinli bo'lgan barcha  $a$  va  $b$  butun sonlarni va  $g(x)$  ni toping:  $f(x) = x^4 + ax^3 + bx - 8x + 1$

5.  $\frac{x+5}{(x-1)(x-2)(x-3)} = \frac{a}{x-1} + \frac{b}{x-2} + \frac{c}{x-3}$  shartni qanoatlantiruvchi  $a, b, c$  butun sonlarni toping.

6. Berilgan ko'phadlar berilgan halqalarda funktsional tengligini isbotlang:

$$6.1. f(x) = x^3 - 2x^2 \text{ va } g(x) = x^2 - 2x, Z_2[x];$$

$$6.2. f(x) = 2x^2 + x + 1 \text{ va } g(x) = 2x^3 + 2x^2 + 2x + \bar{1}, Z_2[x];$$

$$6.3. f(x) = x^{10} + 4x^2 \text{ va } g(x) = 4x^5 + x, Z_2[x];$$

7.  $f(x)$  ko'phad  $g(x)$  ko'phadga berilgan halqada bo'linadimi?

$$7.1. f(x) = (3x^2 - 2x - 1)(10x^3 + 10x - 20) + 2x^4 - 2$$

$$g(x) = 5x^2 - 5, Z[x] \text{ va } Q[x];$$

$$7.2. f(x) = x^2 + x + \bar{1}, g(x) = x - \bar{1}, Z_3[x];$$

$$7.3. f(x) = x^{100} + x^{98} + x^{96} + \dots + x^4 + x^2 + x - i, g(x) = x^{2+1}, C[x].$$

8.  $f(x)$  ni  $g(x)$  ga bo'linishining zarur va etarli shartini aniqlang:

$$8.1. f(x) = x^3 + bx + c, g(x) = x^2 + ax - 1, R[x];$$

$$8.2. f(x) = x^3 + bx + c, g(x) = x^2 + 1, Z[x];$$

$$8.3. f(x) = x^4 + bx^2 + c, g(x) = x^2 + ax + 1, Z[x].$$

9. Qoldiqni toping:

$$9.1. f(x) = x^{10000} + x^{1000} + x^{100} + x^{10} + x - 1, g(x) = ix - 1; (Z[i])[x];$$

$$9.2. f(x) = x^{30} + x^{26} + x^{20} + x^{15} + x^{10} + x + 1, g(x) = x^5 - 1; Z[x];$$

$$9.3. f(x) = x^{1982} + x^{991} + 1, g(x) = x^2 - 1; C[x];$$

$$9.4. f(x) = x^{1982} + x + 1, g(x) = x^2 - (1+i)x + i; C[x];$$

$$9.5. f(x) = x^{100} + x^{99} - 2x^{98} - 3x^3 + 2x + 5, g(x) = x^2 + x - 2; Z[x].$$

10. Qoldiqni bo'lishni bajaring:

$$10.1. f(x) = 3x^5 + x^4 - 10x^3 + 12x^2 + 10x - 8, g(x) = 3x^2 + x - 1; Q[x];$$

$$10.2. f(x) = (2i + 3)x^4 - 4ix + i - 2; g(x) = x^2 + i, C[x];$$

$$10.3. f(x) = 4x^4 + 2x^2 - x + 1, g(x) = 2x + 3; Z_7[x];$$

$$10.4. f(x) = 10x^7 - 36x^6 + 13x^5 + 38x^4 - 6x^3 + 3x^2 - 20x - 13,$$

$$g(x) = 2x^2 - 4x - 3, R[x].$$

11.  $Z[x]$  da  $f(x)$  ni  $g(x)$  ga bo'lganda  $r(x) = 3x^2 - 4x + 1$  qoldi. Agar deg  $g(x) = 5$  bo'lsa, u holda  $(f(x))^2$  ni  $g(x)$  ga bo'lgandagi qoldiqni toping.

12.  $R[x]$  da  $f(x)$  ni  $g(x)$  ga bo'lganda 3,  $(f(x))^2$  ni  $(g(x))^2$  bo'lganda 9 qoldiq qoldi.  $f(x)$  ni  $(g(x))^2$  ga bo'lgandagi qoldiqni toping.

13. Berilgan halqada  $f(x)$  ni  $g(x)$  ga bo'ling:

$$13.1. f(x) = 10x^4 - 23x^3 + 26x^2 - 9x - 2; g(x) = 2x - 3; Z[x]$$

$$13.2. f(x) = (2 + 2i)x^4 - 6x^3 + (2 - 4i)x^2 + (1 + 11i)x + 2 - 5i; g(x) = (1 - i)x + 3i; C[x]$$

$$13.3. f(x) = 2x^5 + 12,5x^3 - 4x^2 + 5,5x - 2,5; g(x) = 4x^2 + 1; Q[x]$$

$$13.4. f(x) = -5x^6 + 5x^5 - 2x^3 + 3x^2 - 2x + 5; g(x) = 6x^3 + 4; Z_7[x]$$

14.  $f(x)$  ko'phadning  $x_0$  nuqtadagi qiymatini toping:

$$14.1. f(x) = x^4 - 3x^3 + 6x^2 - 10x + 16; x_0 = 4, Z[x];$$

$$14.2. f(x) = x^5 + (1 + 2i)x^4 - (1 + 3i)x^2 + 7, x_0 = -2 - i, C(x);$$

$$14.3. f(x) = x^5 + x^4 + 3x^2 + 1, x_0 = 3, Z_5[x];$$

$$14.4. f(x) = x^3 - (1 + \sqrt{2})x^2 + (1 + \sqrt{2}), x_0 = 1 - \sqrt{2}, Q[(\sqrt{2})][x]$$

15.. Har qanday  $a \in K$  uchun  $f(x) \in K[x]$  ko'phad  $g(x) = x - a$  ikkihadga bo'linishini isbotlang.

$$15.1. f(x) = x^7 - x; K = Z_7,$$

$$15.2. f(x) = x^{10} - x^5; K = Z_5,$$

$$15.3. f(x) = x^p - x; K = Z_p,$$

16.  $R[x]$  halqada  $n$  - toq natural sonlar uchun  $f(x) = (x+a+b)^n - x^n - a^n - b^n$  ko'phad  $g(x) = x + a$  va  $g_1(x) = x + b$  ikkihadlarga bo'linishini isbotlang.

17.  $f(x)$  ko'phadni  $x - a$  darajalariga yoying:

$$17.1. f(x) = x^4 - 2x^3 + 3x^2 - 5x + 1, a = 1; Q[x].$$

$$17.2. f(x) = 2x^4 + x^3 + x^2 + 2, a = 1; Z_3[x].$$

$$17.3. f(x) = x^5 - 3ix^3 - 4x^2 + 5ix - 1, a = -i; C[x].$$

18. Berilgan ko'phadlarning EKUBini toping:

$$18.1. f(x) = x^3 + 3x^2 - 2, g(x) = x^3 + 3x^2 - x - 3;$$

$$18.2. f(x) = x^4 + x^3 - 3x^2 - 2x - 2; g(x) = -x^3 + 3x^2 + 2x + 2;$$

$$18.3. f(x) = 2x^3 + x^2 + 4x + 2, g(x) = 2x^3 + x^2 + 6x + 3.$$

19. Evklid algoritmi yordamida berilgan ko'phadlar EKUBini toping.

$$19.1. f(x) = x^3 + x^2 - x - 6; g(x) = x^3 + x^2 - 10x - 6, Q[x];$$

$$19.2. f(x) = x^4 + x^3 + x^2 + x + 1, g(x) = 4x^3 + 3x^2 + 2x + 1, Q[x];$$

$$19.3. f(x) = x^3 + 3x^2 + 2x + 1, g(x) = x^3 + 2x^2 + x + 2, Z_5[x];$$

$$19.4. f(x) = x^4 + 2ix^3 - 2x^2 - 2ix + 1; g(x) = x^3 + (i+1)x^2 + ix, C[x].$$

20. Berilgan ko'phadlar EKUKini toping.

$$20.1. f(x) = 2x^3 + 7x^2 + 4x - 3, g(x) = x^3 + x^2 - 3x + 1, Q[x];$$

$$20.2. f(x) = x^3 + 6x^2 + 4x + 1, g(x) = x^3 + x^2 + 3x - 4, Z_7[x];$$

$$20.3. f(x) = x^3 - x^2 + 3x - 3, g(x) = x^4 + 2x^3 + 2x - 1, R[x];$$

$$20.4. f(x) = x^4 + 2ix^3 - 2x^2 - 2ix + 1, g(x) = x^3 + (i+1)x^2 + ix, C[x].$$

21. Berilgan ko'phadlar EKUBining chiziqli ifodasidagi  $u(x)$  va  $v(x)$  larni toping.

$$21.1. f(x) = x^3 + 5x^2 + 6x + 2, g(x) = x^2 + 6x + 5; Q[x];$$

$$21.2. f(x) = x^3 + 3x^2 + 2x + 1, g(x) = x^3 + 2x^2 + x + 2, Z_3[x];$$

$$21.3. f(x) = 4x^4 - 2x^3 - 16x^2 + 5x + 9, g(x) = 2x^3 - x^2 - 5x + 4, Q[x].$$

22  $K[x]$  halqada  $f(x)u(x) + g(x)v(x) = \varphi(x)$  tenglamaning barcha yechimlarini toping

22.1  $f(x) = x^2 - 1, g(x) = x^2 - 2x + 1, \varphi(x) = x^3 - 1, K = \mathbb{Q};$

22.2  $f(x) = x^2 - 1, g(x) = x^2 + 3, \varphi(x) = x^3 - 2x^2 + x, K = \mathbb{Z}_5;$

23  $[x]$  halqada  $f(x)u(x) + g(x)v(x) + h(x)\varphi(x) = S(x)$  tenglama yechimga ega ekanligini tekshiring. Bu erda  $f(x) = x^4 + 2x^2 + 1; g(x) = x^3 - x^2 + x - 1; h(x) = x^2 + (i+1)x + i, S(x) = x^2 + (i-1)x - i.$

24.  $K$  maydon ustida berilgan ko'phadlar uchun quyidagi xossalarni isbotlang:

24.1.  $((f(x) : \varphi(x)) \wedge (\varphi(x) : \psi(x))) \Rightarrow (f(x) : \psi(x)), (\varphi(x) \neq 0, \psi(x) \neq 0).$

24.2.  $(f_i(x) : \varphi(x)) \Rightarrow (f_1(x) \pm f_2(x) \pm \dots \pm f_m(x)) : \varphi(x) (i = \overline{1, m}), (\varphi(x) \neq 0).$

24.3.  $f_i(x) (i = \overline{1, m})$  ko'phadlarning kamida bittasi  $\varphi(x) \neq 0$  ga bo'linsa, u holda ularning ko'paytmasi ham  $\varphi(x)$  ga bo'linadi.

24.4. Agar  $f_i(x) (i = \overline{1, m})$  ko'phadlarning xar biri  $\varphi(x) \neq 0$  ga bo'linib,  $g(x)$  lar ixtiyoriy ko'phadlar bo'lsa, u holda  $(f_1(x)g_1(x) \pm f_2(x)g_2(x) \pm \dots \pm f_m(x)g_m(x)) : \varphi(x)$  bo'ladi.

24.5. Har qanday  $f(x)$  ko'phad ixtiyoriy nolinchi darajali ko'phadga bo'linadi.

24.6.  $f(x) : \varphi(x) \Rightarrow f(x) : a\varphi(x) (0 \neq a \in K \text{ maydon}, \varphi(x) \neq 0)$

24.7.  $f(x) \neq 0$  va  $\varphi(x) \neq 0$  ko'phadlar bir-biriga bo'linsa, u holda ular bir-biridan o'zgarmas  $a \neq 0$  ko'paytuvchi bilan farq qiladi.



### Takrorlash uchun savollar

1. Ko'phad darajasining xossalarini ayting.
2. Ko'phadni ikkihadga bo'lishni tushuntiring.
3. Ko'phad ildizi va uning xossalarini ayting.
4. Ko'phadlarning tengligi (algebraic, funksional).
5. Qoldiqli bo'lish haqidagi teoremani tushuntiring.
6. Evklid algoritmi qanday tuziladi?

## 31-§. Ko'p o'zgaruvchili ko'phadlar

**Asosiy tushunchalar:** ko'p o'zgaruvchili ko'phad, ko'phadning darajasi, bir jinsli ko'phad, leksikografik yozilgan ko'phad, simmetrik ko'phad, asosiy (elementar) simmetrik ko'phadlar, ko'phadlarning rezultanti.

Kamida ikkita o'zgaruvchiga bog'liq bo'lgan ko'phad ko'p o'zgaruvchili ko'phad deyiladi.

$n$  ta noma'lumli ko'phad  $x_1^a x_2^b \dots x_n^c$  ko'rinishdagi chekli sondagi hadlarning algebraik yig'indisidan iborat.

$n$  ta  $x_1, x_2, \dots, x_n$  o'zgaruvchili ko'phad  $f(x_1, x_2, \dots, x_n) = \sum_{i=1}^m a_i x_1^{\alpha_i} x_2^{\beta_i} \dots x_n^{\gamma_i}$  ko'rinishda bo'ladi. Bunda  $a_i \in K$ .

$n$  ta noma'lumli ko'phadning darajasi deb, bu ko'phaddagi qo'shiluvchilar darajalarining kattasiga aytiladi.

Barcha kushiluvchilarining darajalari bir xil bo'lgan ko'phadga bir jinsli ko'phad yoki forma deyiladi.

$f(x_1, x_2, \dots, x_n)$  ko'phad berilgan bo'lib, uning ikkita hadidan qaysi birida  $x_1$  ning darajasi katta bo'lsa, usha had yuqori had deb yuritiladi. Agar bu hadlardagi  $x_1$  ning darajasi teng bo'lib, qaysi birida  $x_2$  ning darajasi katta bo'lsa o'sha had yuqori deb xisoblanadi va x.k.

$f(x_1, x_2, \dots, x_n)$  ko'phadni birinchi o'rinda eng yuqori hadni, ikkinchi o'rinda qolgan hadlar orasida eng yuqori bo'lgan hadni va shu jarayon oxirgi had uchun yozilgan bo'lsa, u holda  $f(x_1, x_2, \dots, x_n)$  ko'phad leksikografik yozilgan deyiladi.

Agar ko'p noma'lumli ko'phaddagi ixtiyoriy ikkita noma'lumning o'rinlarini almashtirganda ko'phad o'zgarmasa, u holda bunday ko'phad simmetrik ko'phad deyiladi.

$x_1, x_2, \dots, x_n$  o'zgaruvchilardan tuzilgan

$$\begin{cases} f = \tau_1 + \tau_2 + \dots + \tau_n \\ \tau_1 = \tau_1 \tau_2 + \tau_1 \tau_3 + \dots + \tau_1 \tau_n \\ \dots \\ \tau_n = \tau_1 \tau_2 \dots \tau_n \end{cases}$$

(1)

sistemadagi simmetrik ko'phadlar asosiy (elementar) simmetrik ko'phadlar deyiladi.

Simmetrik ko'phadlar haqidagi asosiy teorema. F maydon ustidagi xar qanday simmetrik ko'phad shu F maydon ustidagi elementar simmetrik ko'phadlar orqali yagona usulda ifodalanadi.

**Misol.**  $f(x, y, z) = (x + 2y)(z^2 - 1) + (y - z)^2 - (x + z)(y - 2)$  ko'phadni bir jinsli ko'phadlar yirindisiga keltiring.

**Yechish:** Berilgan ko'phaddagi qavslarni ochib, guruhlasak  $f(x, y, z) = (xz^2 + 2yz^2) + (-xy + y^2 - 3yz + z^2) + (x - 2y + 2z)$  hosil bo'ladi.

**Misol.**  $f(x, y, z) = 3x^3 + 3y^3 + 3z^3 + 5xyz + 2x^2 + 2y^2 + 2z^2$  simmetrik ko'phadni elementar simmetrik ko'phadlarga yoying.

**Yechish:** Berilgan ko'phaddan  $2x^2 + 2y^2 + 2z^2 = 2((x + y + z)^2 - 2(xy + yz + xz)) = 2\sigma_1^2 - 4\sigma_2$

ko'phadning  $3x^3 + 3y^3 + 3z^3 + 5xyz$  qismini noma'lum koeffitsientlar usulida elementar simmetrik ko'phadlarga yoyamiz. Buning uchun quyidagi jadvalni tuzamiz:

| Yuqori had darajalari |   |   | Yuqori had | Elementar simmetrik ko'phadlar yoyilmasi              |
|-----------------------|---|---|------------|---|
| x                     | y | z |            |   |
| 3                     | 0 | 0 | $3x^3$     | $3\sigma_1^3 - 3\sigma_1\sigma_2^2 = 3\sigma_1^3$     |
| 2                     | 1 | 0 | $ax^2y$    | $a\sigma_1^2 - 2\sigma_1\sigma_2 = a\sigma_1\sigma_2$ |
| 1                     | 1 | 1 | $bxyz$     | $b\sigma_1 - \sigma_2 = b\sigma_3$                    |

Bu jadvaldan  $3x^3 + 3y^3 + 3z^3 + 5xyz = 3\sigma_1^3 + a\sigma_1\sigma_2 + b\sigma_3$  ga ega bo'lamiz. O'zgaruvchilarga qiymat berish yordamida a, b parametrlarni topamiz:

| x | y | z  | $\sigma_1$ | $\sigma_2$ | $\sigma_3$ | $3x^3 + 3y^3 + 3z^3 + 5xyz = 3\sigma_1^3 + a\sigma_1\sigma_2 + b\sigma_3$ |
|---|---|----|------------|------------|------------|---|
| 1 | 1 | 0  | 2          | 1          | 0          | $6 = 24 + 2a$   |
| 1 | 1 | -2 | 0          | -3         | -2         | $-28 = -2b$   |

Demak,  $a = -9, b = 14$ , ya'ni  $f(x, y, z) = 3\sigma_1^3 - 9\sigma_1\sigma_2 + 14\sigma_3 + 2\sigma_1^2 - 4\sigma_2$ .

**Misol.**  $x_1^2 x_2^2$  hadning orbitasini elementar simmetrik ko'phadlar yordamida ifodani toping.

**Yechish:**  $x_1^2 x_2^2$  hadning orbitasi yoki  $x_1^2 x_2^2$  had yordamida hosil qilingan monogen ko'phad  $\sigma(x_1, x_2, \dots, x_n) = x_1^2 x_2^2 + \dots + x_{n-1}^2 x_n^2$  ko'rinishda bo'lib, uni topish uchun quyidagi jadvalni tuzamiz:

| Yuqori had darajalari |       |       |       |     |       | Yuqori had         | Hosil bo'lgan elementar ko'phadlar                                     |
|-----------------------|-------|-------|-------|-----|-------|--------------------|--|
| $x_1$                 | $x_2$ | $x_3$ | $x_4$ | ... | $x_n$ |                    |  |
| 2                     | 2     | 0     | 0     | ... | 0     | $x_1^2 x_2^2$      | $\sigma_1^{2-2} \sigma_2^{2-0} = \sigma_2^2$                           |
| 2                     | 1     | 1     | 0     | ... | 0     | $ax_1^2 x_2 x_3$   | $a\sigma_1^{2-1} \sigma_2^{1-1} \sigma_3^1 = a\sigma_1 \sigma_3$       |
| 1                     | 1     | 1     | 1     | ... | 0     | $bx_1 x_2 x_3 x_4$ | $b\sigma_1^{1-1} \sigma_2^{1-1} \sigma_3^{1-1} \sigma_4^1 = b\sigma_4$ |

Demak,  $\sigma(x_1, x_2, \dots, x_n) = \sigma_2^2 + a\sigma_1 \sigma_3 + b\sigma_4$ .

O'zgaruvchilarga qiymatlar beramiz.

| $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | ... | $x_n$ | $\sigma_1$ | $\sigma_2$ | $\sigma_3$ | $\sigma_4$ | $\sigma_5$ | ... | $\sigma_n$ | $\sigma(x_1, x_2, \dots, x_n) = \sigma_2^2 + a\sigma_1 \sigma_3 + b\sigma_4$ |
|-------|-------|-------|-------|-------|-----|-------|------------|------------|------------|------------|------------|-----|------------|--|
| 1     | 1     | 1     | 0     | 0     | ... | 0     | 3          | 3          | 1          | 0          | 0          | ... | 0          | $3 = 9 + 3a$   |
| 1     | 1     | 0     | 1     | 0     | ... | 0     | 4          | 6          | 3          | 1          | 0          | ... | 0          | $6 = 36 + 16a + b$   |

Bundan  $a = -2, b = 2$  qiymatlarni hosil qilamiz.

Demak,  $\sigma(x_1, x_2, \dots, x_n) = \sigma_2^2 - 2\sigma_1 \sigma_3 + 2\sigma_4$ .

