D. YUNUSOVA,
A. YUNUSOV

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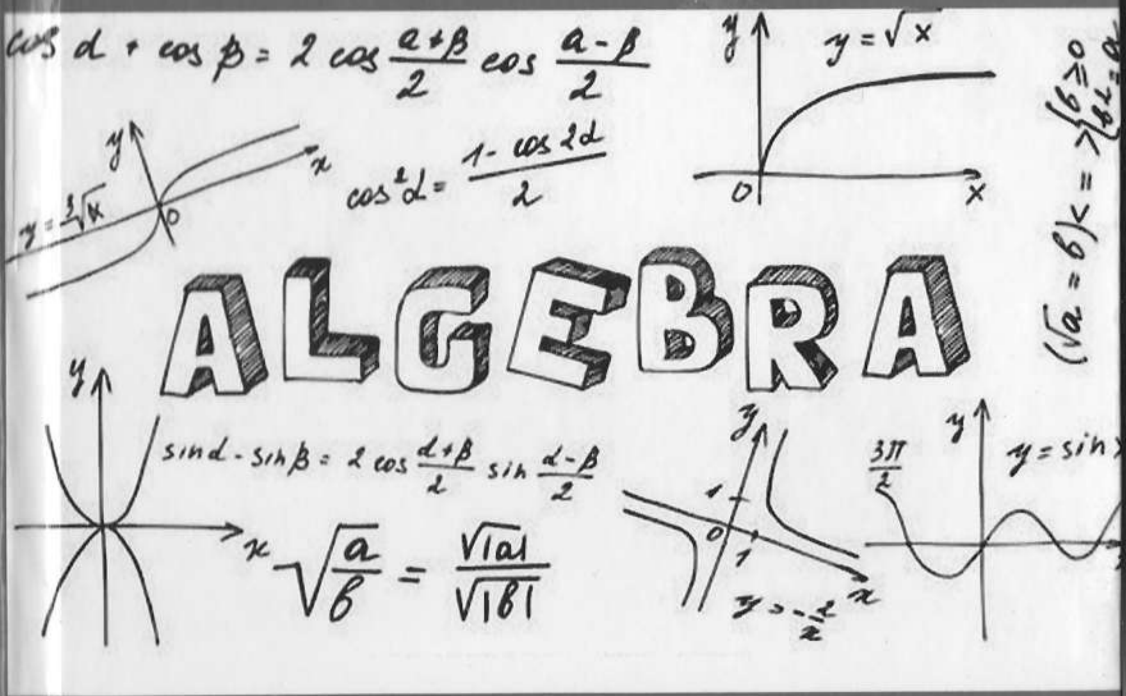

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D.YUNUSOVA, A.YUNUSOV

ALGEBRA VA SONLAR NAZARIYASI MODUL TEXNOLOGIYASI ASOSIDA TAYYORLANGAN MISOL VA MASHQLAR TO'PLAMI

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O'quv qo'llanma «Algebra va sonlar nazariyasi» fani davlat ta'lim standarti hamda dasturiga to'la mos keladi. Tavsiya etilayotgan nazariy savollar va amaliy topshiriqlar modullarda jamlangan bo'lib, ularda keltirilgan metodik tavsiyalar ushbu o'quv qo'llanmadan foydalanuvchilarga keng imkoniyatlar yaratadi.

Keltirilgan misol va mashqlar nazariy bilimlarni chuqurlashtirish, tadbiqini kengaytirishga qaratilgan bo'lib, talabalarni mustaqil, ijodiy izlanishga yo'naltiradi.

Umumiy o'rta, o'rta maxsus matematika t'limiga ushbu fanning keng tatbiqlarini e'tiborga olsak, o'quv qo'llanmadan akademik litsey, kasb-hunar kollejlari o'qituvchilari, o'quvchilari; o'qituvchilar malakasini oshirish va qayta tayyorlash kurslari tinglovchilari foydalanishlari mumkin.

Данное учебное пособие соответствует учебной программе дисциплины «Алгебра и теория чисел» педагогических высших учебных заведений.

Примеры и задачи собраны в модулях, что создает некоторые удобства пользователям. Собранные в учебнике задачи углубляют и расширяют теоретичекие знания студентов.

Учитывая широкое применение курса «Алгебра и теория чисел» в математике школ и лицеев, данное учебное пособие рекомендуем преподавателям школ, лицеев, слушателям курсов повышения квалификации и переподготовки педагогических кадров.

The given manual meets to the curriculum of discipline « Algebra and the theory of numbers» pedagogical higher educational institutions. Examples and tasks are collected in modules that creates some convenience to users. The problems{tasks} collected in the textbook deepen and expand knowledge of students.

Taking into account wide application of a rate « Algebra and the theory of numbers » in mathematics of schools and the licea, the given manual we recommend teachers of schools, licea, students of courses of improvement of qualification and retrainings of the pedagogical staff.

Taqrizchilar: t.f.d., professor N.Sherboyev, f.-m.f.n., dotsent A.Dusumbetov

O`zbekiston Respublikasi Oliy va o'rta maxsus ta'lim vazirligi tomonidan nashrga tavsiya etilgan.

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I MODUL. MATEMATIK MANTIQ ELEMENTLARI

1-§. Mulohaza. Mulohazalar ustida mantiq amallari

Asosiy tushunchalar: mulohaza, rost mulohaza, yolg'on mulohaza, kon'yunksiya, diz'yunksiya, implikasiya, ekvivalensiya, inkor, rostlik jadvali.

Mulohaza matematik mantiqning asosiy tushunchalaridan bo'lib, u rost yoki yolg'onligi bir qiymatli aniqlanadigan darak gapdir. Masalan, «Kvadrat to'g'ri to'rtburchakdir», «2>5» kabi tasdiqlar mulohazalar bo'lib, birinchi mulohaza rost, ikkinchi mulohaza esa yolg'on mulohazadir.

Berilgan A mulohaza rost bo'lganda yolg'on, A mulohaza yolg'on bo'lganda rost bo'ladigan mulohaza A mulohazaning inkori deyiladi va $\rceil A$ yoki $\bar{A}$ orqali belgilanadi.

A va B mulohazalar rost bo'lgandagina rost bo'lib, qolgan hollarda yolg'on bo'ladigan mulohaza A va B mulohazalarning kon'yunksiyasi deyiladi va $A \wedge B$ yoki $A \,\&\, B$ ko'rinishda belgilanadi

A va B mulohazalar diz'yunksiyasi deb, A va B mulohazalarning ikkalasi ham yolg'on bo'lgandagina yolg'on, qolgan hollarda rost bo'ladigan $A \vee B$ mulohazaga aytiladi.

A va B mulohazalar implikasiyasi deb, A mulohaza rost va B mulohaza yolg'on bo'lgandagina yolg'on, qolgan hollarda rost bo'ladigan $A \Rightarrow B$ mulohazaga aytiladi.

A va B mulohazalar ekvivalensiyasi deb, A va B mulohazalarning ikkalasi ham yolg'on yoki rost bo'lganda rost, qolgan hollarda yolg'on bo'ladigan $A \Leftrightarrow B$ mulohazaga aytiladi

Yuqorida ta'riflangan amallar rostlik jadvali quyidagi ko'rinishda bo'ladi:

A	B	$\rceil$A	A $\wedge$ B	A $\vee$ B	A $\Rightarrow$ B	A $\Leftrightarrow$ B
1	1	0	1	1	1	1
1	0	0	0	1	0	0
0	1	1	0	1	1	0
0	0	1	0	0	1	1

Misol. $\forall(x,y,z \in Z)(x\vdots y \wedge y\vdots z \Rightarrow x\vdots z)$ mulohazaning rost yoki yolg'onligini aniqlang.

Yechish. Berilgan mulohaza kon'yunksiya hamda implikatsiya amallari yordamida hosil qilingan. Bu mantiq amallarining ta'riflariga ko'ra qaralayotgan $x\vdots y \wedge y\vdots z \Rightarrow x\vdots z$ mulohaza $x\vdots y \wedge y\vdots z$ rost va $x\vdots z$ yolg'on bo'lganda yolg'on, boshqa hollarda rost. Har bir mulohazaning rostlik qiymatini aniqlaymiz.

$x\vdots y$ predikat butun sonlar to'plamida olingan har qanday (x,y) juftlikda rost mulohaza bo'lmaydi. Masalan, $x=1, y=2$.

$y\vdots z$ predikat butun sonlar to'plamida olingan har qanday (y,z) juftlikda rost mulohaza bo'lmaydi. Masalan, $y=2, z=3$.

$x\vdots z$ predikat butun sonlar to'plamida olingan har qanday (x,z) juftlikda rost mulohaza bo'lmaydi. Masalan, $x=1, z=3$.

Quyidagi holatlarni qarab chiqamiz:

1) $\forall(x,y,z \in N)(x\vdots y \wedge y\vdots z \Rightarrow x\vdots z)$ mulohazadagi $\forall(x,y,z \in N)(x\vdots y)$ mulohaza yolg'on. U holda kon'yunksiya va implikatsiya amallari ta'rifiga ko'ra $\forall(x,y,z \in N)(x\vdots y \wedge y\vdots z \Rightarrow x\vdots z)$ mulohaza rost.

2) $\forall(x,y,z \in N)(x\vdots y \wedge y\vdots z \Rightarrow x\vdots z)$ mulohazadagi $\forall(x,y,z \in N)(y\vdots z)$ mulohaza yolg'on. U holda kon'yunksiya va implikatsiya amallari ta'rifiga ko'ra $\forall(x,y,z \in N)(x\vdots y \wedge y\vdots z \Rightarrow x\vdots z)$ mulohaza rost.

3) $\forall(x,y,z \in N)(x\vdots y \wedge y\vdots z \Rightarrow x\vdots z)$ mulohazadagi $\forall(x,y,z \in N)(x\vdots z)$, $\forall(x,y,z \in N)(y\vdots z)$ yolg'on. U holda kon'yunksiya va implikatsiya amallari ta'rifiga ko'ra $\forall(x,y,z \in N)(x\vdots y \wedge y\vdots z \Rightarrow x\vdots z)$ mulohaza rost.

4) $\forall(x,y,z \in N)(x\vdots y \wedge y\vdots z)$ rost bo'lsa $\forall(x,y,z \in N)(x\vdots y)$ va $\forall(x,y,z \in N)(y\vdots z)$ lar bir vaqtda rost. $x\vdots y$ va $y\vdots z$ bo'lsa, u holda shunday $k,l \in N$ sonlar topiladiki, $x = y\cdot k$ va $y = z\cdot 1$. Bunday $x = y\cdot k = (z\cdot 1)\cdot k = z\cdot(1\cdot k)$. Demak $x\vdots z$ implikatsiya ta'rifiga ko'ra, bu holda ham berilgan $\forall(x,y,z \in N)(x\vdots y \wedge y\vdots z \Rightarrow x\vdots z)$ mulohaza rost.

Demak, berilgan mulohaza rost mulohaza.

Misol va mashqlar

1. Quyidagi gaplarning qaysilari mulohaza bo'ladi?

1.1. ABCD to'rtburchakning yuzi A'B'C'D' to'rtburchak yuziga teng.

1.2. Tomonlari teng parallelogramm rombdir.

1.3. Berilgan uchburchaklar o'xshash.

1.4. Har qanday tub son toq.

1.5. $\sqrt{3}$ - irrasional son.

1.6. Yashasin O'zbekiston yoshlari!

1.7. 2 ga qarama-qarshi son mavjud emas.

1.8. 5ning butun bo'luvchilari 4 ta.

1.9. -1 kompleks son.

1.10. 6 3ga karrali son.

1.11. Oyda hayot mavjud.

1.12. Ertaga qor yog'adi.

1.13. Guruhdagi talabalar soni 20 nafar.

1.14. Sirdaryo Orol dengiziga quyiladi.

1.15. Siz qaysi oliygohda o'qiysiz?

1.16. O'zbekiston Mustaqilligining 15 yilligi muborak bo'lsin!

1.17. Har qanday son musbat.

1.18. 0 har qanday haqiqiy songa bo'linadi.

1.19. 2, 3, 5 sonlari tub sonlar.

1.20. Barcha insonlar yoshi 20 da.

1.21. Galaktikamizda shunday sayyora bor-ki, unda hayot mavjud.

1.22. 5 soni 25 va 70 sonlarining eng katta umumiy bo'luvchisi.

1.23. $3x^3 - 5y + 9$.

2. Mulhazaning rost yoki yolg'onligini aniqlang:

2.1. $2 \in \{ x | 2x^3 - 3x^2+1=0, x\in R\}$.

2.2. 1966 yil Toshkentda er qimirlagan.

2.3. 8-mart dam olish kuni.

2.4. $3 \in \{ n| \frac{2n+1}{3n-2}, n \in N\}$.

2.5. $\{1; 1,2\} \subset \{ x | x^3 + x^2 - x - 1 = 0, x \in Z\}$.

2.6. $2 \leq 3$.

2.7. 10 ning natural bo'luvchilari 2 ta.

2.8. $2 \cdot 2 \leq 4$.

2.9. $[4,12,24]=24$.

2.10. Gipotenuza to'g'ri burchakli uchburchakning eng uzun tomoni.

3. Quyidagi mulohazalarning inkorini ifodalang:

3.1. 15 soni 5songa bo'linadi.

3.2. Oy Erning yo'ldoshi.

3.3. $2>3$.

3.4. $5+3<10$.

3.5. i -mavhum son.

3.6. ABCD to'rtburchak – romb.

3.7. n – juft natural son.

3.8. Shunday haqiqiy son mavjudki, u juft son.

3.9. Barcha natural sonlar musbat.

3.10. Barcha natural sonlar birdan katta.

4. Biri ikkinchisining inkori bo'lgan mulohazalar juftligini aniqlang:

4.1. $2<3$; $3<2$.

4.2. $5 \leq 4$; $5>4$.

4.3. «4-murakkab son», «4-tub son».

4.4. «Shunday natural son mavjudki, u tub son», «Barcha natural sonlar murakkab».

4.5. «6 ning barcha natural bo'luvchilari tub sonlar», «6ning kamida bitta natural bo'luvchisi murakkab son».

4.6. «ABC to'g'ri burchakli uchburchak», «ABC o'tmas burchakli uchburchak».

4.7. «f- funksiya – toq» , «f – funksiya – juft».

4.8. «Barcha tub sonlar toq» , «Shunday tub son mavjud-ki, u juft».

4.9. «Irratsional sonlar mavjud», «Barcha sonlar rasional».

5. Quyidagi mulohazalarning rostlik shartlarini mulohazalar diz'yunksiyasi yoki kon'yunksiyasi orqali ifodalang:

5.1. $x \cdot y \neq 0$.

5.2. $x \cdot y = 0$.

5.3. $x^2 + y^2 = 0$.

5.4. $\frac{x}{y} = 0$.

5.5. $|x| < 6$.

5.6. $|x| = 4$.

6. Quyidagi mulohazalarning rostlik qiymatlarini aniqlang:

6.1. Har qanday natural son yo tub, yo murakkab.

6.2. Shunday natural son mavjudki u ham tub, ham juft son.

6.3. 2 ga teng bo'lmagan son yoki 2dan katta yoki 2dan kichik bo'ladi.

6.4. Agar uchburchak teng tomonli bo'lsa, u teng yonli bo'ladi.

6.5. Agar to'rtburchak romb bo'lsa, u kvadrat bo'ladi.

6.6. Agar $15 \vdots 5$, u holda $15 \vdots 4$.

6.7. Agar $x^2 = 4$ bo'lsa, u holda $x = 2$ va $x = -2$.

6.8. Natural son 6 ga bo'linadi faqat va faqat shu holdaki, agar u 2 ga va 3ga bo'linsa.

6.9. Ikkita uchburchak teng bo'ladi faqat va faqat shu holdaki, agar ularning mos tomonlari teng bo'lsa yoki ularning mos burchaklari teng bo'lsa.

6.10. Berilgan butun sonning butun bo'luvchilari kamida to'rtta bo'lsa, u murakkab son bo'ladi.

7. A orqali «10 soni 5 soniga bo'linadi», B orqali «10 soni 3 soniga bo'linadi» mulohazalar belgilangan bo'lsa, u holda quyidagi mulohazalarni o'qing va ularning rostlik qiymatlarini aniqlang:

7.1. $A \wedge B$.

7.2. $A \vee B$.

7.3. $\neg A \wedge B$.

7.4. $A \wedge \neg B$.

7.5. $\neg A \wedge \neg B$.

7.6. $A \Rightarrow B$.

7.7. $B \Rightarrow A$.

7.8. $\neg A \Rightarrow B$.

7.9. $\neg B \Rightarrow A$.

7.10. $A \Leftrightarrow B$.

7.11. $\neg A \Leftrightarrow \neg B$.

7.12. $\neg B \Leftrightarrow \neg A$.

8. Quyidagi mulohazalarni sodda mulohazalarga ajrating. Sodda mulohazalarni harflar yordamida belgilab, berilgan mulohazalarni ular yordamida ifodalang:

8.1. Agar berilgan funksiya juft ham toq ham emas bo'lsa, u holda u yoki juft funksiya yoki toq funksiya bo'ladi.

8.2. Agar berilgan son 3 ga bo'linsa va 5 ga bo'linmasa, u holda bu son 15 ga bo'linmaydi.

8.3. Ketma-ket kelgan uchta natural sonlarning kamida bittasi toq son bo'ladi.

8.4. Har qanday natural sonni 3 ga bo'lganda yoki 0 yoki 1 yoki 2 qoldiq qoladi.

9. Bir vaqtda $A \wedge B$-rost, $A \wedge C$-yolg'on, $(A \wedge B) \wedge \neg C$ – yolg'on bo'luvchi A, B, C mulohazalar mavjudmi?

10. Berilgan shartlar asosida quyidagi mulohazalarning rostlik qiymatini aniqlash mumkin-mi? Agar mumkin bo'lsa, mulohazaning rostlik qiymatini aniqlang.

10.1. $(A \Rightarrow B) \Rightarrow C$, C-rost mulohaza.

10.2. $A \wedge (B \Rightarrow C)$, $(B \Rightarrow C)$- yolg'on mulohaza.

10.3. $A \vee (B \Rightarrow C)$, B-yolg'on mulohaza.

10.4. $\neg(A \vee B) \Leftrightarrow (\neg A \wedge \neg B)$, A-rost mulohaza.

10.5. $(A \Rightarrow B) \Rightarrow (\neg B \Rightarrow \neg A)$, B-rost mulohaza.

10.6. $(A \wedge B) \Rightarrow (A \vee C)$, A-yolg'on mulohaza.

Takrorlash uchun savollar

1. Mulohaza deb qanday gapga aytiladi? Har qanday o'tgan zamon darak gapi mulohaza bo'la oladimi? Kelasi zamon darak gaplari-chi?

2. Mulohazalar kon'yunksiyasi nima? Qanday o'qiladi? Rost kon'yunksiyaga, yolg'on kon'yunksiyaga misollar keltiring.

3. Mulohazalar diz'yunksiyasi nima? Qanday o'qiladi? Rost diz'yunksiyaga, yolg'on diz'yunksiyaga misollar keltiring.

4. Mulohazalar implikasiyasi nima? Qanday o'qiladi? Rost implikasiya, yolg'on implikasiyaga misollar keltiring.

5. Mulohazalar ekvivalensiyasi nima? Qanday o'qiladi? Rost ekvivalensiyaga, yolg'on ekvivalensiyaga misollar keltiring.

6. Mulohaza inkori nima? Qanday o'qiladi? Rost inkorga, yolg'on inkorgaga misollar keltiring.

7. Mantiqiy amallarning bajarilish tartibini ayting.

8. Rostlik jadvali nima?

2-§. Formula. Teng kuchli formulalar. Mantiq qonunlari

Asosiy tushunchalar: mulohazaviy formula, rostlik qiymatlar tizimi, formulaning rostlik jadvali, teng kuchli formulalar, aynan rost formula, tavtologiya, mantiq qonuni, aynan yolg'on formula, ziddiyat, bajariluvchi formula.

1) Har qanday mulohaza formuladir.

2) Agar A, B lar formula bo'lsa, u holda

($\rceil$A), (A $\wedge$ B), (A $\vee$ B), (A $\Rightarrow$ B), (A $\Leftrightarrow$ B) lar ham formuladir.

A formula faqat $A_1,\ldots, A_n$ mulohazalardan hosil qilingan bo'lsin, u holda A formulani A ($A_1,\ldots, A_n$) ko'rinishida yozib olamiz va $A_1\ldots A_n$ – mulohazalarni elementar mulohazalar deymiz. Har bir A_k $(k=\overline{1,n})$ mulohaza 0 yoki 1 qiymatlarni qabul qilishi mumkin. A_k mulohazaning qabul qiladigan qiymati i_k bo'lsin, u holda $(i_1,\ldots,i_n)$ - n lik $A_1,\ldots, A_n$ – mulohazalarning qabul qiladigan qiymatlari tizimi deyiladi.

A va B formulalar tarkibiga kirgan barcha mulohazalar $A_1,\ldots, A_n$ lardan iborat bo'lsin. Agar $A_1,\ldots, A_n$ mulohazalarning barcha $(i_1,\ldots,i_n)$ qiymatlari tizimida A va B formulalar bir xil qiymatlar qabul qilsalar, u holda bu formulalar teng kuchli formulalar deyiladi va $\mathcal{A}\equiv\mathcal{B}$ ko'rinishida belgilanadi.

Formulada qatnashgan mantiq amallari soni formulaning rangi deyiladi.

1. A formula - A mulohazadan iborat bo'lsa, uning formulaosti faqat uning o'zidan iborat.

2. Agar formulaning ko'rinishi A $*$ B dan iborat bo'lsa, u holda uning formulaostilari A, B, A $*$ B lar hamda A va B larning barcha formulaostilaridan iborat bo'ladi. Bu erda $*$ - $\wedge$, $\vee$, $\Rightarrow$, $\Leftrightarrow$ amallaridan biri.

Agar formulaning ko'rinishi $\rceil$ A bo'lsa, uning formulaostilari A formula, A formulaning barcha formulaostilari va $\rceil$ A ning o'zidan iborat.

A formula, shu formula tarkibiga kirgan barcha mulohazalarning qabul qilishi mumkin bo'lgan barcha qiymatlari tizimida rost bo'lsa, bu formula aynan rost formula yoki mantiq qonun yoki tovtologiya; mulohazalarning kamida bitta

qiymatlari tizimida rost qiymat qabul qilsa, bajariluvchi formula; barcha qiymatlari tizimida yolg'on qiymat qabul qilsa, aynan yolg'on formula yoki ziddiyat deyiladi.

Misol. (A $\wedge$ B$\Rightarrow$A $\wedge$ C) formulaning turini aniqlang.

Yechish. Berilgan formulada uchta A, B, C mulohazalar qatnashganligi sababli, ularning qiymatlar tizimlari 2^3 = 8 ta bo'ladi. Formulaning rostlik jadvaliga 8 ta tizimni tartib bilan joylashtiramiz. Mantiq amallarining bajarilish tartibiga ko'ra avval A $\wedge$ B kon'yunksiyani, keyin A $\vee$ C diz'yunksiyani va nihoyat hosil qilingan formulalarning implikasiyasini bajaramiz. Ya'ni amallarning ta'riflariga ko'ra mos ustunlarni to'ldiramiz. Natijada quyidagi rostlik jadvali xosil bo'ladi:

A	B	C	A $\wedge$ B	A $\vee$ C	A $\wedge$ B $\rightarrow$ A $\vee$ C
1	1	1	1	1	1
1	1	0	1	1	1
1	0	1	0	1	1
1	0	0	0	1	1
0	1	1	0	1	1
0	1	0	0	0	1
0	0	1	0	1	1
0	0	0	0	0	1

Formulaning rostlik jadvalidagi oxirgi ustun - formulaning rostlik qiymatlar ustuni faqat rost qiymatlardan iborat bo'lganligi uchun berilgan formula aynan rost (tavtologiya, mantiq qonuni) degan xulosaga kelamiz.

Misol. Berilgan $\rceil$ (A $\wedge$ B), $\rceil$ A $\vee$ $\rceil$ B formulalar tengkuchli ekanligini isbotlang.

Yechish. Berilgan formulalar teng kuchli ekanligini isbotlash uchun rostlik jadvallari tuzamiz:

A	B	$A \wedge B$	$\rceil(A \wedge B)$
1	1	1	0
1	0	0	1
0	1	0	1
0	0	0	1

A	B	$\rceil A$	$\rceil B$	$\rceil A \vee \rceil B$
1	1	0	0	0
1	0	0	1	1
0	1	1	0	1
0	0	1	1	1

Formulalarning rostlik jadvallaridagi formulalar rostlik qiymatlari ustunlari mos tizimlarda bir hil ekanligidan berilgan formulalarning teng kuchli ekanligi kelib chiqadi.

Formulalarning teng kuchli ekanligini isbotlash uchun bitta rostlik jadvalini tuzish ham mumkin:

A	B	$A \wedge B$	$\rceil(A \wedge B)$	$\rceil A$	$\rceil B$	$\rceil A \vee \rceil b$
1	1	1	0	0	0	0
1	0	0	1	0	1	1
0	1	0	1	1	0	1
0	0	0	1	1	1	1

hosil bo'lgan 4- va 7- ustunlardagi rostlik qiymatlarini solishtirib, berilgan formulalarning teng kuchli ekanligiga ishonch hosil qilamiz.

Misol. $A \Leftrightarrow B \equiv A \wedge B \vee A \wedge B$ tengkuchlilikni asosiy teng kuchliliklar yordamida isbotlang.

Yechish. Asosiy teng kuchliliklardan foydalanib quyidagi teng kuchli formulalar ketma – ketligini hosil qilamiz:

$$A \Leftrightarrow B \equiv (A \Rightarrow B) \wedge (B \Rightarrow A) \equiv (\rceil A \vee B) \wedge (\rceil B \vee A) \equiv$$
$$\equiv ((\rceil A \vee B) \wedge \rceil B) \vee (\rceil A \vee B) \wedge A) \equiv (\rceil A \wedge \rceil B) \vee$$
$$\vee (B \wedge \rceil B) \vee (\rceil A \wedge A) \vee (B \wedge A) \equiv (\rceil A \wedge \rceil B) \vee 0 \vee$$
$$\vee 0 \vee (B \wedge A) \equiv (\rceil A \wedge \rceil B) \vee (B \wedge A).$$

Misol va mashqlar

1. Quyidagi ifodalarning qaysilari mulohazaviy formula bo'ladi?

1.1. $A(B \Rightarrow C)$.

1.2. $((A \vee (B \wedge C)) \Leftrightarrow (\rceil D))$

1.3. $A \Rightarrow (B \wedge D \Rightarrow C)$.

1.4. $(A \vee (B \wedge C)) \Rightarrow D$.

1.5. $(A \Leftrightarrow D \vee (B \wedge C)) \Rightarrow D \wedge \rceil A$.

1.6. $(((A \Leftrightarrow (\rceil D)) \vee (B \wedge C)) \Rightarrow (D \wedge (\rceil A)))$.

2. Quyidagi ifodalarga qavslarni turli hil joylashtirish yordamida mulohazaviy formulalar hosil qiling:

2.1. $A \wedge B \Rightarrow C$.

2.2. $A \Rightarrow B \wedge C \Rightarrow \rceil C$.

2.3. $\rceil A \Leftrightarrow \rceil B \vee C \wedge B$.

2.4. $\rceil A \wedge B \Rightarrow C$.

Berilgan formulalarning barcha qism formulalarini aniqlang:

3.1. $(((A \Leftrightarrow B) \wedge (\rceil C)) \Rightarrow (((A \vee B) \Rightarrow A) \Rightarrow (\rceil C)))$.

3.2. $(((A \vee B) \vee (\rceil C)) \wedge ((\rceil A) \vee ((\rceil B) \vee C)))$.

3.3. $(((\rceil A) \Leftrightarrow (\rceil B)) \vee (C \wedge B))$.

3.4. $((\rceil(A \Leftrightarrow (\rceil(B \vee C)))) \wedge B)$.

3.5. $(((A \Rightarrow B) \wedge (C \Rightarrow A)) \vee (B \wedge (\rceil C)))$.

3.6. $((\neg((\neg A) \Leftrightarrow C) \wedge B)) \vee ((A \vee C) \Leftrightarrow C))$.

3.7. $(((A \Leftrightarrow C) \Rightarrow B) \vee ((\neg A) \wedge (\neg C)))$.

3.8. $(((B \Rightarrow (((\neg B) \vee A) \wedge C)) \Leftrightarrow (\neg A))$.

4. Quyidagi formulalarning turini aniqlang (formulalarning tashqi qavslari tushirib qoldirilgan):

4.1. $(\neg(X \vee Y) \Rightarrow \neg(X \wedge Y))$.

4.2. $(X \Rightarrow Y) \Rightarrow (\neg Y \Rightarrow \neg X)$.

4.3. $\neg(X \Rightarrow (Y \Rightarrow X) \wedge Z$.

4.4. $\neg X \Rightarrow (X \Rightarrow Y) \vee Z$.

4.5. $\neg(X \Rightarrow Y) \Rightarrow ((X \wedge Z) \Rightarrow (Y \wedge Z))$.

4.6. $(X \wedge Y) \Rightarrow Z \Leftrightarrow X \Rightarrow (Y \Rightarrow Z)$.

4.7. $(X \wedge Y) \Rightarrow Z \Leftrightarrow (X \wedge \neg Z) \Rightarrow \neg Y$.

4.8. $\neg(X \Rightarrow Y) \Leftrightarrow X \wedge \neg Y$.

4.9. $(X \Rightarrow Y) \wedge \neg Y \Rightarrow \neg X$.

4.10. $(X \Rightarrow Y) \Rightarrow (X \wedge Z \Rightarrow Y \wedge Z)$.

4.11. $(X \Rightarrow Y) \wedge (Z \Rightarrow T) \Rightarrow (X \wedge Z \Rightarrow Y \wedge T)$.

4.12. $\neg(X \Leftrightarrow Y) \Leftrightarrow (\neg(X \Rightarrow Y) \vee \neg(Y \Rightarrow X))$.

4.13. $(X \wedge Y) \Rightarrow (Z \wedge \neg Z \Rightarrow X \vee Z)$.

4.14. $(X \Leftrightarrow Y) \Leftrightarrow (X \Rightarrow Y) \wedge (Y \Rightarrow X)$.

5. 4-misolda keltirilgan formulalar ranglarini aniqlang.

6. Quyidagi formulalarning aynan rost ekanligini isbotlang:

6.1. $(A \Leftrightarrow B) \Leftrightarrow (\neg A \Leftrightarrow \neg B)$;

6.2. $(A \Rightarrow B) \Rightarrow ((A \Rightarrow (B \Rightarrow C)) \Rightarrow (A \Rightarrow C))$;

6.3. $(A \Rightarrow B) \Rightarrow ((B \Rightarrow A) \Rightarrow (A \Leftrightarrow B))$;

6.4. $(A \Rightarrow C) \Rightarrow ((A \vee B) \Rightarrow (C \vee B))$.

7. Quyidagi formulalarning aynan yolg'on ekanligini isbotlang:

7.1. $A \wedge (B \wedge (\neg A \vee \neg B))$;

7.2. $\neg(\neg(A \vee B) \Rightarrow \neg(A \wedge B))$;

7.3. $\neg(A \Rightarrow (B \Rightarrow A))$;

7.4. $\neg(A \Rightarrow C) \Rightarrow ((B \Rightarrow C) \Rightarrow (A \vee B \Rightarrow C))$;

7.5. $\neg(A \Rightarrow B) \Rightarrow ((A \wedge C) \Rightarrow (B \wedge A))$.

8. Quyidagi formulalarning qaysilari bajariluvchi ekanligini aniqlang:

8.1. $\neg(A \Rightarrow \neg A)$;

8.2. $(A \Rightarrow B) \Rightarrow (B \Rightarrow A)$;

8.3. $(B \Rightarrow (A \wedge C)) \wedge \neg((A \vee C) \Rightarrow B)$;

8.4. $\neg((A \Leftrightarrow \neg B) \vee C) \wedge B$;

8.5. $(A \wedge B) \Rightarrow ((C \vee B) \Rightarrow (B \wedge \neg B))$.

9. Rostlik jadvali yordamida quyidagi formulalar tavtologiya ekanligini isbotlang:

9.1. $(A \vee (\neg A))$ (uchinchisini inkor qilish qonuni).

9.2. $(\neg(A \wedge (\neg A)))$ (ziddiyatni inkor qilish qonuni).

9.3. $((\neg(\neg A)) \Leftrightarrow A)$ (qo'sh inkor qonuni).

9.4. $(A \Rightarrow A)$ (ayniyat qonuni).

9.5. $((A \wedge A) \Leftrightarrow A)$ (kon'yunksiyaning idempotentlik qonuni).

9.6. $((A \vee A) \Leftrightarrow A)$ (diz'yunksiyaning idempotentlik qonuni).

9.7. $((A \Rightarrow B) \Leftrightarrow ((\neg B) \Rightarrow (\neg A)))$ (kontrapozisiya qonuni).

9.8. $((A \Rightarrow B) \Leftrightarrow ((\neg A) \vee B))$.

9.9. $((A \Leftrightarrow B) \Leftrightarrow ((A \Rightarrow B) \wedge (B \Rightarrow A)))$.

9.10. $((A \wedge (B \vee A)) \Leftrightarrow A)$ (yutilish qonuni).

9.11. $((A \vee (B \wedge A)) \Leftrightarrow A)$ (yutilish qonuni).

9.12. $((\neg(A \wedge B)) \Leftrightarrow ((\neg A) \vee (\neg B)))$ (de Morgan qonuni).

9.13. $((\neg(A \vee B)) \Leftrightarrow ((\neg A) \wedge (\neg B)))$ (de Morgan qonuni).

9.14. $((A \vee B) \Leftrightarrow ((\neg A) \Rightarrow B))$.

9.15. $(((A \Rightarrow B) \wedge (B \Rightarrow C)) \Rightarrow (A \Rightarrow C))$ (tranzitiv hulosa qoidasi).

9.16. $((A \Leftrightarrow B) \Leftrightarrow ((\neg A) \Leftrightarrow (\neg B)))$ (qarama-qarshilik qonuni).

9.17. $((A \wedge B) \Leftrightarrow (B \wedge A))$ (kon'yunksiyaning kommutativlik qonuni).

9.18. $((A \vee B) \Leftrightarrow (B \vee A))$(diz'yunksiyaning kommutativlik qonuni).

9.19. $(((A \wedge B) \wedge C) \Leftrightarrow (A \wedge (B \wedge C)))$ (kon'yunksiyaning assosiativlik qonuni).

9.20. $(((A \vee B) \vee C) \Leftrightarrow (A \vee (B \vee C)))$ (diz'yunksiyaning assosiativlik qonuni).

9.21. $((A \wedge (B \vee C)) \Leftrightarrow ((A \wedge B) \vee (A \wedge C)))$ (kon'yunksiyaning diz'yunksiyaga nisbatan distributivlik qonuni).

9.22. $((A \vee (B \wedge C)) \Leftrightarrow ((A \vee B) \wedge (A \vee C)))$ (diz'yunksiyaning kon'yunksiyaga nisbatan distributivlik qonuni).

10. Quyidagi tengkuchliliklarni isbotlang (mulohazaviy formulalarning tashqi qavslari tashlab yuborilgan):

10.1. $A \wedge A \equiv A$.

10.2. $A \vee A \equiv A$.

10.3. $A \vee \neg A \equiv 1$.

10.4. $A \wedge \neg A \equiv 0$.

10.5. $A \vee 0 \equiv A$.

10.6. $A \vee 1 \equiv 1$.

10.7. $A \wedge 0 \equiv 0$.

10.8. $A \wedge 1 \equiv A$.

10.9. $\neg\neg A \equiv A$.

10.10. $A \wedge B \equiv B \wedge A$.

10.11. $A \vee B \equiv B \vee A$.

10.12. $A \wedge (B \vee A) \equiv A$.

10.13. $A \vee (B \wedge A) \equiv A$.

10.14. $A \Rightarrow B \equiv \neg A \vee B$.

10.15. $A \Leftrightarrow B \equiv (A \Rightarrow B) \wedge (B \Rightarrow A)$.

10.16. $\neg(A \vee B) \equiv \neg A \wedge \neg B$.

10.17. $\neg(A \wedge B) \equiv \neg A \vee \neg B$.

10.18. $(A \wedge B) \wedge C \equiv A \wedge (B \wedge C)$.

10.19. $(A \vee B) \vee C \equiv A \vee (B \vee C)$.

10.20. $A \wedge (B \vee C) \equiv (A \wedge B) \vee (A \wedge C)$.

10.21. $A \vee (B \wedge C) \equiv (A \vee B) \wedge (A \vee C)$.

11. 10-misoldagi tengkuchliliklar yordamida quyidagi formulalarni soddalashtiring (mulohazaviy formulalarning tashqi qavslari tashlab yuborilgan):

11.1. $\neg(\neg A \vee B) \Rightarrow ((A \vee B) \Rightarrow A)$.

11.2. $\neg(\neg A \wedge \neg B) \vee ((A \Rightarrow B) \wedge A)$.

11.3. $(A \Rightarrow B) \wedge (B \Rightarrow A) \wedge (A \vee B)$.

11.4. $(A \Rightarrow B) \wedge (B \Rightarrow \neg A) \wedge (C \Rightarrow A)$.

11.5. $(A \wedge C) \vee (A \wedge \neg C) \vee (B \wedge C) \vee (\neg A \wedge B \wedge C)$.

11.6. $\neg((A \Rightarrow B) \wedge (B \Rightarrow \neg A))$.

12. Teng kuchli almashtirishlar yordamida quyidagi formulalarni shunday almashtiring-ki, natijada hosil bo'lgan formulalarda faqat $\neg$ va $\wedge$ amallari qatnashsin:

12.1. $(A \vee B) \Rightarrow (\neg A \Rightarrow C)$;

12.2. $(\neg A \Rightarrow B) \vee \neg(A \Rightarrow B)$;

12.3. $((A \vee B \vee C) \Rightarrow A) \vee C$;

12.4. $((A \Rightarrow B) \Rightarrow C) \Rightarrow \neg A$;

12.5. $(A \vee (B \Rightarrow C)) \Rightarrow A$.

13. Teng kuchli almashtirishlar yordamida quyidagi formulalarni shunday almashtiring–ki, natijada hosil bo'lgan formulalarda faqat $\neg$ va $\vee$ amallari qatnashsin:

13.1. $(A \Rightarrow B) \Rightarrow (B \wedge C)$;

13.2. $(\neg A \wedge \neg B) \Rightarrow (A \wedge B)$;

13.3. $((\neg A \wedge \neg B) \vee C) \Rightarrow (C \wedge \neg B)$;

13.4. $((A \Rightarrow (B \wedge C)) \Rightarrow (\neg B \Rightarrow \neg A)) \Rightarrow \neg B$;

13.5. $((A \Rightarrow B) \wedge (B \Rightarrow C)) \Rightarrow (A \Rightarrow C)$.