**Misol.**  $x \frac{19-x}{x+1} \left( x + \frac{19-x}{x+1} \right) = 84$  tenglamani yeching.

**Yechish:**  $u = x \frac{19-x}{x+1}, v = x + \frac{19-x}{x+1}$  belgilashlarni kiritib  $\begin{cases} u+v=19 \\ u \cdot v=84 \end{cases}$

... sistemani tuzamiz. Bu sistemaning  $u_1 = 7, v_1 = 12; u_2 = 12, v_2 = 7$  yechimlari mavjud. U lar yordamida



... yechimlari  $x_1 = 6 \pm \sqrt{29}$  va  $x_2 = 4, x_3 = 3$ .

B maydonda  $f(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$ ,

$$g(x) = b_m x^m + b_{m-1} x^{m-1} + \dots + b_1 x + b_0$$

ko'phadlar berilgan va  $a_n b_m \neq 0 \wedge \alpha_1, \alpha_2, \dots, \alpha_n$  lar  $f(x)$  ning ildizlari bo'lsin.

$f(x), g(x)$  ko'phadlarning rezultanti  $R(f, g) = a_n^m g(\alpha_1)g(\alpha_2)\dots(g(\alpha_n))$  dan iborat.

Agar  $\beta_1, \beta_2, \dots, \beta_m - g(x)$  ning ildizlari bo'lsa, u holda

$$R(f, g) = a_n^m b_m^n \prod_{\substack{1 \leq i \leq n \\ 1 \leq j \leq m}} (\alpha_i - \beta_j)$$

$R(g, f) = (-1)^{mn} R(f, g)$  lar o'rinli.

Silvestr formulasi yordamida rezultant quyidagicha topiladi:

$$R(f, g) = \begin{vmatrix} a_n & a_{n-1} & \dots & a_1 & a_0 & 0 & \dots & 0 \\ 0 & a_n & \dots & a_2 & a_1 & a_0 & \dots & 0 \\ \dots & \dots \\ 0 & 0 & \dots & a_n & a_{n-1} & a_{n-2} & \dots & a_0 \\ b_m & b_{m-1} & \dots & b_1 & b_0 & 0 & \dots & 0 \\ \dots & \dots \\ 0 & 0 & \dots & b_m & b_{m-1} & b_{m-2} & \dots & 0 \end{vmatrix}$$

$f(x)$  ko'phadning diskriminanti:

$$D(f) = (-1)^{\frac{n(n-1)}{2}} a_n^{-1} R(f, f') \text{ yoki } D(f) = a_n^{2n-2} \prod_{1 \leq i < j \leq n} (\alpha_i - \alpha_j)^2.$$

Misol.  $f(x) = x^2 - 3x + 6$  va  $g(x) = x^3 + x^2 - x - 1$  uchun rezultant

Yechish: 1-usul. Berilgan  $f(x)$  ning ildizlari kompleks sonlar  $\alpha_1, \alpha_2$  ning ildizlari  $x = \pm 1$ .

$$R(f, g) = (-1)^{3 \cdot 2} R(g, f) = R(g, f) \text{ bo'lganligi uchun}$$

$$R(f, g) = f(-1)f(-1)f(1) = 10 \cdot 10 \cdot 4 = 400.$$

2-usul.  $\alpha_1$  va  $\alpha_2$   $f(x)$  ning ildizlari bo'lsin. U holda

$$R(f, g) = (\alpha_1^3 + \alpha_1^2 - \alpha_1 - 1)(\alpha_2^3 + \alpha_2^2 - \alpha_2 - 1) = (\alpha_1 \alpha_2)^3 + (\alpha_1 \alpha_2)^2 (\alpha_1 \alpha_2)^{-1} - \alpha_1 \alpha_2 (\alpha_1^2 + \alpha_2^2) - (\alpha_1^3 + \alpha_2^3) + (\alpha_1 \alpha_2)^2 - \alpha_1 \alpha_2 (\alpha_1 + \alpha_2) - (\alpha_1^2 + \alpha_2^2) + (\alpha_1 + \alpha_2) + 1$$

Viet formulalariga ko'ra

$$\alpha_1 + \alpha_2 = 3 \text{ va } \alpha_1 \alpha_2 = 6 \text{ bo'lib,}$$

$$\alpha_1^2 + \alpha_2^2 = (\alpha_1 + \alpha_2)^2 - 2\alpha_1 \alpha_2 = 3 \text{ va}$$

$$\alpha_1^3 + \alpha_2^3 = (\alpha_1 + \alpha_2)((\alpha_1 + \alpha_2)^2 - 3\alpha_1 \alpha_2) = -27.$$

Demak,  $R(f, g) = 400$ .

3-usul.

$$R(f, g) = \begin{vmatrix} 1 & -3 & 6 & 0 & 0 \\ 0 & 1 & -3 & 6 & 0 \\ 0 & 0 & 1 & -3 & 6 \\ 1 & 1 & -1 & -1 & 0 \\ 0 & 1 & 1 & -1 & -1 \end{vmatrix} = \dots = 400$$

$f(x)$  ko'phad karrali ildizga ega bo'lishi uchun  $R(f, f') = 0$  bo'lishi zarur va etarli.

Ikki o'zgaruvchili yuqori darajali tenglamalar sistemasini rezultant yordamida yechish mumkin.

Misol.  $\begin{cases} y^2 - y + x^2 - 3x = 0 \\ y^2 + (11-6x)y - x^2 + 7x - 12 = 0 \end{cases}$  sistemaning yechimlarini toping.

Yechish: Sistemani  $y$  o'zgaruvchiga nisbatan qaraymiz va uning rezultantini tuzamiz:

$$R(x) = \begin{vmatrix} 1 & -1 & x^2 - 3x & 0 \\ 0 & 1 & -1 & x^2 - 3x \\ 1 & 11 - 6x & x^2 + 7x - 12 & 0 \\ 0 & 1 & 11 - 6x & -x^2 + 7x - 12 \end{vmatrix} = \dots =$$

$$= 2x(x-2)(20x^2 - 100x + 120) = 40x(x-2)^2(x-3)$$

Bundan  $x_1 = 0$ ,  $x_2 = 2$ ,  $x_3 = 0$  yechimlarni topamiz.  $x$  o'zgaruvchining qiymatlarini sistemaga qo'yib  $y$  ning qiymatlarini topamiz.

- 1)  $x = 0$  da  $y = 1$  bo'lib,  $(0; 1)$  yechim.
- 2)  $x = 2$  da  $y = -1 \wedge y = 2$  bo'lib,  $(2; -1) \wedge (2; 2)$  yechimlar.
- 3)  $x = 3$  da  $y = 0$  bo'lib,  $(3; 0)$  yechim.



### Misol va mashqlar

1. Berilgan ko'phadlarni kanonik shaklga keltiring:

1.1.  $f(x, y) = (x - y)^2(x^2 + xy + y^2)(x + 2y) + x^2 - 1$

1.2.  $f(x, y) = (x - y)(xy - z)(x - z)xyz$ .

2. Berilgan ko'phadlarni leksiografik tartibda yozing va uning yuqori hadini toping:

2.1.  $f(x, y, z) = (\bar{2}x + \bar{3}y)^2 z - x(y + z - \bar{3}xz)$ ,  $Z_5[x, y, z]$ .

2.2.  $f(x, y, z, t) = (x + y)(z + y) + \bar{2}x(y + t + \bar{1}) + (y + z)^3$ ,  $Z_3[x, y, z, t]$ .

3. Quyidagi ko'phadlarni simmetrik ko'phadga to'ldiring:

3.1.  $f(x, y) = x^2 + 2y$ .

3.2.  $f(x, y) = x^3 + x^2y + xy$ .

3.3.  $f(x, y, z) = x^3 + 2xy + 2yz + 5$ .

3.4.  $f(x, y, z) = (x + y)^2 + 2xz + xyz$ .

4. Quyidagi ko'phadlarning yuqori hadini toping:

4.1.  $f(x_1, y_2) = 5\zeta_1^2 \zeta_2 \zeta_3$ .

4.2.  $f(x_1, y_2) = 5\zeta_1^2 + 2\zeta_2 \zeta_3 - 3\zeta_3^2$ .

5. Quyidagi ko'phadlarni elementar ko'phadlar yordamida ifodalang:

5.1.  $f(x, y) = x^3y + y^3x + 2x^2 + 2y^2$ .

5.2.  $f(x, y) = 2x^4y - 5x^2y + 2xy^4 - 5xy^2$ .

5.3.  $f(x, y, z) = x^2y + xy^2 + x^2z + x^2 + y^2z + yz^2$ .

5.4.  $f(x, y, z) = x^4 + y^4 + z^4 - 2x^2y^2 - 2x^2z^2 - 2y^2z^2$ .

5.5.  $f(x, y, z, t) = (xy + zt)(xz + yt)(xt + yz)$ .

5.6.  $f(x, y, z) = (xy + z)(xz + y)(yz + x)$ .

6. Agar  $n \in \mathbb{N}$  uchun  $S_n(x, y) = x^n + y^n$  bo'lsa, barcha  $k > 2$  uchun  $S_k = \zeta_1 S_{k-1} - \zeta_2 S_{k-2}$  ekanligini isbotlang.

7. 6-misoldagi formula yordamida quyidagilarni o'rinli ekanligini tekshiring:

7.1.  $S_2 = \zeta_1^2 - 2\zeta_2$ .

7.2.  $S_3 = \zeta_1^3 - 3\zeta_1 \zeta_2$ .

7.3.  $S_4 = \zeta_1^4 - 4\zeta_1^2 \zeta_2 + 2\zeta_2^2$ .

7.4.  $S_7 = \zeta_1^7 - 7\zeta_1^5 \zeta_2 + 14\zeta_1^3 \zeta_2^2 - 7\zeta_1 \zeta_2^3$ .

8. Agar  $n \in \mathbb{N}$  uchun  $S_n(x, y, z) = x^n + y^n + z^n$  bo'lsa, barcha  $k > 3$  uchun  $S_k = \sigma_1 S_{k-1} - \sigma_2 S_{k-2} + \sigma_3 S_{k-3}$  ekanligini isbotlang.

9. 8-misoldan foydalanib quyidagilar o'rinli ekanligini tekshiring:

9.1.  $S_3 = \sigma_1^3 - 3\sigma_1 \sigma_2 + 3\sigma_3$ .

9.2.  $S_4 = \sigma_1^4 - 4\sigma_1^2 \sigma_2 + 2\sigma_2^2 + 4\sigma_1 \sigma_3$ .

9.3.  $S_5 = \sigma_1^5 - 5\sigma_1^3 \sigma_2 + 5\sigma_1 \sigma_2^2 + 5\sigma_1^2 \sigma_3 - 5\sigma_2 \sigma_3$ .

9.4.  $S_6 = \sigma_1^6 - 6\sigma_1^4 \sigma_2 + 9\sigma_1^2 \sigma_2^2 - 2\sigma_2^3 + 6\sigma_1^3 \sigma_3 - 12\sigma_1 \sigma_2 \sigma_3 + 3\sigma_3^2$ .

10. Berilgan ratsional kasrlar surat va mahrajini elementar simmetrik ko'phadlar orqali ifodalab qiymatini toping:

10.1.  $\frac{f(x, y)}{g(x, y)} = \frac{(x - y)^4}{x + y}$  va  $\zeta_1 = 2$ ,  $\zeta_2 = 1$ .

$$10.2 \quad \frac{f(x, y, z)}{g(x, y, z)} = \frac{1}{1} \cdot \frac{1}{z} \cdot \frac{1}{x} \cdot \frac{2}{xy} \cdot \frac{2}{xz} \cdot \frac{2}{yz} \quad \text{va } \xi_1 = 0, \xi_2 = 1, \xi_3 = 2.$$

11. Quyidagi hadlar orbitasini elementar simmetrik ko'phadlar yordamida ifodalang:

1.1.  $x_1^3 x_1, P[x_1, x_2, x_3].$

1.2.  $x_1 x_2 x_3, P[x_1, x_2, x_3, x_4].$

1.3.  $x_1^3, P[x_1, x_2, \dots, x_n].$

12. Isbotlang:

12.1.  $x^4 + y^4 + (x - y^4) = 2(x^2 + xy + y^2)^2.$

12.2.  $(x + y)^3 + 3xy(1 - x - y) - 1 = (x + y - 1)(x^2 + y^2 - xy + x + y + 1).$

12.3.  $x(y + z)^2 + y(x + z)^2 + z(x + y)^2 = (y + z)(x + z)(x + y) + 4xyz.$

12.4.  $(xy + xz + yz)^3 + (x^2 - yz)^2 + (y^2 - zx)^2 + (z^2 - xy)^2 = (x^2 + y^2 + z^2)^2.$

13. Agar  $x + y + z = 0$  bo'lsa, quyidagi tengliklarni isbotlang:

13.1.  $x^4 + y^4 + z^4 = 2(xy + xz + yz)^2.$

13.2.  $2(x^4 + y^4 + z^4) = (x^2 + y^2 + z^2)^2.$

13.3.  $2(x^5 + y^5 + z^5) = 5xyz(x^2 + y^2 + z^2).$

14. Berilgan ko'phadlar rezultantini hisoblang:

1.1.  $f(x) = 6x^2 + x - 2, g(x) = 3x^2 - 4x + 2.$

1.2.  $f(x) = x^4 - 2x^2 + 3, g(x) = x^2 - x + 1.$

1.3.  $f(x) = x^2 - 2x + 2, g(x) = 2x^2 + x - 5.$

1.4.  $f(x) = x^3 + 2x^2 + 4x + 1, g(x) = 3x^2 + 4x + 4.$

15.  $a$  ning qanday qiymatlarida berilgan ko'phadlar umumiy ildizga ega:

15.1.  $f(x) = 2x^2 + ax - 3, g(x) = ax^2 + x - 2.$

15.2.  $f(x) = x^3 - 5x^2 + 4ax - 4, g(x) = 3x^2 - 5ax + 8.$

15.3.  $f(x) = x^3 - ax + 2, g(x) = x^2 + ax + 2.$

15.4.  $f(x) = x^3 + ax^2 - 9, g(x) = x^3 + ax - 3.$

16. Berilgan ko'phadlar diskriminantini hisoblang:

16.1.  $f(x) = x^3 + 6x + 2.$

16.2.  $f(x) = x^3 - 9x^2 + 21x - 5.$

16.3.  $f(x) = x^5 + 2.$

16.4.  $f(x) = ax^2 + bx + c.$

16.5.  $f(x) = x^3 + px + q.$

16.6.  $f(x) = x^3 + ax^2 + bx + c.$

17. Isbotlang:

17.1.  $R(f, g_1 \pm g_2) = R(f, g_1) \pm R(f, g_2), \deg f = 1.$

17.2.  $R(f, g_1 \cdot g_2) = R(f, g_1) \cdot R(f, g_2).$

17.3.  $R(f_1 \cdot f_2, g_1 \cdot g_2) = R(f_1, g_1) \cdot R(f_1, g_2) \cdot R(f_2, g_1) \cdot R(f_2, g_2).$

17.4.  $D((x - a) \cdot f(x)) = D(f(x)) \cdot (f'(a))^2.$

17.5.  $D(f \cdot g) = D(f) \cdot D(g) \cdot (R(f, g))^2.$

18.  $a$  ning qanday qiymatlarida ko'phad karrali ildizga ega?

18.1.  $f(x) = x^3 - 3x + a.$

18.2.  $f(x) = x^4 - 4x + a.$

18.3.  $f(x) = 4x^3 - ax + 1.$

18.4.  $f(x) = x^3 + (2 - 3i)x^2 - ax - 2.$

19. Tenglamalar sistemasini yeching:

19.1.  $\begin{cases} x^2 + 2y^2 = 17 \\ 6x^2 - xy - 12y^2 = 0 \end{cases}$

19.2.  $\begin{cases} y^2 - 5y + 4x - 4 = 0 \\ 2y^2 + y - x^2 + 1 = 0 \end{cases}$

19.3.  $\begin{cases} 5x^2 - 5y^2 - 3x + 9y = 0 \\ 5x^2 - 15x^2 - 13xy - y^2 = 0 \end{cases}$

$$19.4. \begin{cases} (y-1)x^2 + xy - 3 = 0 \\ (y-1)x^2 - 2x + y - 1 = 0 \end{cases}$$

20.  $R$  maydonida quyidagi sistemalarni yeching:

$$20.1. \begin{cases} x^2 + y^2 = 35 \\ x + y + 5 \end{cases}$$

$$20.2. \begin{cases} x^2 + xy + y^2 = 49 \\ x^4 + x^{2y} + y = 931 \end{cases}$$

$$20.3. \begin{cases} x + y + z = 1 \\ x^2 + y^2 + z^2 = 9 \\ x^3 + y^3 + z^3 = 1 \end{cases}$$

$$20.4. \begin{cases} x - y + z = 6 \\ x^2 + y^2 + z^2 = 14 \\ x^3 - y^3 + z^3 = 36 \end{cases}$$

$$20.5. \begin{cases} \sqrt{x} + \sqrt{y} = 9 \\ \sqrt[3]{x} + \sqrt[3]{y} = 5 \end{cases}$$

$$20.6. \begin{cases} \sqrt{\frac{x}{y}} + \sqrt{\frac{y}{x}} = \frac{61}{\sqrt{xy}} + 1 \\ \sqrt[4]{x^3y} + \sqrt[4]{xy^3} = 78 \end{cases}$$

21. Quyidagi tenglamalarni yeching.

$$21.1. x + \sqrt{17-x^2} + x\sqrt{17-x^2} = 9.$$

$$21.2. \sqrt[3]{10-x} - \sqrt[3]{3-x} = 1.$$

$$21.3. \sqrt[4]{8-x} + \sqrt[4]{89+x} = 5.$$

$$21.4. \sqrt[4]{78 + \sqrt[3]{24 + \sqrt{x}}} - \sqrt[4]{84 - \sqrt[3]{30 - \sqrt{x}}} = 0.$$



### Takrorlash uchun savollar

1. Ko'p o'zgaruvchili ko'phadlar halqasi.

2. Ko'phad darajasi, xossalari.
3. Ko'phad hadlarining leksikografik tartibi.
4. Simmetrik ko'phadlar.
5. Ikki ko'phad rezultanti.
6. Ikki o'zgaruvchili yuqori darajali tenglamalar.

### 32-§. Maydon ustida ko'phadlar

**Asosiy tushunchalar:** keltiriladigan ko'phad, keltirilmaydigan ko'phad.

Agar  $F$  maydon ustida berilgan va darajasi nolga teng bo'lmagan  $f(x)$  ko'phadni shu maydon ustidagi va darajalari  $f(x)$  ning darajasidan kichik ikkita  $g(x), h(x)$  ko'phadlar ko'paytmasi shaklida ifodalash mumkin bo'lsa, u holda  $f(x)$  ko'phadni  $F$  maydon ustida keltiriladigan ko'phad, aksincha, agar bunday ko'paytma shaklida ifodalash mumkin bo'lmasa, u holda  $f(x)$  ni  $F$  maydon ustida keltirilmaydigan ko'phad deyiladi.

Algebraning asosiy teoremasi. Darajasi 1 dari kichik bo'lmagan kompleks koeffitsientli xar qanday ko'phad kamida bitta kompleks ildizga ega.

Agar  $d(x)$  ko'phad  $f(x)$  va  $\varphi(x)$  ko'phadlarning umumiy bo'luvchisi bo'lib,  $d(x)$  ko'phad  $f(x)$  va  $\varphi(x)$  larning ixtiyoriy umumiy bo'luvchisiga bo'linsa, u holda  $d(x)$  bo'luvchini  $f(x)$  va  $\varphi(x)$  ko'phadlarning eng katta umumiy bo'luvchisi (EKUB) deyiladi va uni  $(f(x); \varphi(x))$  ko'rinishda belgilanadi.

$$f(x) = a_0 + a_1x + \dots + a_nx^n, \quad f(y) = a_0 + a_1y + \dots + a_ny^n \text{ bo'lsin.}$$

$$f(x) - f(y) = \sum_{k=1}^n a_k(x^k - y^k) = (x-y) \sum_{k=1}^n a_k(x^{k-1} + x^{k-2}y + \dots + y^{k-1}) = (x-y)F(x,y),$$

bu erda  $F(x,y) = \sum_{k=1}^n a_k(x^{k-1} + x^{k-2}y + \dots + y^{k-1})$ . Aytaylik  $x=y$  bo'lsin. U holda

$$F(x;x) = \sum_{k=1}^n ka_kx^{k-1} = a_1 + 2a_2x + \dots + na_nx^{n-1} \text{ bo'lib, } F(x;x) \text{ ni } f(x) \text{ ko'phadning formal xosilasi deyiladi va uni } f'(x) \text{ yoki } f' \text{ orqali belgilanadi.}$$

$$f(x) = a_0x^n + a_1x^{n-1} + \dots + a_{n-1}x + a_n \text{ ko'phadni } x\text{-s ning darajalari buyicha}$$

$$f(x) = f(c) + f'(c)(x-c) + \frac{f''(c)}{2!}(x-c)^2 + \dots + \frac{f^{(n)}(c)}{n!}(x-c)^n$$

ko'rinishda yoziladi.

Kompleks sonlar maydoni ustida  $f(z) = z^n + c_1 z^{n-1} + \dots + c_{n-1} z + c_n$  ko'phad berilgan bo'lib,  $\alpha_1, \alpha_2, \dots, \alpha_n$  lar  $f(z)$  ko'phadning ildizi bo'lsa, u holda ushbu

$$\begin{cases} c_1 = -(\alpha_1 + \alpha_2 + \dots + \alpha_n); \\ c_2 = \alpha_1 \alpha_2 + \alpha_1 \alpha_3 + \dots + \alpha_{n-1} \alpha_n; \\ c_3 = -(\alpha_1 \alpha_2 \alpha_3 + \alpha_1 \alpha_2 \alpha_4 + \dots + \alpha_{n-2} \alpha_{n-1} \alpha_n); \\ \dots \\ c_n = (-1)^n \alpha_1 \alpha_2 \dots \alpha_n \end{cases}$$

munosabatlar o'rinli bo'ladi.

Kompleks sonlar maydoni C ustidagi ushbu  $ax^3 + bx^2 + cx + d = 0$  ( $a \neq 0$ ) ko'rinishdagi tenglama 3-darajali bir noma'lumli tenglama deyiladi. Uning xar ikki qismini  $a$  ga bo'lib

$$\tilde{a}^3 + \frac{b}{a} \tilde{a}^2 + \frac{\tilde{c}}{a} \tilde{a} + \frac{d}{a} = 0$$

tenglamani xosil qilamiz. Unda  $x = y - 3 \frac{b}{a}$  almashtirish bajarib, soddalashtirgandan so'ng  $y^3 + ry + q = 0$  tenglamani xosil qilamiz. Bunda  $y = u + v$  almashtirishdan so'ng  $u$  va  $v$  larni shunday tanlab olamizki, natijada  $3uv + r = 0$  shart bajarilsin. U holda

$$\begin{cases} u^3 + v^3 = -q \\ u^3 v^3 = -\frac{p^3}{27} \end{cases}$$

sistemaga ega bo'lamiz. Sistemadan ko'rinadiki  $u^3$  va  $v^3$  lar Viet teoremasiga ko'ra qandaydir  $z^2 + qz - \frac{p^3}{27} = 0$  tenglamaning ildizi bo'ladi. Bu kvadrat tenglamani yechib  $z_1 = u^3$  dan  $u = \sqrt[3]{-\frac{q}{2} + \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}}$ ,  $v = \sqrt[3]{-\frac{q}{2} - \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}}$  larni xosil qilamiz.  $u$  va  $v$  ning xar biriga uchta qiymat,  $u$  o'zgaruvchi uchun  $v$  to'qqizta qiymat topiladi.

Agar  $u, \xi u, \xi^2 u$  (bunda  $\xi$  son 1 dan chiqarilgan 3-darajali ildiz)  $z_1$  ning uchinchi darajali ildizlarining qiymatlari bo'lsa, unga mos  $z_2$  ning uchinchi darajali ildizlari qiymatlari  $v, \xi^2 v, \xi v$  bo'ladi. Natijada keltirilgan tenglama  $y_1 = u + v, y_2 = \xi u + \xi^2 v, y_3 = \xi^2 u + \xi v$  ildizlarga ega bo'lib, unda  $\xi = -\frac{1}{2} + i \frac{\sqrt{3}}{2}$  bo'lgani uchun

$y_1 = u + v, y_2 = -\frac{1}{2}(u + v) + i \frac{\sqrt{3}}{2}(u - v), y_3 = -\frac{1}{2}(u + v) - i \frac{\sqrt{3}}{2}(u - v)$  bo'ladi. Bu erda  $x = y - \frac{3\xi}{a}$  ni e'tiborga olib berilgan tenglamaning  $x_1 = y_1 - \frac{3\xi}{a}, x_2 = y_2 - \frac{3\xi}{a}, x_3 = y_3 - \frac{3\xi}{a}$  ildizlari topiladi.

Kub tenglamani bu usulda echish uni Kardano usuli bilan echish deyiladi.

Agar  $x^3 + px + q = 0$  tenglamada  $r, q$  lar haqiqiy sonlar bo'lib,  $\Delta = \frac{q^2}{4} + \frac{p^3}{27}$  bo'lsa, u holda quyidagi mulohazalar o'rinli:

- 1) Agar  $\Delta > 0$  bo'lsa, tenglama bitta haqiqiy va ikkita o'zaro qo'shma mavxum ildizlarga ega bo'ladi;
- 2) Agar  $\Delta = 0$  bo'lsa, tenglamaning barcha ildizlari haqiqiy va kamida bitta ildizi karrali bo'ladi;

Agar  $\Delta < 0$  bo'lsa, tenglamaning barcha ildizlari haqiqiy va turlicha Agar  $a$  butun son koeffitsientlari butun bo'lgan  $a_0 x^n + a_1 x^{n-1} + \dots + a_{n-1} x + a_n = 0$  tenglamaning ildizi bo'lsa, u holda  $\frac{f(1)}{a-1}$  va  $\frac{f(-1)}{a+1}$  sonlar xam butun sonlar bo'ladi.

Agar  $r/q$  ( $q > 0$ ) qisqarmas kasr koeffitsientlari butun bo'lgan  $a_0 x^n + a_1 x^{n-1} + \dots + a_{n-1} x + a_n = 0$  tenglamaning ildizi bo'lsa, u holda  $r$  son  $a_n$  ozod hadning  $q$  son esa  $a_0$  bosh koeffitsientning bo'luvchisi bo'ladi. Eyzenshteyn kriteriyasi. Butun koeffitsientli  $f(x) = c_n x^n + c_{n-1} x^{n-1} + \dots + c_1 x + c_0$  ko'phadning bosh koeffitsienti  $c_n$  dan boshqa barcha koeffitsientlari  $r$  tub songa bo'linib, ozod had  $c_0$  esa  $r^2$  ga bo'linmasa, u holda  $f(x)$  ko'phad ratsional sonlar maydoni ustida keltirilgan ko'phad bo'ladi.

Kasrning maxrajidagi irratsionallikni yo'qotish mumkin, ya'ni  $F_1$  sonlar maydoni ustida keltirilmaydigan  $n$ -darajali  $r(x) = x^n + a_1x^{n-1} + \dots + a_{n-1}x + a_n$  ( $n \geq 2$ ) ko'phad berilgan bo'lib,  $x = a$  uning ildizi bo'lsa, u holda  $\frac{f(a)}{g(a)}$  ( $g(a) \neq 0$ ) kasr-ratsional ifodani shunday o'zgartirish mumkinki, natijada uning maxraji butun ratsional ifodaga aylanadi.

**Misol.**  $f(x) = x^4 + 2x^3 - 3x^2 - 5x + 2$  ko'phad  $Q$  maydonda keltiriluvchimi?

**Yechish.**  $f(x)$  ko'phad darajasi 4 ga teng bo'lganligi uchun, agar u  $Q$  maydonda keltiriluvchi bo'lsa, u holda  $f(x)$  ko'phadni ikkita 2-darajali yoki 1- va 3-darajali ko'phadlar ko'paytmasiga yoyish mumkin.

Agar  $f(x) = (ax^2 + bx + c)(dx^2 + mx + n)$  deb faraz qilsak bu tenglamaning yechimlari na butun sonlar, na kasr sonlarda mavjud emasligiga ishonch hosil qilish mumkin.

Agar  $f(x) = (ax + b)(cx^3 + dx^2 + mx + n)$  deb faraz qilsak, u holda tenglamadan

$$\begin{cases} ac = 1 \\ ad + bc = 2 \\ am + bd = -3 \\ an + bm = -5 \\ bn = 2 \end{cases}$$

sistemani hosil qilamiz. Bu sistemaning yechimlaridan biri  $a = c = n = 1$ ,  $b = 2$ ,  $d = 0$ ,  $m = -3$ . Demak,  $f(x) = (x + 2)(x^3 - 3x + 1)$  bo'lib berilgan  $f(x)$  ko'phad  $Q$  maydonda keltiriluvchi ekan.

**Misol.**  $Q[x]$  da berilgan  $f(x) = (x - 2)(x - 3)^2(x + 1)$  va  $g(x) = x^3 - 3x^2 - 2x + 6$  ko'phadlarning EKUB va EKUKini toping.

**Yechish.**  $f(x)$  ko'phad kanonik yoyilma ko'rinishida berilganligi uchun  $g(x)$  ko'phadni keltirilmaydigan ko'phadlar kanonik yoyilmasiga keltiramiz:

$$f(x) = (x^3 - 3x^2) - (2x - 6) = x^2(x - 3) - 2(x - 3) = (x - 3)(x^2 - 2). \quad \text{Demak,}$$

$$(f, g) = x - 3; \quad (f, g) = (x - 2)(x^2 - 2)(x - 3)^2(x + 1)$$

**Misol.**  $Q[x]$  halqada berilgan  $f(x) = x^4 - 2x^3 + 3x^2 - 5x + 1$  ko'phad hosilalarining  $x_0 = 1$  nuqtadagi hosilalarini toping va berilgan ko'phadni  $x - 1$  ikkihad darajalariga yoying.

**Yechish.**

1-usul.  $f'(x) = 4x^3 - 6x^2 + 6x - 5;$

$$f''(x) = 12x^2 - 12x + 6;$$

$$f'''(x) = 24x - 12;$$

$$f^{IV}(x) = 24.$$

U holda  $f'(1) = 4 \cdot 1^3 - 6 \cdot 1 - 5 = -1;$

$$f''(1) = 12 \cdot 1 - 12 \cdot 1 + 6 = 6;$$

$$f'''(1) = 24 \cdot 1 - 12 = 12;$$

$$f^{IV}(1) = 24$$

Berilgan ko'phadning  $(x - 1)$  darajalariga yoyilmasini Teylor formulasidan foydalanib topamiz:

$$f(x) = f(1) + f'(1)(x - 1) + \frac{f''(1)}{2!}(x - 1)^2 + \frac{f'''(1)}{3!}(x - 1)^3 + \frac{f^{IV}(1)}{4!}(x - 1)^4$$

Bu erda  $f(1) = -2$  bo'lganligi uchun

$$f(x) = -2 - (x - 1) + 3(x - 1)^2 + 2(x - 1)^3 + (x - 1)^4.$$

2-usul. Gornor sxemasi yordamida yoyilmani topamiz:

|   |   |    |   |    |    |
|---|---|----|---|----|----|
|   | 1 | -2 | 3 | -5 | 1  |
| 1 | 1 | -1 | 2 | -3 | -2 |
| 1 | 1 | 0  | 2 | -1 |    |
| 1 | 1 | 1  | 3 |    |    |
| 1 | 1 | 2  |   |    |    |
| 1 | 1 |    |   |    |    |
| 1 | 1 |    |   |    |    |

Jadvaldan  $f(1) = -2$ ;  $f'(1) = -1$ ;  $\frac{f''(1)}{2!} = 3$ ;  $\frac{f'''(1)}{3!} = 2$ ;  $\frac{f^{(4)}(1)}{4!} = 1$  larni aniqlaymiz.

Bundan,  $f(x) = (x-1)^4 + 2(x-1)^3 + 3(x-1)^2 - (x-1) - 2$  va  $f'(1) = -1$ ;  $f''(1) = 6$ ;  $f'''(1) = 12$ ; larni topamiz.

**Misol.**  $a$  ning qanday qiymatlarida  $f(x) = x^3 + x^2ax + 3$  ko'phad karrali ildizga ega bo'ladi?

**Yechish.** Berilgan ko'phadning karrali ildizi  $\alpha$  bo'lsin. U holda  $f(x) = (x-\alpha)^2 \cdot h(x)$ . Bu ko'phadning hosilasi

$f'(x) = 2(x-\alpha)h(x) + h'(x)(x-\alpha)^2$  bo'lib,  $f'(\alpha) = 0$  bo'ladi.

$f(\alpha) = 0$ ,  $f'(\alpha) = 0$  lardan quyidagi tenglamalar sistemasini tuzamiz:

$$\begin{cases} \alpha^3 + \alpha^2 + a\alpha + 3 = 0 \\ 3\alpha^2 + 2\alpha + a = 0 \end{cases}$$

Bundan  $a = -3\alpha^2 - 2\alpha$  yordamida  $\alpha^3 + \alpha^2 - 3\alpha^3 - 2\alpha^2 + 3 = 0$ , ya'ni  $-2\alpha^3 - \alpha^2 + 3 = 0$  tenglamaga ega bo'lamiz. Uning  $\alpha_1 = -\frac{3}{2}$ ;  $\alpha_2 = 1$  yechimlari mavjud bo'lib,  $a_1 = -\frac{15}{4}$ ,  $a_2 = -5$  ga ega bo'lamiz.

Demak,  $a = -5$  da berilgan ko'phad karrali ildizga ega.

**Misol.** Agar  $Q[x]$  halqada 2 son  $f(x)$  ko'phadning 3 karrali ildizi bo'lsa, u holda

$g(x) = f'(x)(x^2 + 3) + (x+3)f''(x)$  ko'phadning nycha karrali ildizi bo'ladi?

**Yechish.** 2 soni  $f(x)$  ko'phadning 3 karrali ildizi bo'lganligi uchun  $f(2) = f'(2) = f''(2) = 0$  va  $f'''(2) \neq 0$ . 2 sonning  $g(x)$  uchun tekshiramiz:  $g(2) = 0$  va  $g'(2) \neq 0$ .

Demak, 2 soni  $g(x)$  uchun bir karrali ildiz.

**Misol.**  $f(x)$  ko'phadni karrali ko'paytuvchilarga yiting.

**Yechish.**  $f(x)$  ko'phadni keltirilmaydigan ko'phadlarga kanonik yoyilmasini quyidagi jadvaldan foydalanib topamiz.

C maydonda  $f(x) = \varphi_1\varphi_2^2 \dots \varphi_m^n$ ,  $1 \leq i \leq m$  bo'lsin,  $\varphi_i$  ko'phadlarni quyidagicha aniqlaymiz.

|  |  |                               |
|--|--|-------------------------------|
| $f = \varphi_1\varphi_2^2 \dots \varphi_m^n$                 | $q_1 = \frac{1}{d_1} = \varphi_1\varphi_2 \dots \varphi_m$   | $\varphi_1 = \frac{q_1}{q_2}$ |
| $d_1 = (f, f') = \varphi_2\varphi_3^2 \dots \varphi_m^{n-1}$ | $q_2 = \frac{d_1}{d_2} = \varphi_2\varphi_3 \dots \varphi_m$ | $\varphi_2 = \frac{q_2}{q_3}$ |
| $d_2 = (d_1, d_1') = \varphi_3 \dots \varphi_m^{n-2}$        | $q_3 = \frac{d_2}{d_3} = \varphi_3 \dots \varphi_m$          | .....                         |
| .....  | .....  | $\varphi_m = q_m$             |
| $d_{m-1} = (d_{m-2}, d_{m-2}') = \varphi_m$                  | $q_m = \frac{d_{m-1}}{d_m} = \varphi_m$                      |                               |
| $d_m = 1$  |  |                               |

Berilgan  $f(x)$  ko'phad uchun Evklid algoritmi yordamida  $d_1, d_2, \dots, d_m$  larni topamiz:  $f'(x) = 4x^3 - 6ix^2 - 2i$ .

$$d_1 = (f, f') = (x-i)^2;$$

$$d_1' = 2x - 2i = 2(x-i);$$

$$d_2 = x - i;$$

$$d_2' = 1;$$

$$d_3 = (d_2, d_2') = 1;$$

$$q_1 = \frac{f}{d_1} = x^2 + 1;$$

$$q_2 = \frac{d_1}{d_2} = x - i;$$

$$q_1 = \frac{d_1}{d_2} = x - i,$$

Bulardan,  $\varphi_1 = \frac{q_1}{q_2} = x + i$ ;  $\varphi_2 = \frac{q_2}{q_1} = 1$ ;  $\varphi_3 = q_3 = x - i$  lar kelib chiqadi.

Demak,  $f(x) = (x + i)(x - i)^3$ .

**Misol.**  $Q$  maydonda  $\frac{f(x)}{g(x)} = \frac{x^2 + 4x + 3}{(x^2 - 4x + 4)(x + 3)^2}$  kasrni elementar kasrlarga yoying.

**Yechish.**  $f(x)$  va  $g(x)$  ko'phadlarning o'zaro tub yoki tubmasligini tekshiramiz. Buning uchun ularning EKUBini topamiz.  $(f, g) = x + 3$ .  $f(x)$  va

$g(x)$  larni o'zaro tub holga keltiramiz va qisqarmas kasr  $\frac{f(x)}{g(x)} = \frac{x + 1}{(x - 2)^2(x + 3)}$  ni hosil qilamiz. Bundan, A, V, S parametrlar yordamida

$$\frac{x + 1}{(x - 2)^2(x + 3)} = \frac{A}{(x - 2)} + \frac{B}{(x - 2)^2} + \frac{C}{x + 3}$$

yoyilmani tuzamiz. Natijada

$$x + 1 = A(x - 2)(x + 3) + B(x + 3) + C(x - 2)^2$$

tenglama kelib chiqadi va uning yechimlari

- a)  $x = 2$  da  $B = \frac{3}{5}$
- b)  $x = -3$  da  $C = -\frac{2}{25}$
- v)  $A + C = 0$  va  $A = \frac{2}{25}$

$$\text{Demak, } \frac{f(x)}{g(x)} = \frac{x + 1}{(x - 2)^2(x + 3)} = \frac{2}{25(x - 2)} + \frac{3}{5(x - 2)^2} - \frac{2}{25(x + 3)}$$



**Misol va mashqlar**

- 1.  $f(x)$  ko'phad berilgan maydonda keltirilmasligini isbotlang:
- 1.1.  $f(x) = x^3 - 2$ ,  $Q$ .

1.2.  $f(x) = x^2 + x + 1$ ,  $Q$ .

1.3.  $f(x) = x^2 + x + 1$ ,  $Z$ .

1.4.  $f(x) = x^6 + x^3 + 1$ ,  $Q$ .

2.  $Q$  maydonda berilgan quyidagi ko'phadlarni keltirilmaydigan ko'phadlar ko'paytmasiga yoying.

2.1.  $f(x) = 2x^5 - x^4 - 6x^3 + 3x^2 + 4x - 2$

2.2.  $f(x) = 3x^5 + x^4 - 15x^3 - 5x^2 + 12x + 4$

3.  $f(x) = 2x^5 - x^4 - 2x^3 + x^2 - 4x + 2$  ko'phadning 2 juft bir-biriga qarama-qarshi ildizlari mavjudligi ma'lum bo'lsa, uni  $Q, R, C$  maydonlardagi keltirilmaydigan ko'phadlarga yoyilmasini toping.

4.  $Q$  maydonda berilgan 3-darajali ko'phad keltiriluvchi bo'lishi uchun uning bitta ildizi ratsional son bo'lishi zarur va etarli ekanligini isbotlang.

5.  $Z[x]$  halqada quyidagi ko'phadlar keltirilmasligini isbotlang.

5.1.  $f(x) = x^5 - x^2 + 1$ .

5.2.  $f(x) = x^5 + x^4 + x^3 + x^2 + 1$ .

5.3.  $f(x) = x^3 - x^2 + x + 1$ .

6.  $f(x) = x^4 + 4$  ko'phad  $Z_5, Q, R, C$  maydonlarning qaysi birida keltiriluvchi?

7. Quyidagi ko'phadlarni keltirilmaydigan ko'phadlarga yoying.

7.1.  $f(x) = x^4 - 6x^3 + 11x^2 - 6x + 1$ ,  $R[x]$ .

7.2.  $f(x) = x^4 + 4$ ,  $C[x]$ .

7.3.  $f(x) = (x^2 + x - 1)^2 + 3x(x^2 + x - 1) + 2x^2$ .

7.4.  $f(x) = x^4 + 4$ ;  $C[x]$ .

7.5.  $f(x) = x^2(x - 3)^2 + 4x^2 - 12x + 4$ .

7.6.  $f(x) = x^6 + 27$ ;  $C[x]$ .