14. Quyidagi formulalarning inkorini toping:

14.1. $(A \wedge (B \vee \neg C)) \vee (\neg A \wedge B)$;

14.2. $((\neg A \wedge \neg B \wedge \neg C) \vee D) \wedge \neg Q \wedge \neg R \wedge \neg P$;

14.3. $(((\neg A \wedge (\neg B \vee C)) \vee D) \wedge \neg Q) \vee (\neg R \wedge (P \vee \neg F))$;

14.4. $((A \wedge (\neg B \vee (\neg C \wedge D))) \vee \neg Q) \wedge R$.

15. Teng kuchli almashtirishlar yordamida quyidagi formulalarning ziddiyat ekanligini isbotlang:

15.1. $(A \Rightarrow B) \wedge (B \Rightarrow A) \wedge ((A \wedge \neg B) \vee (\neg A \wedge B))$;

15.2. $((A \wedge \neg B) \Rightarrow (\neg A \vee (A \wedge B))) \wedge ((\neg B \vee (A \wedge B)) \Rightarrow (A \wedge \neg B))$;

15.3. $((A \Rightarrow B) \wedge (B \Rightarrow C)) \Rightarrow \neg(A \Rightarrow C)$;

15.4. $(A \Rightarrow B) \wedge (A \Rightarrow \neg B) \wedge A$;

15.5. $(A \wedge \neg B) \vee (A \wedge \neg C)) \Leftrightarrow ((A \Rightarrow B) \wedge (A \Rightarrow C))$.

Takrorlash uchun savollar

1. Mulohazaviy formula ta'rifini ayting va misol keltiring.
2. Mantiqiy amallarni bajarilish tartibi qanday?
3. Mulohazalarning qabul qiladigan qiymatlar tizimi nima? Ularning soni nimaga bog'liq?
4. Formulaning rostlik jadvali qanday tuziladi?
5. Teng kuchli formulalarga ta'rif bering.
6. Formulalarning teng kuchli ekanligi qanday isbotlanadi?
7. Aynan rost, aynan yolg'on, bajariluvchi formulalar ta'riflarini ayting.
8. Tavtologiya, ziddiyat, mantiq qonuni ta'rifini ayting.
9. Asosiy tengkuchliliklardan qaysilarini eslab qoldingiz?
10. Teng kuchli formula bilan mantiq qonuni orasida qanday bog'lanish bor?

3-§. Predikatlar. Kvantorlar

Asosiy **tushunchalar**: Predikat, bir o'zgaruvchili predikatning qiymatlar sohasi, predikatning rostlik sohasi, predikatlar kon'yunksiyasi, diz'yunksiyasi, implikasiyasi, ekvivalensiyasi, predikat inkori, umumiylik kvantori, mavjudlik kvantori, predikatli formula.

M to'plamning a elementi haqida aytilgan tasdiqqa a ning o'rniga M ning aniq bitta elementini qo'ysak mulohaza hosil bo'lsa,bunday tasdiqlarni bir o'zgaruvchili mulohazaviy formula yoki bir o'zgaruvchili predikat deb ataymiz. n ta $x_1, \ldots, x_n$ o'zgaruvchilarga bog'liq R $(x_1, \ldots, x_n)$-tasdiq berilgan bo'lsin. U holda $x_1, \ldots, x_n$ o'zgaruvchilarning mazmunga ega bo'ladigan qiymatlar to'plami, shu o'zgaruvchilarning yo'l qo'yiladigan qiymatlari sohasi deyiladi. Agar $R(x_1, \ldots, x_n)$ tasdiq $x_1, \ldots, x_n$ o'zgaruvchilarning yo'l qo'yilishi mumkin bo'lgan har qanday qiymatlarida mulohazaga aylansa, n- o'zgaruvchili predikat yoki n o'zgaruvchili mulohazaviy formula deyiladi. Bu erda n - 0, 1, 2 va hokazo manfiy bo'lmagan butun qiymatlar qabul qiladi. 0- o'rinli predikat sifatida mulohaza tushuniladi.

$M \neq \varnothing$ to'plamda aniqlangan bir o'rinli R(x) - predikat berilgan bo'lsin, u holda R(x) - predikatning inkori deb har qanday $x \in M$ element uchun R(x)-predikat rost bo'lganda yolg'on bo'ladigan; R(x) yolg'on bo'lganda rost bo'ladigan $\neg R(x)$ predikatga aytiladi. Ya'ni, M ning ixtiyoriy elementi uchun $(\neg R)(x) = \neg(R(x))$ tenglik o'rinli bo'ladi.

Xuddi shunday $M \neq \varnothing$ to'plamda aniqlangan P(x) va Q(x) bir o'rinli predikatlar uchun $\wedge$, $\vee$, $\Rightarrow$, $\Leftrightarrow$ amallari quyidagi tengliklar yordamida aniqlanadi:

$(R \wedge Q)(x) = R(x) \wedge Q(x)$;

$(R \vee Q)(x) = R(x) \vee Q(x)$;

$(R \Rightarrow Q)(x) = R(x) \Rightarrow Q(x)$;

$(R \Leftrightarrow Q)(x) = R(x) \Leftrightarrow Q(x)$.

$M \neq \varnothing$ to'plamda aniqlangan R(x) predikat berilgan bo'lsin, u holda R(x) predikatni rost mulohazaga aylantiradigan x ning M to'plamga tegishli barcha elementlarini E_r orqali belgilaymiz. E_r-R(x) predikatning rostlik sohasi deyiladi.

$\forall x R(x)$ ifoda, M to'plamning barcha elementlari uchun R(x) rost bo'lganda rost, M to'plamning kamida bitta x_0 elementi uchun $R(x_0)$ yolg'on bo'lganda yolg'on bo'ladigan mulohazadir. Bu erdagi $\forall$ belgi umumiylik kvantorini bildiradi.

$\exists x R(x)$ mulohaza bo'lib, M to'plamning kamida bitta x_0 elementi uchun $R(x_0)$ rost bo'lganda rost qolgan hollarda, ya'ni M to'plamning barcha elementlari uchun R(x)- yolg'on bo'lganda yolg'on bo'ladigan mulohazadir.

R(x,y)- butun sonlar to'plami Z da aniqlangan «x+y>0» mazmunidagi predikat bo'lsin, u holda

$\forall x \forall y R(x,y)$- «ixtiyoriy ikkita butun son yig'inidisi musbat bo'ladi» - yolg'on mulohaza;

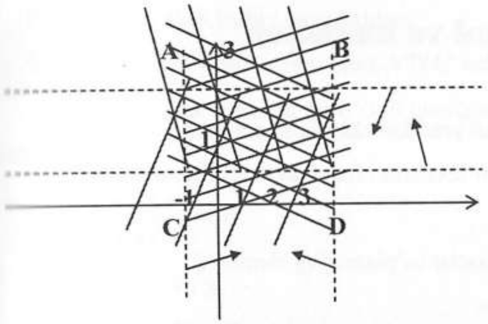
$\forall x \exists y R(x,y)$-«har qanday butun son x uchun shunday y butun son mavjud bo'lib ulraning yig'indisi musbat» - rost mulohaza;

$\exists x \forall y R(x,y)$-«shunday x butun son mavjud bo'lib, uning ixtiyoriy y butun son bilan yig'idisi musbat» - yolg'on mulohaza;

$\exists x \exists y R(x,y)$-«shunday x va y butun sonlar mavjud-ki, ularning yig'indisi musbat» - rost mulohaza bo'ladi.

Misol. Dekart koordinatalar tekisligida $x<3 \wedge x>-1 \wedge y < 3 \wedge y >1$ predikatning rostlik sohasini tasvirlang.

Yechish. Berilgan ikki o'rinli predikat to'rtta bir o'rinli predikatlarning kon'yunksiyasidan tashkil topgan. Kon'yunksiya amalining ta'rifidan, predikatlardagi ikkala o'zgaruvchi o'rniga qiymatlar berganimizda, ularning barchasini rost mulohazaga aylantiruvchi x va y larning qiymatlari berilgan predikatning rostlik sohasi bo'ladi. Buning uchun har bir predikatning rostlik sohalarini aniqlab, ularning kesishmasini topamiz:



hosil bo'lgan chizmadagi ABCD to'rtburchakning ichki nuqtalari berilgan predikatning rostlik sohasi bo'ladi.

Misol. M = { 1, 2, . . . , 20} to'plamda quyidagi predikatlar berilgan:

A(x): «(x : 5)»; B(x): «x – juft son»; C(x): «x – tub son»; D(x): «x 3 ga karrali».

$A(x) \wedge B(x) \Rightarrow C(x) \vee D(x)$ predikatning rostlik sohasini toping.

Yechish. K orqali M to'plamning $A(x) \wedge B(x)$ predikatni rost, $C(x) \vee D(x)$ predikatni yolg'on mulohazaga aylantiradigan elementlarini belgilab olamiz. Mantiq amallarining ta'rifiga ko'ra berilgan predikatning rostlik sohasi M to'plamdan K to'plamni ayirishdan hosil bo'lgan to'plamdan iborat. K to'plamni aniqlaymiz:

1) $A(x) \wedge B(x)$ predikat rost mulohazaga aylanadigan qiymatlar to'plami A(x) va B(x) predikatlarni bir vaqtda rost mulohazaga aylantiradigan M to'plamning elementlari, ya'ni $A_1=\{5,10,15,20\}$ va $B_1=\{2,4,6,8,10,12,14,16,18,20\}$ to'plamlarning kesishmasidan iborat. Bu to'plamni M_1 orqali belgilaymiz:

$M_1= A_1 \cap B_1 = \{10,20\}$.

2)$C(x) \vee D(x)$ predikat C(x) va D(x) predikatlar bir vaqtda yolg'on mulohazaga aylanadigan M to'plamning qiymatlarida yolg'on mulohaza bo'ladi. U $C_1 =\{1,4,6,8,9,10,12,14,15,16,18,20\}$ va

$D_1=\{1,2,4,5,7,8,10,11,13,14,16,17,19,20\}$ to'plamlarning kesishmasidan iborat $M_2 = \{1,4,8,10,14,16,20\}$ to'plamdan iborat.

Demak, K= $M_1 \cap M_2$ = {10,20} to'plamdan iborat. U holda M\K berilgan $A(x) \wedge B(x) \Rightarrow C(x) \vee D(x)$ predikatning rostlik sohasi.

Misol va mashqlar

1. Quyidagi gaplardan qaysilari predikat ekanligini aniqlang:

1.1. Natural sonning natural bo'luvchilari faqat ikkita bo'lsa, u tub son bo'ladi.

1.2. «x-mevali daraxt», (x daraxtlar to'plamining elementi).

1.3. «Nargizaning yoshi 18da».

1.4. «x+y=5», ($x,y \in N$).

1.5. «x+yi –mavhum son», ($x,y \in R$).

1.6. «x^3+3x-4», ($x \in Z$)

1.7. «x va y bir sinfda o'qiydi», (x,y o'quvchilar to'plamining elementlari).

1.8. «x va y o'xshash», (x,y geometrik figuralar to'plamining elementlari).

1.9. «5=2+4».

1.10. «Nodir x ning akasi», (x insonlar to'plamining elementi).

1.11. «x 5 ga bo'linadi», ($x \in N$).

1.12. «$x^2 + 2x + 4$», ($x \in R$).

1.13. «$\mathrm{ctg}\, 45^0 = 1$».

1.14. « x va y lar z ning turli tomonlarida yotadi» (x va y lar tekislikdagi nuqtalar to'plamiga, z esa tekislikdagi to'g'ri chiziqlar to'plamiga tegishli) .

2. Quyidagi mulohazalar uchun shunday predikatlar tuzingki, ulardagi o'zgaruvchilar o'rniga qiymat berganda berilgan mulohaza hosil bo'lsin:

2.1. 2+3>7.

2.2. Feruzaning farzandlari 3 nafar.

2.3. A.Navoiy ko'chasi Toshkent shahridagi markaziy ko'chalardan biri.

2.4. $\log_2 2 = 1$.

2.5. Nargiza Namangan viloyatida tug'ilgan.

2.6. $(5^2-1)=(5-1)(5+1)$.

2.7. 16 - murakkab son.

2.8. (4,12,20)=4.

2.9. Palov-o'zbek milliy taomlaridan.

2.10. Berilgan ABC uchburchak A'B'C' uchburchakka teng.

3. Sonlar o'qida quyidagi bir o'rinli predikatlarning rostlik sohasini tasvirlang:

3.1. $\dfrac{x^2+3x+2}{x^2+4x+3} < 0$.

3.2. $\sqrt{x^2-1} = -3$.

3.3. $2x^2 + x - 30 > 0$.

3.4. $(\mathrm{Sin}x \geq 0)$.

3.5. $(|x+2| < 0)$.

3.6. $\dfrac{x}{x-1} < 0$.

3.7. $3x^2 - 2x + 4 > 0$.

3.8. $|x-1| < |x+3|$.

3.9. $|x+5| \leq 3$.

3.10. $|x| \leq 1$.

4. Dekart koordinatalar tekisligida quyidagi ikki o'rinli predikatlarning rostlik sohasini tasvirlang:

4.1. $((x > 2) \wedge (y \geq 1)) \wedge ((x < -1) \wedge (y < -2))$.

4.2. $x + 3y = 3$.

4.3. $x - y \geq 0$.

4.4. $(x-2)^2 + (y+3)^2 = 4$.

4.5. $\lg x = \lg y$.

4.6. $\rceil(x > 2) \wedge (y < 2)$.

4.7. $(x = y) \vee (|x| \leq 1)$.

4.8. $(x \geq 3) \Rightarrow (y < 5)$.

4.9. $((x-1)^2 + y^2 = 4) \wedge (y = x)$.

4.10. $(x^2 + 2x + 1 = 0) \wedge (y = 2x + 3)$.

5. M = { 1, 2, . . . , 20} to'plamda quyidagi predikatlar berilgan:

A(x): «$\rceil$(x $\vdots$ 5)»; B(x): «x – juft son»; C(x): «x – tub son»; D(x): «x 3 ga karrali». Quyidagi predikatlarning rostlik sohasini toping:

5.1. $A(x) \wedge D(x) \Rightarrow \rceil C(x)$.

5.2. $A(x) \wedge C(x) \Rightarrow \rceil D(x)$.

5.3. $A(x) \Rightarrow B(x)$.

5.4. $D(x) \Rightarrow \rceil C(x)$.

5.5. $C(x) \Rightarrow A(x)$.

5.6. $A)x) \vee B(x) \vee D(x)$.

5.7. $\rceil B(x) \vee \rceil D(x)$.

5.8. $\rceil C(x) \vee D(x) \wedge B(x)$.

5.9. $\rceil B(x) \vee C(x) \Rightarrow D(x)$.

5.10. $A(x) \wedge B(x) \wedge D(x)$.

5.11. $\rceil A(x) \wedge \rceil C(x) \vee B(x)$.

5.12. $\rceil B(x) \wedge \rceil C(x) \wedge D(x)$.

5.13. $A(x) \vee \rceil B(x) \wedge D(x)$.

5.14. $A(x) \wedge C(x) \vee \rceil D(x)$.

5.15. $B(x) \vee C(x) \wedge A(x)$.

5.16. $A(x) \Leftrightarrow \rceil B(x) \wedge D(x)$.

6. Quyidagi mulohazalarni o'qing va ularning rostlik qiymatini aniqlang:

6.1. $\forall x A(x)$, A(x): «x-natural son», $x \in R$.

6.2. $\exists x A(x)$, A(x): «x-butun son», $x \in R$.

6.3. $\forall x(x+3=5)$, $x \in R$.

6.4. $\exists x(4+x=10)$, $x \in R$.

6.5. $\forall x \forall y(x+y<4)$, $x,y \in R$.

6.6. $\forall x \exists y(x+y>4)$, $x,y \in R$.

6.7. $\exists x \forall y(x+y=14)$, $x,y \in R$.

6.8. $\exists x \exists y(x \leq y)$, $x,y \in R$.

6.9. $\forall x \exists y \exists z([x,y]=z)$, $x,y \in R$.

6.10. $\forall x \forall y \exists z([x,y]=z)$, $x,y,z \in R$.

6.11. $\forall x(x<0 \Rightarrow x>0)$, $x \in \{0,1,2\}$.

6.12. $(x \in T)$ $(a^2 + b^2 = c^2)$, T – uchburchaklar to'plami va *a, b, c* - uchburchak tomonlari.

6.13. $\forall x \forall y(\frac{x}{y} \Rightarrow \frac{y}{x})$, $x,y \in N$.

6.14. $\forall x\, (f(x)>0)$, $f(x) = x^2 - 4x + 3$, $x \in R$.

6.15. $\forall x\, (\frac{2x-5}{x} \in R)$, $x \in R$.

6.16. $\forall x\, (x<10)$, $x \in \{1, . . . , 10\}$.

6.17. $\forall x(x+5 \leq 15)$, $x \in \{1, . . . , 10\}$.

6.18. $\forall x \forall y(x - y < 10)$, $x,y \in \{1, . . . , 10\}$.

6.19. $\forall x \exists y\, (\frac{x}{y} \in A)$, $x,y \in \{1, . . . , 10\}$.

6.20. $\forall x \forall y(x<y)$, $x \in \{1, . . . , 5\}$, $y \in \{5 , . . . , 10\}$.

6.21. $\forall x \forall y\, (x \vdots y)$, $x \in \{4k \mid k \in Z\}$, $y \in \{1, 2, 4\}$.

7. Quyidagi predikatlardan kvantorlar yordamida mulohazalar hosil qiling va ularning qiymatlar, rostlik sohalarini toping:

7.1. A(x): «x-talaba».

7.2. A(x): «x-butun son».

7.3. A(x): «x-to'g'ri chiziq».

7.4. A(x): «$x \vdots 5$».

7.5. A(x,y): «x+y=4».

7.6. A(x,y): «x<y».

7.7. A(x,y): «$x \vdots y$».

7.8. A(x,y): «x//y».

7.9. A(x,y,z): «$\frac{x}{y} = z$».

7.10. A(x,y,z): «[x,y]=z».

8. Quyidagi formulalardagi erkli va bog'liq o'zgaruvchilarni aniqlang:

8.1. $\forall x A(x)$.

8.2. $A(y) \Rightarrow \exists x B(x)$.

8.3. $\exists x \forall y (A(x) \wedge B(y)) \Rightarrow \forall y C(t,y)$.

8.4. $\forall x (\exists y (A(x,y)) \Rightarrow B(t,z)$.

Takrorlash uchun savollar

1. Predikatga ta'rif bering.
2. Predikatning qiymatlar sohasi, rostlik sohasi nima? Misollar yordamida tushuntiring
3. Predikatlar diz'yunksiyasi, kon'yunksiyasi, implikasiyasi, ekvivalensiyasiga misollar keltiring.
4. Mantiq amallarini qo'llash natijasida hosil bo'ladigan predikat o'zgaruvchilarining soni haqida nima deyish mumkin?
5. Umumiylik va mavjudlik kvantorlarini qo'llashga misollar keltiring.
6. Predikatli formula qanday hosil qilinadi?
7. Predikatli formulaning qanday turlarini bilasiz?

4-§. Matematik mantiqning tadbiqlari

Asosiy tushunchalar: teorema, to'g'ri teorema, to'g'riga teskari teorema, to'g'riga qarama-qarshi teorema, teskariga qarama-qarshi teorema, teorema isboti. Rele kontakt sxemasi

Matematik mantiq elementlari mavzuning o'qitilishidan qo'yilgan asosiy maqsad–matematik mantiq fanining algebra, geometriya, matematik tahlil kabi bir

qancha matematik fanlarga tadbiqining eng sodda ko'rinishlaridan biri-matematik jumlalar (aksioma, teorema, ta'rif,...)larni mulohazalar va predikatlar algebralari tili orqali ifodalashga o'quvchilarni o'rgatishdir.

Natural sonlar to'plamida qaralgan tub son tushunchasi uchun quyidagi formulani keltirish mumkin :

$(\forall n \in N)((n \text{ - tub son}) \Leftrightarrow (n \neq 1 \wedge n \vdots\ p \Rightarrow p=1 \vee p=n))$.

Yoki quyidagi belgilashlarni kiritsak:

A(x)–«x-tub son», B(x)–«x≠1», C(x)–«x: p», D(x)–«x=1», P(x)–«x=p» , u xolda yuqoridagi formulani quyidagicha ifodalash mumkin :

$(\forall x \in N)\ (A(x) \Leftrightarrow B(x) \wedge C(x) \Rightarrow D(x) \vee P(x))$.

Teorema va uning turlari. Har qanday teorema shart va natijadan iborat. Agar A teoremaning sharti B esa uning hulosasi bo'lsa, u holda teoremani

$A \Rightarrow B$ (1) ko'rinishda yozishimiz mumkin.

$B \Rightarrow A$ (2) teoremaga (1) teoremaga teskari teorema deyiladi.

$\neg A \Rightarrow \neg B$ (3) teoremaga (1) teoremaga qarama-qarshi teorema deyiladi.

$\neg B \Rightarrow \neg A$ (4) teoremaga berilgan (1) teoremaning teskarisiga qarama-qarshi (yoki berilgan (1) teoremaning qarama-qarshisiga teskari) teorema deyiladi.

Rostlik jadvallari orqali $A \Rightarrow B \equiv \neg B \Rightarrow \neg A$ va $B \Rightarrow A \equiv \neg A \Rightarrow \neg B$ tengkuchliliklarni isbot qilib, quyidagi xulosani chiqaramiz:

$A \Rightarrow B$ teorema o'rniga $\neg B \Rightarrow \neg A$ teoremani isbot qilib, $A \Rightarrow B$ rost, ya'ni to'g'ri deb aytishimiz mumkin.

Isbot tushunchasi. $A_1, A_2, \ldots, A_n$ (1) mulohazalar berilgan bo'lib, quyidagi shartlar bajarilsa:

A_1 - aksioma yoki avval isbot qilingan mulohaza bo'lsin;

har bir A_i, $i \geq 2$ yoki o'zidan oldingi mulohazadan keltirib chiqarilsin, yoki avval isbot qilingan mulohaza bo'lsin.

U holda (1) ketma-ketlikni biz A_n mulohazaning isboti deymiz.

Isbot qilish usullari. Teorema shartining rostligidan, xulosaning rostligini to'g'ridan-to'g'ri keltirib chiqarishni bevosita isbot qilish deb tushunamiz. Mantiq qonunlari orqali isbot qilishga, teskarisidan isbot qilish, uchinchisini inkor qilish qonuni orqali isbot qilish, induksiya yordamida isbot qilish va h.k.lar kiradi.

Avtomatik boshqarish qurilmalari va elektron hisoblash mashinalarida ko'plab rele–kontakt sxemalar uchraydi. Har qanday sxemaga mulohazalar algebrasining biror bir formulasini mos qo'yish mumkin va aksincha. RKS bilan mulohazalar algebrasining formulalari orasidagi bunday munosabat murakkab RKSlarni mulohazalar algebrasining formulalari yordamida soddalashtirish imkoniyatini beradi. Kontaktni shartli ravishda

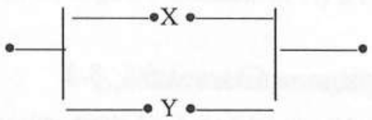
yoki ——•—— , yoki ——— ——— , yoki ——• •——

ko'rinishda belgilaymiz. Kontakt yopiq (tok o'tkazadigan) yoki ochiq (tok o'tkazmaydigan) holatda bo'lishi mumkin. Kontaktning yopiq holatiga 1 ni, ochiq holatiga 0 ni mos qo'yamiz.

Barcha kontaktlar orasida doimo tok o'tkazadigan (doimo yopiq) hamda butunlay tok o'tkazmaydigan (doimo ochiq) kontaktlar mavjuddir. Ularni ham mos ravishda 1 va 0 bilan belgilaymiz va hamda ——•—— , ——— ——— ko'rinishda ifodalaymiz.

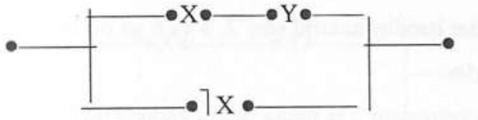
Biz o'zgaruvchi kontaktlar bilan ish ko'rganimiz uchun ularni X, Y, Z, ... harflar bilan belgilaymiz. U holda ikkita X va Y mulohazalarning kon'yunksiyasiga kontaktlarni ketma–ket ulash natijasida hosil bo'ladigan

——•X•——•Y•—— sxemani, X va Y mulohazalarning diz'yunksiyasiga kontaktlarni parallel ulash natijasida hosil bo'ladigan quyidagi

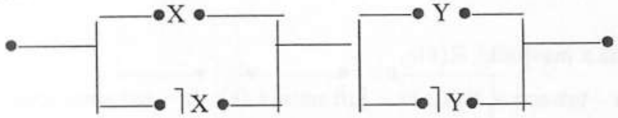


sxemani mos qo'yamiz. Har qanday mulohazviy formulani faqat $\rceil$, $\wedge$, $\vee$ amallar qatnashgan formulaga keltirish mumkin bo'lganligidan har bir formulani RKS orqali ifoda qilish va aksincha, har qanday RKS ni mulohazaviy formula orqali ifodalash mumkin.

Misol. $(X \wedge Y) \vee \rceil X$ - formulaga quyidagi rele–kontakt sxemasi mos keladi:



Misol.



sxemaga $(X \vee \rceil X) \wedge (Y \vee \rceil Y)$ formula mos keladi.

Misol va mashqlar

1. Quyidagi teoremalarga teskari, to'g'riga qarama-qarshi, teskariga qarama-qarshi teoremalarni ifodalang:

1.1. Agar berilgan to'rtburchak kvadrat bo'lsa, u romb bo'ladi.

1.2. Agar berilgan ikki to'g'ri chiziqning har biri uchinchi to'g'ri chiziqqa parallel bo'lsa, u holda berilgan to'g'ri chiziqlar parallel bo'ladi.

1.3. Agar parallelogramm romb bo'lsa, uning diagonallari perpendikulyar bo'ladi.

1.4. Agar ketma-ketlik monoton va chegaralangan bo'lsa, u holda u limitga ega bo'ladi.

1.5. Agar berilgan natural sonning raqamlar yig'indisi 3ga bo'linsa, berilgan son 3ga bo'linadi.

1.6. Agar rasional sonlar ketma-ketligi yaqinlashuvchi bo'lsa, u holda u fundamental ketma-ketlik bo'ladi.

1.7. Agar 3 ta sonning ko'paytmasi nolga teng bo'lsa, u holda ko'paytuvchilarning kamida bittasi nolga teng bo'ladi.

1.8. Uchburchakning ichki burchaklari yig'indisi 180^0ga teng.

2. Quyidagi mulohazalarni predikatlar algebrasi tilida ifodalang:

2.1. «Barcha rasional sonlar haqiqiy».

2.2. «Ayrim rasional sonlar haqiqiy emas».

2.3. «12 ga bo'linuvchi har qanday natural son 2, 4 va 6 ga bo'linadi».

2.4. «Ayrim ilonlar zaharli».

2.5. «Bir to'g'ri chiziqda yotmagan 3 ta nuqta orqali yagona tekislik o'tkazish mumkin».

2.6. «Yagona x mavjudki, R(x)».

3. A(x): «x – tub son », B(x): «x – juft son», C(x): «x – toq son», D(x): «x y ni bo'ladi» kabi xossalarni bildirsa quyidagilarni o'qing:

3.1. A(7).

3.2. $B(2) \wedge A(2)$.

3.3. $\forall x(B(x) \Rightarrow \forall y(D(x, y) \Rightarrow B(y))$.

3.4. $\forall x(C(x) \Rightarrow \forall y(A(y) \Rightarrow \rceil D(x, y)))$.

4. Quyidagi tasdiqlar va ularning inkorlarini predikatlar tilida ifodalang:

4.1. Tartiblangan to'plam chiziqli tartiblangan deyiladi, agar to'plamning ixtiyoriy x, y elementlari uchun yoki $x = y$ yoki $x < y$ yoki $x > y$ bo'lsa.

4.2. $f(x)$ funksiya M to'plamda chegaralangan deyiladi, agar shunday manfiymas L soni mavjud bo'lib, har qanday $x \in M$ uchun $|f(x)| \leq L$ bo'lsa.

4.3. $f(x)$ funksiya M to'plamda o'suvchi deyiladi, agar to'plamning ixtiyoriy x_1, x_2 elementlari uchun, $x_1 < x_2$ ekanligidan $f(x_1) < f(x_2)$ kelib chiqsa.

4.4. $f(x)$ funksiya davriy deyiladi, agar shunday $T \neq 0$ soni mavjud bo'lib, funksiyaning aniqlanish sohasidan olingan har qanday x uchun $x - T$ va $x + T$ lar ham shu sohaga tegishli bo'lib, $f(x \pm T) = f(x)$ shart bajarilsa.

5. Quyidagi formulalar uchun rele-kontakt sxemalarini tuzing:

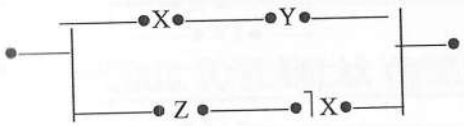
5.1. $(X \wedge Y \wedge Z) \vee \rceil(X \wedge Y \wedge Z) \vee (X \wedge Y)$.

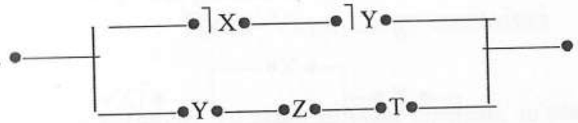
5.2. $(X \Rightarrow Y) \wedge (Y \Rightarrow Z)$.

5.3. $((X \Rightarrow Y) \wedge (Y \Rightarrow Z)) \Rightarrow (X \Rightarrow Z)$.

5.4. $(X \Rightarrow (Y \Rightarrow Z)) \Rightarrow (Y \Rightarrow X)$.

Quyidagi rele–kontakt sxemalariga mos keluvchi mulohazalar algebrasining formulasini aniqlang:

6.1.


6.2.


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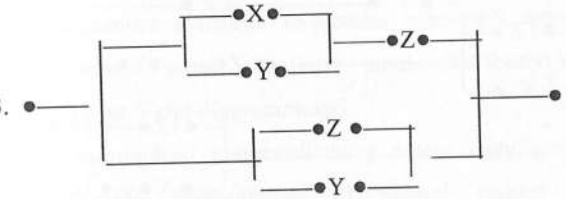
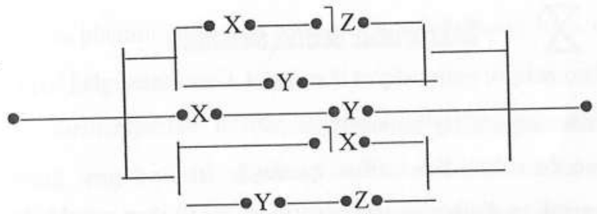
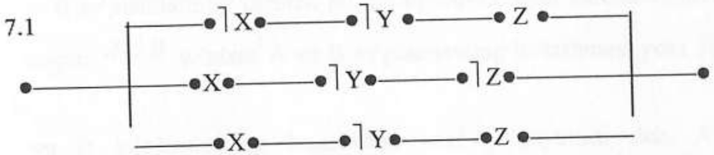
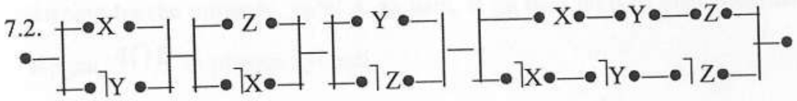


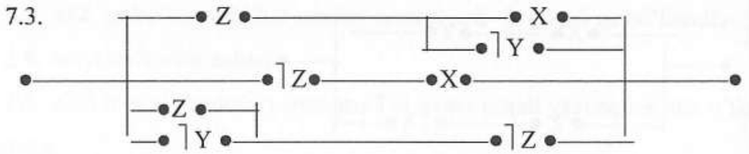
6.4.


7. Quyidagi rele–kontakt sxemalarini soddalashtiring:

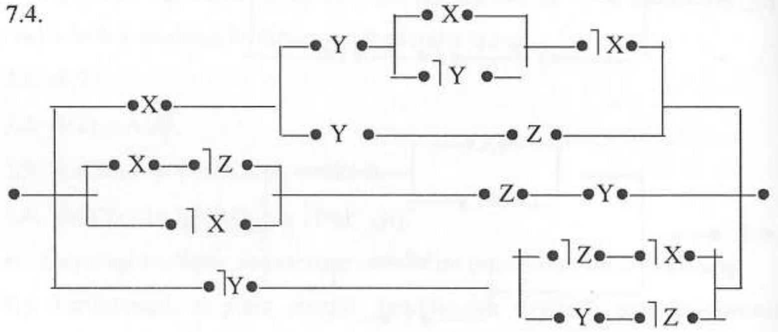
7.1


7.2.


7.3.



7.4.



Takrorlash uchun savollar

1. Teoremaning qanday turlarini bilasiz?
2. Teoremalarni isbotlash usullari qanday?
3. Matematik tasdiqlarni predikatlar tilida ifodalashga misol keltiring.
4. Rele-kontakt sxemalarini soddalashtirishda muloxazaviy formulalarning ahamiyati nimada?

II MODUL. TO'PLAMLAR VA MUNOSABATLAR

5-§. To'plam. To'plamlar ustida amallar. Eyler-Venn diagrammalari

Asosiy tushunchalar: To'plam, to'plam elementi, to'plamlarning tengligi, qismto'plam, bo'sh to'plam, universal to'plam, to'plamlar birlashmasi, to'plamlar kesishmasi, to'plamlar ayirmasi, to'plamlar simmetrik ayirmasi, idempotentlik xossasi, kommutativ amal, assosiativ amal, distributivlik xossasi, to'plam to'ldiruvchisi, Eyler-Venn diagrammalari.

To'plam tushunchasi matematikaning asosiy tushunchalaridan biri bo'lib, misollar yordamida tushuntiriladi. To'plamni tashkil qiluvchi ob'ektlar to'plamining elementlari deyiladi.

Agar A to'plamning xar bir elementi B to'plamning ham elementi bo'lsa, $A \subset B$ orqali belgilanadi va A to'plam B to'plamning to'plamostisi deyiladi.

Bir xil elementlardan tashkil topgan to'plamlar teng deyiladi. A va B to'plamlarning teng bo'lishi uchun $A \subset B$ va $B \subset A$ bo'lishi zarur va etarli ekanligini ko'rish qiyin emas. Bitta ham elementi yo'q to'plamni bo'sh to'plam deb ataymiz, ya'ni $\varnothing$ orqali belgilaymiz.

A va B to'plamlarning kamida biriga tegishli bo'lgan barcha elementlardan tashkil topgan $A \cup B$ to'plam A va B to'plamlarning birlashmasi yoki yig'indisi deyiladi.

A va B to'plamlarning kesishmasi yoki ko'paytmasi deb, A va B to'plamlarning barcha umumiy, ya'ni A ga ham, B ga ham tegishli elementlardan tashkil topgan $A \cap B$ to'plamga aytiladi.

A va B to'plamlarning ayirmasi deb, A to'plamning B to'plamga kirmagan barcha elementlardan tashkil topgan to'plamga aytiladi. A va B to'plamlarning ayirmasi A \ B ko'rinishida belgilanadi.

$(A \setminus B) \cup (B \setminus A)$ to'plam A va B to'plamlarning simmetrik ayirmasi deyilidi va $A \Delta B$ orqali belgilanadi.

Agar $A \subset B$ bo'lsa, B \ A to'plam A to'plamning B to'plamgacha to'ldiruvchi to'plam deyiladi va $\subset A$ yoki A' orqali belgilanadi.

Misol. A,B $\subset$ M $=\{1, \ldots, 20\}$ to'plamlar uchun quyidagilarni aniqlang:

$A \setminus B,\ B \setminus A,\ A \cup B,\ A \cap B,\ A',\ B'$. $A=\{1,3,5,7,9\}$, $B=\{2,4,7,8\}$.

Yechish: Berilgan to'plamlar uchun to'plamlar ustida bajariladigan amallarning ta'riflarini qo'llab quyidagi to'plamlarni hosil qilamiz:

$A \setminus B=\{1,3,5,9\}$; $B \setminus A=\{2,4,8\}$; $A \cup B=\{1,2,3,4,5,7,8,9\}$;

$A \cap B=\{7\}$; $A'=\{2,4,6,8,10,11,12,13,14,15,16,17,18,19,20\}$;

$B'=\{1,3,5,6,9,10,11,12,13,14,15,16,17,18,19,20\}$.

Misol. $(A \cup B) \setminus C = (A \setminus B) \cup (B \setminus C)$ tenglikni isbotlang.

Yechish: To'plamlarning tengligini isbotlash uchun $M=N \Leftrightarrow M \subset N \wedge N \subset M$ tasdiqdan foydalanamiz.

1) $\forall x \in ((A \cup B) \setminus C) \Rightarrow x \in (A \cup B) \wedge x \notin C \Rightarrow x \in A \vee x \in B \wedge x \notin C \Rightarrow$

$\Rightarrow (x \in A \wedge x \notin C) \vee (x \in B \wedge x \notin C) \Rightarrow x \in (A \setminus C) \vee x \in (B \setminus C) \Rightarrow$

$\Rightarrow [x \in ((A \setminus C) \cup (B \setminus C))$. Bundan $(A \cup B) \setminus C \subset (A \setminus C) \cup (B \setminus C)$ ekanligi kelib chiqadi.

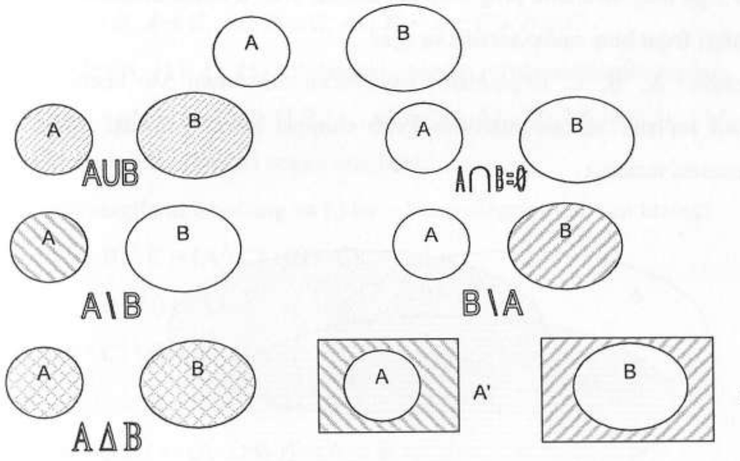
2) $\forall x \in ((A \setminus C) \cup (B \setminus C)) \Rightarrow x \in (A \setminus C) \vee x \in (B \setminus C) \Rightarrow (x \in A \wedge x \notin C) \vee$ $\vee (x \in B \wedge x \notin C) \Rightarrow x \in A \vee x \in B \wedge x \notin C) \Rightarrow x \in (A \cup B) \wedge x \notin C \Rightarrow$

$\Rightarrow x \in ((A \cup B) \setminus C)$. Dundan $(A \setminus C) \cup (B \setminus C) \subset (A \cup B) \setminus C$ ekanligi kelib chiqadi.

Demak $(A \cup B) \setminus C = (A \setminus C) \cup (B \setminus C)$.

To'mlamlar ustida amallarni Eyler-Venn diagrammalari deb ataladigan quyidagi shakllar yordamida ifoda qilish, amallarning xossalarini isbot qilishni ancha engillashtiradi.

Universal to'plam to'g'ri to'rt burchak shaklida, uning to'plamostilarini to'g'ri to'rtburchak ichidagi doiralar orqali ifoda qilinadi. U xolda, ikki to'plam birlashmasi, kesishmasi, ayirmasi, to'lduruvchi to'plamlar, ikki to'plamning simmetrik ayirmasi mos ravishda quyidagicha ifodalanadi:



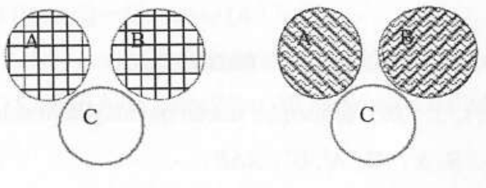
$(A \cup B) \setminus C = (A \setminus C) \cup (B \setminus C)$ tenglikni Eyler – Venn diagrammalarida tasvirlaymiz. Buning uchun tenglikda qatnashgan uchta to'plam uchun biror bir vaziyatni aniqlab, tenglikning ikkala tomonini ikkita diagrammada tasvirlaymiz:

$A \cup B$ ni ; $(A \cup B) \setminus C$ ni orqali;

$A \setminus C$ ni ; $B \setminus C$ ni ;

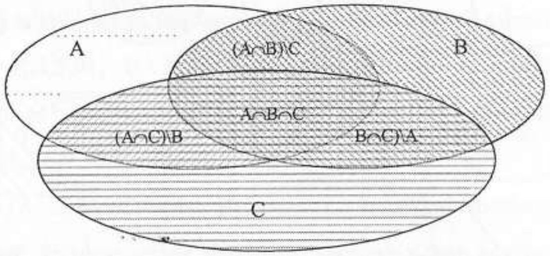
$(A \setminus C) \cup (B \setminus C)$ ni orqali belgilab

quyidagilarni hosil qilamiz:



Masala. 58 nafar mexanizatorlar haydovchi, traktorchi, kombaynchi mutaxassisligiga ega. Ulardan 11 nafari ham haydovchi ham traktorchi; 6 nafari ham traktorchi ham kombaynchi; 5 nafari–haydovchi va kombaynchi; 3 nafari-uchchala mutaxassislikka ega. Agar kombaynchi, haydovchi va traktorchilar soni ayirmasi 7 ga teng arifmetik progressiyani tashkil etsa, u holda mexanizatorlardan necha nafari faqat bitta mutaxassislikka ega?

Yechish. A, B, C to'plamlar- haydovchi, traktorchi va kombaynchilar to'plamlari bo'lsin. Masala shartidan kelib chiqqan holda quyidagi Eyler-Venn diagrammasini tuzamiz



Masala shartiga ko'ra $A\cap B\cap C=3, A\cap B=11, A\cap C=5, B\cap C=6$. U holda $(A\cap B)\setminus C=8, (A\cap C)\setminus B=2, (B\cap C)\setminus A=3$ bo'ladi. Faqat haydovchilar to'plami (A \ B) \ C ni x, faqat traktorchilar to'plami (B \ A) \ C ni y , faqat kombaynchilar to'plami (C \ A) \ B ni z orqali belgilasak, u holda

$x+y+z+3+8+2+3=58$ bo'lib, bundan $x+y+z=42$ kelib chiqadi. z, x, y lar ayirmasi 7 ga teng arifmetik progressiyani tashkil etganligi uchun $x=14, y=7, z=21$.

Javob. Faqat haydovchilar-14, faqat kombaynchilar-7,faqat traktorchilar-21 nafar.

Misol va mashqlar

1. A, B ⊂ M = {1, … , 20} to'plamlar uchun quyidagilarni aniqlang:

A \ B, B \ A , A ∪ B, A ∩ B, A', B' , $A\Delta B$:

1.1. A = {1, 3, 5}, B = {11, 13, 15}.

1.2. A = {3, 5, 7}, B = {8,. . . ,15}.

1.3. A = {1,. . . , 5}, B = {1,. . . ,13}.

1.4. A = {5,. . . , 12}, B = {12,. . . , 15}.

2. Shunday A, B, C to'plamlarni toping-ki, ular uchun quyidagi shartlar bajarilsin: $A\subset B, A\not\subset C, A\subset B\subset C, A\subset B\wedge A\subset C\wedge B\not\subset C$.

3. M={{∅}, {1}, {1,2}} to'plamning barcha to'plamostilarini toping.

4. Agar n∈N uchun M_n={1,2,. . . , n} bo'lsa, M_1, M_2, M_3, M_4, . . . , M_n larning barcha to'plamostilari sonini aniqlang.

5. Quyidagilarni isbotlang va Eyler – Venn diagrammalarini tuzing:

5.1. (A \ B) \ C = (A \ C) \ (B \ C).

5.2. A \ (B \ C) ⊂ A ∪ C.

5.3. (A \ C) \ (B \ A) ⊂ A \ C.

5.4. A \ C ⊂ (A \ B) ∪ (B \ C).

5.5. ((A ∪ B)' ∩ (A' ∪ B'))' = A ∪ B.

5.6. A ⊂ B ⊂ C ≡ A ∪ B = B ∩ C.

5.7. A ⊂ B ⇒ A \ C ⊂ B \ C.

5.8. A ⊂ B ⇒ A ∩ C ⊂ B ∩ C.

5.9. A ⊂ B ⇒ A ∪ C ⊂ B ∪ C.

5.10. B ⊂ A ∧ C = A \ B ⇒ A = B ∪ C.

5.11. A ⊄ B ∧ B ∩ C = ∅ ⇒ A ∪ C ⊄ B ∪ C.

5.12. C = A \ B ⇒ B ∩ C = ∅.

5.13. B ∩ C = ∅ ∧ A ∩ C ≠ ∅ ⇒ A \ B ≠ ∅.

5.14. A ⊂ C ⇒ A ∪ (B ∩ C) = (A ∪ B) ∩ C.

5.15. A \ (B ∩ C) = (A \ B) ∪ (A \ B).

6. L'yuis Kerrol masalasi. Shiddatli jangda 100 nafar qaroqchidan 70 nafari bitta ko'zidan, 75 nafari bitta qulog'idan, 80 nafari bitta qo'lidan va 85 nafari bitta

oyog'idan ayrildi. Bir vaqtning o'zida ham ko'zi, ham qulog'i, ham qo'li va oyog'idan ayrilgan qaroqchilarning eng kam sonini aniqlang.

7. To'plamlar ustida bajariladigan algebraik amallarning quyidagi xossalarini isbotlang:

7.1. $A \cap A = A$ kesishmaning idempotentligi;

7.2. $A \cup A = A$ birlashmaning idempotentligi;

7.3. $A \cap B = B \cap A$ kesishmaning kommutativligi;

7.4. $A \cup B = B \cup A$ birlashmaning kommutativligi;

7.5. $(A \cap B) \cap C = A \cap (B \cap C)$ kesishmaning assosiativligi;

7.6. $(A \cup B) \cup C = A \cup (B \cup C)$ birlashmaning assosiativligi;

7.7. $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ kesishmaning birlashmaga nisbatan distributivligi;

7.8. $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ birlashmaning kesishmaga nisbatan distributivligi;

7.9. $(A \setminus B) \cap C = (A \cap C) \setminus B = (A \cap C) \setminus (B \cap C)$;

7.10. $X \setminus \bigcup_{i=1}^{\infty} A_i = \bigcap_{i=1}^{\infty} (X \setminus A_i)$;

7.11. $X \setminus \bigcap_{i=1}^{\infty} A_i = \bigcup_{i=1}^{\infty} (X \setminus A_i)$.

Такрорлаш учун саволлар

1. To'plam tushunchasiga misollar keltiring.
2. To'plam elementi deb nimaga aytiladi?
3. Qism to'plam ta'rifini ayting.
4. Teng to'plamlar tushunchasiga ta'rif bering.
5. Bo'sh to'plam, universal to'plamlar ta'rifini ayting. Misollar keltiring
6. To'plamlar birlashmasi, kesishmasiga ta'rif bering.
7. To'plamlar ayirmasi, simmetrik ayirmasiga ta'rif bering.
8. To'plamlar birlashmasining qanday xossalarini bilasiz?

9. To'plamlar kesishmasining qanday xossalarini bilasiz?
10. To'plamlar ustida bajariladigan amallarning xossalari qanday tushunchalar yordamida isbotlanadi?
11. Eyler-Venn diagrammalarini tushuntiring.
12. Eyler-Venn diagrammalari yordamida to'plamlarning tengligini isbotlash mumkinmi?

6-§. Dekart ko'paytma. Binar munosabatlar. Ekvivalentlik munosabati

Asosiy tushunchalar: tartiblangan juftlik, kortej, to'plamlarning to'g'ri (Dekart) ko'paytmasi, binar, n-ar munosabatlar, binar munosabatning aniqlanish va qiymatlar sohasi, binar munosabat inversiyasi, binar munosabatlar kompozisiyasi, refleksiv, antirefleksiv, simmetrik, antirefleksiv, tranzitiv binar munosabatlar, ekvivalentlik munosabati, faktor-to'plam.

$A_1, \dots, A_n$ - bo'sh bo'lmgan to'plamlar $\forall a_1 \in A_1, \dots, \forall a_n \in A_n$ -elementlardan tuzilgan barcha $(a_1, \dots, a_n)$ n-liklar to'plami $A_1, \dots, A_n$ to'plamlarning dekart ko'paytmasi deyiladi. $A_1, \dots, A_n$ to'plamlarning dekart ko'paytmasi $A_1 \times \dots \times A_n$ ko'rinishida belgilanadi.

$A \neq \varnothing$ to'plam berilgan bo'lsin. A^n ning ixtiyoriy ρ to'plamostini A to'plamda aniqlangan n-ar munosabat deyiladi. A^2 ning ixtiyoriy to'plamostisi A to'plamida berilgan binar munosabat deyiladi.

Agar R - binar munosabata tegishli barcha juftliklarning, barcha birinchi koordinatalaridan tuzilgan to'plam $Dom\,R$ aniqlanish, barcha ikkinchi koordinatalaridan tuzilgan to'plam esa $\operatorname{Im} R$ o'zgarish sohalari deyiladi.

$R^{\cup} = \{(b,a) \mid ((a,b) \in R\}$ munosabat R munosabatning inversiyasi deyiladi.

P va Q binar munosabatlar bo'sh bo'lmagan A to'plamda berilgan bo'lsin. U holda $P \circ Q = \{(a,c) \mid \exists\, b \in A, (a,b) \in Q \wedge (b,c) \in P\}$ to'plam P va Q binar munosabatlarning kompozisiyasi deyiladi.

A to'plamida R –binar munosabat berilgan bo'lsin.

a) Agar $\forall a \in A$ uchun $(a,a) \in R$ bo'lsa, R –binar munosabat refleksiv munosabat deyiladi;

b) Agar $(a,b) \in R$ bo'lishidan $(b,a) \in R$ bo'lishi kelib chiqsa, ya'ni $R^{-1} = R$ shart bajarilsa, R-simmetrik munosabat deyiladi;

c) Agar $\forall (a,b) \in R$ va $(b,a) \in R$ bo'lishidan $(a,c) \in R$ bo'lishi kelib chiqsa, ya'ni $R \circ R \subset R$ shart bajarilsa, R-tranzitiv munosabat deyiladi;

d) refleksiv, simmetrik va tranzitiv bo'lgan binar munosabat ekvivalentlik munosabati deyiladi.

$\forall a \in A$ uchun $\bar{a}$ orqali A to'plamning a ga ekvivalent bo'lgan barcha elementlarini belgilaymiz va bu to'plamni a element yaratgan ekvivalentlik sinfi deb ataymiz. R ekvivalentlik munosabati bo'yicha aniqlangan barcha ekvivalentlik sinflari to'plamiga A to'plamning R ekvivalentlik munosabati bo'yicha faktor-to'plami deyiladi.

Misol. ***R***, S, T $\subset$ A $\times$ A – binar munosabatlar uchun

$R \circ (S \setminus T) = (R \circ S) \setminus (R \circ T)$ tenglikni isbotlang.

Yechish. Binar munosabatlar tartiblangan juftliklardan iborat to'plamlar ekanligini bilgan holda to'plamlar ayirmasi, to'plamlar tengligi hamda binar munosabatlar kompozisiyasining ta'riflaridan foydalanib berilgan tenglikni isbotlaymiz:

1) $\forall (x,y) \in (R \circ (S \setminus T)) \Rightarrow \exists z \in A, (x,z) \in (S \setminus T) \wedge (z,y) \in R \Rightarrow$

$\Rightarrow (x,z) \in S \wedge (x,z) \notin T \wedge (z,y) \in R \Rightarrow (x,z) \in S \wedge (z,y) \in R \wedge$

$\wedge (x,z) \notin T \wedge (z,y) \in R \Rightarrow (x,y) \in (R \circ S) \wedge (x,y) \notin (R \circ T) \Rightarrow$

$\Rightarrow (x,y) \in ((R \circ S) \setminus (R \circ T))$. Demak, $R \circ (S \setminus T) \subset (R \circ S) \setminus (R \circ T)$;

2) $\forall (x,y) \in ((R \circ S) \setminus (R \circ T)) \Rightarrow (x,y) \in (R \circ S) \wedge (x,y) \in (R \circ T) \Rightarrow$

$\Rightarrow \exists z \in A, ((x,z) \in S \wedge (z,y) \in R) \wedge ((x,z) \notin T \wedge (z,y) \in R) \Rightarrow$

$\Rightarrow (x,z) \in S \wedge (x,z) \notin T \wedge (z,y) \in R \Rightarrow (x,z) \in (S \setminus T) \wedge (z,y) \in R \Rightarrow$

$\Rightarrow (x,y) \in (R \circ (S \setminus T))$. Demak, $(R \circ S) \setminus (R \circ T) \subset R \circ (S \setminus T)$.

Natijada $R \circ (S \setminus T) = (R \circ S) \setminus (R \circ T)$ tenglik isbotlandi.

Misol. M = {1, 2, . . . , 10} to'plamda berilgan

$R = \{\langle x,y\rangle \mid x, y \in M \wedge x = y - 1\}$ binar munosabatning xossalarini tekshiring va grafini chizing.

Yechish. Berilgan binar munosabatni qanday xossalarga bo'ysunishini tekshiramiz:

1) refleksivlik xossasi. $\forall (x \in M)$ $(x = x-1)$ yolg'on, chunki, masalan M to'plamning 2 elementi uchun $2 \neq 2 - 1$. Demak, R- refleksiv emas.

2) Antirefleksivlik xossasi. $\forall (x \in M)$ $\rceil$ $(x = x-1)$ rost. Demak, R- antirefleksiv.

3) Simmetriklik xossasi. $\forall (x,y \in M)$ $(x = y - 1 \Rightarrow y = x - 1)$ yolg'on. Chunki, masalan $3, 4 \in M$ uchun $3 = 4-1 \Rightarrow 4 = 3-1$ da birinchi mulohaza rost va ikkinchi mulohaza yolg'on bo'lganligi uchun implikasiya yolg'on. Demak, R- simmetrik emas.

4) Antisimmetriklik xossasi. $\forall (x,y \in M)$ $(x = y - 1 \wedge y = x - 1 \Rightarrow x = y)$ rost. Chunki, M to'plamning har qanday x, y elementlari uchun $x = y-1$ va $y = x-1$ mulohazalar bir vaqtda rost bo'la olmaydi. Bundan ularning kon'yunksiyasi berilgan to'plam elementlari uchun yolg'on. Birinchi mulohaza yolg'onbo'lgan implikasiya rost ekanligini e'tiborga olsak, R- antisimmetrik binar munosabat ekanligi kelib chiqadi.

5) Tranzitivlik xossasi.

$\forall (x,y,z \in M)$ $(x = y - 1 \wedge y = z - 1 \Rightarrow x = z - 1)$ yolg'onmulohaza. Chunki, masalan M to'plamning 3,4,5 elementlari uchun

$(3 = 4-1) \wedge (4 = 5 - 1) \Rightarrow (3 = 5-1)$ implikasiyada kon'yunksiya rost, lekin implikasiya natijasi yolg'on mulohaza. Implikasiya ta'rifiga ko'ra, $(3 = 4-1) \wedge (4 = 5 - 1) \Rightarrow (3 = 5-1)$ mulohaza yolg'on. Demak, R- tranzitiv emas.

6)R-ekvivalentlik munosabati bo'la olmaydi, chunki refleksivlik, simmetriklik, tranzitivlik xossalariga ega emas.

7)R-tartib munosabati bo'la olmaydi, chunki R antisimmetrik bo'lgani bilan tranzitiv emas.

Misol. $A = \{1,2\}$, $B = \{2,5\}$ to'plamlar uchun $R = A \times B$, $S = B \times A$ binar munosabatlarni topib, $R \circ S$, $S \circ R$, R^2, S^2 larni aniqlang.

Yechish. To'plamlarning to'g'ri ko'paytmasi, binar munosabatlar kompozisiyasi ta'riflaridan foydalanib quyidagi to'plamlarni hosil qilamiz:

$R = A \times B = \{(1,2), (1,5), (2,2), (2,5)\}$;

$S = B \times A = \{(2,1), (2,2), (5,1), (5,2)\}$

$R \circ S = \{(2,2), (2,5), (5,2), (5,5)\}$

$S \circ R = \{(1,1), (1,2), (2,1), 2,2)\}$

$R^2 = R \circ R = \{(1,2), (1,5), (2,2), (2,5)\}$;

$S^2 = S \circ S = \{(2,1), (2,2), (5,1), (5,2)\}$.

Misol. Berilgan A = {lola, shoda, olomon, osmon, olma, boshoq} so'zlaridan iborat to'plam va undagi S binar munosabat :

«x S y» ⇌ «x va y so'zlarda o harfi bir hil sonda qatnashgan» berilgan. A/S faktor-to'plamni aniqlang.

Yechish. Faktor to'plam - bo'sh bo'lmagan to'plamda aniqlangan ekvivalentlik munosabati yordamida hosil qilingan ekvivalentlik sinflaridan tuzilgan to'plam. Berilgan to'plam 6 ta so'zdan iborat to'plam va undagi har qanday ikkita x,y so'zlar berilgan binar munosabatda bo'ladi, agar bu so'zlar tarkibida o harfi bir xil sonda qatnashgan bo'lsa.

To'plamda berilgan S binar munosabat ekvivalentlik munosabati ekanligini isbotlaymiz:

1. S – refleksivlik munosabati, chunki A to'plamning har bir so'zini o'zi bilan solishtirsak, ularda o harfi bir hil sonda qatnashgan.

2. S – simmetriklik munosabati, chunki A to'plamning har qanday x, y so'zlari uchun agar x so'z bilan y so'zda o harfi bir hil sonda qatnashgan bo'lsa, u holda y so'z bilan x so'zlarda ham o harfi bir hil sonda qatnashadi.

3. S – tranzitivlik munosabati, chunki A to'plamning har qanday x, y, z so'zlari uchun agar x so'z bilan y so'zda va y so'z bilan z so'zda o harfi bir hil

sonda qatnashgan bo'lsa, u holda x so'z bilan z so'zlarda ham o harfi bir hil sonda qatnashadi.

Endi S ekvivalentlik munosabati yordamida ekvivalentlik sinflarini tuzamiz. Buning uchun «lola» so'zi bilan ekvivalentlik munosabatida bo'lgan so'zlarni bir to'plamga yig'amiz:

$S/_{lola}$ = {lola, shoda, olma}. Xuddi shunday yo'l bilan qolgan ekvivalentlik sinflarini tuzamiz:

$S/_{ismin}$ = {osmon, boshoq}, $S/_{olomon}$ = {olomon}.

U holda $A/S = \{S/_{lola},\ S/_{ismin},\ S/_{ilimin}\}$.

Misol va mashqlar

1. Quyidagi tengliklarni isbotlang

1.1. $(A \setminus B) \times C = (A \times C) \setminus (B \times C)$.

1.2. $A \times (B \setminus C) = (A \times B) \setminus (A \times C)$.

1.3. $(A \cup B) \times C = (A \times C) \cup (B \times C)$.

1.4. $A \times (B \cap C) = (A \times B) \cap (A \times C)$.

1.5. $(A \cap B) \times (C \cap D) = (A \times C) \cap (B \times D$.

1.6. $A \subset B \Rightarrow A \times C \subset B \times C$.

1.7. $A \cup B \subset C \Rightarrow A \times B = (A \times B) \cap (C \times B)$.

1.5. $(A \times B) \cup (B \times A) = C \times C \Rightarrow A = B = C$.

2. R, S, T – binar munosabatlar uchun quyidagilarni isbotlang:

2.1. $(R \cap S)^{\cup} = R^{\cup} \cap S^{\cup}$.

2.2. $(R \cup S)^{\cup} = R^{\cup} \cup S^{\cup}$.

2.3. $R \circ (S \circ T) = (R \circ S) \circ T$.

2.4. $(R \circ S)^{\cup} = S^{\cup} \circ R^{\cup}$.

2.5. $(R \cup S) \circ T = R \circ T \cup S \circ T$.

2.6. $R \circ (S \cup T) = (R \circ S) \cup (R \circ T)$.

2.7. $(R \cap S) \circ T \subset R \circ T \cap S \circ T$.

2.8. $R \circ (S \cap T) \subset R \circ S \cap R \circ T$.

2.9. Dom $(R^{\cup})$ = Im R ..

2.10. Im $(R^{\cup})$ = Dom R..

2.11. Dom $(R \circ S) \subset$ Dom S.

2.12. Im $(R \circ S) \subset$ Im R.

2.13. $(R \setminus S)^{\cup} = R^{\cup} \setminus S^{\cup}$.

2.14. R, S - tranzitiv $\Rightarrow$ $R \cup S$ – tranzitiv.

2.15. R, S – refleksiv $\Rightarrow$ $R \cup S$, $R^{\cup}$, $S^{\cup}$ – refleksiv.

2.16. R, S - simmetrik $\Rightarrow$ $R \cup S$, $R^{\cup}$, $S^{\cup}$ – simmetrik.

2.17. R, S - ekvivalent $\Rightarrow$ $R \cup S$, $R^{\cup}$, $S^{\cup}$ – ekvivalent.

2.18. R, S - antirefleksiv $\Rightarrow$ $R \cup S$, $R^{\cup}$, $S^{\cup}$ – antirefleksiv.

2.19. R, S - antisimmetrik $\Rightarrow$ $R \cup S$, $R^{\cup}$, $S^{\cup}$ – antisimmetrik.

3. $M_3=\{1,2,3\}$ to'plamda nechta turli ekvivalentlik munosabatlari mavjud? M_4 da-chi?

4. $M = \{1, 2, \ldots, 20\}$ to'plamda berilgan quyidagi binar munosabatlarning xossalarini tekshiring:

4.1. $R = \{ <x,y> \mid x,y \in M \wedge x \leq y + 1 \}$.

4.2. $R = \{ <x,y> \mid x,y \in M \wedge |x| = |y| \}$.

4.3. $R = \{ <x,y> \mid x,y \in M \wedge x \vdots y \}$.

4.4. $R = \{ <x,y> \mid x,y \in M \wedge x < y \}$.

4.5. $R = \{ <x,y> \mid x,y \in M \wedge x \leq y \}$.

4.6. $R = \{ <x,y> \mid x,y \in M \wedge x \neq y \}$.

4.7. $R = \{ <x,y> \mid x,y \in M \wedge x^2 + x = y^2 + y \}$.

4.8. $R = \{ <x,y> \mid x,y \in M \wedge x \vdots y \vee x < y \}$.

4.9. $R = \{ <x,y> \mid x,y \in M \wedge (x - y) \vdots 2 \}$.

4.10. $R = \{ <x,y> \mid x,y \in M \wedge x + y = 12 \}$.

4.11. $R = \{ <x,y> \mid x,y \in M \wedge x + y \leq 7 \}$.

4.12. $R = \{ <x,y> \mid x,y \in M \wedge (x > y \wedge x \vdots 3) \}$.

4.13. $R = \{ <x,y> \mid x,y \in M \wedge x + y \geq 10 \}$.

4.14. $R = \{ <x,y> \mid x,y \in M \wedge x - y = -2 \}$.

5. $R = A \times B$, $S = B \times A$ binar munosabatlar uchun R o S, S o R, R^2, S^2 larni aniqlang:

5.1. $A = \{1, 3, 5\}$, $B = \{1, 3, 15\}$;

5.2. $A = \{3, 6, 9\}$, $B = \{4, 8, 16\}$;

5.3. $A = B = \{1, 2, 3, 4\}$;

5.4. $A = \{0, 2, 4\}$, $B = \{\alpha, \beta, \gamma\}$;

5.5. $A = \{\square, \Diamond\}$, $B = \{\clubsuit, \blacklozenge, \heartsuit, \spadesuit\}$;

5.6. $A = \{\wedge, \vee, \Rightarrow, \Leftrightarrow\}$, $B = \{\cap, \cup, \in, \subset\}$.

6. Berilgan A to'plam va undagi S binar munosabat yordamida A / S faktor-to'plamni aniqlang:

6.1. A - tekislikdagi to'g'ri chiziqlar to'plami, S - parallellik munosabati.

6.2. A - tekislikdagi romblar to'plami, S - o'xshashlik munosabati.

6.3. A - tekislikdagi to'rtburchaklar to'plami, S - o'xshashlik munosabati.

6.4. $A = \{ ax + by + c = 0 \mid a, b, c \in R \}$, S - parallellik munosabati.

6.5. A - tekislikdagi muntazam ko'pburchaklar to'plami, S - o'xshashlik munosabati.

6.6. A - bir ko'chada joylashgan binolar to'plami, S - «qavatlar soni teng» munosabati.

6.7. A - tekislikdagi aylanalar to'plami, S - «radiuslari teng» munosabati.

6.8. A - maktabdagi sinflar to'plami, S - «o'quvchilar soni teng» munosabati.

6.9. A - maktabdagi sinflar to'plami, S - «qizlar soni teng» munosabati.

6.10. A - sinfdagi o'quvchilar to'plami, S - «ismlari bir hil harfdan boshlanadi» munosabati.

6.11. A - sinfdagi o'quvchilar to'plami, S - «ismlarda a harfi bir hil marta qatnashgan» munosabati.

6.12. A - tekislikdagi kesmalar to'plami , S - parallellik munosabati.

6.13. A - tekislikdagi vektorlar to'plami , S - tenglik munosabati.

6.14. A = Z , S - «p tub songa bo'lgandagi qoldiqlari teng» munosabati

Takrorlash uchun savollar

1. Tartiblangan juftlik nima?
2. Tartiblangan juftliklar qachon teng bo'ladi?
3. To'plamlarning to'g'ri (Dekart) ko'paytmasi nima?
4. Tartiblangan n lik qanday hosil qilinadi?
5. Binar munosabatga ta'rif bering. Misollar keltiring.
6. Binar munosabatning aniqlanish sohasiga misol keltiring.
7. Binar munosabatning o'zgarish sohasiga ta'rif bering.
8. Binar munosabat inversiyai qanday xosil qilinadi?
9. Binar munosabatlar kompozisiyasini misol yordamida tushuntiring.
10. Refleksiv binar munosabatni ta'riflang va misol keltiring.
11. Simmetrik binar munosabatni ta'riflang va misol keltiring.
12. Tranzitiv binar munosabatni ta'riflang va misol keltiring.
13. Ekvivalentlik binar munosabatini ta'riflang va misol keltiring.
14. To'plamni bo'laklash deganda nimani tushunasiz?
15. Faktor-to'plamni tushuntiring.

7-§. Akslantirish (funksiya). Tartib munosabati. Graflar

Asosiy tushunchalar: Akslantirish (funksiya), akslantirishning aniqlanish sohasi, akslantirish qiymatlar to'plami, akslantirishlar kompozisiyasi, in'ektiv syur'ektiv, biektiv, teskarilanuvchi akslantirish, tartib munosabati, qisman tartib, qat'iy tartib, chiziqli tartib, tartiblangan to'plam, to'la tartiblangan to'plam, binar munosabat grafi.

$f - A$ to'plamda berilgan binar munosabat bo'lsin. Agar $\forall x, y, z \in A$ lar uchun $(x, y) \in f$ va $(x, z) \in f$ bo'lishidan $y = z$ kelib chiqsa, u holda f binar munosabat akslantirish (funksiya) deyiladi.

$Dom\, f = \{x \,/\, \exists y (x, y) \in f\}$ to'plam funksiyaning aniqlanish sohasi, $\operatorname{Im} f = \{y \,/\, \exists x (x, y) \in f\}$ to'plam funksiyaning o'zgarish sohasi deyiladi.

f va g funksiyalar berilgan bo'lsin, u holda $f \circ g = \{(x, z) \,/ \exists\, t (x,t) \in g \text{ ва } (t, z) \in f\}$ to'plam f va g funksiyalarning kompozisiyasi deyiladi.

Agar $\forall x_1, x_2 \in A$ va $x_1 \neq x_2$ elementlar uchun $f(x_1) \neq f(x_2)$ bo'lsa, $f : A \to B$ -in'ektiv, $\operatorname{Im} f = B$ bo'lsa, syur'ektiv akslantirish deyiladi. Agar f ham syur'ektiv, ham in'ektiv akslantirish bo'lsa, u holda biektiv akslantirish deyiladi.

$f : A \to B,\ \ g : B \to A$ akslantirishlar berilgan bo'lsin, agar $f \circ g = E_B$ bo'lsa f akslantirish g akslantirishga chapdan teskari deyiladi.

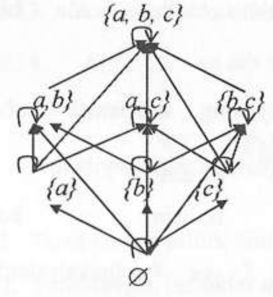
A to'plamda berilgan $R \subset A \times A$ antisimmetrik va tranzitiv munosabat A to'plamdagi tartib munosabati deyiladi. Tartib munosabati refleksiv munosabat bo'lsa, noqat'iy; antirefleksiv munosabat bo'lsa, qat'iy tartib munosabat deyiladi.

A to'plamda R- tartib munosabat berilgan bo'lsin. U holda, agar $\forall a, b \in A$ elementlar uchun xRy yoki $x = y$ yoki yRx munosabatlardan kamida bittasi albatta bajarilsa, bunday munosabot A to'plamdagi chiziqli tartib munosabat deyiladi.

Chiziqli bo'lmagan tartib munosabat, qisman tartib munosabat deyiladi.

Tekislikda chekli sondagi nuqtalardan va shu nuqtalarning ba'zilarini tutashtiruvchi chiziqlardan iborat geometrik figura graf deyiladi. Nuqtalar grafning uchlari, chiziqlar esa grafning qirralari deyiladi.

Misol. $A=\{a,b,c\}$ to'plam va $\mathcal{B}(A)$ -uning barcha to'plamostilari bo'lsin. U holda to'plamosti bo'lish munosabatini quyidagi graf yordamida ifoda qilish mumkin:



Misol va mashqlar

1. Quyidagi munosabatlardan qaysilari akslantirish bo'ladi? Akslantirishlarning aniqlanish va qiymatlar sohalarini aniqlang:

1.1. $R=\{(1,1),(2,1),(3,1)\}$.

1.2. $R=\{(1,1),(1,4),(3,2),(4,4)\}$.

1.3. $R=\{(n,n+1)\mid n\in N\}$.

1.4. $R=\{(1,z)\mid z\in Z\}$.

2. Maktab matematikasidan in'ektiv, syur'ektiv, biektiv, teskari funksiyalarga misollar keltiring.

3. $R=\{(1,1),(2,1),(3,2),(1,5),(4,4)\}$ binar munosabatni M_5 dagi qisman tartib munosabatgacha to'ldiring.

4. M_4 dagi barcha qisman tartib munosabatlarni graflar yordamida keltiring.

5. R, S, – binar munosabatlar uchun quyidagilarni isbotlang:

5.1. R, S - qat'iy tartib $\Rightarrow$ $R\cup S$, $S^{\cup}$ – qat'iy tartib.

5.2. R, S - qisman tartib $\Rightarrow$ $R\cup S$, $R^{\cup}$ – qisman tartib.

5.3. R, S - chiziqli tartib $\Rightarrow$ $R\cup S$, – chiziqli tartib.

6. $M=\{1, 2, \ldots, 20\}$ to'plamda berilgan quyidagi binar munosabatlarning xossalarini tekshiring va grafini chizing:

6.1. $R=\{\langle x,y\rangle \mid x,y\in M\wedge x^2=y^2\}$.

6.2. $R=\{\langle x,y\rangle \mid x,y\in M\wedge x<y\}$.

6.3. $R=\{\langle x,y\rangle \mid x,y\in M\wedge x+y\geq 20\}$.

6.4. $R=\{\langle x,y\rangle \mid x,y\in M\wedge (x+y)\,\vdots\,5\}$.

6.5. $R=\{\langle x,y\rangle \mid x,y\in M\wedge (x>y\wedge x\,\vdots\,3)\}$.

6.6. $R=\{\langle x,y\rangle \mid x,y\in M\wedge x-y=2\}$.

6.7. $R=\{\langle x,y\rangle \mid x,y\in M\wedge (x-y)\,\vdots\,4\}$.

6.8. $R=\{\langle x,y\rangle \mid x,y\in M\wedge x-y=6\}$.

Takrorlash uchun savollar

1. Akslantirishning aniqlanish, qiymatlar sohasiga misol keltiring.
2. Akslantirishlar kompozisiyasini tushuntiring.
3. Akslantirishlar kompozisiyasi xossalarini ayting.
4. In'ektiv akslantirishga maktab matematikasidan misol keltiring.
5. Syur'ektiv akslantirishga maktab matematikasidan misol keltiring.
6. Biektiv akslantirish maktabda qanday nomlangan? Misol keltiring.
7. Tartib munosabatga misollar keltiring.
8. Tartib munosabat turlarini maktab matematikasidan olingan misollar yordamida tushuntiring.
9. Tartiblangan to'plamlarga misollar keltiring.
10. Butun sonlar to'plami to'la tartiblangan to'plam bo'ladi-mi?
11. Qanday binar munosabatni graf yordamida ifodalash mumkin?

III MODUL. ALGEBRA VA ALGEBRAIK SISTEMALAR

8-§. Algebra. Faktor-algebra

Asosiy tushunchalar: Binar algebraik amal, n-ar algebraik amal, kommutativ binar amal, assosiativ binar amal, neytral element, regulyar element, simmetrik element, kongruensiya, algebra, algebra turi, gomomorfizm, izomorfizm, faktor-algebra, algebraik sistema.

A^n to'plamni A ga akslantiradigan har qanday akslantirish A to'plamda berilgan n-ar yoki n-o'rinli algebraik amal deyiladi. Bu erda n-manfiy bo'lmagan butun son bo'lib, algebraik amalning rangi deyiladi. $A \neq \varnothing$ to'plam va A da bajariladigan algebraik amallar to'plami Ω berilgan bo'lsin (A,Ω) - juftlik algebra deyiladi.

a) Agar $\forall a,b \in A$ uchun $a*b=b*a$ bo'lsa, u holda $*$-amali algebraik amal A to'plamida kommutativ deyiladi;

b) Agar $\forall a,b,c \in A$ uchun $a*(b*c)=(c*b)*c$ shart bajarilsa, $*$-A to'plamida assosiativ algebraik amal deyiladi;

c) Agar $\forall a \in A$ uchun shunday $e \in A$ topilib, $e*a=a$ shart bajarilsa, e element $*$ amalga nisbatan chap neytral element, agar $a*e=a$ shart bajarilsa, o'ng neytral element, agar ikkala shart ham bajarilsa neytral element deyiladi.

$a \in A$ element va $\forall b,c \in A$ elementlar uchun $a*b=a*c$ tenglikdan $b=c$ kelib chiqsa, u holda a element chap regulyar element deyiladi.

$a \in A$ element uchun shunday $a' \in A$ element topilib, $a'*a=e$ bo'lsa, a' element a elementga chap simmetrik element deyiladi.

R A to'plamdagi ekvivalentlik munosabati bo'lsin. Agar $a_1,a_2,b_1,b_2 \in A$

R elementlar uchun a_1Rb_1 va a_2Rb_2 shartlardan $(a_1*a_2)R(b_1*b_2)$ kelib chiqsa, u holda R-ekvivalentlik munosabati kongruensiya deyiladi.

Agar (A,Ω) algebra berilgan bo'lsa, Ω-to'plamdagi amallarning ranglaridan iborat to'plam algebraning turi deyiladi.

$A \neq \varnothing$ to'plam uchun Ω-A dan aniqlangan amallar to'plami Ω'-A da aniqlangan munosabatlar to'plami bo'lsin. U holda (A,Ω,Ω'')- tartiblangan uchlik-algebraik sistema deyiladi.

(A,Ω), (B,Ω') algebralar berilgan bo'lsin. Ω dagi barcha amallarni saqlaydigan $\varphi: A \to B$ akslantirish (A,Ω) agebraning (B,Ω) algebraga gomomorfizmi deyiladi.

$\varphi: A \to B$ akslantirish (A,Ω) agebraning (B,Ω') algebraga gomomorfizmi bo'lsin. U holda agar φ- in'ektiv akslantirish bo'lsa, monomorfizm, φ- syur'ektiv akslantirish bo'lsa, epimorfizm, φ- biektiv akslantirish bo'lsa izomorfizm deyiladi.

Algebrani o'zini o'ziga gomomorf akslantirish endomorfizm, algebrani o'zini o'ziga izomorf akslantirish esa avtomorfizm deyiladi.