7.7.  $f(x) = (x + 2)(x + 3)(x + 4)(x + 5) + 1$ .

$$7.8. f(x) = x^{2n} + x^4 + 1; C[x].$$

8. Quyidagi ko'phadlarning EKUB va EKUK larini toping:

$$8.1. f(x)(x-1)^2(x^2-5x+6), g(x) = x^2 - x - 2, Z[x]$$

$$8.2. f(x) = (x^2 - 2x + 3)^2(x^2 + 5x - 6)^2, g(x) = (x^2 - 8x + 12)^2(x^3 - 1), Q[x]$$

$$8.3. f(x) = x^4 + 2x^3 - 2x - 1, g(x) = (x+1)(x^2 - x - 2), Q[x]$$

$$8.4. f(x) = x^5 - x, g(x) = (x^2 + x + 1)^2(2x + 4), Z_5[x]$$

$$8.5. f(x) = x^m - 1, g(x) = x^n - 1$$

$$8.6. f(x) = x^m + 1, g(x) = x^n + 1$$

9. Quyidagi ko'phadlarning hosilasini toping:

$$9.1. f(x) = (x^2 + x - 1)^3(x^3 - 2), Q[x];$$

$$9.2. f(x) = 4x^{10} + 3x^2(x+3), Z_5[x].$$

10. Agar  $Z_3[x]$  halqada  $f'(x) = 2x + 1$  va  $f(1) = 1$  bo'lsa, 6 darajali  $f(x)$  ko'phadni toping.

11. Agar  $Q[x]$  halqada  $f''(x) = 24x + 2$ ,  $f(0) = 1$  va  $f(1) = 5$  bo'lsa,  $f(x)$  ko'phadni toping.

12.  $Z_2[x]$  halqada darajasi 3 dan katta bo'lmagan va o'z hosilasiga bo'linuvchi barcha  $f(x)$  ko'phadlarni toping.

13.  $f(x)$  ko'phadni  $x - a$  darajalariga yoying va hosilalarining  $a$  nuqtadagi qiymatini toping.

$$13.1. f(x) = ix^4 + (1-i)x^3 - (2+i)x^2 + 3x - 3 - 4i, a = 2i, C[x].$$

$$13.2. f(x) = x^5 - 3ix^3 - 4x^2 + 5ix - 1, a = -i, G[x].$$

$$13.3. f(x) = (x-3)(x-2)(x+1)(x+4) + 1, a = -1, Q[x].$$

$$13.4. f(x) = 2x^4 + x^3 + x^2, a = 1, Z_3[x].$$

$$13.5. f(x) = x^4 - 8x^3 + 24x^2 - 50x + 90, a = 2, R[x].$$

$$13.6. f(x) = x^5 - 4x^3 + 6x^2 - 8x + 10, a = 2, R[x].$$

14. Berilgan ildizlar nycha karrali ekanligini aniqlang.

$$14.1. \alpha = 3; f(x) = x^4 - 6x^3 + 10x^2 - 6x + 9, Q[x].$$

$$14.2. \alpha = 2; f(x) = x^5 - 4x^4 + 7x^3 - 11x^2 + 4, Q[x].$$

$$14.3. \alpha = 1+i; f(x) = x^4 - (3+4i)x^3 + (3+3i)x^2 + (8-2i)x - 2 - 2i, C[x].$$

$$14.4. \alpha = 2; f(x) = x^5 - 5x^4 + 7x^3 - 2x^2 + 4x - 8.$$

$$14.5. \alpha = 3; f(x) = x^5 - 6x^4 + 2x^3 + 36x^2 - 27x - 54.$$

15.  $R[x]$  da  $b$  ning qanday qiymatlarida berilgan ko'phad karrali ildizga ega:

$$15.1. f(x) = x^5 - 5x^3 + b,$$

$$15.2. f(x) = x^3 - 4x^2 - 3x + b,$$

$$15.3. f(x) = x^3 + 3x^2 + 3bx - 4,$$

$$15.4. f(x) = x^3 + 5x^2 + 8x + b.$$

16. Berilgan ko'phadlarning karrali ildizga ega bo'lishining zarur va etarli shartlarini aniqlang:

$$16.1. f(x) = x^4 + ax + b.$$

$$16.2. f(x) = x^5 + ax^3 + b.$$

17. Berilgan ko'phadlarni keltirilmaydigan ko'phadlar kanonik yoyilmasini toping:

$$17.1. f(x) = x^5 + 4x^4 + 7x^3 + 8x^2 + 5x + 2.$$

$$17.2. f(x) = x^5 - ix^4 + 5x^3 - ix^2 + 8x + 4i.$$

$$17.3. f(x) = x^5 + 5x^4 + (6-i)x^3 - (4+6i)x^2 - (8+12i)x - 8i.$$

$$17.4. f(x) = x^6 - 6x^4 - 4x^3 + 9x^2 + 12x + 4.$$

18. Quyidagi shartlar asosida kompleks koeffitsientli eng kichik darajali ko'phadni aniqlang:

18.1. 1-ikki karrali, 2,3,  $1+i$  - bir karrali ildizlar.

18.2.  $i$  -ikki karrali,  $-1-i$  - bir karrali ildizlar.

19.  $R[x]$  halqada berilgan kasrlarni qisqarmas kasrga keltiring:

$$19.1. \frac{x^2 - 4x + 3}{x^2 - 5x + 6}$$

$$19.2. \frac{x^8 + x^4 + 1}{x^2 + x + 1}$$

20. Q maydonda berilgan kasrni elementar kasrlarga yoying.

$$20.1. \frac{f(x)}{g(x)} = \frac{x+3}{(x^3-2)(x+1)}$$

$$20.2. \frac{f(x)}{g(x)} = \frac{1}{x^4-2x}$$

$$20.3. \frac{f(x)}{g(x)} = \frac{x^2}{x^4-4}$$

$$20.4. \frac{f(x)}{g(x)} = \frac{1}{x^3+x}$$

21. R maydonda berilgan kasrni elementar kasrlarga yoying:

$$21.1. \frac{f(x)}{g(x)} = \frac{x^3-1}{(x^2+x+1)^2(x^2+1)}$$

$$21.2. \frac{f(x)}{g(x)} = \frac{x^4+2x^3-18x^2+54}{x^5+6x^4+9x^3}$$

$$21.3. \frac{f(x)}{g(x)} = \frac{x^2+3x+2}{(x^4+4)(x+2)}$$

$$21.4. \frac{f(x)}{g(x)} = \frac{x^2}{x^4-4}$$

$$21.5. \frac{f(x)}{g(x)} = \frac{x+3}{(x^3-2)(x+1)}$$

$$21.6. \frac{f(x)}{g(x)} = \frac{x^2}{(x^2+x+2)^2}$$

$$21.7. \frac{f(x)}{g(x)} = \frac{x^2}{x^4-16}$$

$$21.8. \frac{f(x)}{g(x)} = \frac{1}{x^4+4}$$

22. C maydonda berilgan kasrni elementar kasrlarga yoying:

$$22.1. \frac{f(x)}{g(x)} = \frac{x^2}{(x-1)(x+2)(x+3)}$$

$$22.2. \frac{f(x)}{g(x)} = \frac{1}{x^4+4}$$

$$22.3. \frac{f(x)}{g(x)} = \frac{5x^2+6x-23}{(x-1)^3(x+1)^2(x-2)}$$

$$22.4. \frac{f(x)}{g(x)} = \frac{i}{(x-i)(x+2i)}$$

$$22.5. \frac{f(x)}{g(x)} = \frac{2x}{(x-1)(x^2+1)}$$

$$22.6. \frac{f(x)}{g(x)} = \frac{x^2}{x^4-4}$$

23.  $Z_5$  maydonda ( $r$  - tub son)  $\frac{f(x)}{g(x)} = \frac{1}{x^p-x}$  kasrni elementar kasrlarga

yoying.

24. Maydon ustida keltirilmaydigan ko'phadlarning quyidagi xossalari isbotlang:

1°. Agar  $p(x)$  va  $g(x)$  keltirilmaydigan ko'phadlar bo'lib  $r(x):g(x)$  bo'lsa, u holda  $r(x)=ag(x)$  ( $a \neq 0$ ) bo'ladi.

2°. Ixtiyoriy  $f(x)$  ko'phad keltirilmaydigan ixtiyoriy  $r(x)$  ko'phadga bo'linadi yoki  $(f(x):r(x))=1$  bo'ladi.

3°. Agar  $f_i(x)$  ( $i=1,m$ ) ko'phadlarning hech biri keltirilmaydigan  $r(x)$  ko'phadga bo'linmasa, u holda  $f_1(x) \cdot f_2(x) \cdot \dots \cdot f_m(x) \not\equiv r(x)$  bo'ladi.

4°. Agar  $f_1(x) \cdot f_2(x) \cdot \dots \cdot f_m(x) : r(x)$  ( $p(x)$ ) - keltirilmaydigan ko'phad, u holda  $f_i(x)$  ( $i=1,m$ ) ko'phadlarning aqalli bittasi  $r(x)$  ga bo'linadi.

5°.  $p(x)$  keltirilmaydigan ko'phad bo'lsa, u holda  $ap(x)$  ( $0 \neq a \in F$ ) ham keltirilmaydigan ko'phad bo'ladi.

25. Agar  $x_1 = a + bi$  berilgan  $f(x)$  ko'phadning ildizi bo'lsa, uning qolgan yechimlarini toping:

25.1.  $f(x) = x^3 - 4x^2 + 3x + 30$ ;  $x_1 = 3 + i\sqrt{6}$  .  
 25.2.  $f(x) = x^3 - 4x^2 + 3x + 30$ ;  $x_1 = 3 - i\sqrt{6}$  .  
 25.3.  $f(x) = 4x^4 - 24x^3 + 53x^2 + 18x - 42$ ;  $x_1 = 3 - i\sqrt{5}$  .  
 25.4.  $f(x) = x^4 + 2x^3 + 2x^2 + 6x - 3$ ;  $x_1 = -1 - i\sqrt{2}$  .

26. Kardano formulalari yordamida quyidagi tenglamalarni yeching:

26.1.  $-2x^3 - 2x^2 + 12x - 24$ ;      26.6.  $-5x^3 + 8x^2 - 3x - 24$ ;  
 26.2.  $2x^3 + 4x^2 + 4x + 4$ ;      26.7.  $2x^3 + 8x^2 - 12x + 12$ ;  
 26.3.  $2x^3 + 8x^2 - 2x + 5$ ;      26.8.  $2x^3 + 4x^2 + 4x + 4$ ;  
 26.4.  $-5x^3 + 8x^2 - 3x - 3$ ;      26.9.  $5x^3 + x^2 - 3x - 2$ ;  
 26.5.  $6x^3 - 8x^2 + 5x - 3$ ;

27. Ferrari usuli bilan quyidagi tenglamalarni yeching:

27.1.  $x^4 - 2x^3 - 2x^2 + 12x - 24$ ;      27.4.  $x^4 - 5x^3 + 8x^2 - 3x - 24$ ;  
 27.2.  $x^4 + 2x^3 + 4x^2 + 4x + 4$ ;      27.5.  $x^4 - 2x^3 + 8x^2 - 12x + 12$ ;  
 27.3.  $x^4 - 2x^3 + 8x^2 - 12x + 12$ ;      27.6.  $x^4 - 5x^3 + x^2 - 3x - 2$  .

28. Ko'phadning butun ildizlarini toping:

28.1.  $f(x) = x^4 - 3x^2 - 14$ ;      28.5.  $f(x) = x^5 + 3x - 9$ ;  
 28.2.  $f(x) = 4x^4 + 3x^2 - 4$ ;      28.6.  $f(x) = x^5 + 3x - 8$ ;  
 28.3.  $f(x) = x^4 + 4x^3 + 27$ ;      28.7.  $f(x) = x^5 + 3x - 12$ ;  
 28.4.  $f(x) = x^4 - 3x^2 - 24$ ;      28.8.  $f(x) = x^3 + 2x^2 - 3x + 2$ ;

29. Ko'phadning ratsional ildizlarini toping:

29.1.  $f(x) = 4x^5 + 4x^4 + 7x^3 + 8x^2 + 5x + 2$  .  
 29.2.  $f(x) = 5x^5 - x^4 - 2x^3 - 27x^2 - 44x + 7$ ;  
 29.3.  $f(x) = -4x^5 - x^4 - 6x^3 + 11x^2 - 6x + 1$ ;  
 29.4.  $f(x) = 5x^5 + 4x^4 + 4x^3 + 13x^2 + 6x + 9$ ;  
 29.5.  $f(x) = -7x^5 + x^4 - 5x^3 - 8x^2 + 19x - 3$  .

30. Kasr mahrajini irratsionallikdan qutqaring:

30.1.  $\frac{7}{1 - \sqrt{2} + \sqrt{2}}$ ;      30.4.  $\frac{2}{\sqrt[3]{49} - \sqrt[3]{7} + 3}$ ;      30.7.  $\frac{\sqrt{2}}{\sqrt{4} + 2\sqrt{2}}$ ;

30.2.  $\frac{2}{\sqrt[3]{27} - 2\sqrt[3]{9} + \sqrt[3]{3} - 1}$ ;      30.5.  $\frac{2\sqrt{3}}{\sqrt{25} - \sqrt{5} + 6}$ ;      30.8.  $\frac{9}{\sqrt{4} + \sqrt{2} + 3}$ ;  
 30.3.  $\frac{2\sqrt{5}}{\sqrt{6} - \sqrt{3} + 7}$ ;      30.6.  $\frac{9\sqrt{2}}{\sqrt[3]{49} - \sqrt[3]{7} + 3}$ ;      30.9.  $\frac{5}{\sqrt[3]{9} - \sqrt[3]{7}}$ ;



### Takrorlash uchun savollar

1. Ko'phadning butun va ratsional ildizlari.
2. Eyzenshteynning keltirilmaslik alomati.
3. Algebraik elementning minimal ko'phadi.
4. Maydonning oddiy kengaytmasi va uni qurish.
5. Kasr mahrajini algebraik irratsionallikdan qutqarish.
6. Maydonning chekli kengaytmasi. Maydonning murakkab kengaytmasi.
7. Algebraik sonlar maydoni.
8. Tenglamalarni radikallarda echilishi.
9. Uchinchi darajali tenglamalarning kvadrat radikallarda echilish sharti.
10. Kvadrat radikallarda echilmaydigan masalalar.

## JAVOBLAR

### I MODUL. MATEMATIK MANTIQ ELEMENTLARI

#### 1-§. Mulohaza. Mulohazalar ustida mantiq amallari

1. 1.2, 1.5, 1.8, 1.9, 1.10, 1.14, 1.18, 1.19, 1.20, 1.21, 1.22 – mulohaza. 2. 2.1, 2.5, 2.7 – yolg'on. 3. 3.3.  $3 \leq 2$ , 3.8. barcha haqiqiy sonlar toq. 3.10. Shunday natural son mavjudki u birdan katta emas. 4. 4.2, 4.8. 5. 5.1.  $(x \neq 0) \wedge (y \neq 0)$ . 5.2.  $(x = 0) \vee (y = 0)$ . 5.3.  $(x = 0) \wedge (y = 0)$ . 5.4.  $(x = 0) \wedge (y \neq 0)$ . 5.5.  $(x > -6) \wedge (x < 6)$ . 5.6.  $(x = -2) \vee (y = 2)$ . 6. 6.1, 6.5, 6.6, 6.7, 6.9, 6.10 – yolg'on. 7. 7.2, 7.4, 7.7, 7.8, 7.9 – rost. 8. 8.1.  $\neg A \wedge \neg B \Rightarrow A \vee B$ . 8.2.  $A \wedge \neg B \Rightarrow \neg C$ . 8.3.  $A \Rightarrow B \vee C \vee D$ . 8.4.  $A \Rightarrow B \vee C \vee D$ . 9. Mavjud emas. 10. 10.2 – yolg'on, qolganlari rost.

#### 2-§. Formula. Teng kuchli formulalar. Mantiq qonunlari.

2.2.3.  $((\neg A) \Leftrightarrow (\neg B)) \vee (C \wedge B)$ ,  $((\neg A) \Leftrightarrow ((\neg B) \vee C)) \wedge B$ ,  
 $((\neg A) \Leftrightarrow (\neg(B \vee C))) \wedge B$ ,  $((\neg A) \Leftrightarrow ((\neg B) \vee C)) \wedge B$ ,  $(\neg(A \Leftrightarrow (\neg B) \vee C)) \wedge B$ ,  
 $(\neg(A \Leftrightarrow ((\neg B) \vee C))) \wedge B$ ,  $(\neg(A \Leftrightarrow ((\neg B) \vee C) \wedge B))$ ,  
 $(\neg(A \Leftrightarrow (\neg(B \vee (C \wedge B))))$ ,  $(\neg(A \Leftrightarrow (\neg((B \vee C) \wedge B)))$ ,  $(\neg(A \Leftrightarrow (\neg B)) \vee (C \wedge B))$ ,  
 $(\neg(A \Leftrightarrow (\neg(B \vee C))) \wedge B$ ,  $(\neg(A \Leftrightarrow ((\neg(B \vee C)) \wedge B))$ ,  $((\neg(A \Leftrightarrow (\neg B))) \vee (C \wedge B))$ ,  
 $((\neg A) \Leftrightarrow (((\neg B) \vee C) \wedge B))$ ,  $((\neg A) \Leftrightarrow ((\neg(B \vee C)) \wedge B))$ ,  $((\neg(A \Leftrightarrow (\neg(B \vee C)))) \wedge B$ .  
 3.3.1.  $A, B, C, (\neg C), (A \vee B), (A \Leftrightarrow B), ((A \vee B) \Rightarrow A), ((A \Leftrightarrow B) \wedge (\neg C))$ ,  
 $((A \vee B) \Rightarrow A) \Rightarrow (\neg C)$ ,  $((A \Leftrightarrow B) \wedge (\neg C)) \Rightarrow ((A \vee B) \Rightarrow A) \Rightarrow (\neg C)$ . 11. 11.1. 1;  
 11.2.  $A \vee B$ ; 11.3.  $A \wedge B$ ; 11.4.  $\neg A \wedge \neg C$ ; 11.5.  $A \vee (B \wedge C)$ ; 11.6.  $B \Rightarrow \neg A$ . 12.

12.1.  $\neg(\neg A \wedge B \wedge \neg C)$ ; 12.2.  $\neg(\neg A \wedge \neg B)$ ; 12.3.  $\neg(\neg A \wedge B \wedge \neg C)$ ; 12.4.  
 $\neg(A \wedge \neg B) \wedge (A \wedge C)$ ; 12.5.  $\neg(\neg A \wedge \neg B) \wedge (\neg A \wedge C)$ . 13.

13.1.  $\neg(\neg A \vee B) \vee (\neg B \vee \neg C)$ ; 13.2.  $A \vee B \vee (\neg A \vee \neg B)$ ;  
 13.3.  $\neg(\neg(A \vee B) \vee Z) \vee (B \vee \neg C)$ ; 13.4.  $\neg(\neg(\neg A \vee \neg(B \vee \neg C)) \vee (\neg A \vee B)) \vee \neg B$ ;  
 13.5.  $\neg(\neg(\neg A \vee B) \vee (\neg B \vee B \vee C)) \vee (\neg A \vee C)$ . 14. 14.1.  $(\neg A \vee (\neg B \wedge C)) \wedge (A \vee \neg B)$ ;  
 14.2.  $((A \vee B \vee C) \wedge \neg D) \vee Q \vee R \vee P$ ;  
 14.3.  $((A \vee (B \wedge \neg C)) \wedge \neg D) \vee Q \wedge (R \vee (\neg P \wedge F))$ ;  
 14.4.  $((\neg A \vee (B \wedge (C \vee \neg D))) \wedge Q) \vee \neg R$ .

#### 3-§. Predikatlar. Kvantorlar.

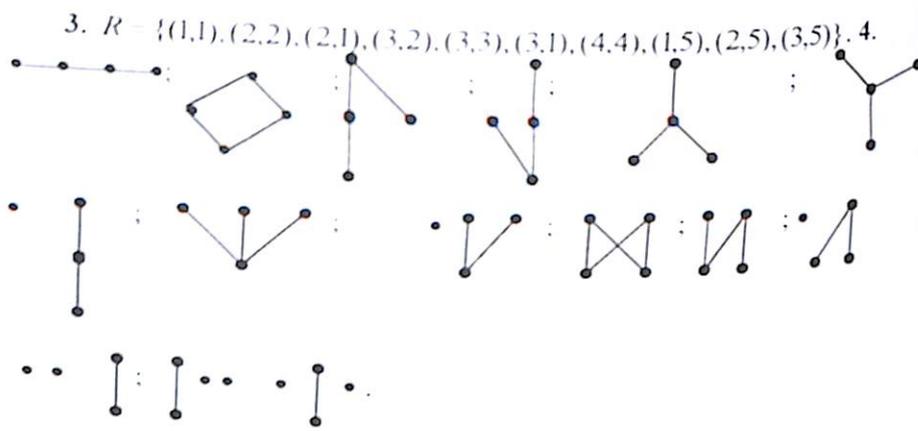
1. 1.2, 1.4, 1.5, 1.7, 1.8, 1.10, 1.11, 1.14. 5. 5.1.  $M \setminus \{3\}$ . 5.2.  $\{1, 2, 3, 4, 5, 6\}$ . 5.3.  
 $\{2, 4, 5, 6, 8, 10, 12, 14, 15, 16, 18, 20\}$ . 5.4.  $M \setminus \{3\}$ . 5.5.  $M \setminus \{5\}$ . 5.6.  $M \setminus \{5\}$ . 5.7.  
 $\{1, 5, 7, 11, 13, 17, 19\}$ . 5.10.  $\{6, 12, 18\}$ .