(A,Ω_1) va (A,Ω_2) bir hil tipli algebralar berilgan bo'lib, $B \subset A$ bo'lsin. Agar $\forall \omega_1 \in \Omega$ n-ar algebraik amalga Ω_2 dan mos keladigan n-ar algebraik amalni ω_2 orqali belgilaymiz. Agar $\forall b_1,\dots,b_n \in B$ uchun $\omega_2(b_1,\dots,b_n)=\omega_1(b_1,\dots,b_n)$ tenglik bajarilsa, u holda ω_2 n-ar algebraik amal ω_1 n-ar algebraik amalning B to'plami bo'yicha cheklangani (B,Ω_1) algebra esa (A,Ω_2) algebraning qism algebrasi yoki algebraosti deyiladi.

(A,Ω) algebra va $\sim$ Ω dagi har bir amalga nisbatan kongruensiya bo'lsin. Ω^* to'plam esa $A/\sim$ faktor-to'plamda aniqlangan va Ω dagi amallar bilan assosirlangan barcha amallar to'plami bo'lsin. U holda $(A/\sim,\Omega^*)$- algebra (A,Ω) algebraning $\sim$- kongruensiya bo'yicha faktor- algebrasi deyiladi.

Misol. Z- butun sonlar to'plami bo'lsin. Z da $a \sim b$ deymiz va a ga $a-b$ juft son bo'lsa, $\sim$- munosabat kongruensiya bo'lishi ravshan. Bu munosabat bo'yicha ekvivalentlik sinflari faqat ikkita bo'lib, ular $[0]$, $[1]$ sinflardan iborat. Bu sinflar to'plamini $Z/\sim$ orqali belgilaylik, $\forall[a],[b]\in Z/\sim$ uchun $\oplus$, Θ amallarini $[a]\oplus[b]=[a+b]$. $[a]\odot[b]=[a\bullet b]$ tengliklar orqali aniqlasak, $(\{[0],[1]\},\oplus,\odot,[0],[1])$ algebra $(Z+,\bullet,0,1)$ algebraning faktor algebrasi bo'ladi.

Misol va mashqlar

1. Berilgan munosabatlarning qaysilari A to'plamda aniqlangan amal bo'lishini tekshiring va uning rangini aniqlang:

1.1. $\circ=\{(a,b,c)\mid a,b,c\in R\wedge a=bc\},A=R.$

1.2. $\circ=\{(a,b)\mid a,b\in R_+\wedge b^3=a\},A=R_+.$

1.3. $\circ=\{(a,b,c)\mid a,b,c\in R\wedge c=a^b\},A=R.$

1.4. $\circ=\{(a,b)\mid a,b\in Z\wedge ab=1\},A=Z.$

1.5. $\circ=\{(a,b,c,d)\mid a,b,c,d\in Z\wedge d=[a,b,c]\},A=Z.$

2. A to'plamda $\circ$ amal quyidagi Keli jadvali yordamida berilgan:

$\circ$	a	b	c	d
a	a	a	a	a
b	b	b	b	b
c	c	c	c	c
d	d	d	d	d

Uning assosiativligini, kommutativ emasligini isbotlang. Neytral element mavjud-mi?

3. Natural sonlar to'plamida har qanday n,m natural sonlar uchun quyidagi amallar kommutativ, assosiativ ekanligini isbotlang. Bu amallarga nisbatan neytral element mavjud-mi?

3.1. $n*m=l$, $l=(n,m)$.

3.2. $n*m=l$, $l=[n,m]$.

4. Quyidagi shartlar asosida algebraga misol keltiring:

4.1. Ikkita binar amal va bitta unar amal aniqlangan.

4.2. Ikkita binar amal va ikkita unar amal aniqlangan.

4.3. Uchta binar amal va bitta binar amal aniqlangan.

5. R^2 to'plamda quyidagi amallarning kommutativ, assosiativ, distributiv ekanligini tekshiring:

5.1. $(a,b)+(c,d)=(a+c,b+d),(a,b)\cdot(c,d)=(ac,ad+bc)$.

5.2. $(a,b)+(c,d)=(a+c,b+d),(a,b)\cdot(c,d)=(ac+bd,ad+bc)$.

6. Quyidagi algebralar orasida gomomorfizm o'rnating va uning turini aniqlang:

6.1. $<\{3^z\mid z\in Z\};\ \bullet,\ ^{-1},1>\ \wedge\ <Z;+,-,0>.$

6.2. $<Z;+,-,\ 0>\ \wedge\ <2Z;+,-,0>.$

6.3. $<\{a+bi\mid a,\ b\in R\wedge i^2=-1\};+,-,0\ >\wedge<R^2;+,-,0>.$

6.4. $<Z;+,-,\ 0>\ \wedge<Z_2,+,-,0>.$

6.5. $<Z^-;+,>\wedge\ <Z^+;+>.$

6.6. $<\{2^z\mid z\in Z\};\ \bullet,\ ^{-1},1>\ \wedge\ <\{3^z\mid z\in Z\};\ \bullet,\ ^{-1},1>.$

6.7. $<2Z;+,-,0>\ \wedge\ <5Z;+,-,0>.$

6.8. $<\{a+b\sqrt{p}\mid a,b\in Q\};+,\bullet>\wedge<\{a-b\sqrt{p}\mid a,b\in Q\};+,\bullet>.$

7. Quyidagi berilgan A to'plam, undagi S binar munosabat tashkil qilgan A/S faktor-to'plamni faktor-algebragacha to'ldirish mumkin-mi:

7.1. $A=\{\ ax+by+c=0\mid a,\ b,\ c\in R\ \}$, S - parallellik munosabati.

7.2. $A=\{\ ax+by=0\mid a,\ b\in R\ \}$, S - tenglik munosabati.

7.3. A - tekislikdagi kesmalar to'plami, S - parallellik munosabati.

7.4. A - tekislikdagi kesmalar to'plami, S - tenglik munosabati.

7.5. A - tekislikdagi vektorlar to'plami, S - parallellik munosabati.

7.6. A - tekislikdagi vektorlar to'plami, S - tenglik munosabati.

7.7. A = Z, S - «p tub songa bo'lgandagi qoldiqlari teng» munosabati.

8. Algebraosti bo'lish munosabati noqat'iy tartib munosabat bo'lishini isbotlang.

9. Agar φ-(A,Ω_1) algebraning (B,Ω_2) algebraga izomorfizmi bo'lsa, u holda φ ga teskari bo'lgan φ^{-1} akslantirish (B,Ω_2) algebraning (A,Ω_1) algebraga izomorfizmi ekanligini isbotlang.

10. $(G_1,\Omega_1),(G_2,\Omega_2),(G,\Omega)$- bir hil turli algebralar berilgan bo'lib, $G_1 \cong G_2$, (G_2,Ω_2)- algebra (G,Ω)- algebraning algebraostisi bo'lsin. U holda (G_1,Ω_1)- qism algebradan iborat qism algebraga ega bo'lgan (G,Ω) algebraga izomorf (G_3,Ω_3) algebra mavjudligini isbotlang.

11. $\varphi: A \to B$ akslantirish (A,Ω_1) algebraning (B,Ω_2) algebraga epimorfizmi, $R=\{(x'x'') \mid \forall x',x'' \in A, \varphi(x')=\varphi(x'')\}$-esa A da aniqlangan ekvivalentlik munosabati bo'lsin. U holda $(A/R,\Omega^*)$ faktor algebra (B,Ω_1) algebraga izomorfligini isbotlang.

Takrorlash uchun savollar

1. Algebra tushunchasiga maktab matematikasidan misollar keltiring.
2. Algebraning turi qanday aniqlanadi?
3. Algebralar gomomorfizmini tushuntiring.
4. Monomorfizm, epimorfizmga misollar keltiring.
5. Izomorfizm, avtomorfizm ta'rifidagi umumiy, farqli shartlarni aniqlang.
6. Gomomorfizmlar kompozisiyasi yana gomomorfizm ekanligini isbotlang.
7. Biektiv akslantirishlar izomorfizm bo'la oladimi?
8. Algebralar izomorfizmi ekvivalentlik munosabati ekanligini asoslang.
9. Algebraostilar kesishmasi yana algebra bo'lishini isbotlang.
10. Faktor-algebra tushunchasini misol yordamida tushuntiring.
11. Akademik litsey, maktab matematikasidan algebraik sistemaga doir misollar keltiring.

9-§. Gruppa. Halqa. Maydon

Asosiy tushunchalar: gruppoid, yarimgruppa, monoid, gruppa, gruppalar gomomorfizmi, gruppaosti, halqa, kommutativ halqa, butunlik sohasi, halqalar gomomorfizmi, qismhalqa.

$A \neq 0$ to'plam va unda aniqlangan $*$ binar algebraik amal berilgan bo'lsin. U holda $(A,*)$ juftlik gruppoid deb ataladi.

$(A,*)$-gruppoidda $*$-assosiativ amal bo'lsa, bunday gruppoid yarimgruppa deyiladi.

Neytral elementga ega bo'lgan yarimgruppa monoid deyiladi.

Bizga $(2,1)$ turli $(G,*,')$ algebra berigan bo'lib quyidagi shartlar bajarilsin.

1. $*$-binar algebraik amal assosiativ, ya'ni $\forall a,b,c \in G$ uchun $(a*b)*c = a*(b*c)$ bo'lsin.

2. G da neytral element mavjud, ya'ni $\forall a \in G$ uchun shunday $e \in G$ topilib, $e*a=a$ shart bajarilsin.

3. Har qanday $a \in G$ uchun $a'*a=e$ bo'lsin.

U holda $(G,*,')$- algebra gruppa deyiladi.

Gruppadagi amal kommutativ, ya'ni $\forall a,b \in G$ uchun $a*b=b*a$ shart bajarilsa, bunday gruppa abel' gruppasi deyiladi.

Gurppadagi elementlar soni uning tartibi deyiladi. Agar gruppa tartibi natural sondan iborat bo'lsa, bunday gruppa chekli tartibli gruppa, aks holda cheksiz tartibli gruppa deyiladi.

Gruppadagi binar algebraik amal "•" bo'lsa, bunday gruppani mul'tiplikativ gruppa, "+" bo'lsa additiv gruppa deymiz.

Gruppalar nazariyasida gomomorfizm, izomorfizm, gruppaosti tushunchalari algebradagi mos tushunchalarning xususiy xollaridir.

Agar $(K,+,-,\bullet)$ $(2,1,2)$ turli algebra uchun quyidagi shartlar bajarilsa

(1) $(K,+,-,)$ abel' gruppasi;

(2) $(K,\bullet)$-yarim gruppa;

(3) $\forall a,b,c \in K$ uchun $a\bullet(b+c)=a\bullet b+a\bullet c$ va $(b+c)\bullet a=b\bullet a+c\bullet a$ u holda $(K,+,-,\bullet)$- algebra halqa deyiladi.

$(K,+,-,)$ additiv gruppaning neytral elementi halqaning noli deyiladi va 0 orqali belgilanadi.

Agar ko'paytirish amali assosiativ bo'lsa, halqa assosiativ halqa, ko'paytirish amaliga nisbatan birlik element mavjud bo'lsa, halqa birlik elementli halqa deyiladi.

Nolning bo'luvchilariga ega bo'lmagan assosiativ, kommutativ halqada $1\neq 0$ shart bajarilsa, bunday halqa butunlik sohasi deyiladi.

$(K,+,-,\bullet)$ halqa berilgan bo'lsin. L esa K ning bo'sh bo'lmagan to'plamostisi bo'lsin. Agar L to'plam K dagi $+,-,\bullet$ amallariga nisbatan algebraik yopiq bo'lsa, ya'ni $\forall a,b\in L$ uchun $a+b\in L,\ \ a\bullet b\in L,\ \ -a\in L$ shartlar bajarilsa $(L,+,-,\bullet)$- algebra $(K,+,-,\bullet)$ halqaning halqaostisi deyiladi.

Misol. $K=\{a+b\sqrt{p}\mid a,b\in R\}$ to'plam maydon tashkil etishini isbotlang.

Yechish. Maydon ta'rifiga ko'ra berilgan to'plamda quyidagi shartlar bajarilishini tekshiramiz:

1) $\forall(z_1,z_2\in K),\exists!(z\in K)\ (z_1+z_2=z)$;

2) $\forall(z_1,z_2\in K)\ (z_1+z_2=z_2+z_1)$;

3) $\forall(z,z_1,z_2\in K),\ ((z+z_1)+z_2)=z+(z_1+z_2))$;

4) $\forall(z\in K),\exists!(e\in K)\ (z+e=z)$;

5) $\forall(z\in K),\exists\ (z'\in K)\ (z+z'=e)$;

6) $\forall(z_1,z_2\in K),\exists!(z\in K)\ (z_1\cdot z_2=z)$;

7) $\forall(z_1,z_2\in K)\ (z_1\cdot z_2=z_2\cdot z_1)$;

8) $\forall(z,z_1,z_2\in K),\ ((z\cdot z_1)\cdot z_2)=z\cdot(z_1\cdot z_2))$;

9) $\forall(z\in K),\exists!(e\in K)\ (z\cdot e=z)$;

10) $\forall(z\in K),\exists\ (z'\in K)\ (z\cdot z'=e)$;

1. To'plamning ixtiyoriy $z_1=a_1+b_1\sqrt{p}$; $z_2=a_2+b_2\sqrt{p}$ elementlari uchun $z_1+z_2=(a_1+b_1\sqrt{p})+(a_2+b_2\sqrt{p})=(a_1+a_2)+(b_1+b_2)\sqrt{p}$ tenglik bilan aniqlanuvchi shu to'plamning $a\ +b\ \sqrt{p}=$ z elementi mavjud. Demak, K to'plamda qo'shish amali aniqlangan. <K;+> - additiv gruppoid.

2. To'plamning ixtiyoriy $z_1=a_1+b_1\sqrt{p},z_2=a_2+b_2\sqrt{p}$ elementlari uchun $z_1+z_2=(a_1+a_2)+(b_1+b_2)\sqrt{p})=(a_2+a_1)+(b_2+b_1)\sqrt{p}=z_2+z_1$. Demak, qo'shish amali kommutativ va <K;+> - additiv abel gruppoid.

3. To'plamning ixtiyoriy $z=a+b\sqrt{p},z_1=a_1+b_1\sqrt{p},z_2=a_2+b_2\sqrt{p}$ elementlari uchun $(z+z_1)+z_2=((a+a_1)+(b+b_1)\sqrt{p})+(a_2+b_2\sqrt{p})=$ $=((a+a_1)+a_2+((b+b_1)+b_2)\sqrt{p})=a+(a_1+a_2)+(b+(b_1+b_2))\sqrt{p}=$ $=z+(z_1+z_2)$. Demak, qo'shish amali assosiativ va <K;+> - additiv abel yarimgruppa.

4. To'plamning ixtiyoriy $z=a+b\sqrt{p}$ elementi uchun $z+e=z$ tenglikni qanoatlantiruvchi elementni aniqlaymiz: $(a+b\sqrt{p})+(x+y\sqrt{p})=a+b\sqrt{p}$ tenglikdan $(a+x)+(b+y)\sqrt{p}=a+b\sqrt{p}$ tenglikni va undan $\begin{cases}a+x=a\\ b+y=b\end{cases}$ tenglamalar sistemasini hosil qilamiz. Uning yechimi $\begin{cases}x=0\\ y=0\end{cases}$ bo'lib, bundan $e=0+0\sqrt{p}=0$ hosil bo'ladi. Demak, K to'plamda qo'shish amaliga nisbatan neytral element mavjud ekan. <K;+> - additiv abel monoidni tashkil etdi..

5. To'plamning ixtiyoriy $z=a+b\sqrt{p}$ elementi uchun $z+z'=e$ tenglikni qanoatlantiruvchi elementni aniqlaymiz: $(a+b\sqrt{p})+(x+y\sqrt{p})=0+0\sqrt{p}$ tenglikdan $(a+x)+(b+y)\sqrt{p}=0+0\sqrt{p}$ tenglikni va undan $\begin{cases}a+x=0\\ b+y=0\end{cases}$ tenglamalar sistemasini hosil qilamiz. Uning yechimi $\begin{cases}x=-a\\ y=-b\end{cases}$ bo'lib, bundan

$z'=-a+(-b)\sqrt{p}=-(a+b\sqrt{p})$ hosil bo'ladi. Demak, K to'plamda qo'shish amaliga nisbatan simmetrik element mavjud ekan. <K;+> - additiv abel gruppani tashkil etdi.

6. To'plamning ixtiyoriy $z_1=a_1+b_1\sqrt{p}$; $z_2=a_2+b_2\sqrt{p}$ elementlari uchun $z_1\cdot z_2=(a_1+b_1\sqrt{p})\cdot(a_2+b_2\sqrt{p})=(a_1\cdot a_2+pb_1b_2)+(a_1b_2+b_1a_2)\sqrt{p}$ tenglik bilan aniqlanuvchi shu to'plamning $a+b\sqrt{p}=$ z elementi mavjud. Demak, K to'plamda ko'paytirish amali aniqlangan. <K;·> - multiplikativ gruppoid.

7. To'plamning ixtiyoriy $z_1=a_1+b_1\sqrt{p}, z_2=a_2+b_2\sqrt{p}$ elementlari uchun

$$z_1\cdot z_2=(a_1a_2+pb_1b_2)+(a_1b_2+b_1a_2)\sqrt{p})=(a_2a_1+pb_2b_1)+$$
$$+(a_2b_1+b_2a_1)\sqrt{p}=z_2\cdot z_1$$

. Demak, ko'paytirish amali kommutativ va <K;·> - multiplikativ abel gruppoid.

8. To'plamning ixtiyoriy $z=a+b\sqrt{p}, z_1=a_1+b_1\sqrt{p}, z_2=a_2+b_2\sqrt{p}$ elementlari uchun $(z\cdot z_1)\cdot z_2=((aa_1+pbb_1)+(ab_1+ba_1)\sqrt{p})\cdot(a_2+b_2\sqrt{p})=$

$$=((aa_1)a_2+p(bb_1)a_2+p(ab_1)b_2+p(ba_1)b_2)+((aa_1)b_2+p(bb_1)b_2+(ab_1)a_2+$$
$$+(ba_1)a_2)\sqrt{p}=(a(a_1a_2)+pb(b_1a_2)+pa(b_1b_2)+pb(a_1b_2))+$$
$$+(a(a_1b_2)+pb(b_1b_2)+a(b_1a_2)+b(a_1a_2))\sqrt{p}=(a+b\sqrt{p})\times$$
$$\times((a_1a_2+pb_1b_2)+(a_1b_2+b_1a_2)\sqrt{p})=(a+b\sqrt{p})((a_1+b_1\sqrt{p})\cdot(a_2+b_2\sqrt{p}))=$$

$=z\cdot(z_1\cdot z_2)$. Demak, ko'paytirish amali assosiativ va <K;·> - multiplikativ abel yarimgruppa.

9. To'plamning ixtiyoriy $z=a+b\sqrt{p}$ elementi uchun $z\cdot e=z$ tenglikni qanoatlantiruvchi elementni aniqlaymiz: $(a+b\sqrt{p})(x+y\sqrt{p})=a+b\sqrt{p}$ tenglikdan $(ax+pby)+(ay+bx)\sqrt{p}=a+b\sqrt{p}$ tenglikni va undan $\begin{cases} ax+pby=a \\ ay+bx=b \end{cases}$ tenglamalar sistemasini hosil qilamiz. Uning yechimi $\begin{cases} x=1 \\ y=0 \end{cases}$ bo'lib, bundan $e=1+0\sqrt{p}=1$ hosil bo'ladi. Demak, K to'plamda ko'paytirish

amaliga nisbatan neytral element mavjud ekan. <K;·> - multiplikativ abel monoidni tashkil etdi..

10. To'plamning ixtiyoriy noldan farqli $z=a+b\sqrt{p}$ elementi uchun $z\cdot z'=e$ tenglikni qanoatlantiruvchi elementni aniqlaymiz: $(a+b\sqrt{p})(x+y\sqrt{p})=1+0\sqrt{p}$ tenglikdan $(ax+pby)+(ay+bx)\sqrt{p}=1+0\sqrt{p}$ tenglikni va undan $\begin{cases} ax+pby=1 \\ ay+bx=0 \end{cases}$ tenglamalar sistemasini hosil qilamiz. Uning yechimi $\begin{cases} x=\dfrac{a}{a^2-pb^2} \\ y=\dfrac{-b}{a^2-pb^2} \end{cases}$ bo'lib, bundan $z'=\dfrac{a}{a^2-pb^2}+\dfrac{-b}{a^2-pb^2}\sqrt{p}$ hosil bo'ladi. Demak, K to'plamda ko'paytirish amaliga nisbatan simmetrik element mavjud ekan.

<K;·,$^{-1}$,1> - multiplikativ abel gruppani tashkil etdi.

K to'plam qo'shish va ko'paytirish amallariga nisbatan abel gruppa shartlariga bo'ysunganligi uchun <K;+,·,-,$^{-1}$,0,1> - maydon bo'ladi.

Misol. $K=\{a+b\sqrt{p}\mid a,b\in R\}$ va $P=\{a+b\sqrt{q}\mid a,b\in R\}$ to'plamlar tashkil etgan maydonlar orasida izomorfizm o'rnating.

Yechish. Algebralar izomorfizmi ta'rifiga ko'ra berilgan <K;+,·,-,$^{-1}$,0,1> maydon elementlarini <R;+,·,-,$^{-1}$,0,1> maydon elementlariga akslantiradigan akslantirish asosiy amallarni saqlashi, in'ektiv va syur'ektiv bo'lishi kerak.

$f:K\to P$ akslantirishni $f(a+b\sqrt{p})=a+b\sqrt{q}$ ko'rinishda olamiz.

K to'plamning ixtiyoriy $z_1=a_1+b_1\sqrt{p}, z_2=a_2+b_2\sqrt{p}$ elementlariga R to'plamning $t_1=a_1+b_1\sqrt{q}, t_2=a_2+b_2\sqrt{q}$ elementlari mos keladi. Tanlab olingan akslantirish izomorfizm ekanligini isbotlaymiz:

1) $\forall z_1,z_2\in K$ uchun $f(z_1+z_2)=f((a_1+b_1\sqrt{p})+(a_2+b_2\sqrt{p}))=$
$=f((a_1+a_2)+(b_1+b_2)\sqrt{p}))=(a_1+a_2)+(b_1+b_2)\sqrt{q}=(a_1+b_1\sqrt{q})+$

$+(a_2+b_2\sqrt{q})=f(a_1+b_1\sqrt{p})+f(a_2+b_2\sqrt{p})$. Demak, qo'shish amali akslantirish natijasida saqlanadi.

2) $\forall z_1,z_2\in K$ uchun $f(z_1\cdot z_2)=f((a_1+b_1\sqrt{p})\cdot(a_2+b_2\sqrt{p}))=$

$$=f((a_1a_2+pb_1b_2)+(a_1b_2+b_1a_2)\sqrt{p}))=(a_1a_2+qb_1b_2)+(a_1b_2+b_1a_2)\sqrt{q}=$$

$$=(a_1+b_1\sqrt{q})+(a_2+b_2\sqrt{q})=f(a_1+b_1\sqrt{p})\cdot f(a_2+b_2\sqrt{p}).$$

Demak, ko'paytirish amali akslantirish natijasida saqlanadi.

3) $f(0)=f(z+(-z))=f((a+b\sqrt{p})+(-a-b\sqrt{p}))=$

$$f((a-a)+(b-b)\sqrt{p}))=(a-a)+(b-b)\sqrt{q}=(a+b\sqrt{q})+$$

$+(-a-b\sqrt{q})=f(a+b\sqrt{p})+(-f(a+b\sqrt{p}))=0$. Demak, qo'shish amaliga nisbatan neytral element neytral elementga, simmetrik element simmetrik elementga o'tdi.

4) $f(1)=f(z\cdot z^{-1})=f((a+b\sqrt{p})\cdot(\frac{a}{a^2-pb^2}+\frac{-b}{a^2-pb^2}\sqrt{p}))=$

$$=1=(a+b\sqrt{q})\cdot(\frac{a}{a^2-qb^2}+\frac{-b}{a^2-qb^2}\sqrt{q})=(a+b\sqrt{q})\cdot(a+b\sqrt{q})^{-1}=+$$

$f(z)\cdot f^{-1}(z)=1$. Demak, ko'paytirish amaliga nisbatan neytral element neytral elementga, simmetrik element simmetrik elementga o'tdi. Aniqlangan akslantirishning gomomorfizm ekanligini isbotladik.

5) $\forall z_1,z_2\in K$ lar uchun $f(z_1)=f(z_2)$ ekanligidan $a_1+b_1\sqrt{q}=a_2+b_2\sqrt{q}$ kelib chiqadi. Bu shart $a_1=a_2, b_1=b_2$ shartlar bajarilganda to'g'ri. Bundan esa, $a_1+b_1\sqrt{p}=a_2+b_2\sqrt{p}$ ni hosil qilamiz. Demak, bir-biriga teng tasvirlarga bir-biriga teng asllar mos keldi. Tekshirilayotgan akslantirish in'ektiv akslantirish ekan.

6) R To'plamdan olingan har qanday $f(z)=a+b\sqrt{q}$ elementga $a+b\sqrt{p}\in K$ element mos keladi. Demak, akslantirish syur'ektiv ekan.

Tekshirilgan xossalarga ko'ra, $f:<K,+,\cdot,-,^{-1},0,1>\to<P,+,\cdot,-,^{-1},0,1>$ akslantirish izomorfizm.

Misol va mashqlar

1. Gruppadagi ixtiyoriy elementga chap teskari element, shu elementga o'ngdan ham teskari bo'lishini isbotlang.

2. Gruppada o'ng birlik element, chap birlik element bo'lishini isbotlang.

3. Gruppaning ixtiyoriy a va b elementlari uchun $ax=b$ va $ya=b$ tenglamalarning har biri yagona yechimga ega bo'lishini isbotlang.

4. Quyidagi to'plamlarni mul'tiplikativ gruppa tashkil etishini isbotlang:

4.1. $G=\{a+b\sqrt{2}\mid a,\ b\in Q,\ a^2+b^2>0\}$.

4.2. $G=\{p^z\mid p$ - tub son $, z\in Z\}$.

5. Quyidagi to'plamlarni additiv gruppa tashkil etishini isbotlang:

5.1. $G=\{a+b\sqrt{3}\mid a,\ b\in Z\}$

5.2. $G=\{\frac{a}{7^k}\mid a\in Z, k\in N\}$

5.3. $G=\{a-b\sqrt{p}\mid a,\ b\in Z$; p - tub son $\}$

6. $(G,\bullet,^{-1}),(H,\bullet,^{-1})$ gruppalar berilgan bo'lsin. $\varphi:G\to H$ -akslantirish gomomorf akslantirish bo'lishi uchun $\forall a,b\in G$, $\varphi(a\bullet b)=\varphi(a)\bullet\varphi(b)$ bo'lishi etarliligini isbotlang.

7. $(G,\bullet,^{-1})$ gruppa berilgan bo'lsin $H\neq\varnothing$ $H\subset G$ to'plamosti gruppaosti bo'lishi uchun $\forall a,b\in H$ elementlari uchun $a\bullet b^{-1}\in H$ bo'lishi zarur va etarli. Isbotlang.

8. $(K,+,-,\bullet)$ halqaning a,b,c lar ixtiyoriy elementlari bo'lsin, quyidagi xossalarni isbotlang:

1) agar $a+b=a$ bo'lsa, $b=0$.

2) agar $a+b=0$ bo'lsa, $a=-b$.

3) $-(-a)=a$.

4) $0\bullet a=a\bullet 0=0$.

5) $(-a)(-b) = a \bullet b$.

6) $(a-b) \bullet c = ca - bc$.

7) $c(a-b) = ca - cb$.

9. Quyidagi to'plamlarni halqa tashkil etishini isbotlang:

9.1. $G = \{a + b\sqrt{2} \mid a, b \in Q\}$.

9.2. $G = \{a - b\sqrt{p} \mid a, b \in Z$; p - tub son $\}$.

9.3. $< Z_{11} ; +, \cdot >$.

10. $(Z, +, -, \bullet)$ butun sonlar halqasi, (K, ⊕, ⊖, ⊙) - $K = \{0, e, a, b\}$ to'plamida amallari quyidagi jadvallar orqali berilgan halqa bo'lsin:

⊙	0	e	a	b
0	0	0	0	0
e	0	e	a	b
a	0	a	0	a
b	0	b	a	e

⊕	0	e	a	b
0	0	e	0	b
e	e	a	a	a
a	a	b	0	e
b	b	a	a	a

Bu halqalar orasida gomomorfizm o'rnating.

11. Butunlik sohasi bo'lmagan kommutativ halqaga misol keltiring.

12. Halqaning barcha qismhalqalari kesishmasi yana shu halqaga qismhalqa bo'lishini isbotlang.

13. Jismga tashkil etadigan algebraga misol tuzing.

14. Quyidagi algebralarni maydon tashkil etishini isbotlang:

14.1. $< \{a - b\sqrt{3} \mid a, b \in Q\}; +, \cdot >$.

14.2. $< Z_7 ; +, \cdot >$.

15. $< \{a + b\sqrt{p} \mid a, b \in Q\}; +, \bullet >$ va $< \{a - b\sqrt{p} \mid a, b \in Q\}; +, \bullet >$ maydonlar orasida izomorfizm o'rnating.

Takrorlash uchun savollar

1. Yarimgruppa deb nimaga aytiladi?
2. Monoidga ta'rif bering va misol keltiring.
3. Gruppa ta'rifini keltiring. Uning asosiy xossalarini ayting.
4. Additiv, mul'tiplikativ gruppalarga algebra, geometriya kursidan misollar keltiring.
5. Gruppalar gomomorfizmining qanday turlarini bilasiz?
6. Har qanday gomomorfizm izomorfizm bo'la oladimi, yoki aksincha?
7. Gruppalar avtomorfizmi nima?
8. Gruppaosti tushunchasiga misollar keltiring.
9. Halqaning qanday turlarini bilasiz?
10. Halqalar gomomorfizmi, izomorfizmiga misollar keltiring.
11. Halqalar avtomorfizmi ta'rifini bayon qiling.
12. Halqaostilar kesishmasi yana halqaosti bo'lishini isbotlang.

IV MODUL. ASOSIY SONLI SISTEMALAR

10-§. Natural sonlar sistemasi. Matematik induksiya prinsipi

Asosiy tushunchalar: natural sonlar sistemasi aksiomalari, matematik induksiya prinsipi, induksiya qadami, natural sonlar yarimhalqasi, tartib munosabati.

N to'plami unda bajariladigan $+,\bullet$ binar algebraik amallar, N to'plamining ajratilgan elementlari 0 va 1 lardan iborat $(N,\bullet,+,0,1)$ algebra uchun quyidagi aksiomalar (shartlar) bajarilsin:

I. $\forall a,b \in N$ uchun $a+b \neq 1$.

II. $\forall a \in N$ uchun yagona shunday a' element mavjud bo'lib, $a+1=a'$.

III. $\forall a,b \in N$ uchun $a+1=b+1$ bo'lsa, $a=b$.

IV. $\forall a,b \in N$ uchun $a+(b+1)=(a+b)+1$.

V. $\forall a \in N$ uchun $a\bullet 1=a$.

VI. $\forall a,b \in N$ uchun $a(b+1)=ab+a$.

VII. Induksiya aksiomasi. N to'plamning ixtiyoriy M to'plamostisi uchun 1) $1\in M$;

2) $\forall a \in M$ uchun $a+1 \in M$ bo'lsa, $M=N$ bo'ladi.

Agar $(N,\bullet,+,0,1)$ algebra uchun yuqorida sanab chiqilgan I-VII shartlar bajrilsa, u holda bu algebra natural sonlar sistemasi deyiladi.

$A(n)$ natural sonlar to'plamida aniqlangan bir o'rinli predikat bo'lib, quyidagi shartlar bajarilsin:

1. $A(1)$-rost mulohaza.

2. $\forall k \in N$ uchun $A(k)$ rost mulohaza bo'lishidan $A(k+1)$-mulohazaning

rost mulohaza bo'lishi kelib chiqsin. U holda $\forall n \in N$ uchun $A(n)$ mulohaza rost mulohaza bo'ladi.

$A(1)$ ni induksiya bazisi, $A(k)$ ni induksiya farazi deb ataymiz.

$A(k)=1$ dan $A(k+1)=1$ kelib chiqishi induksiya qadami deyiladi.

$\forall a,b \in N$ natural sonlar uchun shunday $n \in N$ topilib, $a=b+n$ bo'lsa, a natural son b natural sondan katta deymiz va $a>b$ deb belgilaymiz.

$(A,+,\bullet)$-algebra uchun quyidagi shartlar bajarilsin:

1) $\forall a,b,c$ uchun $(a+b)+c=a+(b+c)$

2) $\forall a,b \in A$ uchun $a+b=b+a$

3) $\forall a,b,x \in A \quad ((a+x=b+x) \Rightarrow (a=b)) \wedge ((x+a=x+b) \Rightarrow (a=b))$

4) $\forall a,b,c \in A \quad (a\cdot b)\cdot c=a\cdot(b\cdot c)$

5) $\forall a,b,c \in A \Rightarrow \quad ((a+b)\cdot c=ac+bc) \wedge (c(a+b)=ca+cb)$

U holda $(A,+,\bullet)$-algebra yarim halqa deyiladi.

Misol. $\dfrac{4^n}{n+1}<\dfrac{(2n)!}{(n!)^2},(n>1)$ ekanligini isbotlang.

Isbot. 1. $n=2$ da $\dfrac{4^2}{2+1}<\dfrac{(2\cdot 2)!}{(2!)^2} \Rightarrow \dfrac{16}{3}<6$ kelib chiqadi, ya'ni tengsizlik o'rinli.

2. Har qanday $k>0$ uchun $\dfrac{4(k+1)}{k+2}<\dfrac{(2k+1)(2k+2)}{(k+1)^2}$ ekanligidan

$\dfrac{4^k}{k+1}\cdot\dfrac{4(k+1)}{k+2}<\dfrac{(2k)!}{(k!)^2}\cdot\dfrac{(2k+1)(2k+2)}{(k+1)^2}$. Bundan $\dfrac{4^{k+1}}{k+2}<\dfrac{(2k+2)!}{((k+1)!)^2}$ kelib chiqadi.

Demak, har qanday $n>1$ natural son uchun $\dfrac{4^n}{n+1}<\dfrac{(2n)!}{(n!)^2}$.

Misol va mashqlar

1. Quyidagi teoremalarni isbotlang:

1.1. $(\forall a,b\in N)\Rightarrow(\exists! c\in N)\wedge(a+b=c)$.

1.2. $\forall a,b,c\in N\Rightarrow(a+b)+c=a+(b+c)$.

1.3. $(\forall a\in N)\Rightarrow(a+1=1+a)$

1.4. $(\forall a,b\in N)\Rightarrow(a+b=b+a)$

1.5. $\forall(a,b,c\in N)\wedge(a+c=b+c)\Rightarrow(a=b)$

1.6. $(\forall a,b\in N)\Rightarrow\exists! p(a=b=p)$

1.7. $(\forall a,b,c\in N)\Rightarrow((a+b)\bullet c=a\bullet c+b\bullet c)$

1.8. $(\forall a\in N)\Rightarrow(a\bullet 1=1\bullet a)$

1.9. $(\forall a,b\in N)\Rightarrow(a\bullet b=b\bullet a)$

1.10. $(\forall a,b,c\in N)\Rightarrow c(a+b)=ca+cb$

1.11. $(\forall a,b,c\in N)\Rightarrow(a\bullet b)c=a(b\bullet c)$

1.12. $(\forall a\in N)\wedge(a\neq 1)\Rightarrow(\exists n\in N)\wedge(a=1+n)$

1.13. $(\forall a,b\in N)\Rightarrow a\neq a+b$.

2. $\forall a,b\in N$ uchun quyidagi tengsizliklardan faqat biri o'rinli ekanligini isbotlang:

1) $a=b$ 2) $\exists n\in N \quad b=a+n$ 3) $\exists l\in N \quad a=b+l$.

3. Natural sonlar to'plamidagi tengsizlik munosabatining quyidagi xossalarini isbotlang:

1°. $(\forall a,b\in N)\wedge(a\neq b)\Rightarrow(a>b)\vee(b>a)$.

2°. $(\forall a\in N)$ учун $\overline{(a>a)}$.

3°. $\forall a,b\in N \quad a>b\Rightarrow\overline{(b>a)}$.

4°. $\forall a,b,c\in N$ учун $(a>b)\wedge(b>c)\Rightarrow(a>c)$.

5°. $\forall a,b,c\in N$ учун $((a>b)\Rightarrow(a+c)>(b+c))$.

6°. $\forall a,b\in N$ учун $a+b>a$.

7°. $\forall a,b,c\in N$ учун $(a>b)\Rightarrow ac>bc$.

4. Tekislikda berilgan bir nuqtadan o'tuvchi n ta turli to'g'ri chiziqlar tekislikni 2n bo'lakka bo'lishini isbotlang.

5. (x_n) ketma-ketlik uchun $x_1=1, x_{n+1}=x_n-\frac{1}{n(n+1)}$ berilgan bo'lsa, x_n ni ifodalang.

6. Quyidagi tasdiqlarni isbotlang:

6.1. $(4^n+15n-1)\vdots 9$.

6.2. $(10^n+18n-1)\vdots 27$

6.3. $(3^{2n+3}+40n-27)\vdots 64$.

6.4. $(6^{2n}+3^{n+2}+3^n)\vdots 11$.

6.5. $(5^n+2\cdot 3^{n-1}+1)\vdots 8$

6.6. $(11^{n+2}+12^{2n+1})\vdots 133$.

6.7. $(9^{n+1}-8n-9)\vdots 16$.

6.8. $(3^{6n}-2^{6n})\vdots 35$.

7. Quyidagi tengliklarni isbotlang:

7.1. $1^3+2^3+\ldots+n^3=\frac{n^2(n+1)^2}{4}$.

7.2. $1\cdot 1!+2\cdot 2!+\ldots+n\cdot n!=(n+1)!-1$.

7.3. $(1-\frac{4}{1})(1-\frac{4}{9})(1-\frac{4}{25})\ldots(1-\frac{4}{(2n-1)^2})=-\frac{2n+1}{2n-2}$.

7.4. $\frac{1}{1\cdot 4}+\frac{1}{4\cdot 7}+\ldots+\frac{1}{(3n-2)(3n+1)}=\frac{n}{3n+1}$.

7.5. $1-2^2+3^2-4^2+\ldots+(-1)^{n-1}n^2=(-1)^{n-1}\frac{n(n+1)}{2}$.

7.6. $\frac{1^2}{1\cdot 3}+\frac{2^2}{3\cdot 5}+\ldots+\frac{n^2}{(2n-1)(2n+1)}=\frac{n(n+1)}{2(2n+1)}$.

7.7. $(x_1+\ldots+x_n)^2=x_1^2+\ldots+x_n^2+2(x_1x_2+\ldots+x_{n-1}x_n)$.

7.8. $1+x+x^2+\ldots+x^n=\frac{x^{n+1}-1}{x-1}$ $(x\neq 1)$.

7.9. $\frac{1}{2!}+\frac{2}{3!}+...+\frac{n}{(n+1)!}=1-\frac{1}{(n+1)!}$.

7.10. $\frac{1}{a\cdot(a+1)}+\frac{1}{(a+1)(a+2)}+...+\frac{1}{(a+n-1)(a+n)}=\frac{n}{a(a+n)}$.

8. Quyidagi tengsizliklarni isbotlang:

8.1. $\frac{1}{n+2}+\frac{1}{n+3}+...+\frac{1}{2n+2}>\frac{13}{24}$.

8.2. $\frac{1}{n+1}+\frac{1}{n+2}+...+\frac{1}{3n+1}>1$.

8.3. $\frac{1}{2}\cdot\frac{3}{4}\cdot\frac{5}{6}\cdot...\cdot\frac{2n-1}{2n}\le\frac{1}{\sqrt{3n+1}}$.

8.4. $|x_1+x_2+...+x_n|\le|x_1|+|x_2|+...+|x_n|$.

8.5. $\frac{1}{1!}+\frac{1}{2!}+\frac{1}{3!}+...+\frac{1}{n!}<2$.

9. Agar $n\ge 2, a>-1$ bo'lsa, n natural son uchun $(1+a)^n\ge 1+na$ o'rinli ekanligini isbotlang.

10. Har qanday n natural son uchun $\frac{n^3}{6}+\frac{n^2}{2}+\frac{n}{3}$ natural son bo'lishini isbotlang.

Takrorlash uchun savollar

1. Natural sonlar sistemasi deb nimaga aytiladi?
2. Matematik induksiya metodini ifodalovchi teoremani ayting.
3. Induksiya bazisi, induksiya farazi, induksiya qadami nima?
4. Natural sonlarni qo'shishning qanday xossalarini bilasiz?
5. Natural sonlarni ko'paytirishning xossalarini ayting.
6. Natural sonlar to'plamida tartib munosabatini aniqlang.
7. Natural sonlar to'plamidagi tengsizlik munosabatining xossalarini bayon qiling.
8. Natural sonlar to'plami kommutativ yarimhalqa bo'lishini isbotlang.

11-§. Butun sonlar halqasi. Ratsional sonlar maydoni. Haqiqiy sonlar sistemasi

Asosiy tushunchalar: butun sonlar halqasi, bo'linish munosabati, maydon, rasional sonlar maydoni, haqiqiy sonlar sistemasi.

$\forall z\in Z$ element ikkita natural son ayirmasi ko'rnishida ifoda qilinadi. $(Z,+,\bullet)$ halqani, butun sonlar halqasi deb ataymiz.

Agar a,b $b\neq 0$ butun sonlar uchun shunday q butun son topilib $a=bq$ tenglik o'rinli bo'lsa, a butun son b butun songa bo'linadi yoki b butun son a butun sonni bo'ladi deyiladi va mos ravishda $a\vdots b$ yoki $b\backslash a$ ko'rinishda belgilanadi.

$\forall a,b\in Z$ sonlar uchun $a\vdots b$ va $b\vdots a$ shartlar bajarilsa a va b sonlar assosiirlangan deyiladi.

Assosiativ, kommutativ, birlik elementga ega, noldan farqli har bir element teskarilanuvchi bo'lgan va $1\neq 0$ shart bajariladigan halqa maydon deyiladi.

Maydonning noldan farqli har qanday elementi teskarilanuvchi bo'lgan halqaosti maydonosti deyiladi.

K-butunlik sohasi P-maydonning halqaostisi bo'lsin. P-maydonning ixtiyoriy p elementi uchun K-butunlik sohasining m,n elementlari topilib, $P=mn^{-1}$ tenglik o'rinli bo'lsa, P-maydon K-butunlik sohasining nisbatlar maydoni deyiladi. Butun sonlar xalqasining nisbatlar maydoni rasional sonlar maydoni deyiladi.

Tartiblangan maydonning ixtiyoriy musbat a,b elementlari uchun shunday n- natural son mavjud bo'lib $n\cdot a>b$ shart bajarilsa, bu maydon Arximedcha tartiblangan maydon deyiladi.

F tartiblangan maydondagi har qanday fundamental ketma-ketlik shu maydonda yaqinlashuvchi bo'lsa, bunday maydon to'liq deyiladi.

Arximedcha turtiblangan rasional sonlar sistemasini o'z ichiga olgan eng kichik to'liq maydon haqiqiy sonlar sistemasi deyiladi.

Misol va mashqlar

1. 5 ga bo'linuvchi butun sonlar to'plami halqa tashkil etishini isbotlang.

2. Z_6 to'plam halqa tashkil etishini isbotlang.

3. Q^2 to'plam $(a,b)+(c,d)=(a+c,b+d)$,

4. $(a,b)\cdot(c,d)=(ac+2bd,ad+bc)$ amallarga nisbatan maydon tashkil etishini isbotlang.

5. Butun sonlar halqasidagi bo'linish munosabatining quyidagi xossalarini isbotlang:

1°. $\forall a\in Z,\ a\neq 0$ uchun $a\vdots a$

2°. $\forall a\in Z,\ a\neq 0$ uchun $0\vdots a$

3°. $\forall a\in Z,\ a\vdots 0$ va $a\vdots(-1)$

4°. $\vdots$ munosabati tranzitiv munosabatdir, ya'ni $\forall a,b,c\in Z,$ uchun $a\vdots b$ va $b\vdots c$ bo'lsa $a\vdots c$

5°. $\forall a,b,c\in Z$ uchun $a\vdots c$ bo'lsa $a\cdot b\vdots c$

6°. $\forall a,b,c\in Z$ uchun $a\vdots c$ va $b\vdots c$ bo'lsa $(a+b)\vdots c$

7°. $\forall a,b,c\in Z$ va $c\neq 0$ uchun $bc\vdots ac$ bo'lsa, $b\vdots a$

8°. $\forall a,b,c\in Z$ uchun $a\vdots c$ va $b\vdots d$ bo'lsa $(a\cdot b)\vdots(c\cdot d)$

9°. $\forall a,b,c\in Z$ va $a\vdots b$ bo'lsa $a\cdot c\vdots b\cdot c$

10°. $\forall a,b,c,m,n,\in Z$ uchun $a\vdots c$ va $b\vdots c$ bo'lsa $(ma+nb)\vdots c$

6. $(P,+,-,\bullet,1)$ maydonning ihtiyoriy a,b,c elementlari uchun quyidagi munosabatlar o'rinli ekanligini isbotlang:

1°. $0\cdot a=a\cdot 0=0$

2°. Agar $ab=1$ bo'lsa, u holda $b=a^{-1}$

3°. $c\neq 0$ bo'lib, $ac=bc$ bo'lsa, $a=b$

4°. $ab=0$ bo'lsa, $a=0$ yoki $b=0$

5°. $a\neq 0$ va $b\neq 0$ bo'lsa, $ab\neq a$

6°. $\frac{a}{b}=\frac{c}{d}$ bo'lsa, $ad=bc$

7°. $\frac{a}{b}+\frac{c}{b}=\frac{ad+bc}{bd}$

8°. $\frac{a}{b}\cdot\frac{c}{d}=\frac{ac}{bd}$

9°. $\frac{-a}{b}=\frac{a}{-b}=-\frac{a}{b}$

10°. Noldan farqli a,b elementlar uchun $\left(\frac{a}{b}\right)^{-1}=\frac{b}{a}$

11°. $c\neq 0$ element uchun $\frac{a}{b}=\frac{ac}{bc}=\frac{ac^{-1}}{bc^{-1}}$

7. $(F,+,-,\cdot,1,<)$ tartiblangan maydon uchun quyidagi hossalarni isbotlang:

1°. $\forall a,b\in F$ uchun $a<b$ bo'lishi uchun $b-a>0$ bo'lishi zarur va etarli;

2°. $\forall a\in F$ uchun $a<0,\ 0<a\ 0=a$ shartlardan bir vaqtda faqat biri o'rinli;

3°. agar $a\geq 0$ va $b\geq 0$ bo'lsa, u holda $ab\geq 0$ va $a+b\geq 0$ bo'ladi;

4°. $a\leq b$ va $c\leq d$ bo'lsa, $a+c<b+d$;

5°. Agar $a<b$ va $c<0$ bo'lsa $ac>bc$.

6°. $\forall a\in F$ element uchun $a^2\geq 0$. Xususan $a\neq 0$ bo'lsa, $a^2>0$;

7°. $\forall n\in N$ uchun $n\cdot 1>0$ xususan $1>0$;

8°. Tartiblangan maydon butunlik sohasidir.

8. $(F,+,-,\cdot,1,<)$ tartiblangan maydonning $\forall a,b$ elementlari uchun quyidagi munosabatlarni isbotlang:

1°. $|a|=|-a|$.

2°. $|a|\geq a$ va $|a|\geq -a$;

3°. $|a+b|\leq|a|+|b|$;

4°. $|a\cdot b|=|a|\cdot|b|$

5°. $|b|>0$.

6°. $\forall a\in F$, $a^2\geq 0$ element uchun $|b|<a$ bo'lishi uchun $-a<b<a$ bo'lishi zarur va etarli.

7°. $\forall a,b,c,F$, $a>0$ elementlar uchun $|b|>a$ bo'lishi uchun $b>a$ yoki $b<-a$ bo'lishi zarur va etarli.

9. Har qanday $a+b\sqrt{7}(a,b\in Q)$ ko'rinishdagi haqiqiy son uchun aniqlangan $f(a+b\sqrt{7})=a-b\sqrt{7}$ akslantirish haqiqiy sonlar maydonining avtomorfizmi bo'lishini isbotlang.

Takrorlash uchun savollar

1. Natural sonlar yarimhalqasini qamrab olgan eng kichik kommutativ halqani quring.
2. Butun sonlar to'plami halqa tashkil etishini isbotlang.
3. Butun sonlar halqasida tartib munosabatini aniqlang.
4. Butun sonlar halqasida bo'linish munosabatining xossalarini ayting.
5. Maydon tushunchasiga ta'rif bering.
6. Maydonning sodda xossalarini ayting.
7. Maydonlar izomorfizmiga misol keltiring.
8. Rasional sonlar maydonida tartib munosabatini aniqlang.

12-§. Kompleks sonlar maydoni

Asosiy tushunchalar: kompleks son, kompleks sonlar maydoni, sonli maydon, o'zaro qo'shma kompleks sonlar, kompleks son moduli, kompleks tekislik, mavhum o'q, kompleks son argumenti, kompleks sonning trigonometrik shakli, Muavr formulalari, n- darajali ildizlar.

$C=\{b+bi/a,b\in\Re\}$ - to'plamni qaraylik. C-da

$(a+bi)+(c+di)=(a+c)+(b+d)i$; $(a+bi)\cdot c+di(=(ac-bd)+(ad+bc)i$;

$-(a+bi)=(-a)+(-b)i$ tengliklar orqali qo'shish, ko'paytirish, qarama-qarshisini olish amallarini aniqlaymiz. Ixtiyoriy $a+b_i\neq 0$ element uchun teskari element $(a+b_i)^{-1}\dfrac{a}{a^2+b^2}-\dfrac{b}{a^2+b^2}i$ formula bilan aniqlanadi.

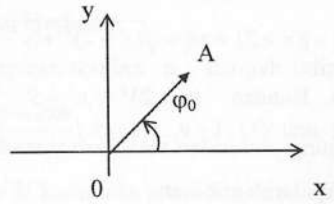
Kompleks sonlar maydonining har qanday maydonostisi sonli maydon deyiladi.

$Z=a+bi$ kompleks son uchun $Z=a-bi$ - qo'shma kompleks son deyiladi.

$|Z|=\sqrt{a^2+b^2}$ son $a+bi(a,b\in\Re)$ kompleks sonining moduli deyiladi.

Har bir $a+bi$ kompleks songa tekislikda (a,b) nuqtani mos qo'ysak, bu nuqta kompleks sonning geometrik tasviri deyiladi. Bu nuqtani koordinatalar boshi bilan tutashtirsak, boshi koordinatalar boshida, uchi esa (a,b) koordinatali nuqtada bo'lgan $\overrightarrow{OA}$ vektor hosil bo'ladi. Bu vektorning uzunligi esa $a+bi$ kompleks sonning moduliga tengligi ayon.

Har bir bi kompleks songa Oy o'qida $(0,b)$ nuqta mos keladi. Bu o'qni mavxum o'q deb ataymiz. Ox o'qni xaqiqiy o'q deymiz.



OA vektorning Ox o'qini musbat yo'nalishi soat strelkasi qarama-qarshi yo'nalishida hosil qilgan φ_q burchagi $a+bi$ kompleks sonning boshlang'ich argumenti deyiladi.

$a+bi$ kompleks son berilgan bo'lib, r uning moduli φ esa argumenti bo'lsin, u holda $b = r\sin\varphi,\ b = r\cos\varphi$ tengliklarni ko'rsatish qiyin emas. Demak, $a+bi = r(\cos\varphi + i\sin\varphi)$ tenglik o'rnili. Bu esa kompleks sonning trigonometrik ko'rinishi deyiladi.

$Z_1 = r_1(\cos\varphi_1 + i\sin\varphi_1),\ z_2 = r_2(\cos\varphi_2 + i\sin\varphi_2)$ kompleks sonlar berilgan bo'lsin. U holda quyidagilar o'rinli:

$1^\circ\ z_1 \cdot z_2 = r_1 \cdot r_2(\cos(\varphi_1+\varphi_2) + i\sin(\varphi_1+\varphi_2))$

$2^\circ\ \frac{z_1}{z_2} = \frac{r_1}{r_2}(\cos(\varphi_1-\varphi_2)) + i\sin(\varphi_1-\varphi_2))$

$z^n = (r(\cos\varphi + i\sin\varphi)^n = r^n(\cos n\varphi + i\sin n\varphi)$ bu formula Muavr formulasi deyiladi.

$z = r(\cos\varphi + i\sin\varphi)$. kompleks son berilgan bo'lsin. Bundan r - kompleks sonning moduli $\varphi = \varphi_0 + 2k\pi$ kompleks sonning argumenti, φ_0 - boshlang'ich argumenti bo'lsin. U holda z - kompleks son n ta xar hil n - darajali kompleks ildizlarga ega bo'ladi va bu ildizlar quyidagi formula yordamida topiladi:

$$U_k = \sqrt[n]{r}\left(\cos\frac{\varphi_0 + 2k\pi}{n} + i\sin\frac{\varphi_0 + 2k\pi}{n}\right),\quad k = 1, \ldots, n\text{-}1$$

Misol. $|z+2| = 3$ tenglamani eching.

Yechish. Kompleks sonning moduli, kompleks sonlarning tengligi ta'riflaridan quyidagi ifodani hosil qilamiz:

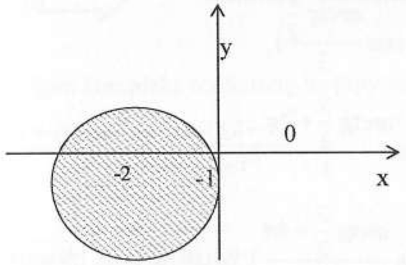
$|z+2| = |x+yi+2| = |(x+2)+yi| = \sqrt{(x+2)^2 + y^2}$ va

$\sqrt{(x+2)^2 + y^2} = 3$. Bundan $(x+2)^2 + y^2 = 9$ tenglamani hosil qilamiz. Xosil bo'lgan tenglamaning yechimlari tekislikdagi markazi (-2; 0) nuqtada radiusi 3 ga teng aylananing nuqtalaridan iborat.

Misol. $|z+2+i| \le 3$ tengsizliklarni eching va yechimlar to'plamini Dekart koordinatalar tekisligida ifodalang.

Yechish. $|z+2+i| \le 2$ tengsizlikka kompleks sonning moduli ta'rifini qo'llasak, $|z+2+i| = |x+yi+2+i| = |(x+2)+(y+1)i| = \sqrt{(x+2)^2 + (y+1)^2}$ ni hosil qilamiz. Bundan $(x+2)^2 + (y+1)^2 \le 4$ tengsizlikni hosil qilamiz. Bu

tengsizlikning yechimlari markazi (-2; -1) nuqtada, radiusi 2 ga teng doiradan iborat. Doirani Dekart koordinatalar tekisligida chizamiz:



Misol. $\sqrt{3+4i}$ ni hisoblang.

Yechish. Kompleks sondan kvadrat ildiz chiqarish formulalari

1) $\sqrt{\underset{b>0}{a+bi}} = \pm\left(\sqrt{\frac{a+\sqrt{a^2+b^2}}{2}} + i\sqrt{\frac{-a+\sqrt{a^2+b^2}}{2}}\right)$ va

2) $\sqrt{\underset{b<0}{a+bi}} = \pm\left(\sqrt{\frac{a+\sqrt{a^2+b^2}}{2}} - i\sqrt{\frac{-a+\sqrt{a^2+b^2}}{2}}\right)$ lardan foydalanamiz. Berilgan misolda $b>0$ bo'lganligi uchun birinchi formulani qo'llaymiz:

$$\sqrt{\underset{4>0}{3+4i}} = \pm\left(\sqrt{\frac{3+\sqrt{3^2+4^2}}{2}} + i\sqrt{\frac{-3+\sqrt{3^2+4^2}}{2}}\right) = \pm\left(\sqrt{\frac{3+5}{2}} + i\sqrt{\frac{-3+5}{2}}\right) =$$

$\pm(2+2i) = \pm 2(1+i)$.

Misol. $\sqrt[3]{2+3i}$ ildizlarni hisoblang.

Yechish. Ixtiyoriy kompleks sondan n - darajali ildizlarni topish formulasi $u_k = |c|^{\frac{1}{n}}\left(\cos\frac{\varphi + 2\pi k}{n} + i\sin\frac{\varphi + 2\pi k}{n}\right), k = 0, \ldots, n-1$ (1) dan foydalanamiz. Buning uchun avval berilgan $z = 2+3i$ kompleks sonni trigonometrik ko'rinishga keltiramiz: kompleks sonning moduli - $|z|=\sqrt{a^2+b^2}=\sqrt{2^2+3^3}=\sqrt{13}$; argumenti $\varphi = arctg\frac{b}{a} = arctg\frac{3}{2}$ dan iborat. U holda $z=2+3i=\sqrt{13}(\cos(arctg\frac{3}{2})+i\sin(arctg\frac{3}{2}))$. Topilgan modul va argumentni (1) formulaga qo'yamiz: $u_k=\sqrt{13}^{\frac{1}{3}}(\cos\frac{arctg\frac{3}{2}+2\pi k}{3}+i\sin\frac{arctg\frac{3}{2}+2\pi k}{3}), k=0,1,2.$

Bundan $u_0=\sqrt[6]{13}(\cos\frac{arctg\frac{3}{2}}{3}+i\sin\frac{arctg\frac{3}{2}}{3})$,

$u_1=\sqrt[6]{13}(\cos\frac{arctg\frac{3}{2}+2\pi}{3}+i\sin\frac{arctg\frac{3}{2}+2\pi}{3})$,

$u_2=\sqrt[6]{13}(\cos\frac{arctg\frac{3}{2}+4\pi}{3}+i\sin\frac{arctg\frac{3}{2}+4\pi}{3})$ ildizlarni hosil qilamiz.

Misol. $\sin x+\sin 2x+\ldots+\sin nx=\frac{\sin\frac{n+1}{2}}{\sin\frac{x}{2}}\sin\frac{nx}{2}$ tenglikni isbotlang.

Isbot. 1. $n=1$ bo'lsin. U xolda $\sin x=\frac{\sin\frac{1+1}{2}}{\sin\frac{x}{2}}\sin\frac{x}{2}\Rightarrow \sin x=\sin x$. Tenglik o'rinli.

2. $n=k$ uchun $\sin x+\sin 2x+\ldots+\sin kx=\frac{\sin\frac{k+1}{2}}{\sin\frac{x}{2}}\sin\frac{kx}{2}$ bo'lsin.

U holda $n=k+1$ da $\sin x+\sin 2x+\ldots+\sin kx+\sin(k+1)x=$

$$=\frac{\sin\frac{k+1}{2}}{\sin\frac{x}{2}}\sin\frac{kx}{2}+\sin(k+1)x=\frac{\sin\frac{k+1}{2}}{\sin\frac{x}{2}}\sin\frac{kx}{2}+2\sin\frac{k+1}{2}x\cos\frac{k+1}{2}x=$$

$$=(2\cos\frac{k+1}{2}x\sin\frac{x}{2}=\sin\frac{k+2}{2}x-\sin\frac{kx}{2})=\frac{\sin\frac{k+2}{2}}{\sin\frac{x}{2}}\sin\frac{k+1}{2}x.$$

Demak, har qanday $n\in N$ uchun $\sin x+\sin 2x+\ldots+\sin nx=\frac{\sin\frac{n+1}{2}}{\sin\frac{x}{2}}\sin\frac{nx}{2}$ tenglik o'rinli.

Misol va mashqlar

1. Berilgan kompleks sonlarning haqiqiy va mavhum qismlarini toping:

1.1. $\frac{(1+i)(2+i)}{2-i}-\frac{(1-i)(2-i)}{2+i}$.

1.2. $\left(\frac{i^7+2}{1+i^{11}}\right)^2$.

1.3. $\left(\frac{1-i}{1+i}\right)^{2007}$.

1.4. $\frac{(1+2i)^3-(1+3i)^2}{(3-i)^3+(1+5i)^2}$.

2. Ixtiyoriy z_1, z_2 - kompleks sonlar uchun quyidagi hossalar o'rinli ekanligini isbotlang:

1^0. $\overline{z_1+z_2}=\overline{z_1}+\overline{z_2}$.

2^0. $\overline{(-z_1)}=-\overline{z_1}$.

3^0. $\overline{z_1\cdot z_2}=\overline{z_1}\cdot\overline{z_2}$.

4^0. $\overline{(\overline{z})}=z$.

5^0. $z_2\neq 0, \overline{\left(\frac{z_1}{z_2}\right)}=\frac{\overline{z_1}}{\overline{z_2}}$.

6^0. $z=\bar{z}$ bo'lishi uchun $z\in R$ bo'lishi zarur va etarli.

7^0. Agar $z=a+bi$ bo'lsa, u xolda $z\cdot\bar{z}=a^2+b^2$

3. Ixtiyoriy z_1, z_2 - kompleks sonlar uchun quyidagi hossalar o'rinli ekanligini isbotlang:

1^0. $|z|^2=z\cdot\bar{z}$.

2^0. $|z|=0$ faqat va faqat shu xoldaki, agar z = 0 bo'lsa.

3^0. $|z_1\cdot z_2|=|z_1|\cdot|z_2|$.

4^0. $|z^{-1}|=|z|^{-1}(z\neq 0)$.

5^0. $|z_1+z_2|\leq|z_1|+|z_2|$.

6^0. $|z_1|-|z_2|\leq|z_1+z_2|$.

7^0. $\left||z_1|-|z_2|\right|\leq|z_1+z_2|$.

4. Tenglamani eching:

4.1. $\bar{z}=5-z$.

4.2. $\bar{z}=-3z-1+2i$

4.3. $z^2+\bar{z}=1$

4.4. $z^2-2z\bar{z}-3=3i$.

4.5. $z^2+5z+5-3i=0$.

4.6. $z^2(1+i)-z+1+2i=0$.

4.7. $z^2+(2+i)z-1+7i=0$.

4.8. $(1+i)z^2+iz+2+4i=0$.

5. Quyidagi tenglamalarni eching:

5.1. $z+|z+1|+i=0$.

5.2. $z|z|-2z+i=0$.

5.3. $z|z|-2iz^2+2i=0$.

6. Quyidagi tenglamalar sistemasini eching:

6.1. $\begin{cases}|z-2i|=|z|,\\ |z-i|=|z-1|\end{cases}$

6.2. $\begin{cases}\left|\dfrac{z-12}{z-8i}\right|=\dfrac{5}{3},\\ \left|\dfrac{z-4}{z-8}\right|=1\end{cases}$.

6.3. $\begin{cases}z_1+2z_2=1+i,\\ 3z_1+iz_2=2-3i\end{cases}$.

6.4. $\begin{cases}4iz_1-5z_2=-4+14i,\\ 3z_1+2iz_2=7+3i\end{cases}$.

6.5. $\begin{cases}|z+1-i|=|z+i|,\\ |3+2i-z|=|z+1|\end{cases}$.

6.6. $\begin{cases}|z+1|=|z+2|,\\ |3z+9|=|5z+10i|\end{cases}$.

7. Quyidagi tengsizliklarning yechimlar to'plamini koordinatalar tekisligida ifodalang:

7.1. $|z+2|\geq|z|$.

7.2. $|z-5+i|<4$.

7.3. $|z+6i|>|z-3|$.

7.4. $|z+2+2i|\leq\sqrt{2}$.

7.5. $|z-1-3i|\leq|z-1|$.

7.6. $|z+5+6i|>0$.

7.7. $|z+i|\geq|z-2|$.

7.8. $|z-3-i|<|z+2i|$.

8. N'yuton binomi va Muavr formulalari yordamida quyidagilarni hisoblang:

8.1. $1-3C_n^2+9C_n^4-27C_n^6+\dots$

8.2. $C_n^1-3C_n^3+9C_n^5-27C_n^7+\dots$

8.3. $\sqrt{3^n}-\sqrt{3^{n-2}}C_n^2+\sqrt{3^{n-4}}C_n^4-\sqrt{3^{n-6}}C_n^6+\dots$

8.4. $C_n^1 - \frac{1}{3}C_n^3 + \frac{1}{9}C_n^5 - \frac{1}{27}C_n^7 + \dots$

9. Har qanday k butun sonlar uchun quyidagilarni isbotlang:

9.1. $\left(\frac{1+i\sqrt{3}}{1-i\sqrt{3}}\right)^{3k} = 1.$

9.2. $\left(\frac{\sqrt{3}+i}{\sqrt{3}-i}\right)^{6k} = 1.$

10. Quyidagi kompleks sonlarni trigonometrik shaklga keltiring:

10.1. $5.$

10.2. $-2i.$

10.3. $-\frac{1}{2}-\frac{1}{2}i.$

10.4. $(\sqrt{12}-2i)4i^{13}.$

10.5. $\sin\alpha + i\cos\alpha.$

11. Quyidagilarni hisoblang:

11.1. $\frac{(\cos\theta - i\sin\theta)(ctg\beta + i)(-\cos\alpha + i\sin\alpha)}{1 - itg(\alpha+\beta)}.$

11.2. $\left(-2+i\sqrt{12}\right)^{2008}.$

11.3. $(1-\cos\alpha + i\sin\alpha)^n, n\in Z.$

11.4. $\frac{(tg30^0 - i)^{15}}{(-1+i)^{2000}}(-\sin 30^0 + i\cos 30^0).$

12. Ildizlarni toping:

12.1. $\sqrt[3]{2-i}.$

12.2. $\sqrt[6]{64}.$

12.3. $\sqrt[4]{2+3i}.$

12.4. $\sqrt[4]{\frac{1}{8}+\frac{iSin60^0}{4}}.$

12.5. $\sqrt[5]{\frac{-2i}{1-\sqrt{3}i}}.$

12.6. $\sqrt[4]{\frac{1}{2}((\sqrt{3}+1)+(\sqrt{3}-1)i)}.$

12.7. $\sqrt[6]{\frac{1-i}{-\sqrt{3}+i}}.$

12.8. $\sqrt[8]{\left(\frac{-5+7i}{i}\right)^3}.$

13. Birning 6-darajali ildizlari to'plami mul'tiplikativ gruppa tashkil etishini isbotlang.

14. Agar n va m o'zaro tub bo'lsa, u holda birning nm – darajali ildizlarini birning n- va m- darajali ildizlari yordamida ifodalash mumkinligini isbotlang.

Takrorlash uchun savollar

1. Haqiqiy sonlar maydonining kompleks kengaytmasini quring.
2. Kompleks sonlar ustida arifmetik amallarni aniqlang.
3. Kompleks sonning geometrik tasviri nimadan iborat?
4. Geometrik ko'rinishdagi kompleks sonlarni qo'shish qanday bajariladi?
5. Kompleks sonning argumenti qanday aniqlanadi?
6. Trigonometrik ko'rinishda berilgan kompleks sonlarni ko'paytirish, bo'lish amallari qanday bajariladi?
7. Kompleks sonning trigonometrik shaklga keltirish qanday amalga oshiriladi?
8. Birning n-darajali ildiziga ta'rif bering.
9. Birning n-darajali ildizlari soni nechta? Javobingizni asoslang.
10. Ixtiyoriy kompleks sondan n-darajali ildiz topish formulasini ifodalang.

V MODUL. ARIFMETIK VEKTOR FAZO. CHIZIQLI TENGLAMALAR SISTEMASI

13-§. Arifmetik vektor fazo

Asosiy tushunchalar: n-o'lchovli arifmetik vektor, vektorlar yig'indisi, skalyarni vektorga ko'paytirish, n-o'lchovli arifmetik vektor fazo, chiziqli vektor fazo, fazoosti, vektorlar sistemasi, chiziqli kombinasiya, chiziqli bog'liq sistema, chiziqli bog'lanmagan sistema, vektorlarning ekvivalent sistemalari, vektorlar sistemasini elementar almashtirishlar, vektorlar sistemasining bazisi, vektorlar sistemasining rangi, vektorlar sistemasining chiziqli qobig'i, chiziqli ko'phillik.

$F=<F;+,\cdot,-,^{-1},0,1>$ ixtiyoriy maydon bo'lib, F uning asosiy to'plami bo'lsin. F^n to'g'ri ko'paytmaning ixtiyoriy $\vec{a}=(a_1,a_2,...,a_n)$ elementi n-o'lchovli arifmetik vektor deyiladi.

F^nning ixtiyoriy ikkita $\vec{a}=(a_1,a_2,...,a_n)$ va $\vec{b}=(b_1,b_2,...,b_n)$ vektorlari uchun $a_1=b_1$, $a_2=b_2,...,a_n=b_n$ bo'lsa, berilgan vektorlar teng deyiladi.

F^nning ixtiyoriy ikkita $\vec{a}=(a_1,a_2,...,a_n)$ va $\vec{b}=(b_1,b_2,...,b_n)$ vektorlarining yig'indisi deb $\vec{a}+\vec{b}=(a_1+b_1,a_2+b_2,...,a_n+b_n)$ vektorga aytiladi.

$\forall\lambda\in F$ skalyarni $\forall\vec{a}\in F^n$ vektorga ko'paytirish deb $\lambda\vec{a}=(\lambda a_1,\lambda a_2,...,\lambda a_n)$ vektorga aytiladi.

F^n to'plam, unda aniqlangan qo'shish binar amali va skalyarni vektorga ko'paytirish unar amallari yordamida hosil qilingan $F^n=<F^n;+,\{\omega_\lambda\mid\lambda\in F\}>$ algebra F maydon ustida qurilgan n–o'lchovli arifmetik vektor fazo deyiladi.

$F^n=<F^n;+,\{\omega_\lambda\mid\lambda\in F\}>$ n-o'lchovli arifmetik vektor fazo berilgan bo'lsin. F^nning ixtiyoriy bo'sh bo'lmagan qism to'plami $F^k(k\le n)$ arifmetik vektor fazo

tashkil qilsa, F^k arifmetik vektor fazoga F^n arifmetik vetor fazoning fazoostisi (qismfazosi) deyiladi.

F^n vektor fazoning vektorlaridan iborat $\vec{a}_1,\vec{a}_2,...,\vec{a}_n,...$ sistemaga vektorlarning cheksiz sistemasi; $\vec{a}_1,\vec{a}_2,...,\vec{a}_n$ sistemaga vektorlarning chekli sistemasi deyiladi.

F^n vektor fazoning $\vec{a}_1,\vec{a}_2,...,\vec{a}_n,...$ sistemasi va F maydonning $\lambda_1,\lambda_2,...,\lambda_n,...$ skalyarlari berilgan bo'lsin. $\lambda_1\vec{a}_1+\lambda_2\vec{a}_2+...+\lambda_n\vec{a}_n+...$ ifodaga $\vec{a}_1,\vec{a}_2,...,\vec{a}_n,...$ vektorlar sistemasining chiziqli kombinasiyasi deyiladi. Agar kamida bittasi noldan farqli shunday $\lambda_1,\lambda_2,...,\lambda_n$ skalyarlar topilib, $\lambda_1\vec{a}_1+\lambda_2\vec{a}_2+...+\lambda_n\vec{a}_n=\vec{0}$ tenglik bajarilsa, u xolda $\vec{a}_1,\vec{a}_2,...,\vec{a}_n$ sistema vektorlarning chiziqli bog'langan sistemasi deyiladi; tenglik $\lambda_1=0,\lambda_2=0,...,\lambda_n=0$ bo'lganda bajarilsa, u xolda $\vec{a}_1,\vec{a}_2,...,\vec{a}_n$ vektorlarning chiziqli bog'lanmagan (chizikli erkli) sistemasi deyiladi.

Agar S va T sistemalarning ixtiyoriy biridan olingan har qanday noldan farqli vektorni ikkinchi sistema vektorlarining chiziqli kombinasiyasi sifatida ifodalash mumkin bo'lsa, bunday sistemalar ekvivalent sistemalar deyiladi va $R\sim T$ ko'rinishda belgilanadi.

Vektorlar chekli sistemasini elementar almashtirishlar deb quyidagi almashtirishlarga aytiladi:

1) sistemaning qandaydir bir vektorini noldan farqli skalyarga ko'paytirish;

2) sistemaning skalyarga ko'paytirilgan bir vektorini ikkinchi vektoriga qo'shish yoki ayirish;

3) nol vektorni sistemadan chiqarish yoki sistemaga kiritish.

Vektorlar chekli sistemasining chiziqli erkli, bo'sh bo'lmagan qism sistemasi yordamida sistemaning har qanday vektorini chiziqli ifodalash mumkin bo'lsa, bunday qism sistemaga berilgan sistemaning bazisi deyiladi.

Vektorlar chekli sistemasining ixtiyoriy bazisidagi vektorlar soniga uning rangi deyiladi.

$\alpha_1\vec{a}_1+\alpha_2\vec{a}_2+...+\alpha_n\vec{a}_n(\alpha_i\in o')$ ko'rinishdagi barcha chiziqli kombinasiyalar to'plamiga $\vec{a}_1,\vec{a}_2,...,\vec{a}_n$ vektorlarning chiziqli qobig'i deyiladi va u $L(\vec{a}_1,\vec{a}_2,...,\vec{a}_n)$ ko'rinishda belgilanadi.

$\vec{o}_0+W=\{\vec{x}_0+\vec{y}\,|\,\vec{x}_0\in F^n\}$ to'plamga W qism fazoning $\vec{o}_0$ vektorga siljitishdan hosil bo'lgan chiziqli ko'phillik deyiladi va u $N=\vec{x}_0+W$ orqali belgilanadi.

Misol. $V=\{ax^2+by+c \mid a,b,c,x,y\in R\}$ to'plam R maydon ustida chiziqli fazo tashkil etishini isbotlang.

Yechish: Chiziqli fazo ta'rifiga ko'ra berilgan V to'plamda qo'shish binar amalini, skalyarni vektorga ko'paytirish unar amallarini aniqlab, ular uchun quyidagi xossalar tekshiriladi:

1^0. $\forall(\vec{a},\vec{b}\in V)\ (\vec{a}+\vec{b}=\vec{b}+\vec{a})$.

2^0. $\forall(\vec{a},\vec{b},\vec{c}\in V)\ (\vec{a}+(\vec{b}+\vec{c})=(\vec{a}+\vec{b})+\vec{c})$.

3^0. $\forall(\vec{a}\in V)\wedge\exists(\vec{e}\in V)\ (\vec{a}+\vec{e}=\vec{a})$.

4^0. $\forall(\vec{a}\in V)\ \wedge\ \exists(\vec{a'}\in V)\ (\vec{a}+\vec{a'}=\vec{0})$.

5^0. $\forall(\alpha\in R)\wedge\forall(\vec{a},\vec{b}\in V)\ (\alpha(\vec{a}+\vec{b})=\alpha\vec{a}+\alpha\vec{b})$.

6^0. $\forall(\alpha,\beta\in R)\wedge\forall(\vec{a}\in V)\ ((\alpha\beta)\vec{a}=\alpha(\beta\vec{a}))$.

7^0. $\forall(\alpha,\beta\in R)\wedge\forall(\vec{a}\in V)\ ((\alpha+\beta)\vec{a}=\alpha\vec{a}+\beta\vec{a})$.

8^0. $\forall(\vec{a}\in V)\ (1\cdot\vec{a}=\vec{a})$.

V to'plamning ixtiyoriy $\vec{z}_1=a_1x^2+b_1y+c_1$ va $\vec{z}_2=a_2x^2+b_2y+c_2$ elementlari berilgan bo'lsin.

$\vec{z}_1+\vec{z}_2=a_1x^2+b_1y+c_1+a_2x^2+b_2y+c_2=(a_1+a_2)x^2+(b_1+b_2)y+(c_1+c_2)\in V$

Demak, qo'shish binar amali V to'plamda aniqlangan va $<V;+>$ additiv gruppoid bo'ladi.

1^0 –isboti: V to'plamning ixtiyoriy $\vec{z}_1,\vec{z}_2$ elementlari berilgan bo'lsin

$\vec{z}_1+\vec{z}_2=a_1x^2+b_1y+c_1+a_2x^2+b_2y+c_2=(a_1+a_2)x^2+(b_1+b_2)y+(c_1+c_2)=$

$|\,R$ da qo'shish amali kommutativ bo'lganligi uchun$\,|=$

$=(a_2+a_1)x^2+(b_2+b_1)y+(c_2+c_1)=a_2x^2+b_2y+c_2+a_1x^2+b_1y+c_1=\vec{z}_2+\vec{z}_1$.

Demak, V da qo'shish amali kommutativ va $<V;+>$ additiv abel gruppoid.