#### 4-§. Matematik mantiqning tabdirlari

4. 4.1.  $\forall x \in M \forall y \in M ((x = y) \vee (x < y) \vee (x > y))$ ;  
 $\exists x \in M \exists y \in M (\neg(x = y) \vee \neg(x < y) \vee \neg(x > y))$ .  
 4.2.  $\exists L \in R, \forall y \in M (|f(x)| \leq L)$ ;  $\forall L \in R, \exists y \in M (|f(x)| > L)$ .  
 4.3.  $\forall x_1 \in M \forall x_2 \in M ((x_1 < x_2) \Rightarrow f(x_1) < f(x_2))$ ;  
 $\exists x_1 \in M \exists x_2 \in M ((x_1 < x_2) \wedge f(x_1) \geq f(x_2))$ ;  
 4.4.  $\exists T \in R \setminus \{0\} \forall x \in M ((x \pm T \in M) \wedge f(x \pm T) = f(x))$ ;  
 $\forall T \in R \setminus \{0\} \exists x \in M ((x \pm T \notin M) \vee f(x \pm T) \neq f(x))$ .

### II MODUL. TO'PLAMLAR VA MUNOSABATLAR

5-§. To'plam. To'plamlar ustida amallar. Eylar-Venn diagrammalari  
 2.  $A = \{1\}$ ,  $B = \{a, \{1\}\}$ ,  $C = \{\{a, \{1\}\}\}$ . 4.  $|R(M_1)| = 2$ ,  $|R(M_2)| = 4$ ,  $|R(M_3)| = 16$ ,  
 $|R(M_n)| = 2^n$ . 6. 10 nafardan kam emas.  
 7-§. Akslantirish (funksiya). Tartib munosabati. Graflar



III MODUL. ALGEBRA VA ALGEBRAIK SISTEMALAR

8-§. Algebra. Faktor-algebra

1. 1.1 amal emas, 1.2.unar amal, 1.3. amal emas, 1.4. amal emas, 1.5 amal emas, agar natural sonlar to'plamida qaralsa, ternar amal. 2. Neytral element mavjud emas. 3. 3.1. Neytral element mavjud emas. 3.2. Neytral element 1. 4. 4.1. Butun sonlar to'plamida qo'shish, ko'paytirish va qarama-qarshi elementni topish. 4.2. Ratsional sonlar to'plamida qo'shish, ko'paytirish, qarama-qarshi va teskari elementlarni topish. 4.3. Mulohazalar to'plamida dizyunkttsiya, konyunkttsiya, implikatsiya, inkor amallari.

9-§. Gruppa. Halqa. Maydon

10.  $4(z) \begin{cases} 0, aazp z = 4k \\ e, aazp z = 4k+1 \\ a, aazp z = 4k+2 \\ b, aazp z = 4k+3 \end{cases}$  akslantirish gomomorfizmdir.

IV MODUL. ASOSIY SONLI SISTEMALAR

12-§. Kompleks sonlar maydoni

1. 1.1.  $Rez=0, Imz=\frac{14}{5}$ . 1.2.  $Rez=2, Imz=\frac{3}{2}$ . 1.3.  $Rez=0, Imz=-1.1.4.$   
 $Rez=\frac{1}{2}, Imz=0$ . 4. 4.1.  $z = 2,5 + yi, y \in R$ . 4.2.  $z = -\frac{1}{4} + i$ . 4.3.  $z_{1,2} = \frac{-1 \pm \sqrt{5}}{2}$ . 4.4.

0. 4.5.  $z_1 = -1 + i, z_2 = -4 - i$ . 4.6.  $z = \frac{3-7i}{8}, z_2 = \frac{1+3i}{8}$ . 4.7.  
 $z_1 = 1-2i, z_2 = -3+i$ . 4.8.  $z_1 = -2i, z_2 = \frac{-1+3i}{2}$ . 5. 5.1.  $1-i$ . 5.2.  $i$ . 5.3. 0. 6.  
6.1.  $1+i$ . 6.2.  $6+8i, 6+17i$ . 6.3.  $z = 1-i, z_2 = i$ . 6.4.  $z_1 = 1+i, z_2 = -2i$ . 6.5.  
 $z = \frac{7}{6} + \frac{5}{6}i$ . 6.6.  $z_1 = -1,5 - 2i, z_2 = -1,5 - 4,25i$ . 8. 8.1.  $2^n \cos \frac{n\pi}{3}$ . 8.2.  
 $\frac{2^n}{\sqrt{3}} \sin \frac{2n\pi}{3}$ . 8.3.  $2^n \cos \frac{5n\pi}{3}$ . 8.4.  $\frac{2^n}{3} \sin \frac{n\pi}{6}$ . 9. 9.1.  $5(\cos 0 + i \sin 0)$ . 9.2.  
 $2(\cos \frac{3}{2}\pi + i \sin \frac{3}{2}\pi)$ . 9.3.  $\frac{1}{2\sqrt{2}} (\cos \frac{5}{4}\pi + i \sin \frac{5}{4}\pi)$ . 9.4.  $16(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3})$ . 9.5.  
 $\cos(\frac{\pi}{2} - \alpha) + i \sin(\frac{\pi}{2} - \alpha)$ . 11. 11.1.  $-\frac{\cos(\alpha + \beta)}{\sin \beta} (\cos(2\beta - \theta) + i \sin(2\beta - \theta))$ . 11.4.  
11.2.  $4^{2008}$ . 11.3.  $2^n \sin^n \frac{\alpha}{2} (\cos n(\frac{\pi}{2} - \alpha) + i \sin n(\frac{\pi}{2} - \alpha))$ . 11.4.  
 $-\frac{3^8}{\sqrt{3} \cdot 2^{971}} (-1 + i\sqrt{3})$ .  
12. 12.1.  $\sqrt{5}(\cos \frac{\varphi + 2k\pi}{3} + i \sin \frac{\varphi + 2k\pi}{3}), k = 0,1,2; \varphi = \arcsin(-\frac{1}{\sqrt{5}})$ .  
12.2.  $2(\cos \frac{k\pi}{6} + i \sin \frac{k\pi}{6}), k = 0,1,2,3,4,5$ .  
12.3.  $\sqrt[3]{13}(\cos \frac{\varphi + 2k\pi}{4} + i \sin \frac{\varphi + 2k\pi}{4}), k = 0,1,2,3$ .  
12.4.  $\sqrt{\frac{1}{2}}(\cos \frac{\frac{\pi}{3} + 2k\pi}{4} + i \sin \frac{\frac{\pi}{3} + 2k\pi}{4}), k = 0,1,2,3$ .  
12.5.  $\cos \frac{11\pi + 2k\pi}{5} + i \sin \frac{11\pi + 2k\pi}{5}, k = 0,1,2,3,4$ .  
12.6.  $\sqrt[3]{2}(\cos \frac{\varphi + 2k\pi}{4} + i \sin \frac{\varphi + 2k\pi}{4}), k = 0,1,2,3; \varphi = \arccos \frac{\sqrt{3}+1}{2\sqrt{2}}$ .

$$12.7. \frac{1}{\sqrt{2}} \left( \cos \frac{11\pi + 2k\pi}{6} + i \sin \frac{11\pi + 2k\pi}{6} \right), k = 0,1,2,3,4,5.$$

$$12.8. \sqrt[7]{74} \left( \cos \frac{3\varphi + 2k\pi}{8} + i \sin \frac{3\varphi + 2k\pi}{8} \right), k = 0,1,2,3,4,5,6,7; \varphi = \arccos \frac{7}{\sqrt{74}}.$$

### V MODUL . ARIFMETIK VEKTOR FAZO.

#### CHIZIQLI TENGLAMALAR SISTEMASI

##### 13-§. Arifmetik vektor fazo.

1. 2. 2.1.  $(8 + \sqrt{2}, 0, -9, 4)$ . 2.2.  $(-2, 12, -17)$ . 2.3.  $\left( \frac{1}{2} \sin \alpha, \frac{1}{2}, \frac{1}{2} \cos 3\alpha \right)$ . 2.4.  $\left( -\frac{31}{6}, \frac{11}{2}, -3 \right)$ .

##### 14-§. Matritsa va uning rangi

3. agar  $\lambda = 1 \wedge \lambda = \frac{1}{4}$  bo'lsa rang 2; agar  $\lambda = 1 \vee \lambda = \frac{1}{4}$  bo'lsa, rang 3ga teng.  
4.  $\lambda = 0$  da rang 2,  $\lambda \neq 0$  da rang 3ga teng.

##### 15-§. Chiziqli tenglamalar sistemasi .

2. 2.1.  $(1, -1, 0)$ . 2.2.  $(1, 2, -1, -2)$ . 2.3.  $\left( \frac{2}{3}, -1, \frac{3}{2}, 0 \right)$ . 2.4. hamjoysiz. 3. 3.1. har qanday  $\lambda$  uchun ChTS hamjoyli. 3.2.  $\lambda \neq -2$  da ChTS hamjoyli. 3.3.  $\lambda = -3$  da ChTS hamjoysiz. 3.4.  $\lambda = 2$  da ChTS hamjoysiz. 4. 4.1.  $x = \frac{1}{4}(-a + b + c + d)$ ,  $y = \frac{1}{4}(a - b + c + d)$ ,  $z = \frac{1}{4}(a + b - c + d)$ ,  
 $t = \frac{1}{4}(a + b + c - d)$ . 4.2.  $x = \frac{1}{A}(ap - bq - cr - ds)$ ,  $y = \frac{1}{A}(aq + bp + cs - dr)$ ,  
 $z = \frac{1}{A}(ar - bs + cp + dp)$ ,  $t = \frac{1}{A}(as + br - cq + dp)$ ,  $A = a^2 + b^2 + c^2 + d^2$ .  
4.3.  $x_k = (-1)^{n+k} \sum_{i=1}^n \frac{b_i f_{ik}}{(a_i - a_1) \dots (a_i - a_{i-1})(a_i - a_{i+1}) \dots (a_i - a_n)}$ , bu erda

$f_a, a_1, \dots, a_{k-1}, a_{k+1}, \dots, a_n$  elementlarning  $n - i$  tadan ko'paytmalari yig'indisi. 4.4.

$$x_k = \frac{\prod_{i \neq k} (b - a_i)}{\prod_{i \neq k} (a_k - a_i)} = \frac{f(b)}{(b - a_k) f'(a_k)}, f(x) = (x - a_1)(x - a_2) \dots (x - a_n). \quad 5. \quad 5.1.$$

$$x_1 = \frac{x_3 - 9x_4 - 2}{11}, x_2 = \frac{-5x_3 + x_4 + 10}{11}. \quad 5.2.$$

$$x_3 = 22x_1 - 33x_2 - 11, x_4 = -16x_1 + 24x_2 + 8. \quad 5.3. \quad (3, 2, 1). \quad 5.4.$$

$$x_1 = \frac{-6 + 8x_4}{7}, x_2 = \frac{1 - 13x_4}{7}, x_3 = \frac{15 - 6x_4}{7}. \quad 5.5. \quad 5.6.$$

$$x_3 = 13, x_4 = 19 - 3x_1 - 2x_2, x_5 = -34.$$

$$x_3 = \frac{4}{3}x_1 + \frac{2}{3}x_2, x_4 = -\frac{14}{3}x_1 - \frac{7}{3}x_2 - 1, x_5 = \frac{4}{3}x_1 + \frac{2}{3}x_2 + 2. \quad 6.$$

8.8.2.  $x_1 = 2x_3 + 8x_4, x_2 = -x_3 - 2x_4, x_3, x_4 \in R, x_5 = 0$ .  $\vec{a}_1 = (2, -1, 1, 0, 0)$ ,  
 $\vec{a}_2 = (8, -2, 0, 1, 0)$ . 8.4.  $x_1 = x_2 = x_3 = x_4 = 0$ ; fundamental sistema mavjud emas.  
8.6.  $x_1 = x_3 + x_4 + 5x_5, x_2 = -2x_3 - 2x_4 - 6x_5, x_3, x_4, x_5 \in R$ ; agar  $\lambda = 0$  bo'lsa, bo'lsa,

$\vec{a}_1 = (1, -2, 1, 0, 0), \vec{a}_2 = (1, -2, 0, 1, 0), \vec{a}_3 = (5, -6, 0, 0, 1)$ . 8.8. agar  $\lambda = 0$  bo'lsa, bo'lsa, fundamental sistema  $\vec{e}_1, \dots, \vec{e}_4 \in R^4$  fundamental sistema bo'lsa,  
 $x_1, x_2, x_3, x_4 \in R$  va ortonormal sistema  $\vec{e}_1, \dots, \vec{e}_4 \in R^4$  fundamental sistema bo'lsa, bo'ladi;

8.10.  $\lambda \neq 0$  bo'ladi;  $x_1 = -3x_3 - 4x_4, x_2 = -2x_3 - 3x_4, x_3, x_4 \in R; \vec{a}_1 = (-3, -2, 1, 0), \vec{a}_2 = (-4, -3, 0, 1)$ ,  
 $x_1 = x_3 = x_4 = 0, x_2 \in R, \vec{a} = (0, 1, 0, 0); \lambda = -2$ ,

8.10.  $\lambda = -1, x_1 = x_3 = x_4 \in R; \vec{a}_1 = (1, -2, 0, 0), \vec{a}_2 = (0, -2, 1, 0); \lambda \in \{-1, -2\}, x_1 = x_2 = x_3 = x_4 = 0$ ,  
 $x_2 = -2x_1 - 2x_3 - 2x_4, x_1, x_3, x_4 \in R;$   
fundamental sistema mavjud emas.

VI MODUL. MATRITSALAR

16-§. Matritsalar va ular ustida amallar

5. 5.1.  $\begin{pmatrix} 0 & 1 & 2 & -1 \end{pmatrix}$ . 5.2.  $\begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$ . 5.3.  $\begin{pmatrix} \cos(\alpha + \beta) & -\sin(\alpha + \beta) \\ \sin(\alpha + \beta) & \cos(\alpha + \beta) \end{pmatrix}$ .

va  $\begin{pmatrix} \cos(\alpha + \beta) & -\sin(\alpha + \beta) \\ \sin(\alpha + \beta) & \cos(\alpha + \beta) \end{pmatrix}$ . 5.4.  $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ .  $\begin{pmatrix} -4 & 6 & 0 & 2 \\ -3 & 2 & -2 & 2 \\ 4 & -1 & 4 & -3 \\ 1 & 6 & 6 & -2 \end{pmatrix}$ . 5.5.

ko'paytma mavjud emas. 5.6.  $\begin{pmatrix} 2 & -3 & -1 \\ 12 & -18 & -6 \\ -4 & 6 & 2 \end{pmatrix}$ . 6. 6.1.  $n$ -juft bo'lsa.  $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}^n$ .

toq bo'lsa,  $\begin{pmatrix} 2 & -1 \\ 3 & -2 \end{pmatrix}$ . 6.2.  $\begin{pmatrix} 2^n & n2^{n-1} \\ 0 & 2^n \end{pmatrix}$ . 6.3.  $\begin{pmatrix} \cos n\alpha & -\sin n\alpha \\ \sin n\alpha & \cos n\alpha \end{pmatrix}$ . 6.4.

$\begin{pmatrix} \lambda^n & n\lambda^{n-1} \\ 0 & \lambda^n \end{pmatrix}$ . 6.5.  $\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 2^n & 0 & 0 \\ 0 & 0 & 3^n & 0 \\ 0 & 0 & 0 & 4^n \end{pmatrix}^n$ . 6.6.  $\left(\frac{1}{2} \sin 2\alpha\right)^n \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$ . 7. 7.1.

$\begin{pmatrix} a & 0 \\ b & a \end{pmatrix}$ . 7.2.  $\begin{pmatrix} a & 2b \\ 3b & a+3b \end{pmatrix}$ . 7.3.  $\begin{pmatrix} a & 3b \\ -5b & a+9b \end{pmatrix}$ . 7.4.  $\begin{pmatrix} a & b & c \\ 0 & a & b \\ 0 & 0 & a \end{pmatrix}$ . 7.5.

$\begin{pmatrix} a & b & c \\ 0 & a & b \\ 0 & 0 & a \end{pmatrix}$ . 7.6.  $\begin{pmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{pmatrix}$ . 7.7.  $\begin{pmatrix} a & 0 & c \\ 0 & b & 0 \\ c & 0 & a \end{pmatrix}$ . 7.8.  $\begin{pmatrix} a & b & c & d \\ 0 & a & b & c \\ 0 & 0 & a & b \\ 0 & 0 & 0 & a \end{pmatrix}$ . 12. 12.1.

$\begin{pmatrix} -2 & 3 \\ 3 & -4 \end{pmatrix}$ . 12.2.  $\frac{1}{ad-bc} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$ . 12.3.  $\begin{pmatrix} \frac{3}{2} & \frac{1}{2} & \frac{3}{2} \\ -\frac{3}{2} & -\frac{1}{2} & -\frac{1}{2} \\ 1 & 0 & 1 \end{pmatrix}$ . 12.4.

$\begin{pmatrix} -8 & 29 & -11 \\ -5 & 18 & -7 \\ 1 & -3 & 1 \end{pmatrix}$ . 12.5.  $\frac{1}{4} \begin{pmatrix} 2 & 0 & 0 & 0 \\ -3 & 2 & 0 & 0 \\ 25 & -14 & 12 & -16 \\ -19 & 10 & -8 & 12 \end{pmatrix}$ . 12.6.  $\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{pmatrix}$ .

12.7.  $\begin{pmatrix} 1 & -1 & 0 & \dots & 0 \\ 0 & 1 & -1 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & 1 \end{pmatrix}$ . 12.8.  $\begin{pmatrix} 1 & -a & 0 & 0 & \dots & 0 \\ 0 & 1 & -a & 0 & \dots & 0 \\ 0 & 0 & 1 & -a & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \dots & 1 \end{pmatrix}$ . 12.9.

$\frac{1}{n+1} \begin{pmatrix} n & n-1 & n-2 & n-3 & \dots & 1 \\ n-1 & 2(n-1) & 2(n-2) & 2(n-3) & \dots & 2 \\ n-2 & 2(n-2) & 3(n-2) & 3(n-3) & \dots & 3 \\ n-3 & 2(n-3) & 3(n-3) & 4(n-3) & \dots & 4 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 2 & 3 & 4 & \dots & n \end{pmatrix}$ . 12.10.

$-\frac{1}{s} \begin{pmatrix} 1+a_1s & 1 & 1 & \dots & \frac{1}{a_1a_n} \\ \frac{1}{a_1^2} & \frac{1}{a_1a_2} & \frac{1}{a_1a_3} & \dots & \frac{1}{a_1a_n} \\ 1 & 1+a_2s & 1 & \dots & \frac{1}{a_2a_n} \\ \frac{1}{a_2a_1} & \frac{1}{a_2^2} & \frac{1}{a_2a_3} & \dots & \frac{1}{a_2a_n} \\ 1 & 1 & 1+a_3s & \dots & \frac{1}{a_3a_n} \\ \frac{1}{a_3a_1} & \frac{1}{a_3a_2} & \frac{1}{a_3^2} & \dots & \frac{1+a_ns}{a_3^2} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \frac{1}{a_na_1} & \frac{1}{a_na_2} & \frac{1}{a_na_3} & \dots & \frac{1+a_ns}{a_n^2} \end{pmatrix}$ . 14. 14.1.