2^0 –isboti: V to'plamning ixtiyoriy $\vec{z}_1=a_1x^2+b_1y+c_1$, $\vec{z}_2=a_2x^2+b_2y+c_2$, $\vec{z}_3=a_3x^2+b_3y+c_3$ elementlari berilgan bo'lsin.
$(\vec{z}_1+\vec{z}_2)+\vec{z}_3=|\ 1^0$ –ko'ra $|=(a_1+a_2)x^2+(b_1+b_2)y+(c_1+c_2)+a_3x^2+b_3y+c_3$
=|haqiqiy sonlar to'plamida qo'shish amali kommutativ, ko'paytirish qo'shishga nisbatan distributivligidan|=
$((a_1+a_2)+a_3)x^2+((b_1+b_2)+b_3)y+(c_1+c_2)+c_3$ =haqiqiy sonlar maydonida qo'shish amali assosiativ bo'lganligi uchun| =

$$=(a_1+(a_2+a_3))x^2+(b_1+(b_2+b_3))y+c_1+(c_2+c_3)=$$
$$=a_1x^2+b_1y+c_1+(a_2+a_3)x^2+(b_2+b_3)y+(c_2+c_3)=$$
$$a_1x^2+b_1y+c_1+(a_2x^2+b_2y+c_2+a_3x^2+b_3y+c_3)=\vec{z}_1+(\vec{z}_2+\vec{z}_3)$$

Demak, V to'plamda qo'shish amali assosiativ va $<V;+>$ additiv abel gruppa.

3^0 –isboti: V to'plamning ixtiyoriy $\vec{z}=ax^2+by+c$ va shunday $\vec{e}=e_1x^2+e_2y+e_3$ elementlari uchun $\vec{z}+\vec{e}=\vec{z}$ ekanligini aniqlaymiz: $\vec{z}+\vec{e}=\vec{z}$ dan $ax^2+by+c+e_1x^2+e_2y+e_3=ax^2+by+c$ tenglamani, bundan $\begin{cases}a+e_1=a\\ b+e_2=b\\ c+e_3=c\end{cases}$ tenglamalar sistemasini hosil qilamiz. R da tenglamalar sistemasi yagona $e_1=0$, $e_2=0$, $e_3=0$ yechimga ega.

Demak, $\vec{e}=0\cdot x^2+0\cdot y+0=\vec{0}\in V$ va $<V;+,0>$ - additiv abel monoid.

4^0 –isboti: V to'plamning ixtiyoriy $\vec{z}=ax^2+by+c$ va shunday $\vec{z'}=a'x^2+b'y+c'$ elementlari uchun $\vec{z}+\vec{z'}=\vec{0}$ ekanligini keltirib chiqaramiz:

$\vec{z}+\vec{z'}=\vec{0}$ dan $ax^2+by+c+a'x^2+b'y+c'=0\cdot x^2+0\cdot y+0$ tenglamani bundan $\begin{cases} a+a'=0 \\ b+b'=0 \\ c+c'=0 \end{cases}$ tenglamalar sistemasini hosil qilamiz.

Tenglamalar sistemasi R maydonda yagona $a'=-a$, $b'=-b$, $c'=-c$ yechimga ega. Demak, $\overline{z'}=-ax^2-by-c=-(ax^2+by+c)\in V$ va $<V;+,-,\bar{0}>$ - additiv abel gruppa.

V to'plamda skalyarni ko'paytirish unar amalini aniqlaymiz: ixtiyoriy $\alpha\in R$ skalyar va ixtiyoriy $\bar{z}=ax^2+by+c\in V$ berilgan bo'lsin. $\omega_\alpha(\bar{z})=\alpha\cdot z=\alpha\cdot(ax^2+by+c)=\alpha\cdot(ax^2)+\alpha\cdot(by)+\alpha\cdot c=|R$ da ko'paytirish assosiativligi uchun$|=(\alpha\cdot a)x^2+(\alpha\cdot b)y+\alpha\cdot c\in V$ ni hosil qilamiz. Demak, V da skalyarni V ning elementiga ko'paytirish unar amallari aniqlangan.

5^0 – isboti. Ixtiyoriy $\alpha\in R$ skalyar va V ning $\bar{z}_1,\bar{z}_2$ elementlari berilgan bo'lsin.

$$\alpha(\bar{z}_1+\bar{z}_2)=\alpha((a_1+a_2)x^2+(b_1+b_2)y+(c_1+c_2))=$$
$$=\alpha(a_1+a_2)x^2+\alpha(b_1+b_2)y+\alpha(c_1+c_2)=$$

$|$R da ko'paytirishning qo'shishga nisbatan distributivligidan$|$=

$$=\alpha\cdot a_1x^2+\alpha\cdot a_2x^2+\alpha\cdot b_1y+\alpha\cdot b_2y+\alpha\cdot c_1+\alpha\cdot c_2=$$

$|$Rda qo'shish amalining kommutativligidan$|$=

$=\alpha\cdot a_1x^2+\alpha\cdot b_1y+\alpha\cdot c_1+\alpha\cdot a_2x^2+\alpha\cdot b_2y+\alpha\cdot c_2=$ $|$Rda ko'paytirishning qo'shishga nisbatan distributivligidan$|$=

$$=\alpha(a_1x^2+b_1y+c_1)+\alpha(a_2x^2+b_2y+c_2)=\alpha\cdot\bar{z}_1+\alpha\cdot\bar{z}_2.$$

Demak, skalyarni vektorlar yig'indisiga ko'paytirish distributiv.

6^0 – isboti. Ixtiyoriy $\alpha,\beta\in R$ skalyarlar va ixtiyoriy $\bar{z}\in V$ element berilgan bo'lsin.

$(\alpha\cdot\beta)\bar{z}=(\alpha\cdot\beta)(ax^2+by+c)=|$Rda ko'paytirishning qo'shishga nisbatan distributivligidan$|=(\alpha\cdot\beta)ax^2+(\alpha\cdot\beta)by+(\alpha\cdot\beta)c=$ $|$ko'paytirish R da assosiativligi va ko'paytirishni qo'shishga nisbatan distributivligidan$|$=

$$=\alpha(\beta\cdot ax^2)+\alpha(\beta\cdot by)+\alpha(\beta\cdot c)=\alpha(\beta(ax^2)+(\beta(by)+(\beta c)))=$$

$$=\alpha(\beta(ax^2+by+c))=\alpha(\beta\cdot\bar{z}).$$

Demak, skalyarlar ko'paytmasini V elementiga ko'paytirish assosiativ.

7^0 – isboti. Ixtiyoriy $\alpha,\beta\in R$ skalyarlar va ixtiyoriy $\bar{z}\in V$ element berilgan bo'lsin.

$(\alpha+\beta)\bar{z}=(\alpha+\beta)(ax^2+by+c)=$ $|$Rda ko'paytirishning qo'shishga nisbatan distributivligidan$|=(\alpha+\beta)ax^2+(\alpha+\beta)by+(\alpha+\beta)c=$

$=|$ R da yig'indini o'ngdan ko'paytirish distributivligidan$|$=

$$=\alpha\cdot ax^2+\beta\cdot ax^2+\alpha\cdot by+\beta\cdot by+\alpha\cdot c+\beta\cdot c=$$

$=|$Rda qo'shish komutativligidan$|$=

$=|\alpha\cdot ax^2+\alpha\cdot by+\alpha\cdot c+\beta\cdot ax^2+\beta\cdot by+\beta\cdot c=|$Rda ko'paytirishning qo'shishga nisbatan distributivligidan$|$=

$=\alpha(ax^2+by+c)+\beta(ax^2+by+c)=\alpha\cdot\bar{z}+\beta\cdot\bar{z}.$

Demak, skalyarlar yig'indisini V elementiga ko'paytirish distributiv.

8^0 – isboti. V to'plamning ixtiyoriy $\bar{z}$ elementi berilgan bo'lsin. Skalyarlar to'plami maydon tashkil etishi va har qanday maydonda 1 mavjud ekanligidan $1\cdot\bar{z}=1\cdot(ax^2+by+c)=ax^2+by+c=\bar{z}$.

Demak, $<V;+,\{\omega_\alpha|\alpha\in R\}>$ algebra chiziqli fazo bo'ladi.

Misol. $\overline{a_1}=(1,2,3,4)$, $\overline{a_2}=(0,1,2,1)$, $\overline{a_3}=(1,-1,2,1)$ vektorlar sistemasini chiziqli bog'liq yoki chiziqli bog'liqmasligini aniqlang. Uning bazisi va rangini toping.

Yechish: 1-usul. Chiziqli tenglamalar sistemasi yordamida. Ixtiyoriy α,β,γ skalyarlar berilgan bo'lsin. Berilgan vektorlar sistemasining chiziqli kombinasiyasini tuzamiz: $\alpha\overline{a_1}+\beta\overline{a_2}+\gamma\overline{a_3}=\bar{0}$, ya'ni $\alpha(1,2,3,4)+\beta(0,1,2,1)+\gamma(1,-1,2,1)=\bar{0}$. Hosil bo'lgan tenglikdan skalyarlar qiymatini topamiz. Buning uchun chiziqli tenglamalar sistemasini tuzamiz:

$$\begin{cases} \alpha+\gamma=0 \\ 2\alpha+\beta-\lambda=0 \\ 3\alpha+2\beta+2\gamma=0 \\ 4\alpha+\beta+\gamma=0 \end{cases}$$

Hosil bo'lgan chiziqli tenglamalar sistemasini Gauss usulida echamiz:

$$\begin{cases}\alpha+\gamma=0\\ \beta-3\gamma=0\\ 2\beta-\gamma=0\\ \beta-3\lambda=0\end{cases} \Rightarrow \begin{cases}\alpha+\gamma=0\\ \beta-3\gamma=0\\ 2\beta-\gamma=0\end{cases} \Rightarrow \begin{cases}\alpha+\gamma=0\\ \beta-3\gamma=0\\ 5\lambda=0\end{cases} \Rightarrow \begin{cases}\alpha=0\\ \beta=0\\ \gamma=0\end{cases}$$

$\alpha\vec{a_1}+\beta\vec{a_2}+\gamma\vec{a_3}=\vec{0}$ tenglikdan skalyarlarning barchasi nolga tengligi kelib chiqdi. Demak, $\vec{a_1},\vec{a_2},\vec{a_3}$ vektorlar sistemasi chiziqli bog'liq emas. Shuning uchun sistema o'ziga bazis, rangi 3 ga teng.

Misol. $(a):\vec{a_1}=(1,-1,3);\vec{a_2}=(-1,1,1)$ va $(b):\vec{b_1}=(-2,2,-6);\vec{b_2}=(-1,0,1)$ sistemalar ekvivalentligini tekshiring.

Yechish: Vektorlarning chekli sistemalari ekvivalentligining ta'rifiga ko'ra (a) sistemaning har bir vektori (b) sistema orqali va (b) sistemaning har bir vektori (a)sistema orqali chiziqli ifodalanishini tekshiramiz. Buning uchun

$\vec{a_1}=\beta_{11}\vec{b_1}+\beta_{12}\vec{b_2}$

$\vec{a_2}=\beta_{21}\vec{b_1}+\beta_{22}\vec{b_2}$

$\vec{b_1}=\alpha_{11}\vec{a_1}+\alpha_{12}\vec{a_2}$

$\vec{b_2}=\alpha_{21}\vec{a_1}+\alpha_{22}\vec{a_2}$ tenglamalardagi skalyarlarning qiymatlarini topamiz. Ya'ni 1) $(1,-1,3)=\beta_{11}(-2,2,-6)+\beta_{12}(-1,0,1)$.

2) $(-1,1,1)=\beta_{21}(-2,2,-6)+\beta_{22}(-1,0,1)$.

3) $(-2,2,-6)=\alpha_{11}(1,-1,3)+\alpha_{12}(-1,1,1)$.

4) $(-1,0,1)=\alpha_{21}(1,-1,3)+\alpha_{22}(-1,1,1)$.

Tenglamalarning har biridan quyidagi tenglamalar sistemalarini hosil qilamiz:

$$1)\ \begin{cases}1=-2\beta_{11}-\beta_{12}\\ -1=2\beta_{11}\\ 3=-6\beta_{11}+\beta_{12}\end{cases} \qquad 2)\ \begin{cases}-1=-2\beta_{21}-\beta_{22}\\ 1=2\beta_{21}\\ 1=-6\beta_{21}+\beta_{22}\end{cases}$$

$$3)\ \begin{cases}-2=\alpha_{11}-\alpha_{12}\\ 2=-\alpha_{11}+\alpha_{12}\\ -6=3\alpha_{11}+\alpha_{12}\end{cases} \qquad 4)\ \begin{cases}-1=\alpha_{21}-\alpha_{22}\\ 0=-\alpha_{21}+\alpha_{22}\\ 1=3\alpha_{21}+\alpha_{22}\end{cases}$$

Tenglamalar sistemalarini echamiz :

$$1)\ \begin{cases}1=-2\beta_{11}-\beta_{12}\\ -1=2\beta_{11}\\ 3=-6\beta_{11}+\beta_{12}\end{cases} \Rightarrow \begin{cases}\beta_{11}=-\dfrac{1}{2}\\ \beta_{12}=0\end{cases}$$

$$2)\ \begin{cases}-1=-2\beta_{21}-\beta_{22}\\ 1=2\beta_{21}\\ 1=-6\beta_{21}+\beta_{22}\end{cases} \Rightarrow \begin{cases}\beta_{21}=\dfrac{1}{2}\\ \beta_{22}=0\end{cases}$$

$$3)\ \begin{cases}-2=\alpha_{11}-\alpha_{12}\\ 2=-\alpha_{11}+\alpha_{12}\\ -6=3\alpha_{11}+\alpha_{12}\end{cases} \Rightarrow \begin{cases}-2=\alpha_{11}-\alpha_{12}\\ -6=3\alpha_{11}+\alpha_{12}\end{cases} \Rightarrow \begin{cases}-2=\alpha_{11}-\alpha_{12}\\ 0=4\alpha_{12}\end{cases} \Rightarrow \begin{cases}\alpha_{12}=0\\ \beta\alpha_{11}=-2\end{cases}.$$

$$4)\ \begin{cases}-1=\alpha_{21}-\alpha_{22}\\ 0=-\alpha_{21}+\alpha_{22}\\ 1=3\alpha_{21}+\alpha_{22}\end{cases} \Rightarrow \begin{cases}-1=\alpha_{21}-\alpha_{22}\\ 0=-\alpha_{21}+\alpha_{22}\\ 0=4\alpha_{21}\end{cases} \Rightarrow \begin{cases}\alpha_{21}=0\\ \alpha_{22}=0\\ -1\neq 0-0\end{cases} \Rightarrow \text{yechim mavjud}$$

emas.

Demak, $\vec{a_1}=-\frac{1}{2}\vec{b_1}$, $\vec{a_2}=\frac{1}{2}\vec{b_1}$, $\vec{b_1}=-2\vec{a_1}$.

Ya'ni $\vec{b_2}$ vektor (a) sistema yordamida chiziqli ifodalanmaydi. Shu sababli (a), (b) sistemalar ekvivalent emas.

Misol. $\vec{x}=(1,\ 2,\ 1,\ 1)$ vektorni (a): $\vec{a_1}=(0,\ 1,\ 1,\ 0)$, $\vec{a_2}=(-1,\ 0,\ 0,\ 1)$, $\vec{a_3}=(2,\ 1,\ 1,\ 4)$ vektorlar orqali chiziqli ifodalang.

Yechish: Xaqiqiy sonlar maydonidan shunday α,β,γ skalyarlarni aniqlashimiz kerak-ki, ular $\vec{x}=\alpha\vec{a_1}+\beta\vec{a_2}+\gamma\vec{a_3}$ tenglikni qanoatlantirsin.

Buning uchun tenglamalar sistemasini tuzamiz:

$$(1,\ 2,\ 1,\ 1)=\alpha(0,\ 1,\ 1,\ 0)+\beta(-1,\ 0,\ 0,\ 1)+\gamma(2,\ 1,\ 1,\ 4)$$

tenglikdan quyidagi sistema kelib chiqadi:

$$\begin{cases}1=-\beta+2\alpha\\ 2=\alpha+\gamma\\ 1=\alpha+\gamma\\ 1=\beta+4\gamma\end{cases}$$

Hosil bo'lgan sistema hamjoyli bo'lsa, $\vec{x}$ vektor $\vec{a_1}, \vec{a_2}, \vec{a_3}$ vektorlar orqali chiziqli ifodalanadi. Lekin sistemaning 2- va 3- tenglamalari birgalikda yechimga ega emas.

Demak, $\vec{x}$ vektor (a) sistema orqali chiziqli ifodalanmaydi.

Misol. $\vec{x} = (1,\ 3,\ 0)$ vektorni (a): $\vec{a_1} = (1,\ 0,\ 2)$, $\vec{a_2} = (2,\ 3,\ 2)$, vektorlar orqali chiziqli ifodalang.

Yechish: $\vec{x} = \alpha\vec{a_1} + \beta\vec{a_2}$ tenglikni qanoatlantiruvchi α, β haqiqiy sonlarni aniqlaymiz:

$(1,\ 3,\ 0) = \alpha(1,\ 0,\ 2) + \beta(2,\ 3,\ 2)$ tenglamadan

$$\begin{cases} 1 = \alpha + 2\beta \\ 3 = 3\beta \\ 0 = 2\alpha + 2\beta \end{cases}$$ tenglamalar sistemasini hosil qilamiz.

Chiziqli tenglamalar sistemasining $\begin{cases} \beta = 1 \\ \alpha = -1 \end{cases}$ yechimi yordamida

$\vec{x} = (-1) \cdot \vec{a_1} + 1 \cdot \vec{a_2}$ ya'ni $\vec{x} = -\vec{a_1} + \vec{a_2}$ ifodani ±zish mumkin. Demak, $\vec{x}$ vektorni $\vec{a_1}, \vec{a_2}$ vektorlar orqali chiziqli ifodasi mavjud.

Misol. $(a): \vec{a_1} = (0,\ 1,\ 1,\ 0)$, $\vec{a_2} = (-1,\ 0,\ 0,\ 1)$, $\vec{a_3} = (2,\ 1,\ 1,\ 4)$ vektorlar sistemasining chiziqli qobig'i chiziqli fazo tashkil etishini isbotlang.

Yechish: Ta'rifga ko'ra $(a): \vec{a_1} = (0,\ 1,\ 1,\ 0)$, $\vec{a_2} = (-1,\ 0,\ 0,\ 1)$, $\vec{a_3} = (2,\ 1,\ 1,\ 4)$ sistemaning chiziqli qobig'i $L(a_1, a_2, a_3) = \{\alpha_1 a_1 + \alpha_2 a_2 + \alpha_3 a_3 \mid \alpha_1, \alpha_2, \alpha_3 \in R\}$ to'plamdan iborat. Uning chiziqli fazo tashkil etishini tekshiramiz:

1. $L(\vec{a}_1, \vec{a}_2, \vec{a}_3)$ to'plamning ixtiyoriy $\vec{z}_1 = \alpha_1\vec{a}_1 + \alpha_2\vec{a}_2 + \alpha_3\vec{a}_3$ va $\vec{z}_2 = \beta_1\vec{a}_1 + \beta_2\vec{a}_2 + \beta_3\vec{a}_3$ elementlari berilgan bo'lsin.

$$\vec{z}_1 + \vec{z}_2 = \alpha_1\vec{a}_1 + \alpha_2\vec{a}_2 + \alpha_3\vec{a}_3 + \beta_1\vec{a}_1 + \beta_2\vec{a}_2 + \beta_3\vec{a}_3 = (\alpha_1 + \beta_1)\vec{a}_1 + (\alpha_2 + \beta_2)\vec{a}_2 + $$
$$+ (\alpha_3 + \beta_3)\vec{a}_3 \in L(\vec{a}_1, \vec{a}_2, \vec{a}_3)$$

Demak, qo'shish binar amali $L(\vec{a}_1, \vec{a}_2, \vec{a}_3)$ to'plamda aniqlangan.

2. $L(\vec{a}_1, \vec{a}_2, \vec{a}_3)$ to'plamning ixtiyoriy $\vec{z}_1, \vec{z}_2$ elementlari berilgan bo'lsin

$$\vec{z}_1 + \vec{z}_2 = \alpha_1\vec{a}_1 + \alpha_2\vec{a}_2 + \alpha_3\vec{a}_3 + \beta_1\vec{a}_1 + \beta_2\vec{a}_2 + \beta_3\vec{a}_3 = (\alpha_1 + \beta_1)\vec{a}_1 + (\alpha_2 + \beta_2)\vec{a}_2 +$$
$$+ (\alpha_3 + \beta_3)\vec{a}_3 = (\beta_1 + \alpha_1)\vec{a}_1 + (\beta_2 + \alpha_2)\vec{a}_2 + (\beta_3 + \alpha_3)\vec{a}_3 = \beta_1\vec{a}_1 + \beta_2\vec{a}_2 + \beta_3\vec{a}_3 +$$
$$+ \alpha_1\vec{a}_1 + \alpha_2\vec{a}_2 + \alpha_3\vec{a}_3 = \vec{z}_2 + \vec{z}_1$$

Demak, $L(\vec{a}_1, \vec{a}_2, \vec{a}_3)$ da qo'shish amali kommutativ.

3. $L(\vec{a}_1, \vec{a}_2, \vec{a}_3)$ to'plamning ixtiyoriy $\vec{z}_1 = \alpha_1\vec{a}_1 + \alpha_2\vec{a}_2 + \alpha_3\vec{a}_3$ $\vec{z}_2 = \beta_1\vec{a}_1 + \beta_2\vec{a}_2 + \beta_3\vec{a}_3$ $\vec{z}_3 = \gamma_1\vec{a}_1 + \gamma_2\vec{a}_2 + y_3\vec{a}_3$ elementlari berilgan bo'lsin.

$(\vec{z}_1 + \vec{z}_2) + \vec{z}_3 = (\alpha_1 + \beta_1)\vec{a}_1 + (\alpha_2 + \beta_2)\vec{a}_2 + (\alpha_3 + \beta_3)\vec{a}_3 + \gamma_1\vec{a}_1 + \gamma_2\vec{a}_2 + y_3\vec{a}_3 =$

=|haqiqiy sonlar to'plamida qo'shish amali kommutativ, ko'paytirish qo'shishga nisbatan distributivligidan|=

$=((\alpha_1 + \beta_1) + \gamma_1)\vec{a}_1 + ((\alpha_2 + \beta_2) + \gamma_2)\vec{a}_2 + ((\alpha_3 + \beta_3) + \gamma_3)\vec{a}_3 =$ |haqiqiy sonlar maydonida qo'shish amali assosiativ bo'lganligi uchun|

$$(\alpha_1 + (\beta_1 + \gamma_1))\vec{a}_1 + (\alpha_2 + (\beta_2 + \gamma_2))\vec{a}_2 + (\alpha_3 + (\beta_3 + \gamma_3))\vec{a}_3 =$$
$$= (\alpha_1\vec{a}_1 + \alpha_2\vec{a}_2 + \alpha_3\vec{a}_3) + ((\beta_1 + \gamma_1)\vec{a}_1 + (\beta_2 + \gamma_2)\vec{a}_2 + (\beta_3 + \gamma_3)\vec{a}_3) =$$
$$= (\alpha_1\vec{a}_1 + \alpha_2\vec{a}_2 + \alpha_3\vec{a}_3) + ((\beta_1\vec{a}_1 + \beta_2\vec{a}_2 + \beta_3\vec{a}_3) + (\gamma_1\vec{a}_1 + \gamma_2\vec{a}_2 + \gamma_3\vec{a}_3)) =$$
$$\vec{z}_1 + (\vec{z}_2 + \vec{z}_3).$$

Demak, $L(\vec{a}_1, \vec{a}_2, \vec{a}_3)$ to'plamda qo'shish amali assosiativ

4. $L(\vec{a}_1, \vec{a}_2, \vec{a}_3)$ to'plamning ixtiyoriy $\vec{z} = \alpha_1\vec{a}_1 + \alpha_2\vec{a}_2 + \alpha_3\vec{a}_3$ elementi uchun $\vec{z} + \vec{e} = \vec{z}$ tenglikni qanoatlantiruvchi shunday $\vec{e} = e_1\vec{a}_1 + e_2\vec{a}_2 + e_3\vec{a}_3$ mavjudligini aniqlaymiz.

$\vec{z} + \vec{e} = \vec{z}$ tenglikdan $(\alpha_1 + e_1)\vec{a}_1 + (\alpha_2 + e_2)\vec{a}_2 + (\alpha_3 + e_3)\vec{a}_3 = \alpha_1\vec{a}_1 + \alpha_2\vec{a}_2 + \alpha_3\vec{a}_3$

tenglamani, bundan $\begin{cases} \alpha_1 + e_1 = \alpha_1 \\ \alpha_2 + e_2 = \alpha_2 \\ \alpha_3 + e_3 = \alpha_3 \end{cases}$ tenglamalar sistemasini hosil qilamiz. R da tenglamalar sistemasi yagona $e_1 = 0$, $e_2 = 0$, $e_3 = 0$ yechimga ega.

Demak, $\vec{e} = 0 \cdot \vec{a}_1 + 0 \cdot \vec{a}_2 + 0 \cdot \vec{a}_3 = \vec{0} \in L(\vec{a}_1, \vec{a}_2, \vec{a}_3)$

5. $L(\vec{a}_1,\vec{a}_2,\vec{a}_3)$ to'plamning ixtiyoriy $\vec{z}=\alpha_1\vec{a}_1+\alpha_2\vec{a}_2+\alpha_3\vec{a}_3$ va shunday $\vec{z'}=\alpha'_1\vec{a}_1+\alpha'_2\vec{a}_2+\alpha'_3\vec{a}_3$ elementlari uchun $\vec{z}+\vec{z'}=\vec{0}$ ekanligini keltirib chiqaramiz:

$\vec{z}+\vec{z'}=\vec{0}$ dan

$(\alpha_1+\alpha'_1)\vec{a}_1+(\alpha_2+\alpha'_2)\vec{a}_2+(\alpha_3+\alpha'_3)\vec{a}_3=0\cdot\vec{a}_1+0\cdot\vec{a}_2+0\cdot\vec{a}_3$

tenglamani, bundan $\begin{cases}\alpha_1+\alpha'_1=0\\ \alpha_2+\alpha'_2=0\\ \alpha_3+\alpha'_3=0\end{cases}$ tenglamalar sistemasini hosil qilamiz.

Tenglamalar sistemasi R maydonda yagona $\alpha_1=-\alpha_1$, $\alpha_2=-\alpha_2$, $\alpha_3=-\alpha_3$ yechimga ega. Demak,

$\vec{z'}=-\alpha_1\vec{a}_1-\alpha_2\vec{a}_2-\alpha_3\vec{a}_3=-(\alpha_1\vec{a}_1+\alpha_2\vec{a}_2+\alpha_3\vec{a}_3)\in L(\vec{a}_1,\vec{a}_2,\vec{a}_3)$ va

$< L(\vec{a}_1,\vec{a}_2,\vec{a}_3);+,-,\vec{0} >$ - additiv abel gruppa.

6. $L(\vec{a}_1,\vec{a}_2,\vec{a}_3)$ to'plamda λ skalyarni $\vec{z}=\alpha_1\vec{a}_1+\alpha_2\vec{a}_2+\alpha_3\vec{a}_3$ vektorga ko'paytirish unar amalini aniqlaymiz.

Ixtiyoriy $\lambda\in R$ skalyar va ixtiyoriy $\vec{z}=\alpha_1\vec{a}_1+\alpha_2\vec{a}_2+\alpha_3\vec{a}_3$ berilgan bo'lsin.

$$\omega_\alpha(\vec{z})=\lambda\cdot\vec{z}=\lambda\cdot(\alpha_1\vec{a}_1+\alpha_2\vec{a}_2+\alpha_3\vec{a}_3)=\lambda\cdot(\alpha_1\vec{a}_1)+\lambda\cdot(\alpha_2\vec{a}_2)+\lambda\cdot(\alpha_3\vec{a}_3)=$$
$$=(\lambda\cdot\alpha_1)\vec{a}_1)+(\lambda\cdot\alpha_2)\vec{a}_2+(\lambda\cdot\alpha_3)\vec{a}_3\in L(\vec{a}_1,\vec{a}_2,\vec{a}_3).$$

Demak, $L(\vec{a}_1,\vec{a}_2,\vec{a}_3)$ da skalyarni $L(\vec{a}_1,\vec{a}_2,\vec{a}_3)$ning elementiga ko'paytirish unar amallari aniqlangan.

7. Ixtiyoriy $\lambda\in R$ skalyar va $L(\vec{a}_1,\vec{a}_2,\vec{a}_3)$ning $\vec{z}_1,\vec{z}_2$ elementlari berilgan bo'lsin.

$$\lambda(\vec{z}_1+\vec{z}_2)=\lambda((\alpha_1+\beta_1)\vec{a}_1+(\alpha_2+\beta_2)\vec{a}_2+(\alpha_3+\beta_3)\vec{a}_3)=\lambda(\alpha_1+\beta_1)\vec{a}_1+$$
$$+\lambda(\alpha_2+\beta_2)\vec{a}_2+\lambda(\alpha_3+\beta_3)\vec{a}_3=\lambda(\alpha_1\vec{a}_1)+\lambda(\alpha_2\vec{a}_2)+\lambda(\alpha_3\vec{a}_3)+\lambda(\beta_1\vec{a}_1)+$$
$$+\lambda(\beta_2\vec{a}_2)+\lambda(\beta_3\vec{a}_3)=\lambda(\alpha_1\vec{a}_1+\alpha_2\vec{a}_2+\alpha_3\vec{a}_3)+\lambda(\beta_1\vec{a}_1+\beta_2\vec{a}_2+\beta_3\vec{a}_3)=\lambda\cdot\vec{z}_1+\lambda\cdot\vec{z}_2.$$

Demak, skalyarni vektorlar yig'indisiga ko'paytirish distributiv.

8. Ixtiyoriy $\lambda,\delta\in R$ skalyarlar va ixtiyoriy $\vec{z}=\alpha_1\vec{a}_1+\alpha_2\vec{a}_2+\alpha_3\vec{a}_3$ element berilgan bo'lsin.

$$(\lambda\cdot\delta)\cdot\vec{z}=(\lambda\cdot\delta)(\alpha_1\vec{a}_1+\alpha_2\vec{a}_2+\alpha_3\vec{a}_3)=(\lambda\cdot\delta)(\alpha_1\vec{a}_1)+(\lambda\cdot\delta)(\alpha_2\vec{a}_2)+$$
$$+(\lambda\cdot\delta)(\alpha_3\vec{a}_3)=((\lambda\cdot\delta)\alpha_1)\vec{a}_1+((\lambda\cdot\delta)\alpha_2)\vec{a}_2+((\lambda\cdot\delta)\alpha_3)\vec{a}_3=$$
$$=(\lambda(\delta\cdot\alpha_1)\vec{a}_1+(\lambda(\delta\cdot\alpha_2)\vec{a}_2+(\lambda(\delta\cdot\alpha_3)\vec{a}_3=\lambda((\delta\cdot\alpha_1)\vec{a}_1+(\delta\cdot\alpha_2)\vec{a}_2+$$
$$+(\delta\cdot\alpha_3)\vec{a}_3)=\lambda(\delta(\alpha_1\vec{a}_1+\alpha_2\vec{a}_2+\alpha_3\vec{a}_3))=\lambda(\delta\cdot\vec{z}).$$

Demak, skalyarlar ko'paytmasini $L(\vec{a}_1,\vec{a}_2,\vec{a}_3)$ elementiga ko'paytirish assosiativ.

9. Ixtiyoriy $\lambda,\delta\in R$ skalyarlar va ixtiyoriy $\vec{z}=\alpha_1\vec{a}_1+\alpha_2\vec{a}_2+\alpha_3\vec{a}_3$ element berilgan bo'lsin.

$$(\lambda+\delta)\cdot\vec{z}=(\lambda+\delta)(\alpha_1\vec{a}_1+\alpha_2\vec{a}_2+\alpha_3\vec{a}_3)=(\lambda+\delta)\alpha_1\vec{a}_1+(\lambda+\delta)\alpha_2\vec{a}_2+$$
$$+(\lambda+\delta)\alpha_3\vec{a}_3=\lambda(\alpha_1\vec{a}_1)+\delta(\alpha_1\vec{a}_1)+\lambda(\alpha_2\vec{a}_2)+\delta(\alpha_2\vec{a}_2)+\lambda(\alpha_3\vec{a}_3)+$$
$$+\beta(\alpha_3\vec{a}_3)=\lambda(\alpha_1\vec{a}_1+\alpha_2\vec{a}_2+\alpha_3\vec{a}_3)+\delta(\alpha_1\vec{a}_1+\alpha_2\vec{a}_2+\alpha_3\vec{a}_3)=$$
$$\lambda\cdot\vec{z}+\delta\cdot\vec{z}.$$

Demak, skalyarlar yig'indisini $L(\vec{a}_1,\vec{a}_2,\vec{a}_3)$ elementiga ko'paytirish distributiv.

10. $L(\vec{a}_1,\vec{a}_2,\vec{a}_3)$ to'plamning ixtiyoriy $\vec{z}=\alpha_1\vec{a}_1+\alpha_2\vec{a}_2+\alpha_3\vec{a}_3$ elementi berilgan bo'lsin. Skalyarlar to'plami maydon tashkil etishi va har qanday maydonda 1 mavjud ekanligidan

$$1\cdot\vec{z}=1\cdot(\alpha_1\vec{a}_1+\alpha_2\vec{a}_2+\alpha_3\vec{a}_3)=1\cdot(\alpha_1\vec{a}_1)+1\cdot(\alpha_2\vec{a}_2)+1\cdot(\alpha_3\vec{a}_3)=$$
$$=\alpha_1\vec{a}_1+\alpha_2\vec{a}_2+\alpha_3\vec{a}_3=\vec{z}.$$

Demak, $< L(\vec{a}_1,\vec{a}_2,\vec{a}_3);+,\omega_\lambda\mid\lambda\in R >$ algebra chiziqli fazo bo'ladi.

$\overline{a_1},\overline{a_2},\overline{a_3}$ vektorlar sistemasi chiziqli qobig'i tashkil etgan chiziqli fazo bazisini topish uchun $\overline{a_1},\overline{a_2},\overline{a_3}$ vektorlar sistemasining bazisini topamiz.

$\overline{a_1}=(0,\ 1,\ 1,\ 0),\overline{a_2}=(-1,\ 0,\ 0,\ 1),\overline{a_3}=(2,\ 1,\ 1,\ 4)$ sistema vektorlarini matrisaning satrlari sifatida olamiz va matrisaning bazisini topimiz:

$$\begin{array}{c}\vec{a}_1\\ \vec{a}_2\\ \vec{a}_3\end{array}\begin{pmatrix}0&1&1&0\\-1&0&0&1\\2&1&1&4\end{pmatrix}\sim\begin{array}{c}\vec{a}_2\\ \vec{a}_1\\ \vec{a}_3+\vec{a}_2\end{array}\begin{pmatrix}-1&0&0&1\\0&1&1&0\\0&1&1&6\end{pmatrix}\sim\begin{array}{c}\vec{a}_2\\ \vec{a}_1\\ (\vec{a}_3+\vec{a}_2)-\vec{a}_1\end{array}\begin{pmatrix}-1&0&0&1\\0&1&1&0\\0&0&0&6\end{pmatrix}$$

Matrisaning satr vektorlari chiziqli erkli bo'lganligi uchun, $\overline{a_1}=(0,\ 1,\ 1,\ 0)$, $\overline{a_2}=(-1,\ 0,\ 0,\ 1)$, $\overline{a_3}=(2,\ 1,\ 1,\ 4)$ vektorlar sistemasining bazisi sistemaning o'zidan iborat. Demak, L=$< L(\vec{a}_1,\vec{a}_2,\vec{a}_3);+,\omega_\lambda \mid \lambda \in R >$ chiziqli fazoning bazisi berilgan vektorlar sistemasidan iborat.

Misol va mashqlar

1. $\vec{x}=\vec{a}+\vec{b}-\vec{c}$ vektorni toping:

1.1. $\vec{a}=(4,2,3),\vec{b}=(2,3,7),\vec{c}=(1,7,11)$.

1.2. $\vec{a}=(2,4-,2,0),\vec{b}=(-1,3,17,3),\vec{c}=(0,-7,1,4)$.

1.3. $\vec{a}=(4,2,3)+(1,1,1),\vec{b}=(2,3,7),\vec{c}=2(1,7,11)$.

1.4. $\vec{a}=(-1)(4,2,3),\vec{b}=\vec{c},\vec{c}=(1,7,11)$.

2. $\vec{x}$ vektorni toping:

2.1. $\vec{x}=2\vec{a}-3\vec{b}+\vec{c},\vec{a}=(1,2,3,0),\vec{b}=(-2,1,5,-1),\vec{c}=(\sqrt{2},-1,0,1)$.

2.2. $-3\vec{a}-\vec{x}=2\vec{b},\vec{a}=(0,-2,1),\vec{b}=(1,-3,7)$.

2.3. $2\vec{x}+3\vec{a}-4\vec{b}=\vec{0},\vec{a}=(\sin\alpha,0,-\cos\alpha),\vec{b}=(\sin^3\alpha,\frac{1}{4},-\cos^3\alpha)$.

2.4. $\frac{1}{2}\vec{a}+3\vec{b}-\vec{x}=6\vec{c},\vec{a}=(1,-3,2),\vec{b}=(\frac{1}{9},\frac{11}{3},-2),\vec{c}=(1,\frac{2}{3},-2)$.

3. Vektorlarning quyidagi sistemalari chiziqli bog'liq yoki erkliligini aniqlang hamda uning bazisi va rangini toping:

3.1. $\vec{a}_1$=(-1,2,-3,4); $\vec{a}_2$=(-1,1,-1,1).

3.2. $\vec{a}_1$=(0,2,0,4); $\vec{a}_2$=(0,-2,-3,0); $\vec{a}_3$=(-1,1,-1,1).

3.3. $\vec{a}_1$ = (1,2,3); $\vec{a}_2$=(2,-3,4); $\vec{a}_3$=(-1,-1,1); $\vec{a}_4$=(3,4,2).

3.4. $\vec{a}_1$=(1,2,3,4,-3); $\vec{a}_2$=(-1,-2,3,4,1); $\vec{a}_3$=(-5,1,-7,1,2); $\vec{a}_4$=(0,4,1,2,0).

3.5. $\vec{a}_1$=(2,3); $\vec{a}_2$=(-1,-3); $\vec{a}_3$=(1,-1); $\vec{a}_4$=(3,1).

3.6. $\vec{a}_1$=(1,1,-1); $\vec{a}_2$=(4,1,2); $\vec{a}_3$=(-2,4,7) .

3.7. $\vec{a}_1$=(0,2,3,4); $\vec{a}_2$=(5,-2,-3,-4); $\vec{a}_3$=(3,1,2,-3).

3.8. $\vec{a}_1$=(0,2,0); $\vec{a}_2$=(0,-2,-3); $\vec{a}_3$=(-1,1,-1) .