$s = \frac{1}{a_1} + \frac{1}{a_2} + \dots + \frac{1}{a_n}$ .

$$\begin{pmatrix} -3 & -6 \\ 2 & 4 \end{pmatrix} \quad 14.2. \quad \begin{pmatrix} -1 & -1 \\ 2 & 3 \end{pmatrix} \quad 14.3. \quad \begin{pmatrix} 3 & 1 & 3 \\ 2 & 2 & 2 \\ -3 & -1 & -1 \\ 1 & 0 & 1 \end{pmatrix} \quad 14.4. \quad \begin{pmatrix} 3 & -2 \\ 5 & -4 \end{pmatrix}$$

$$14.5. \frac{1}{17} \begin{pmatrix} 2 & 5 \\ 9 & 14 \end{pmatrix} \quad 14.6. \quad \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \quad 14.7. \quad \begin{pmatrix} 6 & 4 & 5 \\ 2 & 1 & 2 \\ 3 & 3 & 3 \end{pmatrix} \quad 14.8. \quad \begin{pmatrix} 35 & 32 & 49 \\ 15 & 14 & 22 \\ 31 & 29 & 48 \end{pmatrix} \quad 14.9.$$

$$\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix} \quad 14.10. \quad \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 2 \end{pmatrix} \quad 14.11. \quad \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix} \quad 14.12.$$

$$\frac{1}{3} \begin{pmatrix} -101 & 55 & -176 & 217 \\ 22 & -11 & 40 & -47 \\ 8 & -4 & 14 & -19 \\ -3 & 3 & -6 & 6 \end{pmatrix} \quad 15. \quad 15.1. \quad (1, -2, 3). \quad 15.2. \quad \left(15, \frac{59}{2}, -\frac{22}{5}\right) \quad 15.3.$$

$$(1, 2, 2, 0). \quad 15.4. \quad (1, 2, 3, 4). \quad 15.5. \quad (2, 0, 2, 2). \quad 15.6. \quad \left(2, -\frac{37}{2}, 2, -\frac{9}{2}\right). \quad 15.7. \quad (-1, 0, 2, -1).$$

$$15.8. \quad (0, 0, 0, 0). \quad 16. \quad 16.1. \quad X = \begin{pmatrix} 2 & 3 \\ 0 & 2 \end{pmatrix}, Y = \begin{pmatrix} -1 & -2 \\ 0 & -1 \end{pmatrix} \quad 16.2.$$

$$X \in R^{2 \times 2}, Y = 2X + \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}.$$

## VII MODUL. DETERMINANTLAR

### 17-§. O'rniqa qo'yishlar.

$$1. \quad 1.1. \quad \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 1 & 3 & 4 & 5 \end{pmatrix} \quad 1.2. \quad \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 5 & 3 & 2 & 1 & 4 \end{pmatrix} \quad 2. \quad 2.1. \\ (1 \ 5 \ 3)(2 \ 4 \ 7). \quad 2.5. \quad (1 \ 2)(3 \ 4) \dots (2n-1 \ 2n). \quad 2.6. \\ (1 \ n+1)(2 \ n+2) \dots (n \ 2n). \quad 4. \quad 4.1. \quad \alpha=2, \beta=4. \quad 4.2. \quad \alpha=10, \beta=6. \quad 5.$$

$$(10 \ 9 \ 8 \ 7 \ 6 \ 5 \ 4 \ 3 \ 2 \ 1), C_{10}^2 = 45 \text{ ta inversiya. } 8. \quad 8.1. \quad \frac{3n(n-1)}{2}. \quad 8.2. \quad \frac{3n(n+1)}{2}. \quad 8.3. \quad \frac{n(3n+1)}{2}. \quad 8.4. \quad \frac{n(3n-1)}{2}.$$

### 18-§. Determinantlar

$$1. \quad 1.1. \quad 6. \quad 1.2. \quad -1. \quad 1.3. \quad 2. \quad 1.4. \quad 0. \quad 1.5. \quad -\sin \alpha. \quad 1.6. \quad 1. \quad 1.7. \quad 1. \quad 1.8. \quad 0. \quad 1.9. \\ 3abc - a^3 - b^3 - c^3. \quad 1.10. \quad (ab + bc + ca)x + abc. \quad 1.11. \quad 1. \quad 1.12. \quad -\frac{3}{2} - \frac{3i\sqrt{3}}{2}. \quad 1.13. \\ 1 + \alpha^2 + \beta^2 + \gamma^2. \quad 1.14. \quad \text{Birinchi qatorga 2- va 3-qatorlarni qo'shib Viet} \\ \text{formularidan foydalaning.} \quad 4. \quad 4.1. \quad n!. \quad 4.2. \quad (-1)^{n-1} n!. \quad 4.3. \\ x_1 x_2 \dots x_n \left(1 + \frac{1}{x_1} + \dots + \frac{1}{x_n}\right). \quad 4.4. \quad x_1(x_2 - a_{12})(x_3 - a_{23}) \dots (x_n - a_{n-1,n}). \quad 4.5. \\ (x-1)(x-2) \dots (x-n+1). \quad 4.6. \quad (-1)^n (a-1)(a-2) \dots (a-n). \quad 4.7. \quad n+1. \quad 4.8. \\ 2^{n+1} - 1. \quad 4.9. \quad x^n + (a_1 + a_2 + \dots + a_n)x^{n-1}. \quad 4.10. \quad \prod_{k=1}^n (1 - a_k x). \quad 5. \quad 5.1. \quad (1, 3, 2). \quad 5.2. \\ (2, 1, 3). \quad 5.3. \quad \left(\frac{5}{3}, 0, -\frac{2}{3}\right). \quad 5.4. \quad \left(\frac{1}{3}, 0, -\frac{1}{6}\right). \quad 5.5. \quad (-1, -1, 0, 1). \quad 5.6. \quad \left(-\frac{3}{5}, \frac{6}{5}, 3, 2\right). \quad 5.7. \\ (-3, 0, -\frac{1}{2}, \frac{2}{3}). \quad 5.8. \quad (-1, 2, 0, 1). \quad 5.9. \quad (1, 2, 3, 4). \quad 5.10. \quad (1, 0, -1, 0). \quad 8. \quad 8.1. \quad 2. \quad 8.2. \quad 2. \quad 8.3. \\ 3. \quad 8.4. \quad 3. \quad 8.5. \quad 3. \quad 8.6. \quad 2. \quad 8.7. \quad 5. \quad 8.8. \quad n, \text{ agar } n\text{-toq bo'lsa, } n-1, \text{ agar } n\text{-juft bo'lsa. } 9. \\ 9.1. \quad \begin{pmatrix} -2 & 1 \\ \frac{3}{2} & -\frac{1}{2} \end{pmatrix} \quad 9.2. \quad \begin{pmatrix} 7 & -4 \\ -5 & 3 \end{pmatrix} \quad 9.3. \quad \begin{pmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{pmatrix} \quad 9.4. \\ \frac{1}{18-5i} \begin{pmatrix} 5+4i & -3-i \\ -1+i & 2-3i \end{pmatrix} \quad 9.5. \quad \begin{pmatrix} 1 & -1 & 1 \\ -38 & 41 & -34 \\ 27 & -29 & 24 \end{pmatrix} \quad 9.6. \quad -\frac{1}{34} \begin{pmatrix} -42 & -15 & 58 \\ 24 & 11 & -38 \\ 22 & 3 & -32 \end{pmatrix} \\ 9.7. \quad \frac{1}{4} \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{pmatrix} \quad 9.8. \quad -\frac{1}{4} \begin{pmatrix} -1 & -1 & -1 & -1 \\ -1 & -i & 1 & -i \\ -1 & 1 & -1 & 1 \\ -1 & -i & 1 & i \end{pmatrix}$$

### VIII MODUL. VEKTOR FAZOLAR

19-§ Vektor fazo. Fazoostilar kesishmasi, yigindisi.

1. 1.1.  $\dim V = 1$ , bazislardan biri  $\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$ . 1.2.  $\dim V = 4$ , bazislardan biri

$$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}. \quad 1.3. \dim V = 9, \text{ bazislardan biri } \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \quad 1.7. \dim V = 6, \text{ bazislardan biri } \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

6.1.  $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$ . 3. 3.1.  $\dim V = n-1$ , bazis

$\vec{a}_1 = (1, 0, 0, \dots, 0), \dots, \vec{a}_{n-1} = (0, 0, 0, \dots, 1, 0)$ . 3.2.  $\dim V = n-1$ , bazislardan biri

$\vec{a}_1 = (1, -1, 0, \dots, 0), \vec{a}_2 = (0, 1, -1, \dots, 0), \dots, \vec{a}_{n-1} = (0, 0, 0, \dots, 1, -1)$ . 3.3.  $n$ -juft son

bo'lsa,  $\dim V = \frac{n}{2}$ , bazislardan biri  $\vec{a}_1 = (1, 0, 0, \dots, 0), \vec{a}_2 = (0, 0, 1, \dots, 0), \dots$

$\vec{a}_{\frac{n}{2}} = (0, 0, 0, \dots, 1, 0)$ ;  $n$ -toq son bo'lsa,  $\dim V = \frac{n+1}{2}$ , bazislardan biri

$\vec{a}_1 = (1, 0, 0, \dots, 0), \vec{a}_2 = (0, 0, 1, \dots, 0), \dots, \vec{a}_{\frac{n+1}{2}} = (0, 0, 0, \dots, 0, 1)$ . 4. 4.1. tashkil etmaydi.

4.2. tashkil etmaydi. 4.3. to'g'ri chiziq koordinatalar boshidan o'tgan bo'lsa tashkil etadi, aks holda tashkil etmaydi. 4.4. tashkil etadi. 4.5. tashkil etmaydi. 4.6. tashkil etadi.

6. 6.1. o'lchovi 3, bazislardan biri  $\vec{a}_1, \vec{a}_2, \vec{a}_3$ . 6.2. o'lchovi 3, bazislardan biri  $\vec{a}_1, \vec{a}_2, \vec{a}_3$ .

6.3. o'lchovi 3, bazislardan biri  $\vec{a}_1, \vec{a}_2, \vec{a}_3$ . 6.4. o'lchovi 3, sistema o'ziga bazis.

### 20-§. Skalyar ko'paytmali vektor fazolar.

Evklid vektor fazolar. Vektor fazolar izomorfizmi.

5. 5.2. vektorlar sistemasi chiziqli bog'liq bo'lganligi uchun uni ortogonalab bo'lmaydi. 5.3.  $\vec{b} = \vec{a}, \vec{b}_2 = \vec{a}_1, \vec{b}_3 = (1, -3, 3, 4)$ . 5.4.

$\vec{b}_1 = \vec{a}_1, \vec{b}_2 = (-2, 1, 2), \vec{b}_3 = (8, 32, -8)$ . 5.5.

$\vec{b}_1 = \vec{a}_1, \vec{b}_2 = (0, 1, 0, 0), \vec{b}_3 = (-1, 0, 0, 1), \vec{b}_4 = (-1, 0, 2, -1)$ . 5.6.

$\vec{b}_1 = \vec{a}_1, \vec{b}_2 = (2, 1, -2, -2), \vec{b}_3 = (2, 6, 4, 1)$ . 11. 11.1.  $\vec{b}_1 = (2, 1, 0), \vec{b}_2 = (-3, 0, 1)$ . 11.2.  $\emptyset$ .

11.3.  $\vec{b}_1 = (-1, 1, -1, 0), \vec{b}_2 = (2, 0, 5, 1)$ . 11.4.  $\vec{b}_1 = (-1, 1, 1, 0), \vec{b}_2 = (1, 2, 0, 1)$ . 11.5.

$\vec{b}_1 = (1, -3, -2, 0), \vec{b}_2 = (0, -5, -3, 1)$ . 12. 12.1.  $\vec{b}_1 = (1, 0, 1, 0), \vec{b}_2 = (0, 1, 0, 1)$ . 12.2.

### IX MODUL. CHIZIQLI AKSLANTIRISHLAR

21-§. Chiziqli akslantirish. Chiziqli operator yadrosi va obrazi. Chiziqli operator matritsasi.

5. 5.1.  $\begin{pmatrix} 2 & -11 & 6 \\ 1 & -7 & 4 \\ 2 & -1 & 0 \end{pmatrix}$ . 5.2.  $\frac{1}{3} \begin{pmatrix} -6 & 11 & 5 \\ -12 & 13 & 10 \\ 6 & -5 & -5 \end{pmatrix}$ . 9. 9.1.  $r=1, d=1$ . 9.2.

$r=2, d=0$ . 9.3.  $r=1, d=2$ . 9.4.  $r=2, d=1$ . 9.5.  $r=3, d=0$ . 9.6.  $r=1, d=3$ . 9.7.  $r=2, d=2$ . 9.8.  $r=3, d=1$ .

22-§. Chiziqli operatorlar ustida amallar. Chiziqli algebralar. Teskari operator. Xos vektorlar va xos qiymatlar.

1. 1.1.  $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ . 1.2.  $\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$ . 2.  $\begin{pmatrix} 2 & -2 \\ -10 & 10 \end{pmatrix}$ . 6. 6.1.  $\begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix}$ . 6.2.

$\begin{pmatrix} 2 & 0 & 2 \\ -8 & 1 & -5 \\ -1 & 0 & -1 \end{pmatrix}$ . 6.3.  $\begin{pmatrix} -16 & -5 & 7 & -1 \\ 14 & 5 & -6 & 1 \\ 11 & 3 & -5 & 1 \\ 3 & 1 & -1 & 0 \end{pmatrix}$ . 6.4.  $\begin{pmatrix} 1 & -1 & 0 & 0 \\ 10 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$ . 7. 7.1.

$$\lambda_1 = 0, \bar{a} = c_1(1, -2), \lambda_2 = 4, \bar{a} = c_2(1, 2), c_1, c_2 \neq 0. \quad 7.2.$$

$$\lambda_1 = \lambda_2 = 2, \bar{a} = c_1(1, 0), c_1 \neq 0. \quad 7.3. \quad \lambda_1 = \lambda_2 = \lambda_3 = 1, \bar{a} = c(1, 3, -3), c \neq 0. \quad 7.4.$$

$$\lambda = 2, \bar{a} = c(1, 0, 0), c \neq 0.$$

### X MODUL. CHIZIQLI TENGSIKLIKLAR SISTEMASI

#### 23-§. Chiziqli tengsizliklar sistemasi. Qavariq konus.

3. 3.1. ikkalasi ham natija emas. 3.2. 2) sistema 1)ning natijasi. 3.3. 2) sistema 1)ning natijasi. 3.4. 1) sistema 2)ning natijasi. 3.5. teng kuchli. 3.6. 2) sistema 1)ning natijasi.

13. 13.1.  $f_{\min} = 0, x_1 = 3, x_2 = 3.$  13.2.  $f_{\min} = -3\frac{1}{2}, x_1 = \frac{3}{2}, x_2 = \frac{5}{4}.$

13.3.  $f_{\min} = -6, x_1 = 0, x_2 = 6.$  13.4.  $f_{\min} = 3, x_1 = 0, x_2 = 0.$  14. 14.1.

$f_{\max} = 14\frac{1}{2}, x_1 = 1\frac{1}{4}, x_2 = 3.$  14.2.  $f_{\max} = 17, x_1 = 5, x_2 = 1.$  14.3.

$f_{\max} = 6, x_1 = 0, x_2 = \frac{3}{2}.$  14.4.  $f_{\max} = 9\frac{1}{7}, x_1 = 3\frac{6}{7}, x_2 = 1\frac{3}{7}.$

### XI MODUL. BUTUN SONLAR HALQASIDA BO'LINISH MUNOSABATI

#### 24-§. Tub va murakkab sonlar. EKUB. EKUK.

7. 7.1. 12, 168. 7.2. 9, 217. 7.3. 24, 1170. 7.4. 8, 624. 7.5. 30, 2418. 7.6. 8, 1440. 7.7. 12, 1960. 7.8. 24, 2808. 7.9. 16, 2340. 7.10. 30, 3844. 7.11. 8, 3096. 7.12. 24, 8736. 8. 8.1. 88. 8.2. 357. 8.3. 1, 8.4. 113. 8.5. 3109. 8.6. 3911. 8.7. 382. 8.8. 2011. 8.12. 490. 9. 9.1. 30, 120; 60, 90; 90, 60; 120, 30. 9.2. 20, 420; 60, 140; 420, 20; 140, 60. 9.3. 552, 115; 435, 232; 232, 435; 115, 552. 9.4. 495, 315; 315, 495.

#### 25-§. Chekli zanjir kasrlar. Munosib kasrlar.

2. 2.1. [1,9]. 2.2. [0;2,15]. 2.3. [-2;1,30,2]. 2.4. [1;2,3,4]. 2.5. [0;1,4,3,2]. 2.6. [-3;1,1,2]. 2.7. [2;2,3,1,5]. 2.8. [0;1,2,5,2]. 2.9. [1;4,2,1,7]. 2.10. [1;1,2,1,2,1,2]. 2.11. [0;1,2,3,4,5]. 2.12. [0;1,1,38]. 3. 3.1. [3;(3,6)]. 3.2. [3;(2,6)]. 3.3. [3;(1,1,1,6)]. 3.4. [5;(3,2,3,10)]. 3.5. [5;(2,10)]. 3.6. [7;(1,2,7,2,1,14)]. 3.7. [(2)]. 3.8. [(1,2)]. 3.9. [(2,2,2,1,12,1)]. 3.10. [2;(1)]. 3.11. [1;(1,1,4,1)]. 3.12. [2;(18,2)].

$$4.4.1. \frac{20}{31}. 4.2. \frac{131}{583}. 4.3. \frac{7}{23}. 4.4. \frac{97}{113}. 4.5. \frac{17}{83}. 4.6. \frac{359}{113}. 4.7. \frac{9+2\sqrt{39}}{5}. 4.8.$$

$$1+\sqrt{3}. 4.9. \sqrt{11}. 4.10. \sqrt{3}. 4.11. 5-\sqrt{15}. 4.12. \frac{245-\sqrt{85}}{74}. 5. 5.1.$$

$$x=-8360-117t, y=2717+38t, t \in \mathbb{Z}. 5.3. x=-2+4t, y=-4+7t, t \in \mathbb{Z}. 5.9.$$

$$x=-125-114t, y=45+41t, t \in \mathbb{Z}. 5.7. x=1-9t, y=39+49t, t \in \mathbb{Z}. 5.13.$$

$$x=9+31t, y=2-12t, t \in \mathbb{Z}. 5.11. x=75+23t, y=-120-37t, t \in \mathbb{Z}. 5.17.$$

$$x=4+17t, y=-11-53t, t \in \mathbb{Z}. 5.15. x=-15-39t, y=-25-64t, t \in \mathbb{Z}. 5.19. x=1270-559t, y=-2020-571t, t \in \mathbb{Z}.$$

#### 26-§. Sistematik sonlar va ular ustida amallar

1. 1.1. 11000<sub>2</sub> 1.2. 10001111<sub>2</sub>; 1.3. 101011<sub>2</sub>; 1.4. 111<sub>2</sub>; 1.5. 2255025<sub>7</sub>; 1.6. 3(10)94913<sub>12</sub>; 1.7. 30413<sub>7</sub>; 1.8. 190000<sub>12</sub>; 1.9. 56<sub>7</sub> va koldik 202<sub>7</sub>; 1.10. (10)<sub>9412</sub> va koldik 87<sub>12</sub>; 1.11. 2, 4<sub>8</sub>. 2. 2.1. 10000<sub>12</sub>; 2.2. 11, 1110<sub>12</sub>; 2.3. 100000, 1100<sub>12</sub>; 2.4. 0, 1337<sub>8</sub>; 2.5. 11, 14<sub>8</sub>. 3. 3.1. 1<sub>8</sub>; 3.2. 1<sub>6</sub>; 3.3. 1<sub>8</sub>; 3.4. 1<sub>5</sub>; 3.5. 1<sub>5</sub>; 3.6. 1<sub>8</sub>; 3.7. 1<sub>3</sub>; 3.8. 1<sub>7</sub>; 3.9. 1<sub>7</sub>; 3.10. 1<sub>8</sub>; 3.11. 1<sub>4</sub>; 3.12. 1<sub>8</sub>; 3.13. 1<sub>6</sub>. 4. 4.1. 39; 4.2. 205; 4.3. 229; 4.4. 2617; 4.5. 704; 4.6. 8387; 4.7. 1668; 4.8. 1523; 4.9. 6871; 4.10. 5669;

4.11. 42923. 5. 5.1. 0,875; 5.2. 0,75; 5.3. 25,9365; 5.4. 287,388671875; 5.5. 0,044921875. 6. 6.1. 4(10)2(11)<sub>12</sub>; 6.2. 230578<sub>9</sub>; 6.3. 11202102120100<sub>3</sub>; 6.4. 367341<sub>8</sub>; 6.5. 1000100011011011101100<sub>2</sub>; 6.6. 2121311<sub>5</sub>; 6.7. 4126<sub>8</sub>. 7. 7.1. 11111111010 = 2210122<sub>3</sub> = 31132<sub>5</sub>; 7.2. 1010111000010<sub>2</sub> = 10211012<sub>3</sub> = 42121<sub>5</sub>; 7.3. 2061<sub>7</sub>; 7.4. 1653212<sub>7</sub>; 7.5. 55173<sub>8</sub>; 7.6. 42167<sub>8</sub>. 8. 8.1. 4; 8.2. 5; 8.3. 9; 8.4. 9; 8.5. 5; 8.6. 9; 8.7. 7; 8.8. 6; 8.9. 5. 9. 9.1. 5; 9.2. 8; 9.3. 6; 9.4. 7; 9.5. 7; 9.6. 7; 9.7. 7; 9.8. 9; 9.9. 6; 9.10. g, g ≥ 2.