3.9. $\vec{a}_1$=(-4,2,3); $\vec{a}_2$=(2,0,4); $\vec{a}_3$=(-1,-1,-1).

3.10. $\vec{a}_1$=(-4,2,3,0); $\vec{a}_2$=(2,0,0,4); $\vec{a}_3$=(-1,-1,0,-1).

4. Vektorlarning (*a*) va (*b*) sistemalari ekvivalent ekanligini tekshiring:

4.1. $\vec{a}_1$=(2,3); $\vec{a}_2$=(-1,-3); $\vec{a}_3$=(1,-1); $\vec{a}_4$=(3,1);

$\vec{b}_1$=(-2,-6); $\vec{b}_2$=(1,-1); $\vec{b}_3$=(4,0) .

4.2. $\vec{a}_1$=(1,-3,4); $\vec{a}_2$=(-1,-2,-3); $\vec{a}_3$=(8,1,-1); $\vec{a}_4$ =(-3,-4,-1);

$\vec{b}_1$ = (-1, 3, -4); $\vec{b}_2$ = (0, -5, 1); $\vec{b}_3$ = (5, -3, -2).

4.3. $\vec{a}_1$=(0,1,2,3,4); $\vec{a}_2$=(-2,-1,2,-3,4); $\vec{a}_3$=(3,-1,1,-1,1); $\vec{a}_4$=(9,3,4,1,2);

$\vec{b}_1$ = (0, -1, -2, -3, -4); $\vec{b}_2$ = (-2, 0, 4, 0, 8) .

4.4. $\vec{a}_1$=(1,2,3,4); $\vec{a}_2$=(-1,2,-3,4); $\vec{a}_3$=(-1,1,-1,1); $\vec{a}_4$=(3,4,1,2);

$\vec{b}_1$ = (5, 6, 7, 8); $\vec{b}_2$ = (0, 4 , 0, 8); $\vec{b}_3$ = (2, 3, 2, 1).

4.5. $\vec{a}_1$=(0,2,0,4); $\vec{a}_2$=(0,-2,-3,0); $\vec{a}_3$=(-1,1,-1,1);

$\vec{b}_1$ = (0, 0, -3, 4); $\vec{b}_2$ = (6, -8 , 3, -6); $\vec{b}_3$ = (-1, 3, -1, 5) .

4.6. $\vec{a}_1$=(0,1,4); $\vec{a}_2$=(2,-3,4); $\vec{a}_3$=(-1,1,-1);

$\vec{b}_1$ = (6, -3, 8); $\vec{b}_2$ = (3, -4 , 5); $\vec{b}_3$ = (-2, 2, -2).

4.7. $\vec{a}_1$=(-1,2,-3,4); $\vec{a}_2$=(1,2,3,-4); $\vec{a}_3$=(1,1,-1,1);

$\vec{b}_1$ = (-1, 6, -3, 4); $\vec{b}_2$ = (0, 4 , 0, 0); $\vec{b}_3$ = (2, 3, 2, -3).

4.8. $\vec{a}_1$=(1,-2,3,-4); $\vec{a}_3$=(-1,-1,-1,-1); $\vec{a}_3$=(-3,4,1,2);

$\vec{b}_1$ = (0, -3, 2, -5); $\vec{b}_2$ = (-4, 3 , 0, 1) .

4.9. $\vec{a}_1$=(0,2,3,4); $\vec{a}_2$=(-1,0,-1,1); $\vec{a}_3$=(3,4,1,0);

$\vec{b}_1$ = (-1, 2, 2, 5); $\vec{b}_2$ = (2, 4, 0, -1); $\vec{b}_3$ = (-6, -8, -2, 0) .

4.10. $\vec{a}_1$=(-1,1,-3,3); $\vec{a}_2$=(-4,1,-3,0);

$\vec{b}_1$ = (-3, 0, 0, -3); $\vec{b}_2$ = (4, 3 , 2, 2) .

5. $\vec{x}$ vektorning (*a*) sistemadagi chiziqli ifodasini toping :

5.1. $\vec{x}$ = (1,1,1) ; $\vec{a}_1$=(1,2,3); $\vec{a}_2$=(2,-3,4); $\vec{a}_3$=(-1,-1,1); $\vec{a}_4$=(3,4,2).

5.2. $\vec{x}$ = (4,7,1,-1) ; $\vec{a}_1$=(-1,7,-3,9); $\vec{a}_2$=(-1,6,-1,1); $\vec{a}_3$=(3,-4,-1,2).

5.3. $\vec{x}=(-4,9)$; $\vec{a}_1=(3,4)$; $\vec{a}_2=(-2,-3)$; $\vec{a}_3=(-1,6)$.

5.4. $\vec{x}=(1,1,1,1,1)$; $\vec{a}_1=(0,1,2,3,4)$; $\vec{a}_2=(-2,-1,2,-3,4)$; $\vec{a}_3=(3,-1,1,-1,1)$.

5.5. $\vec{x}=(1,-3,0)$; $\vec{a}_1=(0,1,4)$; $\vec{a}_2=(2,-3,4)$; $\vec{a}_3=(-1,1,-1)$.

5.6. $\vec{x}=(-6,-1,0,0,1)$; $\vec{a}_1=(1,2,3,4,-3)$; $\vec{a}_2=(-1,-2,3,4,1)$; $\vec{a}_3=(-5,1,-7,1,2)$.

5.7. $\vec{x}=(-9,1)$; $\vec{a}_1=(2,3)$; $\vec{a}_2=(-1,-3)$; $\vec{a}_3=(1,-1)$; $\vec{a}_4=(3,1)$.

5.8. $\vec{x}=(-2,-1,0)$; $\vec{a}_1=(1,-3,4)$; $\vec{a}_2=(-1,-2,-3)$; $\vec{a}_3=(8,1,-1)$; $\vec{a}_4=(-3,-4,-1)$.

5.9. $\vec{x}=(4,-1,1)$; $\vec{a}_1=(5,3,-4)$; $\vec{a}_2=(0,2,4)$; $\vec{a}_3=(1,5,2)$.

6. 5-misoldagi (*a*) sistema chiziqli qobig'ining chiziqli fazo tashkil etishini tekshiring.

7. F^n da aniqlangan qo'shish va skalyarni vektorga ko'paytirish amallarining quyidagi xossalarini isbotlang:

1°. $\forall(\vec{a},\vec{b}\in F^n)(\vec{a}+\vec{b}=\vec{b}+\vec{a})$ -qo'shishning kommutativlik xossasi;

2°. $\forall(\vec{a},\vec{b},\vec{c}\in F^n)((\vec{a}+\vec{b})+\vec{c}=\vec{a}+(\vec{b}+\vec{c}))$ -qo'shishning assosiativlik xossasi;

3°. $\forall(\vec{a}\in F^n)(\vec{a}+\vec{0}=\vec{a})$ (qo'shishga nisbatan neytral element mavjud);

4°. $\forall(\vec{a}\in F^n)(\vec{a}+(-\vec{a})=\vec{0})$ (qo'shish amaliga nisbatan simmetrik element mavjud);

5°. $\forall(\lambda\in F)\wedge\forall(\vec{a}\in F^n)(\lambda(\vec{a}+\vec{b})=\lambda\vec{a}+\lambda\vec{b})$ (skalyarni vektorlar yig'indisiga ko'paytirish distributiv);

6°. $\forall(\lambda,\mu\in F)\wedge\forall(\vec{a}\in F^n)((\lambda\cdot\mu)\vec{a}=\lambda(\mu(\vec{a})))$ (skalyarlar ko'paytmasini vektorga ko'paytirish assosiativ);

7°. $\forall(\lambda,\mu\in F)\wedge\forall(\vec{a}\in F^n)((\lambda+\mu)(\vec{a})=(\lambda\vec{a}+\mu\vec{a}))$ (skalyarlar yig'indisini vektorga ko'paytirish distributiv;

8°. $\forall(\vec{a}\in F^n)(1\cdot\vec{a}=\vec{a})$.

8. Quyidagi to'plamlar R maydon ustida chiziqli fazo tashkil etishini isbotlang:

8.1. V = $\{\frac{a}{b}\mid a,b\in Z\wedge b\neq 0\}$.

8.2. V = $\{(a,0)\mid a\in \mathrm{R}\}$.

8.3. V = $\{(a,0,0)\mid a\in \mathrm{R}\}$.

8.4. V = $\{(a,0,c)\mid a,c\in \mathrm{R}\}$.

8.5. V = $\{(a,0,b,0)\mid a,b\in \mathrm{R}\}$.

8.6. V = $\{(0,a,b,0)\mid a,b\in \mathrm{R}\}$.

8.7. V = $\{ax+by)\mid a,b\in \mathrm{R}\}$.

8.8. V = $\{y=ax+b)\mid a,b\in \mathrm{R}\}$.

8.9. V = $\{ax+b\sqrt{2})\mid a,b\in \mathrm{Q}\}$.

8.10. V = $\{ax-by+cz)\mid a,b,c\in \mathrm{R}\}$.

9. Har qanday α_{ij} sonlar uchun quyidagi vektorlar sistemasi chiziqli erkli bo'lishini isbotlang:

$$\vec{a}_1=(1,0,0,\ldots,0,0,\alpha_{11},\alpha_{12},\ldots,\alpha_{1k}),$$
$$\vec{a}_2=(0,2,0,\ldots,0,0,\alpha_{21},\alpha_{22},\ldots,\alpha_{2k}),$$
$$\ldots\ldots\ldots\ldots\ldots\ldots$$
$$\vec{a}_m=(0,0,0,\ldots,0,1,\alpha_{m1},\alpha_{m2},\ldots,\alpha_{mk})$$

10. Har qanday $\alpha_i\,(i=1,2,\ldots,n)$ sonlar uchun quyidagi n-o'lchovli vektorlar sistemasi chiziqli bog'liq bo'lishini isbotlang:

$$\vec{e}_1=(1,0,0,\ldots,0,0),$$
$$\vec{e}_2=(0,1,0,\ldots,0,0),$$
$$\ldots\ldots\ldots$$
$$\vec{e}_n=(0,0,0,\ldots,0,1),$$
$$\vec{a}=(\alpha_1,\alpha_2,\ldots,\alpha_n)$$

11. Kamida bitta nol vektorga ega vektorlarning $\vec{a}_1,\vec{a}_2,\ldots,\vec{a}_n$ chekli sistemasi chiziqli bog'langan sistema bo'lishin isbotlang.

12. Chekli vektorlar sistemasining biror-bir qismi chiziqli bog'langan bo'lsa, sistemaning o'zi ham chiziqli bog'langan bo'lishini isbotlang.

13. Vektorlarning chiziqli bog'lanmagan sistemasining har qanday qism sistemasi chiziqli bog'lanmagan sistema bo'lishini isbotlang.

14. Agar $\vec{a}_2,...,\vec{a}_n$ vektorlardan kamida bittasi o'zidan oldingi vektorlarning chiziqli kombinasiyasidan iborat bo'lsa, u holda $\vec{a}_1 \neq \vec{0}$ bo'lgan $\vec{a}_1,\vec{a}_2,...,\vec{a}_n$ vektorlardan iborat sistema chiziqli bog'langan bo'lishini isbotlang.

15. Agar $\vec{a}_1,\vec{a}_2,...,\vec{a}_n$ vektorlarning sistemasi chiziqli bog'lanmagan bo'lib, $\vec{a}_1,\vec{a}_2,...,\vec{a}_n\vec{b}$ sistema chiziqli bog'langan bo'lsa, u holda $\vec{b}$ vektor $\vec{a}_1,\vec{a}_2,...,\vec{a}_n$ vektorlar sistemasi orqali yagona usulda chiziqli ifodalanishini isbotlang.

16. Agar $\vec{a}$ vektor $\vec{b}_1,\vec{b}_2,...\vec{b}_n$ orqali va $\vec{b}_i(i=\overline{1,n})$ vektorlar $\vec{c}_1,\vec{c}_2,...,\vec{c}_m$ vektorlar orqali chiziqli ifodalansa, u holda $\vec{a}$ vektor $\vec{c}_1,\vec{c}_2,...,\vec{c}_m$ vektorlar orqali chiziqli ifodalanishini isbotlang.

17. Agar $\vec{a}_1,...,\vec{a}_{n+1}$ vektorlar $\vec{b}_1,\vec{b}_2,...,\vec{b}_n$ vektorlar orqali chiziqli ifodalansa, u holda $\vec{a}_1,...,\vec{a}_{n+1}$ sistema chiziqli bog'langan bo'lishini isbotlang.

18. Agar $\vec{a}_1,\vec{a}_2,...,\vec{a}_n$ vektorlar $\vec{b}_1,\vec{b}_2,...\vec{b}_m$ sistema orqali chiziqli ifodalansa va $n > m$ bo'lsa, u holda $\vec{a}_1,\vec{a}_2,...,\vec{a}_n$ sistema chiziqli bog'langan bo'lishini isbotlang.

19. Agar $\vec{a}_1,\vec{a}_2,...,\vec{a}_n$ vektorlar $\vec{b}_1,\vec{b}_2,...\vec{b}_m$ sistema orqali chiziqli ifodalansa va $\vec{a}_1,\vec{a}_2,...,\vec{a}_n$ sistema chiziqli bog'lanmagan bo'lsa, u holda $n \leq m$ bo'lishini isbotlang.

20. Agar vektorlarning har qanday chiziqli erkli ikkita chekli sistemalari ekvivalent bo'lsa, ulardagi vektorlar soni teng bo'lishini isbotlang.

21. Agar vektorlarning har qanday chiziqli erkli ikkita chekli sistemalari ekvivalent bo'lsa, ulardagi vektorlar soni teng bo'lishini isbotlang.

22. Agar vektorlarning bir chekli sistemasi ikkinchi sistemani elementar alamashtirishlar natijasida hosil qilingan bo'lsa, bunday sistemalar ekvivalent bo'lishini isbotlang.

23. Kamida bitta noldan farqli vektorga ega bo'lgan har qanday chekli sistema bazisga ega. Vektorlar chekli sistemasining har qanday ikkita bazisi bir hil sondagi vektorlardan iborat bo'lishini isbotlang.

24. $\vec{a}_2,...,\vec{a}_n$ vektorlar sistemasi $\vec{b}_1,\vec{b}_2,...\vec{b}_m$ vektorlar sistemasi orqali chiziqli ifodalansa, u holda $\vec{a}_2,...,\vec{a}_n$ sistemaning rangi $\vec{b}_1,\vec{b}_2,...\vec{b}_m$ sistemaning rangidan katta emasligini ko'rsating.

25. Vektorlar chekli sistemasining har qanday qism sistemasining rangi sistema rangidan katta emasligini isbotlang.

26. Vektorlar ekvivalent chekli sistemalarining ranglari teng bo'lishini isbotlang.

27. n-o'lchovli arifmetik vektor fazoni har qanday chekli sistemasining rangi n dan katta emasligini ko'rsating.

28. Agar vektorlar chekli sistemasining rangi n ga teng bo'lsa, u holda uning k ta vektordan iborat har qanday qism sistemasi $k > n$ bo'lganda chiziqli bog'langan bo'lishini isbotlang.

29. Agar $\vec{a}_2,...,\vec{a}_n$ vektorlarning sistemasining rangi $\vec{a}_2,...,\vec{a}_n,\vec{b}$ vektorlar sistemasining rangiga teng bo'lsa, u holda $\vec{b}$ vektorni $\vec{a}_2,...,\vec{a}_n$ vektorlar sistemasining chiziqli kombinasiyasi ko'rinishida ifodalash mumkinligini ko'rsating.

30. L($\vec{a}_1,\vec{a}_2,...,\vec{a}_n$) chiziqli qobiq vektor fazo tashkil etishini isbotlang.

31. Agar $\vec{b}_1,\vec{b}_2,...,\vec{b}_m$ sistemaning xar bir vektori $\vec{a}_1,\vec{a}_2,...,\vec{a}_n$ sistema orqali

32.chiziqli ifodalansa, u holda L($\vec{b}_1,\vec{b}_2,...,\vec{b}_m$)$\subset$L($\vec{a}_1,\vec{a}_2,...,\vec{a}_n$) bo'lishini isbotlang.

33. . Agar $\vec{a}_1,\vec{a}_2,...,\vec{a}_n$ sistemaning rangi k bo'lsa, u holda L($\vec{a}_1,\vec{a}_2,...,\vec{a}_n$) chiziqli qobiq k o'lchovli bo'lishini ko'rsating.

34. N chiziqli ko'phillik F^n fazoning qismfazosini ifodalashi uchun $\vec{x}_0 \in$ W, ya'ni N=W munosabat bajarilishi zarur va etarli ekanligini isbotlang.

35. Ixtiyoriy ikkita $\vec{x}_0$+W va $\vec{y}_0$+W chiziqli ko'philliklar umumiy elementga ega bo'lmaydi yoki ular ustma-ust tushadi. Isbotlang.

36. F^n vektor fazoning qismfazolari W va W' berilgan bo'lsin. U holda $N_1=\vec{x}_1$+W, $N_2=\vec{x}_2$+W' ko'philliklarning ustma-ust tushishi va $\vec{x}_1-\vec{x}_2 \in$ W bo'lishi zarur va etarli. Isbotlang.

Takrorlash uchun savollar

1. n-o'lchovli vektor deb nimaga aytiladi?
2. n-o'lchovli vektorlarning yig'indisi va skalyarni vektorga ko'paytmasi deb nimaga aytiladi?
3. n-o'lchovli arifmetik vektor fazo deb nimaga aytiladi?
4. Vektorlarning chiziqli bog'liq sistemasi deb nimaga aytiladi?.
5. Vektorlarning chiziqli bog'liq bo'lmagan sistemasi ta'rifini ayting.
6. Vektorlarning ekvivalent sistemalari deb nimaga aytiladi?
7. Vektorlarning sistemasida qanday elementar almashtirishlar bajariladi?
8. Elementar almashtirishlar natijasida qanday sistema hosil bo'ladi?
9. Vektorlar chekli sistemasining bazisi va rangiga ta'rif bering.
10. Vektorlar sistemacining chiziqli qobig'i deb nimaga aytiladi?
11. Chiziqli qobiqning asosiy xossalarini bayon eting.
12. Chiziqli ko'phillikka ta'rif bering.
13. Chiziqli ko'phillikning asosiy xossalarini ayting.
14. Chiziqli ko'phillikka maktab matematikasidan misol keltiring.

14-§. Matrisa va uning rangi

Asosiy tushunchalar: matrisa, nomdosh matrisa, teng matrisalar, matrisaning satr rangi, matrisaning ustun rangi, transponirlangan matrisa, matrisani elementar almashtirishlar, pog'onasimon matrisa.

$F =< F;+,\cdot,-,^{-1},0,1 >$ maydon berilgan bo'lsin.

F maydonning mn ta a_{ij} ($i=\overline{1,m}$, $j=\overline{1,n}$) elementlaridan tuzilgan ushbu

$$A=\begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{m1} & a_{m3} & \dots & a_{mn} \end{pmatrix}$$

ko'rinishdagi jadval F maydon ustidagi $m\times n$ tartibli matrisa deyiladi.

A va B matrisalar berilgan bo'lib, ularning mos ravishda satrlari va ustunlari soni teng bo'lsa, u xolda A va B matrisalarni nomdosh matrisalar deb yuritiladi.

A matrisaning xar bir a_{ij} elementi V matrisaning unga mos b_{ij} elementiga teng bulsa, u xolda A va B nomdosh matrisalar teng (aks xolda teng emas) matrisalar deyiladi.

Bitta satrli matrisalarni satr vektorlar, bitta ustunli matrisalarni ustun vektorlar deb qarash mumkin.

$$A=\begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{pmatrix}$$ matrisada $\overline{A}_1,\dots,\overline{A}_m$ satr vektorlar va $\overline{A}^1,\dots,\overline{A}^n$ ustun vektorlar mavjud.

Matrisadagi satr vektorlar sistemasining rangiga matrisaning satr rangi, ustun vektorlar sistemasining rangiga uning ustun rangi deyiladi. A matrisaning satr rangini $r(A)$, ustun rangini $\rho(A)$ ko'rinishda belgilaymiz.

Matrisa rangini aniqlash uchun matrisa ustida elementar almashtirishlar bajariladi. Ular quyidagilar:

1. Matrisadagi ixtiyoriy ikkita satr yoki ustun o'rinlarini almashtirish.
2. Matrisadagi ixtiyoriy satr yoki ustun elementlarini noldan farqli songa kupaytirish.
3. Matrisaning satr yoki ustun elementlarini noldan farqli songa ko'paytirib, boshqa satr yoki ustunning mos elementlariga qo'shish.
4. Barcha elementlari nollardan iborat bulgan satr yoki ustunni matrisadan chiqarish.

Matrisa satrining boshlovchi elementi deb uning birinchi (chapdan o'ngga qaraganda) noldan farqli elementiga aytiladi.

Matrisa pog'onasimon deyiladi, agar uning nol qatorlari barcha nolmas qatorlardan keyin joylashgan va $\alpha_{1k_1},\alpha_{2k_2},\dots,\alpha_{rk_r}$ boshlovchi elementlari uchun $k_1<k_2<\dots<k_r$ bo'lsa.

A^t matrisa A matrisaning transponirlangani deyiladi, agar A^t matrisa A matrisa satrlarini ustunlar orqali yozishdan hosil bo'lgan bo'lsa, ya'ni

$$A=\begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{pmatrix}; \quad A^t=\begin{pmatrix} a_{11} & a_{21} & \dots & a_{m1} \\ a_{12} & a_{22} & \dots & a_{m2} \\ \dots & \dots & \dots & \dots \\ a_{1n} & a_{2n} & \dots & a_{mn} \end{pmatrix};$$

Misol. $A=\begin{pmatrix} 2 & 1 & 3 & 1 \\ -1 & 0 & 2 & 2 \\ 5 & 4 & 3 & 4 \end{pmatrix}$ matrisaning ustun va satr ranglarini toping.

Yechish: Matrisaning satr rangini topish uchun satr elementar almashtirishlar bajarilib, matrisaning satr vektorlari sistemasi rangi aniqlanadi:

$$A=\begin{pmatrix} 2 & 1 & 3 & 1 \\ -1 & 0 & 2 & 2 \\ 5 & 4 & 3 & 4 \end{pmatrix} \sim \begin{pmatrix} -1 & 0 & 2 & 2 \\ 0 & 1 & 7 & 5 \\ 0 & 4 & 13 & 14 \end{pmatrix} \sim \begin{pmatrix} -1 & 0 & 2 & 2 \\ 0 & 1 & 7 & 5 \\ 0 & 0 & -15 & -6 \end{pmatrix}$$

Hosil bo'lgan pog'onasimon matrisada noldan farqli satrlar 3 ta, demak $r(A)=3$.

Endi matrisaning ustun rangini aniqlash uchun unda ustun elementar almashtirishlar bajaramiz:

$$A=\begin{pmatrix} 2 & 1 & 3 & 1 \\ -1 & 0 & 2 & 2 \\ 5 & 4 & 3 & 4 \end{pmatrix} \sim \begin{pmatrix} 0 & 0 & 0 & 1 \\ -5 & -2 & -4 & 2 \\ -3 & 0 & -9 & 4 \end{pmatrix} \sim \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 2 \\ -3 & -9 & 0 & 4 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ 4 & 0 & -3 & 0 \end{pmatrix}.$$

Hosil bo'lgan ustunli pog'onasimon matrisada 3 ta nolmas ustun mavjud, ya'ni $\rho(A)=3$.

Misol va mashqlar

1. Matrisa rangini aniqlang:

1.1. $\begin{pmatrix} 1 & 2 \\ 1 & 2 \\ 0 & 1 \end{pmatrix}$. 1.2. $\begin{pmatrix} 3 & 0 & 2 \\ 6 & 0 & 4 \\ 9 & 0 & 6 \end{pmatrix}$.

1.3. $\begin{pmatrix} 3 & 0 & 2 \\ 1 & -1 & 3 \\ 4 & -1 & 6 \end{pmatrix}$. 1.4. $(3 \quad 0 \quad 2)$

2. Matrisaning ustun va satr ranglarining tengligini tekshiring:

2.1. $\begin{pmatrix} 6 & 0 & 9 \\ -4 & 5 & 3 \\ 7 & 1 & 2 \end{pmatrix}$. 2.2. $\begin{pmatrix} -1 & 2 & 3 \\ -6 & 5 & 3 \\ 7 & -1 & 4 \end{pmatrix}$.

2.3. $\begin{pmatrix} 4 & 0 & 7 \\ -8 & 5 & 3 \\ 4 & 11 & 4 \end{pmatrix}$. 2.4. $\begin{pmatrix} 13 & 10 & 3 \\ -6 & 15 & 3 \\ 27 & 1 & 14 \end{pmatrix}$.

2.5. $\begin{pmatrix} 2 & 4 & -7 & 0 \\ -2 & 3 & 5 & -1 \\ 0 & 6 & 7 & 9 \\ 11 & 2 & 5 & 7 \end{pmatrix}$. 2.6. $\begin{pmatrix} 3 & 11 & -7 & 4 \\ -1 & 13 & -5 & 8 \\ 10 & 9 & 11 & 2 \\ 7 & 4 & 0 & 11 \end{pmatrix}$

2.7. $\begin{pmatrix} -3 & 1 & 7 & 4 \\ -1 & 3 & 5 & 8 \\ 0 & 9 & 11 & 2 \\ 7 & -4 & 0 & 1 \end{pmatrix}$. 2.8. $\begin{pmatrix} 1 & 1 & 1 \\ \sin\alpha & \cos\alpha & tg\alpha \\ \sin^2\alpha & \cos^2\alpha & tg^2\alpha \end{pmatrix}$.

3. λ ning turli qiymatlarida $\begin{pmatrix} \lambda & 1 & 3 & 4 & 1 \\ 1 & \lambda & -1 & 1 & 5 \\ \lambda & \lambda & 4 & 3 & -4 \\ 1 & 1 & 7 & 7 & -3 \end{pmatrix}$ matrisa rangini toping.

4. λ ning qanday qiymatida $\begin{pmatrix} 3 & 1 & 1 & 4 \\ \lambda & 4 & 10 & 1 \\ 1 & 7 & 17 & 3 \\ 2 & 2 & 4 & 3 \end{pmatrix}$ matrisa rangi eng kichik bo'ladi?

Takrorlash uchun savollar

1. Matrisa deb nimaga aytiladi?
2. Nomdosh matrisalarga ta'rif bering.
3. Qanday matrisalar teng deyiladi?
4. Matrisaning satr (ustun) vektorlari sistemasi nima?
5. Matrisaning satr (ustun) rangi deb nimaga aytiladi?
6. Matrisani elementar almashtirishlar deb qanday almashtirishlarga aytiladi?

15-§. Chiziqli tenglamalar sistemasi

Asosiy tushunchalar: chiziqli tenglamalar sistemasi, ChTSning yechimi, hamjoyli ChTS, hamjoyli bo'lmagan ChTS, ChTSning natijasi, ChTSning chiziqli kombinasiyasi, teng kuchli ChTSlari, ChTSni elementar almashtirishlar, bir jinsli ChTS, BChTSning fundamental yechimlar sistemasi.

$F =< F;+,\cdot,-,^{-1},0,1 >$ maydon berilgan bo'lsin.

Barcha noma'lumlarining darajasi birdan katta bo'lmagan tenglamaga chiziqli tenglama deyiladi. $a_1x_1 + ... + a_nx_n = b$ tenglamani to'g'ri sonli tenglikka aylantiruvchi $\bar{\xi} = (\xi_1,...,\xi_n), \xi_i \in F, i = \overline{1,n}$ vektorga berilgan tenglamaning yechimi deyiladi.

Ushbu $\begin{cases} a_{11}x_1 + a_{12}x_2 + ... + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + ... + a_{2n}x_n = b_2 \\ \dots\dots\dots\dots\dots\dots\dots\dots \\ a_{m1}x_1 + a_{m2}x_2 + ... + a_{mn}x_n = b_m \end{cases}$ (1) sistemaga F maydon ustida berilgan n ta noma'lumli m ta chiziqli tenglamalar sistemasi deyiladi, bunda $a_{ij}, b_i \in F$ $(i = \overline{1,m}; j = \overline{1,n})$ sistemaning koeffisientlari, a_{ij} no'malumlar koeffisientlari, b_j ozod hadlar bo'lib, x_i lar esa no'malumlardan iborat.

n ta noma'lumli m ta chiziqli tenglamalar sistemasining yechimi deb shunday

$\bar{\xi} = (\xi_1,...,\xi_n), \xi_i \in F, i = \overline{1,n}$ vektorga aytiladiki, u sistemaning barcha tenglamalarini to'g'ri tenglikka aylantiradi.

ChTS kamida bitta yechimga ega bo'lsa, u hamjoyli, yechimga ega bo'lmasa, hamjoyli bo'lmagan ChTS deyiladi.

Yagona yechimga ega bo'lgan sistema aniq sistema, cheksiz ko'p yechimga ega bo'lgan sistema aniqmas sistema deyiladi.

Berilgan ikkita ChTS uchun birinchisining har bir yechimi ikkinchisi uchun ham yechim bo'lsa, ikkinchi ChTS birinchi ChTSning natijasi deyiladi.

Ikkita ChTS teng kuchli deyiladi, agar birinchisining har bir yechimi ikkinchisiga yechim bo'lsa va aksincha.

ChTSning noma'lumlari oldidagi koeffisientlardan tuzilgan A = $\begin{pmatrix} a_{11} & a_{12} & ... & a_{1n} \\ a_{21} & a_{22} & ... & a_{2n} \\ ... & ... & ... & ... \\ a_{m1} & a_{m2} & ... & a_{mn} \end{pmatrix}$ matrisa (1)ning asosiy matrisasi, noma'lumlar oldidagi koeffisientlar va ozod hadlardan iborat B= $\begin{pmatrix} a_{11} & a_{12} & ... & a_{1n} & b_1 \\ a_{21} & a_{22} & ... & a_{2n} & b_2 \\ ... & ... & ... & ... & ... \\ a_{m1} & a_{m2} & ... & a_{mn} & b_m \end{pmatrix}$ matrisa (1)ning kengaytirilgan matrisasi deyiladi.

Kroneker-Kapelli teoremasi. chiziqli tenglamalar sistemasi hamjoyli bo'lishi uchun uning asosiy va kengaytirilgan matrisalari ranglarining teng bo'lishi zarur va etarli.

$\begin{cases} a_{11}x_1 + a_{12}x_2 + ... + a_{1n}x_n = 0 \\ a_{21}x_1 + a_{22}x_2 + ... + a_{2n}x_n = 0 \\ \dots\dots\dots\dots\dots\dots\dots\dots \\ a_{m1}x_1 + a_{m2}x_2 + ... + a_{mn}x_n = 0 \end{cases}$ (1*) chiziqli tenglamalar sistemasiga bir jinsli chiziqli tenglamalar sistemasi (BChTS) deyiladi.

F^n arifmetik vektor fazoning W qism fazosining bazisini tashkil etuvchi istalgan vektorlar sistemasi (1*) sistemaning fundamental (asosiy) yechimlari sistemasi deyiladi.

Misol. Tenglamalar sistemasini Kroneker-Kapelli teoremasi asosida tekshiring va yechimlarini toping:

$$\begin{cases} 2x_1+3x_2-x_3+x_4=1 \\ -x_1-x_2+3x_3+2x_4=3 \\ 3x_1+4x_3-4x_4=5 \\ 5x_1+x_2+2x_3=6 \end{cases}$$

Yechish: Kroneker-Kapelli teoremasiga ko'ra bir jinsli bo'lmagan chiziqli tenglamalar sistemasi xamjoyli bo'lishi uchun uning asosiy A va kengaytirilgan B matrisalarning satr ranglari teng bo'lishi kerak.

Berilgan chiziqli tenglamalar sistemasining asosiy va kengaytirilgan matrisalari ranglarini topamiz. Buning uchun chiziqli tenglamalar sistemasining no'malumlari oldidagi koeffisientlardan A matrisani, unga ozod hadlar ustunini qo'shib B matrisani hosil qilamiz:

$$A=\begin{pmatrix} 2 & 3 & -1 & 1 \\ -1 & -1 & 3 & 2 \\ 3 & 0 & 4 & -4 \\ 5 & 1 & 2 & 0 \end{pmatrix} \qquad B=\left(\begin{array}{cccc|c} 2 & 3 & -1 & 1 & 1 \\ -1 & -1 & 3 & 2 & 3 \\ 3 & 0 & 4 & -4 & 5 \\ 5 & 1 & 2 & 0 & 6 \end{array}\right)$$

Matrisaning satr rangini topish uchun satr elementar almashtirishlar bajarib, uni pog'onasimon matrisa ko'rinishiga keltiramiz. Elementar almashtirishlar natijasida berilgan matrisaga ekvivalent matrisa hosil bo'ladi:

$$\left(\begin{array}{cccc|c} 2 & 3 & -1 & 1 & 1 \\ -1 & -1 & 3 & 2 & 3 \\ 3 & 0 & 4 & -4 & 5 \\ 5 & 1 & 2 & 0 & 6 \end{array}\right) \sim$$

birinchi va ikkinchi satrlar o'rnini almashtiramiz:

$$\sim \left(\begin{array}{cccc|c} -1 & -1 & 3 & 2 & 3 \\ 2 & 3 & -1 & 1 & 1 \\ 3 & 0 & 4 & -4 & 5 \\ 5 & 1 & 2 & 0 & 6 \end{array}\right) \sim$$

birinchi ustun birinchi qator elementi -1 ni qoldirib birinchi ustun boshqa elementlarini 0 ga aylantiramiz:

$$\sim \left(\begin{array}{cccc|c} -1 & -1 & 3 & 2 & 3 \\ 0 & 1 & 5 & 5 & 7 \\ 0 & -3 & 13 & 2 & 14 \\ 0 & -4 & 17 & 10 & 21 \end{array}\right) \sim$$

birinchi va ikkinchi satrlarni o'zgartirmaymiz, ikkichi satr yordamida uchinchi, to'rtinchi satrlarning ikkinchi ustunidagi elementlarni nolga aylantiramiz:

$$\sim \left(\begin{array}{cccc|c} -1 & -1 & 3 & 2 & 3 \\ 0 & 1 & 5 & 5 & 7 \\ 0 & 0 & 28 & 17 & 35 \\ 0 & 0 & 37 & 30 & 49 \end{array}\right) \sim$$

1-, 2-, 3- satrlarni qoldirib, 4-satrning 3-ustundagi elementini nolga aylantiramiz:

$$\sim \left(\begin{array}{cccc|c} -1 & -1 & 3 & 2 & 3 \\ 0 & 1 & 5 & 5 & 7 \\ 0 & 0 & 28 & 17 & 35 \\ 0 & 0 & 0 & -211 & -77 \end{array}\right)$$

Hosil bo'lgan pog'onasimon matrisadan asosiy matrisaning rangi $r(A)=4$ va kengaytirilgan matrisaning rangi $r(B)=4$ ekanligini aniqlaymiz (noldan farqli satrlar soni).

Demak, chiziqli tenglamalar sistemasining asosiy va kengaytirilgan matrisalarining ranglari teng, ya'ni teoremaga asosan berilgan chiziqli tenglamalar sistemasi hamjoyli. Endi chiziqli tenglamalar sistemasini yechimlarini topamiz. Buning uchun to'g'ridan-to'g'ri pog'onasimon matrisa yordamida berilgan chiziqli tenglamalar sistemasiga teng kuchli chiziqli tenglamalar sistemasini tuzamiz:

$$\begin{cases} -x_1-x_2+3x_3+2x_4=3 \\ x_2+5x_3+5x_4=7 \\ 28x_3+17x_4=35 \\ -211x_4=-77 \end{cases}$$

Bundan $x_1 = \frac{165}{211}$; $x_2 = \frac{7}{211}$; $x_3 = \frac{217}{211}$; $x_4 = \frac{77}{211}$ yechimni topamiz.

Misol. Berilgan tenglamalar sistemasini Gauss usulida eching:

$$\begin{cases} 3x_1 + x_2 - 2x_3 + x_4 = 2 \\ 2x_1 + x_3 - 2x_4 = 3 \\ 4x_1 + 8x_2 - 12x_3 = 4 \end{cases}$$

Yechish: Chiziqli tenglamalar sistemasini Gauss usuli bilan Yechish deganda sistemadagi noma'lumlarni ketma-ket yo'qotish tushuniladi. Ya'ni, tenglamalar sistemasida elementar almashtirishlar bajarib, tanlab olingan tenglama yordamida qolganlaridagi noma'lumlardan biri oldidagi koeffisientini nolga aylantiramiz. Bu jarayonni davom ettirib, berilgan chiziqli tenglamalar sistemasiga teng kuchli chiziqli tenglamalar sistemasini hosil qilamiz. Noma'lumlar soni eng kam bo'lgan tenglamadan boshlab, noma'lumlar topiladi.

Berilgan chiziqli tenglamalar sistemasidagi 3-tenglamani 4 ga bo'lib, birinchi o'ringa joylashtiramiz va uning yordamida qolgan tenglamalardan x_1 noma'lumni yo'qotamiz:

$$\begin{cases} 3x_1 + x_2 - 2x_3 + x_4 = 2 \\ 2x_1 + x_3 - 2x_4 = 3 \\ 4x_1 + 8x_2 - 12x_3 = 4 \end{cases} \Leftrightarrow \begin{cases} x_1 + 2x_2 - 3x_3 = 1 \\ -5x_2 + 7x_3 + x_4 = -1 \\ -4x_2 + 7x_3 - 2x_4 = 1 \end{cases} \Leftrightarrow$$

Hosil bo'lgan chiziqli tenglamalar sistemasidagi 1-, 2- tenglamalarni o'z o'rnida o'zgarishsiz qoldirib, 3-tenglamaning x_2 noma'lumini 2-tenglama yordamida yo'qotamiz:

$$\Leftrightarrow \begin{cases} x_1 + 2x_2 - 3x_3 = 1 \\ -5x_2 + 7x_3 + x_4 = -1 \\ 7x_3 - 12x_4 = 9 \end{cases}$$

Chiziqli tenglamalar sistemasidagi uchinchi tenglama ikki noma'lumli bitta tenglama bo'lib, uning cheksiz ko'p yechimlari mavjud, ya'ni x_3 yoki x_4 noma'lumni ixtiyoriy haqiqiy son qabul qiladi deb olib, ikkinchisini u orqali chiziqli ifodalaymiz.