### XII MODUL. TAQQOSLAMALAR

27-§. Butun sonlar halqasida taqqoslamalar. Eylar va Ferma teoremlari

2. 2.1. 1; 2.2. 1; 2.3. 4; 2.4. 0; 2.5. 1; 2.6. 1; 2.7. 0; 2.8. 0; 2.9. 1; 2.10.

2.11. 3; 2.12. 11. 4.4.1. 88; 4.2. 67; 4.3. 24; 4.4. 9; 4.5. 27; 4.6. 36; 4.7.  
 $9^{10} \equiv 1 \pmod{100}$ ,  $9^{10q+r} \equiv 9^r \pmod{100}$   $9^9 \equiv 9 \pmod{10}$   
 $9^{99} \equiv 9^9 \equiv 9 \pmod{100}$ ;  
 4.8.  $7^4 = 2401 \equiv 1 \pmod{100}$ ,  $7^{100} \equiv 1 \pmod{100}$   $7^{99} \equiv 7^{100q+89} \equiv 7^{89} \pmod{100}$   
 $7^{88} \equiv 1 \pmod{100}$ ,  $7^{89} \equiv 7 \pmod{100}$ . 8. 8.1. 7; 8.2. 1; 8.3. 22; 8.4. 5; 8.5. 32;  
 8.6. 29; 8.7. 19; 8.8. 1. 8.9. 1; 8.10. 1. 9. 9.1. 2; 9.2. 1, 4, 1, 4; 9.3. 13; 9.4. 7;  
 9.5. 14; 9.6. 14; 9.7. 65; 9.8. 49. 10. 10.1. 21; 10.2. 22; 10.3. 64; 10.4. 21;  
 10.5. 375.  
 10.6. 4; 10.7. 24; 10.8. 1; 10.9. 23; 10.10. 8; 10.11. 8; 10.12. 60; 10.13. 147;  
 10.14. 48; 10.15. 127; 10.16. 5. 11. 11.1. 2; 11.2. 6; 11.3. 1; 11.4. 5; 11.5. 2;  
 11.6. 0; 11.7. 2; 11.8. 2; 11.9. 70; 11.10. 7. 11.11. 19; 11.12. 30; 11.13. 20;  
 11.14. 1; 11.15. 12; 11.16. 10; 11.17. 6; 11.18. 70. 12. 12.1. 01; 12.2. 67; 12.3.  
 31; 12.4. 97; 12.5. 01; 12.6. 61; 12.7. 61; 12.8. 97; 12.9. 76; 12.10. 92; 12.11.  
 84.

28-§. Birinchi darajali va tub modul bo'yicha yuqori darajali taqqoslamalar

2. 2.1.  $x \equiv 2 \pmod{3}$ ; 2.2.  $\emptyset$ ; 2.3.  $x \equiv 2 \pmod{5}$ ; 2.4.  $x \equiv 5 \pmod{7}$ ; 2.5.  
 $x \equiv 4, 9 \pmod{10}$ ; 2.6.  $x \equiv 3 \pmod{7}$ ; 2.7.  $x \equiv 8 \pmod{11}$ ; 2.8.  
 $x \equiv 2, 5, 8, 11 \pmod{12}$ . 3. 3.1.  $x \equiv 3 \pmod{13}$ ; 3.2.  $\emptyset$ ; 3.3.  $x \equiv 3, 10 \pmod{14}$ ;  
 3.4.  $x \equiv 2 \pmod{27}$ ; 3.5.  $x \equiv 6 \pmod{23}$ ; 3.6.  $x \equiv 3 \pmod{37}$ ; 3.7.  
 $x \equiv 11 \pmod{41}$ ; 3.8.  $x \equiv 38 \pmod{51}$ . 4. 4.1.  $x \equiv 4 \pmod{13}$ ; 4.2.  
 $x \equiv 3 \pmod{12}$ ; 4.3.  $x \equiv 10 \pmod{12}$ ; 4.4.  $x \equiv 14 \pmod{19}$ ; 4.5.  
 $x \equiv 13 \pmod{34}$ ;  
 4.6.  $\emptyset$ . 4.7.  $x \equiv 3 \pmod{22}$ ; 4.8.  $x \equiv 1, 14, 27 \pmod{39}$ . 5. 5.1.  $x \equiv 9 \pmod{98}$ ;  
 5.2.  $x \equiv 28 \pmod{119}$ ; 5.3.  $\emptyset$ ; 5.4.  $x \equiv 11 \pmod{169}$ ; 5.5.  $x \equiv 73 \pmod{117}$ ;  
 5.6.  $x \equiv 29 \pmod{201}$ ; 5.7.  $x \equiv 29, 138, 247 \pmod{327}$ ; 5.8.  
 $x \equiv 17, 96, 175, 254, 333 \pmod{395}$ ; 5.9.  $x \equiv 153, 461, 769 \pmod{924}$ ;  
 5.10.  $x \equiv 1630 \pmod{2413}$ ; 5.11.  $x \equiv 200, 751, 1302, 1853, 2404 \pmod{2755}$ .

5.12.  $\emptyset$ . 6. 6.1.  $x \equiv 3 \pmod{23}$ ; 6.2.  $x \equiv 11 \pmod{24}$ ; 6.3.  $x \equiv 11 \pmod{24}$ ;  
 6.4.  $x \equiv 23 \pmod{30}$ ; 6.5.  $\emptyset$ ; 6.6.  $x \equiv 2, 7, 12, 17, 22, 27 \pmod{30}$ ;  
 6.7.  $x \equiv 2 \pmod{41}$ ; 6.8.  $x \equiv 21 \pmod{50}$ . 7. 7.1.  $x \equiv a+b \pmod{ab}$ ;  
 7.2.  $x \equiv (a-b)^{a(ab)-1} \pmod{ab}$ ; 7.3.  $x \equiv (a-b)(a+b)^{a(ab)-1} \pmod{ab}$ ;  
 7.4.  $x \equiv (a-b) \pmod{ab}$ ; 7.5.  $x \equiv \frac{1+p}{2} \pmod{p}$ ; 7.6.  $x \equiv m-1 \pmod{m}$ ;  
 7.7.  $x \equiv a \pmod{m}$ ; 7.8.  $x \equiv a^{p-2} \pmod{p}$ . 8. 8.1.  $x=2+3t, y=-2t, t \in \mathbb{Z}$ ;  
 8.2.  $x=2+3t, y=2+4t, t \in \mathbb{Z}$ ; 8.3.  $x=3+4t, y=1-3t, t \in \mathbb{Z}$ ;  
 8.4.  $x=3+4t, y=-3-5t, t \in \mathbb{Z}$ ; 8.5.  $x=7+8t, y=-2-3t, t \in \mathbb{Z}$ ;  
 8.6.  $x=-3+13t, y=4-17t, t \in \mathbb{Z}$ ; 8.7.  $x=-7+15t, y=12-23t, t \in \mathbb{Z}$ ;  
 8.8.  $x=-1+16t, y=-8+17t, t \in \mathbb{Z}$ ; 8.9.  $x=1+4t, y=2+13t, t \in \mathbb{Z}$ ;  
 8.10.  $\emptyset$ . 8.11.  $x=20+21t, y=23+25t, t \in \mathbb{Z}$ ; 8.12.  
 $x=47+105t, y=21+47t, t \in \mathbb{Z}$ ; 8.13.  $x=94+111t, y=39+47t, t \in \mathbb{Z}$ . 9.  
 9.1.  $x \equiv 18 \pmod{35}$ ; 9.2.  $\emptyset$ ; 9.3.  $x \equiv 12 \pmod{35}$ ; 9.4.  $x \equiv 105 \pmod{225}$ ;  
 9.5.  $x \equiv 170b_1 + 52b_2 \pmod{221}$ ; 9.6.  $x \equiv 100 \pmod{143}$ ,  $y \equiv 111 \pmod{143}$ ;  
 9.7.  $x \equiv 1 \pmod{5}$ ,  $y \equiv 2 \pmod{5}$ ; 9.8.  $\emptyset$ . 10. 10.1.  $x \equiv 91 \pmod{120}$ ;  
 10.2.  $x \equiv 59 \pmod{160}$ ; 10.3.  $x \equiv 33 \pmod{90}$ ; 10.4.  $x \equiv 86 \pmod{315}$ ;  
 10.5.  $x \equiv 256 \pmod{1547}$ ; 10.6.  $\emptyset$ ; 10.7.  $x \equiv 47 \pmod{420}$ ; 10.8.  
 $x \equiv 49 \pmod{420}$ ; 10.9.  $x \equiv 125 \pmod{1496}$ ; 10.10.  
 $x \equiv 11151b_1 + 11800b_2 + 16875b_3 \pmod{39825}$ ; 10.11.  $x \equiv 8479 \pmod{15015}$ . 11.  
 11.1.  $x \equiv 17 \pmod{90}$ ; 11.2.  $x \equiv 4 \pmod{105}$ ; 11.3.  $\emptyset$ ; 11.4.  
 $x \equiv 299 \pmod{385}$ ;  
 11.5.  $\emptyset$ ; 11.6.  $x \equiv 9573 \pmod{13923}$ ; 11.7.  $x \equiv 85056 \pmod{130169}$ . 12. 12.1.  
 $a \equiv 5 \pmod{6}$ ; 12.2.  $a \equiv 0 \pmod{4}$ ; 12.3.  $a \equiv 1 \pmod{7}$ ; 12.4.  $a \equiv 1 \pmod{6}$ .  
 13. 13.1.  $x^3 + 2x^2 + 3 = 0 \pmod{11}$ ; 13.2.  $x^3 + 18x^2 + 4x - 17 \equiv 0 \pmod{59}$ ;  
 13.3.  $x^6 + 4x^5 + 22x^4 + 76x^3 + 70x^2 + 52x + 39 \equiv 0 \pmod{101}$ ;  
 13.4.  $x^n + a_1x^{n-1}h + \dots + a_nh \equiv 0 \pmod{m}$ , bu erda  $a_0h \equiv 1 \pmod{m}$ . 14. 14.1.

- $2x^4 + 3 = 0 \pmod{5}$ ; 14.2.  $3x^4 + 2x^3 + 3x^2 + 2x = 0 \pmod{5}$ ;  
 14.3.  $3x^2 + x - 2 = 0 \pmod{7}$ ; 14.4.  $5x^6 + x^5 + 5x^4 + 3x^2 + 3x + 4 = 0 \pmod{7}$ ;  
 14.5.  $6x^6 + 7x^5 + 3x^4 + 3x^3 + x^2 + 3 = 0 \pmod{11}$ . 15. 15.1.  $x \equiv 2 \pmod{3}$ ;  
 15.2.  $\emptyset$ ; 15.3.  $x \equiv 1 \pmod{3}$ ; 15.4.  $x \equiv 1 \pmod{3}$ ; 15.5.  $x \equiv 1 \pmod{3}$ ;  
 15.6.  $x \equiv 4 \pmod{5}$ ; 15.7.  $x \equiv 3 \pmod{5}$ ; 15.8.  $x \equiv 2 \pmod{5}$ ;  
 15.9.  $\emptyset$ ; 15.10.  $x \equiv 1 \pmod{5}$ ; 15.11.  $x \equiv 1, 2 \pmod{5}$ . 16. 16.1.  
 $x \equiv 2 \pmod{7}$ ;  
 16.2.  $x \equiv 4 \pmod{7}$ ; 16.3.  $x \equiv 1, 2, 3, 4, 5, 6 \pmod{7}$ ; 16.4.  $x \equiv 4, 5 \pmod{7}$ ;  
 16.5.  $x \equiv 4 \pmod{11}$ ; 16.6.  $\emptyset$ . 16.7.  $x \equiv 7, 9 \pmod{11}$ ; 16.8.  
 $x \equiv 12 \pmod{13}$ ;  
 16.9.  $x \equiv 7, 13 \pmod{23}$ . 17. 17.1.  $(x-3)(x-4)^2 \equiv 0 \pmod{5}$ ;  
 17.2.  $(x-1)(x-2)^2 \equiv 0 \pmod{5}$ ; 17.3.  $(x-1)(x-2)(x-3)(x-4) \equiv 0 \pmod{5}$ ;  
 17.4.  $3(x-1)(x-2)(x-3) \equiv 0 \pmod{5}$ ; 17.5.  
 $(x-1)(x-2)(x-3)(x-6) \equiv 0 \pmod{7}$ ;  
 17.6.  $5(x-1)(x-3)(x-5) \equiv 0 \pmod{7}$ ; 17.7.  $6(x-1)(x-2)(x-9) \equiv 0 \pmod{11}$ ;  
 17.8.  $(x-2)(x-3)(x-9) \equiv 0 \pmod{17}$ ; 17.9.  
 $(x-1)(x-13)(x-21) \equiv 0 \pmod{23}$ ;  
 17.10.  $(x-2)^2(x-11)(x-28) \equiv 0 \pmod{29}$ ; 17.11.  
 $(x-17)(x-28)(x-30) \equiv 0 \pmod{31}$ .

29-§. Tub modul bo'yicha boshlang'ich ildizlar va indekslar

1. 1.1. -1; 1.2. 1; 1.3. 1; 1.4. 1; 1.5. -1; 1.6. -1; 1.7. -1; 1.8. -1; 1.9.  
 1.2. 2.1. -1; 2.2. 1; 2.3. -1; 2.4. -1; 2.5. -1; 2.6. 1; 2.7. -1; 2.8. 1; 2.9. 1.  
 3. 3.1. 0; 3.2. 2; 3.3. 0; 3.4. 0; 3.5. 0; 3.6. 2; 3.7. 0; 3.8. 0; 3.9. 0; 3.10. 0. 5.  
 5.1. 4; 5.2. 2; 5.3. 2; 5.4. 6; 5.5. 2; 5.6. 4; 5.7. 8; 5.8. 4; 5.9. 10; 5.10. 6; 5.11.  
 18; 5.12. 18. 6. 6.1. 12, 3 va 2; 6.2. 8, 8 va 4; 6.3. 10, 10, 2 va 5; 6.4. 6, 2 va 12;  
 6.5. 5, 10, 2 va 10. 7. 7.1. 2, 6, 7, 8; 7.2. 2, 6, 7, 11; 7.3.  $\emptyset$ ; 7.4. 2, 3, 10, 13, 14,  
 15; 7.5. 3, 5, 10, 12, 17, 19, 24, 26, 38, 40, 45, 47; 7.6. 2, 5, 11, 14, 20, 23, 29, 32,

- 38, 41, 47, 50, 56, 59, 65, 68, 74, 77. 8. 8.1. 2, 3; 8.2. 2, 5; 8.3. 6, 2; 8.4. 8, 3;  
 8.5. 12, 2, 9, 9.1. 3; 9.2. 3; 9.3. 5; 9.4. 6; 9.5. 2; 9.6. 27; 9.7. 5; 9.8. 7; 9.9. 7;  
 9.10. 3; 9.11. 3; 9.12. 2. 12. 12.1.  $x \equiv 13 \pmod{17}$ ; 12.2.  $x \equiv 8 \pmod{27}$ ; 12.3.  
 $x \equiv 31 \pmod{37}$ ; 12.4.  $x \equiv 30 \pmod{73}$ ; 12.5.  $x \equiv 32 \pmod{79}$ ; 12.6.  
 $x \equiv 74 \pmod{79}$ ; 12.7.  $x \equiv 44 \pmod{83}$ ; 12.8.  $x \equiv 51 \pmod{97}$ ; 12.9.  
 $x \equiv 30 \pmod{221}$ . 13. 13.1.  $x \equiv 7, 10 \pmod{17}$ ; 13.2.  $x \equiv 8, 19 \pmod{27}$ ; 13.3.  
 $x \equiv 10, 43 \pmod{53}$ ; 13.4.  $x \equiv 27, 34 \pmod{61}$ ; 13.5.  $x \equiv 27, 40 \pmod{67}$ ;  
 13.6.  $x \equiv 21, 46 \pmod{67}$ ; 13.7.  $x \equiv 14, 57 \pmod{71}$ ; 13.8.  $x \equiv 17, 66 \pmod{83}$ ;  
 13.9.  $x \equiv 2, 7 \pmod{11}$ ; 13.10.  $x \equiv 5, 20 \pmod{43}$ ; 13.11.  $x \equiv 3, 31 \pmod{47}$ ;  
 13.12.  $x \equiv 1634, 1847 \pmod{59^2}$ ; 13.13.  $x \equiv 253, 4076 \pmod{73^2}$ . 14. 14.1. 3;  
 14.2. 4;  
 14.3. 0; 14.4. 1; 14.5. 0; 14.6. 10; 14.7. 0; 14.8. 7; 14.9. 3; 14.10. 1; 14.11. 0.  
 15. 15.1.  $x \equiv 4, 33 \pmod{37}$ ; 15.2.  $x \equiv 17 \pmod{41}$ ; 15.3.  $\emptyset$ ; 15.4.  
 $x \equiv 2, 18, 23, 39 \pmod{41}$ ; 15.5.  $x \equiv 7 \pmod{43}$ ; 15.6.  $\emptyset$ ; 15.7.  
 $x \equiv 17 \pmod{67}$ ;  
 15.8.  $x \equiv 8, 28, 31, 36, 39, 59 \pmod{67}$ ; 15.9.  $x \equiv 30, 53 \pmod{83}$ ; 15.10.  $\emptyset$ .  
 16. 16.1.  $x \equiv 3, 5, 6 \pmod{7}$ ; 16.2.  $x \equiv 2, 3, 10, 11 \pmod{13}$ ; 16.3.  
 $x \equiv 10, 13 \pmod{23}$ ; 16.4.  $\emptyset$ ; 16.5.  $x \equiv 11, 27, 36 \pmod{37}$ ; 16.6.  
 $x \equiv 25, 30, 31, 36 \pmod{61}$ ; 16.7.  $x \equiv 17 \pmod{73}$ ; 16.8.  
 $x \equiv 12, 23, 35, 38, 50, 61 \pmod{73}$ ; 16.9.  $x \equiv 17, 63, 66 \pmod{73}$ ;  
 16.10.  $x \equiv 3, 24, 46 \pmod{73}$ ; 16.11.  $x \equiv 6, 14, 20, 59, 65, 73 \pmod{79}$ .

XIII MODUL. KO'PHADLAR

30-§. Bir o'zgaruvchili ko'phadlar.

1.  $f_1(x) = f_3(x)$ ;  $f_2(x) = f_4(x)$ . 2. 2.1.  $a = -5, b = -1, c = 6$ . 2.2.  
 $a = 2, b = 5, c = 7$ . 3. 3.1.a)  $a = 6, g_1(x) = x^2 + 3x + 1, g_2(x) = -x^2 - 3x - 1$ ; 3.2.  
 $a = 3, g_1(x) = 2x + 2, g_2(x) = 3x + 3$ ;  $a = 2, g_1(x) = 2x + 3, g_2(x) = 3x + 2$ ; 3.3.

- $a = 4, g_1(x) = 3x^2 - 2x - 2, g_2(x) = -3x^2 + 2x - 2$ . 4.  
 $a = -8, b = 18, g_1(x) = x^2 - 4x + 1, g_2(x) = -x^2 + 4x - 1$ .  
 $a = 8, b = 14, g_1(x) = x^2 + 4x - 1, g_2(x) = -x^2 - 4x + 1$ . 5.  $a = 3, b = -7, c = 4$ . 7. 7.1.  
 $Z[x]$  da  $f(x) \mid g(x), Q[x]$  da  $f(x) \mid g(x)$ . 7.2. bo'linadi. 7.3. bo'linmaydi. 8. 8.1.  
 $b = -1 - a, a = c$ ; 8.2.  $b = -1, c = 0$ ; 8.3. agar  $a = 0$  bo'lsa, u holda  $b = c + 1$  va  
 $c \in z$ ; agar  $a \in z \setminus \{0\}$  bo'lsa, u holda  $b = 2 - a^2$  va  $c = 1$ . 9. 9.1.  $r = 1 - i$ ; 9.2.  
 $r = 7$ . 9.3.  $r(x) = x + 2$ ; 9.4.  $r(x) = (2 + i)x + (1 - i)$ ; 9.5.  $r(x) = -7x + 11$ . 10.  
10.1.  $f(x) = g(x)(x^3 - 3x + 5) + 2x - 3$ ; 10.2.  
 $f(x) = g(x)(3 + 2i)x + (2 - 7i)x + (-2 + i)$ ;  
10.3.  $f(x) = g(x)(2x^2 + 3x) + \bar{1}$ ;  
10.4.  $f(x) = g(x)(5x^5 - 8x^4 - 2x^3 + 3x^2 + 6) + 4x + 5$ . 11.  
 $r(x) = (3x^2 - 4x + 1)^2$ . 12.  $r = 3$ . 13. 13.1.  $h(x) = 5x^3 - 4x^2 + 7x + 6$ ;  $r(x) = 16$ ;  
13.2.  $h(x) = 2ix^3 + (3 - i)x - 2$ ;  $r(x) = 2 + i$ ; 13.3.  $h(x) = 0, 5x^3 + 3x - 1$ ;  
 $r(x) = 2, 5x - 1, 5$ ; 13.4.  $h(x) = \bar{5}x^3 + \bar{2}x^2 + \bar{1}$ ;  $r(x) = \bar{2}x^2 - \bar{2}x + 1$ . 14. 14.1. 136;  
14.2.  $-1 - 46i$ ; 14.3.  $\bar{2}$ ; 14.4.  $9 - 5\sqrt{2}$ .  
17. 17.1.  $f(x) = (x - 1)^4 + 2(x - 1)^3 + 3(x - 1)^2 - (x - 1) - 2$ ;  
17.2.  $f(x) = \bar{2}(x - \bar{1})^4 + (x - \bar{1})^2 + (x - \bar{1})$ ; 17.3.  $f(x) = (x + i)^5 - 5i(x + i)^4 -$   
 $-(3i + 10)(x + i)^3 + (-13 + 10i)(x + i) + (22i + 5)(x + i) + 11 - i$ . 18. 18.1.  
 $(f, g) = x + 1$ . 18.2.  $(f, g) = 1$ . 18.3.  $(f, g) = 2x + 1$ . 19. 19.1.  
 $(f, g) = x - 3$ ; 19.2.  $(f, g) = 1$ ; 19.3.  $(f, g) = x + \bar{3}$ ; 19.4.  $(f, g) = x^2(1 + i)x + i$ . 20.  
20.1.  $[f, g] = (2x^3 + 7x^2 + 4x - 3)(x - 1)$ ;  
20.2.  $[f, g] = (x^3 + \bar{6}x^2 + \bar{4}x + 1)(x^3 + x^2 + \bar{3}x - \bar{4}) : (x + \bar{2})$ ;  
 $[f, g] = (x^3 - x^2 + 3x - 3)(x^4 + 2x^3 + 2x - 1)$ ; 20.4.  
 $[f, g] = x^5 + 2ix^4 - 2x^3 - 2ix^2 + x$ . 20.3.

21. 21.1.  $u(x) = 1, v(x) = -x + 1$ ; 21.2.  $u(x) = -x - \bar{1}, v(x) = x + \bar{2}$ ; 21.3.  
 $u(x) = -\frac{x-1}{3}, v(x) = x^2 - x - \frac{3}{2}$ . 22. 22.1.  $u(x) = \frac{1}{2}(x^2 + x + 1) + (x-1)h(x)$ ,  
 $v(x) = \frac{1}{2}(x^2 + x + 1) - (x+1)h(x), h(x) \in Q[x]$ ; 22.2.  $u(x) = \frac{x^2 + \bar{4}x}{3} + (x + \bar{2})h(x)$ ,  
 $v(x) = \frac{\bar{4} + \bar{1}}{3}(x^2 + \bar{4}x) - (x^2x + \bar{1})h(x), h(x) \in Z_3[x]$ . 23.  $S(x); (f, g, h)$  bo'lganligi  
uchun tenglama echimga ega.

### 31-§. Ko'p o'zgaruvchili ko'phadlar.

- 1.1.1.  $f(x, y) = x^5 + x^4y - 2x^3y^2 - xy^4 + 2y^5 + x^2 - 1$ . 1.2.  
 $f(x, y, z) = x^3y^2z + y^3z^2x + xz^2y - xy^2z^3 - yz^2x^3 - xz^2y^3$ . 2. 2.1. yuqori hadi  $\bar{2}x^2z$ .  
2.2. yuqori hadi  $xz$ . 4. 4.1.  $5x^4y^2z$ . 4.2.  $-3^3y^2z^2$ . 5. 5.1.  
 $f(x, y) = \sigma_1^2\sigma_2 + 2\sigma_1^2 - 2\sigma_2^2 - 4\sigma_2$ . 5.2.  $f(x, y) = 2\sigma_1^2\sigma_2 - 6\sigma_1\sigma_2^2 - 5\sigma_1\sigma_2$ .  
5.3.  $f(x, y, z) = \sigma_1^4 - 4\sigma_1^3\sigma_2 + 8\sigma_1^2\sigma_3$ . 5.5.  
 $f(x, y, z, t) = \sigma_1^2\sigma_4 + \sigma_3^2 - 4\sigma_2\sigma_4$ . 10. 10.1. 0. 10.2.  $\frac{1}{2}$ . 11. 11.1.  
 $o(x_1, x_2, x_3, x_4) = \sigma_2$ . 11.3.  
 $o(x_1, x_2, x_3) = \sigma_1^2 2\sigma_2 - \frac{7}{3}\sigma_1^2$ . 11.2.  
 $o(x_1, x_2, \dots, x_n) = \sigma_1^3 - 3\sigma_1\sigma_2 + 3\sigma_3$ . 14. 14.1. 162. 14.2. 10. 14.3. 41. 14.4. 59.  
15. 15.1.  $a = 1$ . 15.2.  $a = 2$ . 15.3.  $a = 3 \wedge a = -1$ . 15.4.  $a = \pm i\sqrt{2}$  va  $a = \pm 2i\sqrt{3}$ .  
16. 16.1.  $-108$ . 16.2.  $-27036$ . 16.3. 50000. 16.4.  $a(b^2 - 4ac)$ . 16.5.  
 $-27q^2 - 4p^3$ . 16.6.  $-2c^2 + 18abc - 4a^3c - 4b^3 + a^2b^2$ . 18. 18.1.  $a = \pm 2$ . 18.2.  
18.3.  $a = 3$ . 18.4.  $a = 2 + 4i$ . 19.  
 $a \in \left\{ 3, 3 \left( -\frac{1}{2} \pm i \frac{\sqrt{3}}{2} \right) \right\}$ .  
19.1.  $\left\{ (3, 2), (-3, -2), \left( 2\sqrt{2}, -\frac{3\sqrt{2}}{2} \right), \left( -2\sqrt{2}, \frac{3\sqrt{2}}{2} \right) \right\}$ . 19.2.

$$\left\{ (1,0), (2,1), \left( \frac{-19 - \sqrt{177}}{2}, \frac{9 + \sqrt{177}}{2} \right) \right\}. \quad 19.3. \quad \{(0,0), (2,-1), (1,2), (1,68), (2,52)\}.$$

19.4. (1,2). 20. 20.1 (2,3), (3,2). 20.2. (1,2); (2,1). 20.4.

20.3. (1,2,-2), (1,-2,2), (2,1,-2), (2,-2,1), (-2,2,1), (-2,1,2). 20.6.

(1,-2,3), (1,-3,2), (2,-1,3), (2,-3,1), (3,-1,2), (3,-2,1). 20.5. (1,64), (64,1). 20.6.

(16,81), (81,16), (-16,-81), (-81,-16). 21. 21.1. {1,4}. 21.2. {2,11}. 21.3.

{-8,-73}. 22.4.  $x = 0$ .

32-§. Maydon ustida ko'phadlar

2. 2.1.  $f(x) = (x-1)(x+1)(x^2-2)(2x-1)$ . 2.2.  $f(x) = (x^2-1)(x^2-4)(3x+1)$ .

3.  $f(x) = (x^2+1)(x^2-2)(2x-1) = (x^2+1)(x-\sqrt{2})(x+\sqrt{2})(2x-1) = (x-i)(x+i)(x-\sqrt{2})(x+\sqrt{2})(2x-1)$ .

6.  $f(x) = (x+\bar{1})(x+\bar{2})(x+\bar{3})(x+\bar{4})$ ,  $Z_3$ ;  $Q$  da keltirilmaydi,

$f(x) = (x^2 - \sqrt{2}x + 2)(x^2 + \sqrt{2}x + 2)$ ,  $R$ ;  $C$  da keltiriladi. 7. 7.1.

$f(x) = \left(x - \frac{3-\sqrt{5}}{2}\right)^2 \left(x - \frac{3+\sqrt{5}}{2}\right)^2$ . 7.2.  $f(x) = (x-1)(x-2)(x-3)$ .

7.3.  $f(x) = \left(x - \frac{-1-\sqrt{2}}{2}\right) \cdot \left(x - \frac{-1-\sqrt{2}}{2}\right) \cdot \left(x - \frac{-3+\sqrt{3}}{2}\right) \cdot \left(x - \frac{-3-\sqrt{3}}{2}\right)$ .

7.4.  $f(x) = (x-1-i)(x-1+i)(x+1-i)(x+1+i)$ .

7.5.  $f(x) = (x-1)^2(x-2)^2$ .

7.6.  $f(x) = (x-i\sqrt{3})(x+i\sqrt{3}) \left(x - \frac{3}{2} - \frac{\sqrt{3}}{2}i\right) \left(x + \frac{3}{2} - \frac{\sqrt{3}}{2}i\right) \left(x + \frac{3}{2} + \frac{\sqrt{3}}{2}i\right)$ .

7.7.  $f(x) = \left(x - \frac{-7-\sqrt{5}}{2}\right)^2 \left(x - \frac{-7+\sqrt{5}}{2}\right)^2$ .

7.8.  $\prod_{k=1}^{3n-1} \left(x - \cos \frac{2\pi k}{3n} - i \sin \frac{2\pi k}{3n}\right)$ ,  $(k,3)=1$ . 8. 8.1.

$(f,g) = x^2 - x - 2$ ;  $[f,g] = (x-1)^2(x^2+1)(x^2-5x+6)$ .

8.2.  $(f,g) = x-1$ ;  $[f,g] = (x^2-2x+3)^2(x+6)^2(x-1)^2(x-2)^2(x-6)^2(x^2+x+1)$ .

8.3.  $(f,g) = (x+1)^2$ ;  $[f,g] = (x+1)^3(x-1)(x-2)$ .

8.4.  $(f,g) = x + \bar{2}$ ;  $[f,g] = x(x+\bar{1})(x+\bar{3})(x+\bar{4})(x^2+x+\bar{1})^2$ .

8.5.  $(f,g) = x^{(m,n)} - 1$ . 8.6. Agar  $\frac{m}{(m,n)}, \frac{n}{(m,n)}$  lar toq son bo'lsa,

$(f,g) = x^{(m,n)} + 1$ , qolgan hollarda  $(f,g) = 1$ .

9.9.1.  $f'(x) = 3(x^2+x-1)^2(2x+1)(x^3-2) + 3x^2(x^2+x-1)$ ;

9.2.  $f'(x) = x(x+3) + 3x^2$ . 10.  $f(x) = 2x^6 + x^3 + x^2 + x + \bar{2}$ . 11.

$f(x) = 4x^3 + x^2 - x + 1$ . 12.  $f_1(x) = \bar{1}$ ,  $f_2(x) = x^2$ ,  $f_3(x) = x^2 + \bar{1}$ ,

$f_4(x) = x^3 + x^2 + x$  lardan tashqari barcha ko'phadlar. 13. 13.1.

$f(x) = i(x-2i)^4 + (-7-i)(x-2i)^3 + (4-19i)(x-2i)^2 + (27+4i)(x-2i) - 3 + 14i$  va

$f'(2i) = 27 + 4i$ ;  $f''(2i) = 8 - 38i$ ;  $f'''(2i) = -42 - 6i$ ;  $f^{IV}(2i) = 24i$ .

13.2.  $f(x) = (x-i)^5 + 5i(x+i)^4 - (3i+10)(x+i)^3 + (10i-13)(x+i)^2 +$

$+ (5+22i)(x+i) + 11 - i$ ; va  $f'(-i) = 22i + 5$ ;  $f''(-i) = 20; -26$ ;

$f^{IV}(-i) = -120i$ ;  $f^V(-i) = 120$ . 13.3.

$f^{III}(-i) = -60 - 18i$ ;

$f(x) = (x+1)^4 - 4(x+1)^3 - 9(x+1)^2 + 36(x+1) + 1$

$f'(-1) = 36$ ;  $f''(-1) = -18$ ;  $f'''(-1) = -24$ ;  $f^{IV}(-1) = 24$ .

13.4.  $f(x) = 2(x-1)^4 + (x-\bar{1})^2 + (x-\bar{1})$ ,  $f'(\bar{1}) = \bar{1}$ ;  $f''(\bar{1}) = \bar{2}$ ;  $f'''(\bar{1}) = \bar{0}$ ;

$f^{IV}(\bar{1}) = \bar{0}$ ; 13.5.  $f(x) = (x-2)^4 - 18(x-2) + 38$ ;  $f'(2) = -18$ ,

$f''(2) = f'''(2) = 0$ ,  $f^{IV}(2) = 24$ . 13.6.

$f(x) = (x-2)^5 + 10(x-2)^4 + 36(x-2)^3 + 62(x-2)^2 + 48(x-2) + 18$ ;

$f'(2) = 48$ ,  $f''(2) = 124$ ,  $f'''(2) = 216$ ,  $f^{IV}(2) = 240$ ,  $f^V(2) = 120$ . 14. 14.1.

2. 14.2. 1. 14.3. 0. 14.4. 3. 14.5. 3. 15. 15.1.  $b=0$  da  $\alpha=0$ ;  $b=-6\sqrt{3}$  da  $\alpha=-\sqrt{3}$ ;  $b=6\sqrt{3}$  da  $\alpha=\sqrt{3}$ . 15.2.  $b \in \left\{-\frac{14}{27}, 18\right\}$ . 15.3.  $b=0$ . 15.4.  $b=4$  da  $\alpha=-2$ ;  $b=\frac{102}{27}$  da  $\alpha=-\frac{4}{3}$ . 16. 16.1.  $27a^4 = 256b^3$ . 16.2.  $3125b^2 + 108a^5 = 0$ .

$$17. \quad 17.1. \quad f(x) = (x^2 + x + 1)^2(x + 2). \quad 17.2. \quad f(x) = (x + i)^3(x - 2i)^2. \quad 17.3.$$

$$f(x) = (x + 2)^4(x^2 - x - i). \quad 18. \quad 18.1. \quad f(x) = (x - 1)^2(x - 2)(x - 3)(x - 1 - i). \quad 18.2.$$

$$f(x) = (x - i)^2(x + 1 + i). \quad 19. \quad 19.1. \quad \frac{x - 1}{x - 2}. \quad 19.2. \quad (x^2 - x + 1)(x^4 - x^2 + 1). \quad 20. \quad 20.1.$$

$$\frac{f(x)}{g(x)} = -\frac{2}{3(x + 1)} + \frac{2x^2 - 2x + 5}{3(x^3 - 2)}; \quad 20.2. \quad \frac{f(x)}{g(x)} = -\frac{1}{2x} + \frac{x^2}{2(x^3 - 2)}; \quad 20.3.$$

$$\frac{f(x)}{g(x)} = \frac{1}{2(x^2 - 2)} + \frac{1}{2(x^3 + 2)}; \quad 20.4. \quad \frac{f(x)}{g(x)} = \frac{1}{x} - \frac{x}{x^2 + 1}. \quad 21. \quad 21.1.$$

$$\frac{f(x)}{g(x)} = \frac{x + 1}{x^2 + 1} - \frac{x + 2}{x^2 + x + 1}; \quad 21.2. \quad \frac{f(x)}{g(x)} = \frac{6}{x^3} - \frac{4}{x^2} + \frac{1}{x + 3} + \frac{3}{(x + 3)^2};$$

$$21.3. \quad \frac{f(x)}{g(x)} = \frac{x}{8(x^2 + 2x + 2)} - \frac{x - 4}{8(x^2 - 2x + 2)};$$

$$21.4. \quad \frac{f(x)}{g(x)} = \frac{\sqrt{2}}{8(x - \sqrt{2})} - \frac{\sqrt{2}}{8(x + \sqrt{2})} + \frac{1}{2(x^2 + 2)};$$

$$21.5. \quad \frac{f(x)}{g(x)} = -\frac{2}{3(x + 1)} + \frac{\frac{3}{2} + \frac{5\sqrt[3]{2}}{12} - \frac{1\sqrt[3]{4}}{3}}{x - \sqrt[3]{2}} + \frac{\left(\frac{4}{3} - \frac{5\sqrt[3]{2}}{12} + \frac{1\sqrt[3]{4}}{3}\right)x - \frac{3}{2} + \frac{2\sqrt[3]{2}}{3} - \frac{5\sqrt[3]{4}}{3}}{x^2 + \sqrt[3]{2}x + \sqrt[3]{4}};$$

$$21.6. \quad \frac{f(x)}{g(x)} = \frac{1}{x^2 + x + 2} - \frac{x - 2}{(x^2 + x + 2)^2};$$

$$21.7. \quad \frac{f(x)}{g(x)} = \frac{1}{8(x - 2)} - \frac{1}{8(x + 2)} + \frac{1}{2(x^2 + 4)};$$

$$21.8. \quad \frac{f(x)}{g(x)} = \frac{1}{8} \left( \frac{x + 2}{x^2 + 2x + 2} - \frac{x - 2}{x^2 - 2x + 2} \right); \quad 22. \quad 22.1.$$

$$\frac{f(x)}{g(x)} = \frac{1}{12(x - 1)} - \frac{4}{3(x + 2)} + \frac{9}{4(x + 3)};$$

$$22.2. \quad \frac{f(x)}{g(x)} = -\frac{1}{16} \left( \frac{1 + i}{x - 1 - i} + \frac{1 - i}{x - 1 + i} + \frac{-1 + i}{x + 1 - i} + \frac{-1 + i}{x + 1 + i} \right);$$

$$22.3. \quad \frac{f(x)}{g(x)} = \frac{3}{(x - 1)^3} - \frac{4}{(x - 1)^2} - \frac{1}{x - 1} - \frac{1}{(x + 1)^2} - \frac{2}{x + 1} + \frac{1}{x - 2};$$

$$22.4. \quad \frac{f(x)}{g(x)} = \frac{1}{3(x - i)} - \frac{1}{3(x + 2i)};$$

$$22.5. \quad \frac{f(x)}{g(x)} = \frac{1}{x - 1} - \frac{1 + i}{2(x - i)} + \frac{i - 1}{2(x + i)};$$

$$22.6. \quad \frac{f(x)}{g(x)} = \frac{\sqrt{2}}{8(x - \sqrt{2})} - \frac{\sqrt{2}}{8(x + \sqrt{2})} + \frac{\sqrt{2}}{8(x - \sqrt{2}i)} - \frac{\sqrt{2}}{8(x + \sqrt{2}i)};$$

$$23. \quad \frac{f(x)}{g(x)} = \sum_{\alpha=0}^{p-1} \frac{i}{(p-1)(x + \alpha)}$$

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## MUNDARIJA

|  |     |
|--|-----|
| IX MODUL. CHIZIQLI AKSLANTIRISHLAR.....                              | 3   |
| 21-§. Chiziqli akslantirish. Chiziqli operator yadrosi va obrazi.    | 3   |
| Chiziqli operator matritsasi. ....                                   | 13  |
| 22-§. Chiziqli operatorlar ustida amallar. Chiziqli algebralar. .... | 23  |
| Teskari operator. Xos vektorlar va xos qiymatlar.....                | 23  |
| X MODUL. CHIZIQLI TENGSIZLIKLAR SISTEMASI.....                       | 35  |
| 23-§. Chiziqli tengsizliklar sistemasi. Qavariq konus.....           | 35  |
| XI MODUL. BUTUN SONLAR HALQASIDA BO'LINISH                           | 35  |
| MUNOSABATI.....  | 46  |
| 24-§. Tub va murakkab sonlar. EKUB. EKUK.....                        | 53  |
| 25-§. Chekli zanjir kasrlar. Munosib kasrlar.....                    | 61  |
| 26-§. Sistematik sonlar va ular ustida amallar.....                  | 61  |
| XII MODUL. TAQQOSLAMALAR.....  | 70  |
| 27-§. Butun sonlar halqasida taqqoslamalar. Eyler va Ferma           | 70  |
| teoremlari.....  | 88  |
| 28-§. Birinchi darajali va tub modul bo'yicha yuqori darajali        | 88  |
| taqqoslamalar.....   | 101 |
| 29-§. Tub modul bo'yicha boshlang'ich ildizlar va indekslar.....     | 101 |
| XIII MODUL. KO'PHADLAR.....  | 113 |
| 30-§. Bir o'zgaruvchili ko'phadlar.....                              | 123 |
| 31-§. Ko'p o'zgaruvchili ko'phadlar.....                             | 138 |
| J A V O B L A R .....  | 162 |
| ADABIYOTLAR.....   | 162 |

D.YUNUSOVA, A.YUNUSOV

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