Tenglamalar sistemasining yechimlari x_4 noma'lum orqali ifodalanuvchi to'rt o'lchovli arifmetik vektorlardan iborat bo'lgan, cheksiz ko'p elementga ega bo'lgan to'plam bo'ladi. Uni topamiz. Buning uchun $x_4 \in R$ deb olib, x_3 noma'lumni x_4 yordamida ifodalaymiz:

$$7x_3 - 12x_4 = 9 \Leftrightarrow 7x_3 = 12x_4 + 9 \Leftrightarrow x_3 = \frac{12}{7}x_4 + \frac{9}{7}.$$

Oxirgi chiziqli tenglamalar sistemasining ikkinchi tenglamasidagi x_3 noma'lum o'rniga uning topilgan ifodasini qo'yamiz:

$$-5x_2 + 7(\frac{12}{7}x_4 + \frac{9}{7}) + x_4 = -1 \Leftrightarrow -5x_2 + 84x_4 + 63 + x_4 = -1 \Leftrightarrow -5x_2 + 85x_4 + 63 = -1.$$

Hosil bo'lgan tenglamadagi x_2 noma'lumni x_4 orqali ifodasini topib, birinchi tenglamaga qo'yamiz:

$$-5x_2 = -85x_4 - 64 \Leftrightarrow x_2 = 17x_4 - \frac{64}{5}.$$

Birinchi tenglamadagi x_1 ni topamiz:

$$x_1 - 2x_2 - 3x_3 = 1 \Leftrightarrow x_1 = 2x_2 + 3x_3 + 1 = 34x_4 - \frac{128}{5} + \frac{36}{7}x_4 + \frac{27}{7} = \frac{274}{7}x_4 - \frac{761}{35};$$

Demak, berilgan chiziqli tenglamalar sistemasining yechimlar to'plami

$\left\{ (\frac{274}{7}x_4 - \frac{761}{35}; 17x_4 - \frac{64}{5}; \frac{12}{7}x_4 + \frac{9}{7}; x_4) \middle| x_4 \in R \right\}$ to'plamdan iborat.

Misol. Berilgan $\begin{cases} 2x_1 + 3x_2 - x_3 + x_4 = 1 \\ -x_1 - x_2 + 3x_3 + 2x_4 = 3 \\ 3x_1 + 4x_3 - 4x_4 = 5 \\ 5x_1 + x_2 + 2x_3 = 6 \end{cases}$

chiziqli tenglamalar sistemasi yordamida ko'phillik tuzing.

Yechish: 1) Berilgan bir jinsli bo'lmagan chiziqli tenglamalar sistemasi hamjoyli bo'lsa, bitta a_0 yechimini topamiz;

2) Chiziqli tenglamalar sistemasiga assosirlangan bir jinsli chiziqli tenglamalar sistemasining yechimlar fazosi W aniqlanadi.

3) Chiziqli ko'hillik ta'rifiga ko'ra $\overrightarrow{a_0} + W$ bir jinsli bo'lmagan hamjoyli

chiziqli tenglamalar sistemasi va unga assosirlangan bir jinsli chiziqli tenglamalar sistemalari yordamida hosil qilingan chiziqli ko'phillik bo'ladi.

1) $\begin{cases} 2x_1+3x_2-x_3+x_4=1 \\ -x_1-x_2+3x_3+2x_4=3 \\ 3x_1+4x_3-4x_4=5 \end{cases} \Leftrightarrow \begin{cases} x_1+x_2-3x_3-2x_4=-3 \\ x_2+5x_3+5x_4=7 \\ -3x_2+13x_3+2x_4=14 \end{cases} \Leftrightarrow$

$\Leftrightarrow \begin{cases} x_1+x_2-3x_3-2x_4=-3 \\ x_2+5x_3+5x_4=7 \\ 28x_3+17x_4=35 \end{cases}$

Noma'lumlarni ketma-ket yo'qotish natijasida 4 ta noma'lumli 3 ta tenglamadan iborat sistema hosil bo'ldi.

Demak, chiziqli tenglamalar sistemasi cheksiz ko'p yechimga ega. Umumiy yechim $(\frac{115}{14}x_4-\frac{25}{4};-\frac{225}{28}x_4+7;-\frac{17}{28}x_4+\frac{5}{4};x_4), x_4\in R$ ko'rinishda bo'lib,

uning bitta $\overrightarrow{a_0}$ yechimini $x_4=0$ qiymatni qo'yib topamiz;

$\overrightarrow{a_0}=(-\frac{25}{4};7;\frac{5}{4};0)$.

2) Berilgan chiziqli tenglamalar sistemasiga assosirlangan berilgan chiziqli tenglamalar sistemasi

$$\begin{cases} 2x_1+3x_2-x_3+x_4=0 \\ -x_1-x_2+3x_3+2x_4=0 \\ 3x_1+4x_3-4x_4=0 \end{cases}$$

ko'rinishda bo'lib, uning yechimlari cheksiz ko'p. Chunki,

$$\begin{cases} 2x_1+3x_2-x_3+x_4=0 \\ -x_1-x_2+3x_3+2x_4=0 \\ 3x_1+4x_3-4x_4=0 \end{cases} \Leftrightarrow \begin{cases} x_1+x_2-3x_3-2x_4=0 \\ x_2+5x_3+5x_4=0 \\ 28x_3+17x_4=0 \end{cases}$$

Bundan, $x_4\in R$ desak, yechimlar to'plami $W=\left\{(\frac{115}{14}x_4;-\frac{225}{28}x_4;-\frac{17}{28}x_4;x_4)\middle| x_4\in R\right\}$ ko'rinishda bo'ladi. Bir jinsli chiziqli tenglamalar sistemasining yechimlar to'plami chiziqli fazo tashkil etadi ($W=<W;+,\{\omega_\lambda|\lambda\in R\}>$).

3) Hosil bo'lgan $\overrightarrow{a_0}$ vektor va W to'plam yordamida $H=\overrightarrow{a_0}+W$ chiziqli ko'phillikni hosil qilamiz. Chiziqli ko'phillik H ning elementlari berilgan bir jinsli bo'lmagan chiziqli tenglamalar sistemasining yechimlar to'plamini tashkil etadi.

Misol. Berilgan 1) $\begin{cases} x_1+3x_2-5x_3+x_4=1 \\ x_1-2x_2+3x_3-2x_4=3 \end{cases}$ va 2) $\begin{cases} 2x_1+x_2-2x_3-x_4=4 \\ 3x_1-x_2+x_3-3x_4=7 \end{cases}$ chiziqli tenglamalar sistemalari uchun 2-sistema 1-sistemaning natijasi ekanligini tekshiring.

Yechish. 2-chiziqli tenglamalar sistemasi 1-chiziqli tenglamalar sistemasining natijasi bo'lishi uchun ta'rifga ko'ra 1-chiziqli tenglamalar sistemasining Har bir yechimi 2-chiziqli tenglamalar sistemasining Ham yechimi bo'lishi kerak.

1-chiziqli tenglamalar sistemasining yechimlarini Gauss usulidan foydalanib topamiz:

$$\begin{cases} x_1+3x_2-5x_3+x_4=1 \\ x_1-2x_2+3x_3-2x_4=3 \end{cases} \Leftrightarrow \begin{cases} x_1+3x_2-5x_3+x_4=1 \\ -5x_2+8x_3-3x_4=2 \end{cases}.$$

Hosil bo'lgan teng kuchli chiziqli tenglamalar sistemasidagi 2-tenglamada $x_3, x_4\in R$ deb olib, qolgan noma'lumlarni aniqlaymiz:

$$\begin{cases} x_1=\frac{1}{5}x_3+\frac{4}{5}x_4+\frac{11}{5}; \\ x_2=\frac{8}{5}x_3-\frac{3}{5}x_4-\frac{2}{5}; \\ x_3, x_4\in R. \end{cases}$$

Demak, berilgan 1-chiziqli tenglamalar sistemasining cheksiz ko'p yechimlari mavjud bo'lib, umumiy yechim quyidagi ko'rinishda bo'ladi: $(\frac{1}{5}x_3+\frac{4}{5}x_4+\frac{11}{5};\frac{8}{5}x_3-\frac{3}{5}x_4-\frac{2}{5};x_3;x_4), x_3, x_4\in R$.

Topilgan umumiy yechimni 2-chiziqli tenglamalar sistemasiga qo'yamiz:

$$\begin{cases} 2(\frac{1}{5}x_3+\frac{4}{5}x_4+\frac{11}{5})+\frac{8}{5}x_3-\frac{3}{5}x_4-\frac{2}{5}-2x_3-x_4=4 \\ 3(\frac{1}{5}x_3+\frac{4}{5}x_4+\frac{11}{5})-(\frac{8}{5}x_3-\frac{3}{5}x_4-\frac{2}{5})+x_3-3x_4=7 \end{cases} \Leftrightarrow \begin{cases} 4=4 \\ 7=7 \end{cases}.$$

Demak, 1-chiziqli tenglamalar sistemasining Har bir yechimi 2-chiziqli nglamalar sistemasining Ham yechimi bo'ladi. Ta'rifga ko'ra 2-sistema 1-sistemaga natija ekan.

Misol. Berilgan bir jinsli chiziqli tenglamalar sistemasi $\begin{cases} x_1+3x_2-5x_3+x_4=0 \\ x_1-2x_2+3x_3-2x_4=0 \end{cases}$ ning yechimlar to'plami fundamental sistemasini toping.

Yechish. Har qanday bir jinsli chiziqli tenglamalar sistemasi hamjoyli hamda yechimlar to'plami chiziqli vektor fazo tashkil etadi.

Agar bir jinsli chiziqli tenglamalar sistemasi yagona nol yechimga ega bo'lsa, u holda yechimlar fazosi nol o'lchovli chiziqli vektor fazo bo'lib, uning fundamental sistemasi mavjud emas.

Agar bir jinsli chiziqli tenglamalar sistemasi cheksiz ko'p yechimlarga ega bo'lsa, u holda yechimlar to'plami tashkil etgan chiziqli vektor fazoning bazisi fundamental sistema bo'ladi.

$$\begin{cases} x_1+3x_2-5x_3+x_4=0 \\ x_1-2x_2+3x_3-2x_4=0 \end{cases} \Leftrightarrow \begin{cases} x_1+3x_2-5x_3+x_4=0 \\ -5x_2+8x_3-3x_4=0 \end{cases}.$$

Hosil bo'lgan teng kuchli bir jinsli chiziqli tenglamalar sistemasidagi 2-tenglamada $x_3, x_4 \in R$ deb olib, qolgan noma'lumlarni aniqlaymiz:

$$\begin{cases} x_1=\frac{1}{5}x_3+\frac{4}{5}x_4; \\ x_2=\frac{8}{5}x_3-\frac{3}{5}x_4; \\ x_3, x_4 \in R. \end{cases}$$

Demak, berilgan bir jinsli chiziqli tenglamalar sistemasining cheksiz ko'p yechimlari mavjud bo'lib, umumiy yechim quyidagi ko'rinishda bo'ladi: $(\frac{1}{5}x_3+\frac{4}{5}x_4;\frac{8}{5}x_3-\frac{3}{5}x_4;x_3;x_4), x_3, x_4 \in R$.

Umumiy yechimdagi $x_3, x_4 \in R$ erkli o'zgaruvchilarga kamida bittasi noldan farqli qiymatlar beramiz. Masalan, $x_3=1, x_4=0; x_3=0, x_4=1$.

Hosil bo'lgan $\vec{a}_1=(\frac{1}{5};\frac{8}{5};1;0), \vec{a}_2=(\frac{4}{5};-\frac{3}{5};0;1)$ yechimlar yechimlar to'plamining ihtiyoriy yechimini chiziqli ifodalaydi. Demak, berilgan bir jinsli

chiziqli tenglamalar sistemasi yechimlar to'plamining fundamental sistemasi $\vec{a}_1=(\frac{1}{5};\frac{8}{5};1;0), \vec{a}_2=(\frac{4}{5};-\frac{3}{5};0;1)$ vektorlardan iborat.

Misol va mashqlar

1. 2-sistema 1-sistema uchun natija bo'lishini tekshiring:

1.1. 1) $\begin{cases} x_1-3x_2+4x_3+2x_4=1 \\ 2x_1+4x_2-3x_3+3x_4=-1 \end{cases}$; 2) $3x_1+x_2+x_3+5x_4=0$.

1.2. 1) $\begin{cases} 2x_1+4x_2-3x_3+3x_4=-1 \\ 3x_1+x_2+2x_3-x_4=0 \end{cases}$; 2) $-x_1+3x_2-5_3+4x_4=-1$.

1.3. 1) $\begin{cases} 3x_1+x_2+2x_3-x_4=0 \\ 12x_1+4x_2+7x_3+2x_4=2 \end{cases}$; 2) $-8x_1-3x_2-5x_3-3x_4=-2$.

1.4. 1) $\begin{cases} 3x_1-3x_2-6x_3-x_4=1 \\ x_1-6x_1+5x_2-12x_4=2 \\ x_1-7x_2+x_3+4x_4=23 \\ x_2+23x_4=23 \\ -2x_1-7x_2-2x_3+2x_4=14 \end{cases}$; 2) $\begin{cases} 4x_1-9x_2-x_3-13x_4=3 \\ -x_1-14x_2-x_3+6x_4=37 \end{cases}$

1.5. 1) $\begin{cases} 6x_1-5x_3+2x_4=41 \\ 3x_1+5x_2-x_3-3x_4=11 \\ x_1+2x_2+2x_3+13x_4=10 \\ 2x_1+4x_2+x_3-x_4=3 \end{cases}$; 2) $\begin{cases} 3x_1-5x_2-4x_3+5x_4=30 \\ -x_1-2x_2+x_3+14x_4=7 \end{cases}$.

2. Quyidagi elementar almashtirishlar yordamida berilgan ChTSga teng kuchli ChTS hosil bo'lishini isbotlang:

1) sistemadagi tenglamalar o'rnini almashtirish;

2) sistemani qandaydir tenglamasining ikkala qismini noldan farqli skalyarga ko'paytirish;

3) bir tenglamaning ikkala qismiga skalyarga ko'paytirilgan boshqa tenglamaning mos qismlarini qo'shish yoki ayirish.

3. Kroneker-Kapelli teoremasi yordamida quyidagi ChTSlarini tekshiring va yechimlar to'plamini aniqlang:

3.1. $\begin{cases} 5x_1+4x_2+3x_3=1, \\ 2x_1+x_2+4x_3=1, \\ -3x_1-2x_2-x_3=-1, \\ x_1+3x_2+2x_3=-2. \end{cases}$

3.2. $\begin{cases} x_1+2x_2+3x_3-2x_4=6, \\ 2x_1-x_2-2x_3-3x_4=8, \\ 3x_1+2x_2-x_3+2x_4=4, \\ 2x_1-3x_2+2x_3+x_4=-8. \end{cases}$

3.3. $\begin{cases} 6x_1+5x_2+2x_3+4x_4=-4, \\ 9x_1+x_2+4x_3-x_4=-1, \\ 3x_1+4x_2+2x_3-2x_4=-5, \\ 3x_1-9x_2+2x_4=11. \end{cases}$

3.4. $\begin{cases} x_1+x_2+3x_3-2x_4+3x_5=1, \\ 2x_1+2x_2+4x_3-x_4+3x_5=2, \\ 3x_1+3x_2+5x_3-2x_4+3x_5=1, \\ 2x_1+2x_2+8x_3-3x_4+9x_5=2. \end{cases}$

4. λ ning qanday qiymatlarida ChTS hamjoyli bo'lishini aniqlang:

4.1. $\begin{cases} 2x_1-x_2+3x_3+4x_4=5, \\ 4x_1-2x_2+5x_3+6x_4=7, \\ 6x_1-3x_2+7x_3+8x_4=9, \\ \lambda x_1-4x_2+9x_3+10x_4=11. \end{cases}$

4.2. $\begin{cases} (\lambda+3)x_1+2x_2-x_3+4x_4=\lambda, \\ \lambda x_1+(\lambda-1)x_2+2x_3-x_4=2, \\ x_1+3x_2-x_3+11x_4=-10, \\ x_1+4x_2-x_3+18x_4=-18. \end{cases}$

4.3. $\begin{cases} \lambda x_1+x_2+x_3+x_4=1, \\ x_1+\lambda x_2+x_3+x_4=1, \\ x_1+x_2+\lambda x_3+x_4=1, \\ x_1+x_2+x_3+\lambda x_4=1. \end{cases}$

4.4. $\begin{cases} -x_1+(1+\lambda)x_2+(2-\lambda)x_3+\lambda x_4=3, \\ \lambda x_1-x_2+(2-\lambda)x_3+\lambda x_4=2, \\ \lambda x_1+\lambda x_2+(2-\lambda)x_3+\lambda x_4=2, \\ \lambda x_1+\lambda x_2+(2-\lambda)x_3-x_4=2. \end{cases}$

5. Quyidagi ChTSlarini mos usul tanlab eching:

5.1. $\begin{cases} x+y+z+t=a, \\ x-y+z+t=b, \\ x+y-z+t=c, \\ x+y+z-t=d. \end{cases}$

5.2. $\begin{cases} ax+by+cz+dt=p, \\ -bx+ay+dz-ct=q, \\ -cx-dy+az+bt=r, \\ -dx+cy-bz+at=s. \end{cases}$

5.3. $\begin{cases} x_1+a_1x_2+\ldots+a_1^{n-1}x_n=b_1, \\ x_1+a_2x_2+\ldots+a_2^{n-1}x_n=b_2 \\ \ldots\ldots\ldots\ldots\ldots\ldots \\ x_1+a_nx_2+\ldots+a_n^{n-1}x_n=b_n. \end{cases}$, $a_1\neq a_2\neq\ldots\neq a_n$.

5.4. $\begin{cases} x_1+x_2+\ldots+x_n=1, \\ a_1x_1+a_2x_2+\ldots+a_nx_n=b, \\ a_1^2x_1+a_2^2x_2+\ldots+a_n^2x_n=b^2, \\ \ldots\ldots\ldots\ldots\ldots\ldots \\ a_1^{n-1}x_1+a_2^{n-1}x_2+\ldots+a_n^{n-1}x_n=b^{n-1} \end{cases}$, $a_1\neq a_2\neq\ldots\neq a_n$

6. ChTSning umumiy yechimi va bitta xususiy yechimini toping:

6.1. $\begin{cases} 2x_1+7x_2+3x_3+x_4=6, \\ 3x_1+5x_2+2x_3+2x_4=4, \\ 9x_1+4x_2+x_3+7x_4=2. \end{cases}$

6.2. $\begin{cases} 2x_1-3x_2+5x_3+7x_4=1, \\ 4x_1-6x_2+2x_3+3x_4=2, \\ 2x_1-3x_2-11x_3-15x_4=1. \end{cases}$

$$6.3.\quad \begin{cases} 2x_1+5x_2-8x_3=8,\\ 4x_1+3x_2-9x_3=9,\\ 2x_1+3x_2-5x_3=7,\\ x_1+8x_2-7x_3=12. \end{cases}$$

$$6.4.\quad \begin{cases} 3x_1+2x_2+2x_3+2x_4=2,\\ 2x_1+3x_2+2x_3+5x_4=3,\\ 9x_1+x_2+4x_3-5x_4=1,\\ 2x_1+2x_2+3x_3+4x_4=5,\\ 7x_1+x_2+6x_3-x_4=7. \end{cases}$$

$$6.5.\quad \begin{cases} 6x_1+4x_2+5x_3+2x_4+3x_5=1,\\ 3x_1+2x_2+4x_3+x_4+2x_5=3,\\ 3x_1+2x_2-2x_3+x_4=1,\\ 9x_1+6x_2+x_3+3x_4+2x_5=2. \end{cases}$$

$$6.6.\quad \begin{cases} 6x_1+3x_2+2x_3+3x_4+4x_5=5,\\ 4x_1+2x_2+x_3+2x_4+3x_5=4,\\ 4x_1+2x_2+3x_3+2x_4+x_5=0,\\ 2x_1+x_2+7x_3+3x_4+2x_5=1. \end{cases}$$

7. Gauss usulida tenglamalar sistemasining yechimlarini toping:

$$7.1.\quad \begin{cases} 7x_1-3x_2-2x_4=-1,\\ -x_1+3x_2-2x_3+x_4=-6,\\ 2x_2+2x_3+13x_4=14,\\ 2x_1-4x_2-3x_3+2x_4=-3. \end{cases}$$

$$7.2.\quad \begin{cases} 3x_1+4x_2+2x_3-x_4=-1,\\ -x_1+4x_2-x_3+3x_4=-13,\\ -3x_1+2x_2+3x_3+x_4=10,\\ 22x_1-5x_2+7x_3-2x_4=20. \end{cases}$$

$$7.3.\quad \begin{cases} 3x_1-8x_2+x_3+4x_4=-5,\\ x_1-7x_2+2x_3+14x_4=3,\\ -x_1-x_2-4x_3+5x_4=-13. \end{cases}$$

$$7.4.\quad \begin{cases} -4x_1+3x_2+x_3-2x_4=0,\\ 2x_1+x_2-3x_3+3x_4=6,\\ x_1-3x_2+2x_3+5x_4=10,\\ x_1-x_2-6x_3+2x_4=1. \end{cases}$$

$$7.5.\quad \begin{cases} x_1-13x_2+x_3-3x_4=0,\\ x_1-x_2-5x_3+3x_4=1,\\ 3x_1+x_2+2x_3-11x_4=10,\\ x_1-4x_2-3x_3+2x_4=0. \end{cases}$$

$$7.6.\quad \begin{cases} 3x_1+3x_2-6x_3-2x_4=-1,\\ 6x_1+x_2-2x_4=-2,\\ 6x_1-7x_2+21x_3+4x_4=3,\\ 9x_1+4x_2-x_3+4x_4=3. \end{cases}$$

$$7.7.\quad \begin{cases} 11x_1-5x_3+2x_4=5,\\ 3x_1-4x_2-x_3=-1,\\ x_1+2x_2-12x_3+13x_4=7,\\ 5x_1-4x_2+3x_3-2x_4=20. \end{cases}$$

$$7.8.\quad \begin{cases} 5x_1-13x_2+x_3+23x_4=11,\\ x_1+3x_2-5x_3+3x_4=1,\\ 13x_1+x_2+2x_3-11x_4=0,\\ 12x_1+4x_2-17x_3+2x_4=20. \end{cases}$$

8. Quyidagi ChTSning umumiy yechimini va yechimlar fundamental sistemasini toping:

$$8.1.\quad \begin{cases} 3x_1-3x_2+17x_3-25x_4+7x_5=0,\\ x_1+2x_2-7x_3+x_4-11x_5=0. \end{cases}$$

$$8.2.\quad \begin{cases} x_1+4x_2+2x_3-3x_5=0,\\ 2x_1+9x_2+5x_3+2x_4+x_5=0,\\ x_1+3x_2+x_3-2x_4-9x_5=0. \end{cases}$$

$$8.3.\quad \begin{cases} x_1-3x_2+2x_3+x_4=0,\\ -x_1-x_2+2x_3+4x_4=0,\\ 4x_1+3x_2-4x_3+x_4=0. \end{cases}$$

$$8.4.\ \begin{cases} 2x_1-5x_2+4x_3+3x_4=0, \\ 3x_1-4x_2+7x_3+5x_4=0, \\ 4x_1-9x_2+8x_3+5x_4=0, \\ -3x_1+2x_2-5x_3+3x_4=0. \end{cases}$$

$$8.5.\ \begin{cases} -5x_1+13x_2-3x_3-25x_4+6x_5=0, \\ 2x_1-7x_2+x_3-11x_5=0. \end{cases}$$

$$8.6.\ \begin{cases} x_1+x_2+x_3+x_4+x_5=0, \\ 3x_1+2x_2+x_3+x_4-3x_5=0, \\ x_2+2x_3+2x_4+6x_5=0, \\ 5x_1+4x_2+3x_3+3x_4-x_5=0. \end{cases}$$

$$8.7.\ \begin{cases} -9x_1+3x_2-x_3+4x_4=0, \\ 4x_1-x_2+2x_3-5x_4=0, \\ -8x_1+2x_2-3x_3+15x_4=0. \end{cases}$$

$$8.8.\ \begin{cases} \lambda x_1+\lambda x_2+\lambda x_3+\lambda x_4=0, \\ 2\lambda x_1+3\lambda x_2+4\lambda x_3+5\lambda x_4=0. \end{cases}$$

$$8.9.\ \begin{cases} 25x_1+13x_2-x_3+3x_4=0, \\ -13x_1-4x_2+x_3-5x_4=0, \\ 2x_1+32x_2-3x_3-15x_4=0. \end{cases}$$

$$8.10.\ \begin{cases} -4x_1+(2+2\lambda)x_2+2\lambda x_3+2\lambda x_4=0, \\ \lambda x_1+(1+\lambda)x_2+\lambda x_3+\lambda x_4=0, \\ \lambda x_1+(1+\lambda)x_2-2x_3+\lambda x_4=0, \\ -\lambda x_1-(1+\lambda)x_2-\lambda x_3-(2-2\lambda)x_4=0. \end{cases}$$

Takrorlash uchun savollar

1. n ta noma'lumli m ta chiziqli tenglamalar sistemasi deb nimaga aytiladi?
2. ChTSning yechimi deb nimaga aytiladi?
3. Hamjoyli, hamjoyli bo'lmagan ChTSga ta'rif bering.
4. ChTSning natijasiga ta'rif bering.

5. ChTSning chiziqli kombinasiyasi nima?
6. Teng kuchli ChTSlariga ta'rif bering.
7. ChTSni elementar almashtirishlar deganda qanday almashtirishlar tushuniladi?
8. Kroneker-Kapelli teoremasini bayon eting.
9. Bir jinsli ChTS deb qanday sistemaga aytiladi?
10. ChTS va unga assosirlangan BChTS yechimlar yig'indisi, ayirmasi qanday sistemaga yechim bo'ladi?
11. BChTS yechimlar to'plami vektor fazo tashkil etishini tushuntiring.
12. BChTSning fundamental yechimlari sistemasiga ta'rif bering.
13. ChTSni yechishning Gauss usulini tushuntiring.

VI MODUL. MATRISALAR

16-§. Matrisalar va ular ustida amallar

Asosiy tushunchalar: kvadrat matrisa, matrisalarni qo'shish, skalyarni matrisaga ko'paytirish, matrisalar ko'paytmasi, teskarilanuvchi matrisa, elementar matrisa, matrisali tenglama.

$F =< F;+,\cdot,-,^{-1},0,1 >$ maydon va maydon ustida matrisalar to'plami berilgan bo'lsin. Quyidagi munosabatlarni aniqlaymiz:

$\forall A,B \in F^{mxn} \Rightarrow A=B \Leftrightarrow a_{ij}=b_{ij}\ i=1,...,m; j=1,...,n.$

$\forall A,B \in F^{mxn},\ A+B=C,\ C \in F^{mxn}$

$\forall A \in F^{mxn} \wedge \forall \alpha \in F \Rightarrow \omega_{\alpha}(A) = \alpha A = B \in F^{mxn}.$

$\forall A \in F^{mxn},\ \forall B \in F^{nxk} \Rightarrow A\cdot B=C,\ C \in F^{mxk}.$

$A_i \text{™} B^j = a_{i1} b_{1j}+ a_{i2} b_{2j}+ \ldots +a_{in}b_{nj} = c_{ij},\ i= 1, ..., m;\ j=1, ..., k.$

Shunday X va A n-tartibli kvadrat matrisalar berilgan bo'lib, ular uchun XA = AX = E (E – n-tartibli birlik matrisa) shart bajarilsa, u holda X matrisaga A matrisaga teskari matrisa deyiladi va A^{-1} ko'rinishda belgilanadi.

Teskari matrisaga ega matrisa teskarilanuvchi matrisa deyiladi.

Birlik matrisadan quyidagi elementar almashtirishlarning biri yordamida hosil qilingan matrisaga elementar matrisa deyiladi:

1) birlik matrisa satri (ustuni)ni noldan farqli skalarga ko'paytirish.

2) birlik matrisa biror bir satri (ustuni) ga noldan farqli skalyarga ko'paytirilgan satr (ustun)ni qo'shish yoki ayirish.

E birlik matrisada bajarilgan φ satr elementar almashtirish 1) yoki 2) ko'rinishdagi elementar almashtirish bo'lsa, u holda hosil bo'lgan elementar matrisani E_{φ} ko'rinishda belgilaymiz.

Agar A kvadrat matrisani elementar almashtirishlar zanjiri (ketma-ket bajarilgan elementar almashtirishlar) birlik matrisaga o'tkazsa, u holda A matrisa

teskarilanuvchi va bajarilgan elementar almashtirishlar zanjiri E matrisani A^{-1} matrisaga keltiradi. Ya'ni, $A \in F^{n \times n}$ matrisaga teskari matrisani topish uchun tartibi $n \times 2n$ bo'lgan $A\,|\,E = \left(\begin{array}{cccc|cccc} a_{11} & a_{12} & \cdots & a_{1n} & 1 & 0 & \cdots & 0 \\ a_{21} & a_{22} & \cdots & a_{2n} & 0 & 1 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} & 0 & 0 & \cdots & 1 \end{array}\right)$ matrisani elementar almashtirishlar zanjiri yordamida $\left(\begin{array}{cccc|cccc} 1 & 0 & \cdots & 0 & b_{11} & b_{12} & \cdots & b_{1n} \\ 0 & 1 & \cdots & 0 & b_{21} & b_{22} & \cdots & b_{2n} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & 1 & b_{n1} & b_{n2} & \cdots & b_{nn} \end{array}\right) = E\,|\,B$ ko'rinishga keltiramiz. Xosil bo'lgan B matrisa berilgan A matrisaga teskari matrisa.

R maydon ustida n ta noma'lumli n ta chiziqli tenglamalar sistemasi

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n = b_1, \\ a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n = b_2, \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ a_{n1}x_1 + a_{n2}x_2 + \ldots + a_{nn}x_n = b_n. \end{cases}$$

ko'rinishda berilgan bo'lsin.

Quyidagi belgilashlarni kiritamiz:

$$A = \begin{pmatrix} a_{11} & a_{12} & \ldots & a_{1n} \\ a_{21} & a_{22} & \ldots & a_{2n} \\ \ldots & \ldots & \ldots & \ldots \\ a_{n1} & a_{n2} & \ldots & a_{nn} \end{pmatrix}, \quad B = \begin{pmatrix} b_1 \\ b_2 \\ \ldots \\ b_n \end{pmatrix}, \quad X = \begin{pmatrix} x_1 \\ x_2 \\ \ldots \\ x_n \end{pmatrix}.$$

U holda berilgan ChTSni matrisali tenglama, ya'ni $AX = B$ ko'rinishida yozish mumkin.

Agar A matrisaning satrlari chiziqli erkli bo'lsa, u holda $A^{-1}B$ vektor $AX = B$ tenglamaning yagona yechimi bo'ladi.

Lekin matrisali tenglamani faqat ChTS yordamida hosil qilinmaydi. Balki A., B matrisalar berilgan bo'lsa $A \cdot X = B$ yoki $X \cdot A = B$ ko'rinishdagi matrisali tenglamalarni; A,B,C matrisalar berilgan bo'lsa $A \cdot X \cdot B = C$ ko'rinishdagi matrisali tenglamalarni tuzish mumkin bo'lsa, ularni Yechish uchun o'zgaruvchining chap yoki o'ng tomonidagi A matrisa teskarilanuvchi bo'lsa, uning yechimi $A^{-1}\cdot B$ yoki $B\cdot A^{-1}$ ko'rinishda; o'zgaruvchining o'ng va chap tomonidagi A va B lar teskarilanuvchi bo'lsa, $X=A^{-1}\cdot C\cdot B^{-1}$ ko'rinishda bo'ladi.

Misol. $A=\begin{pmatrix}1 & 2 & 3\\ 4 & 1 & -1\\ -2 & 1 & 0\end{pmatrix}$ va $f(x)=3x-x^2+4$ berilgan bo'lsa,

$f(A)$ ni hisoblang.

Yechish: $f(A)=3\cdot A-A^2+4$ ni hisoblash uchun $3\cdot A$, A^2 va $4\cdot E$ matrisalarni aniqlaymiz:

$$1)\; 3\cdot A=3\cdot\begin{pmatrix}1 & 2 & 3\\ 4 & 1 & -1\\ -2 & 1 & 0\end{pmatrix}=\begin{pmatrix}3 & 6 & 9\\ 12 & 3 & -3\\ -6 & 3 & 0\end{pmatrix};$$

2)

$$A^2=\begin{pmatrix}1 & 2 & 3\\ 4 & 1 & -1\\ -2 & 1 & 0\end{pmatrix}^2=\begin{pmatrix}1 & 2 & 3\\ 4 & 1 & -1\\ -2 & 1 & 0\end{pmatrix}\cdot\begin{pmatrix}1 & 2 & 3\\ 4 & 1 & -1\\ -2 & 1 & 0\end{pmatrix}=\begin{pmatrix}1+8-6 & 2+2+3 & 3-2\\ 4+4+2 & 4+1-1 & 12-1\\ -2+4 & -4+1 & -6-1\end{pmatrix}=$$

$$\begin{pmatrix}3 & 7 & 1\\ 10 & 4 & 11\\ 2 & -3 & -7\end{pmatrix};$$

$$3)\; 4\cdot E=4\cdot\begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix}=\begin{pmatrix}4 & 0 & 0\\ 0 & 4 & 0\\ 0 & 0 & 4\end{pmatrix};$$

$$4)\; f(A)=3\cdot A-A^2+4=\begin{pmatrix}3 & 6 & 9\\ 12 & 3 & -3\\ -6 & 3 & 0\end{pmatrix}-\begin{pmatrix}3 & 7 & 1\\ 10 & 4 & 11\\ 2 & -3 & -7\end{pmatrix}+\begin{pmatrix}4 & -1 & 8\\ 2 & 3 & -14\\ -8 & 6 & 11\end{pmatrix}=$$

$$=\begin{pmatrix}4 & -1 & 8\\ 2 & 3 & -14\\ -8 & 6 & 11\end{pmatrix}.$$

Misol. $A=\begin{pmatrix}1 & 1 & 0\\ 2 & -1 & 1\\ 4 & 3 & 2\end{pmatrix}$ matrisaga teskari A^{-1} matrisani toping.

Yechish: 1) Berilgan matrisani teskarilanuvchi ekanligini tekshirib olamiz, ya'ni $r(A)=3$ ekanligini:

$$\begin{pmatrix}1 & 1 & 0\\ 2 & -1 & 1\\ 4 & 3 & 2\end{pmatrix}\sim\begin{pmatrix}1 & 1 & 0\\ 0 & -3 & 1\\ 0 & -1 & 2\end{pmatrix}\sim\begin{pmatrix}1 & 1 & 0\\ 0 & -1 & 2\\ 0 & 0 & -5\end{pmatrix}$$

Demak, $r(A)=3$ ekan, ya'ni matrisa chiziqli erkli va shu sababli teskarilanuvchi.

2) Teskari matrisani elementar matrisalar yordamida topamiz:

$$\left(\begin{array}{ccc|ccc}1 & 1 & 0 & 1 & 0 & 0\\ 2 & -1 & 1 & 0 & 1 & 0\\ 4 & 3 & 2 & 0 & 0 & 1\end{array}\right)\sim\left(\begin{array}{ccc|ccc}1 & 1 & 0 & 1 & 0 & 0\\ 0 & -3 & 1 & -2 & 1 & 0\\ 0 & -1 & 2 & -4 & 0 & 1\end{array}\right)\sim\left(\begin{array}{ccc|ccc}1 & 1 & 0 & 1 & 0 & 0\\ 0 & -1 & 2 & -4 & 0 & 1\\ 0 & 0 & -5 & 10 & 1 & -3\end{array}\right)\sim$$

$$\sim\left(\begin{array}{ccc|ccc}1 & 1 & 0 & 1 & 0 & 0\\ 0 & 1 & -2 & 4 & 0 & -1\\ 0 & 0 & 1 & -2 & -\frac{1}{5} & \frac{3}{5}\end{array}\right)\sim\left(\begin{array}{ccc|ccc}1 & 1 & 0 & 1 & 0 & 0\\ 0 & 1 & 0 & 0 & -\frac{2}{5} & \frac{1}{5}\\ 0 & 0 & 1 & -2 & -\frac{1}{5} & \frac{3}{5}\end{array}\right)\sim$$

$$\left(\begin{array}{ccc|ccc}1 & 0 & 0 & 1 & \frac{2}{5} & -\frac{1}{5}\\ 0 & 1 & 0 & 0 & -\frac{2}{5} & \frac{1}{5}\\ 0 & 0 & 1 & -2 & -\frac{1}{5} & \frac{3}{5}\end{array}\right)$$

Teskari matrisa to'g'ri topilganligiga tekshirish natijasida ishonch hosil qilamiz.

Tekshirish: $\begin{pmatrix}1 & 1 & 0\\ 2 & -1 & 1\\ 4 & 3 & 2\end{pmatrix}\cdot\begin{pmatrix}1 & \frac{2}{5} & -\frac{1}{5}\\ 0 & -\frac{2}{5} & \frac{1}{5}\\ -2 & -\frac{1}{5} & \frac{3}{5}\end{pmatrix}=\begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix}.$

Demak, $A^{-1}=\begin{pmatrix} 1 & \frac{2}{5} & -\frac{1}{5} \\ 0 & -\frac{2}{5} & \frac{1}{5} \\ -2 & -\frac{1}{5} & \frac{3}{5} \end{pmatrix}.$

Misol va mashqlar

1. Matrisalarni qo'shish amalining quyidagi xossalarini isbotlang:

1.1. $\forall A,B \in F^{mxn} \Rightarrow A + B = B + A$ (kommutativlik).

1.2. $\forall A,B,C \in F^{mxn} \Rightarrow (A+B)+C = A + (B + C)$(assosiativlik).

1.3. $\forall A \in F^{mxn}$, $\exists X \in F^{mxn} \Rightarrow A + X = A$ (X=O-neytral).

1.4. $\forall A \in F^{mxn}$, $\exists A' \in F^{mxn} \Rightarrow A+A' = O$ (A'=-A - simmetrik).

2. Skalyarni matrisaga ko'paytirishning quyidagi xossalarini isbotlang:

2.1. $\forall A \in F^{mxn} \wedge \forall \alpha,\beta \in F \Rightarrow (\alpha+\beta)A = \alpha A + \beta A$.

2.2. $\forall A \in F^{mxn} \wedge \forall \alpha,\beta \in F \Rightarrow (\alpha$™$\beta)A = \alpha(\beta A)$.

2.3. $\forall A, B \in F^{mxn} \wedge \forall \alpha \in F \Rightarrow \alpha(A+B) = \alpha$™$A+\alpha$™$B$.

2.4. $\forall A \in F^{mxn} \wedge \forall \alpha \in F \Rightarrow \alpha$™$A = A$™$\alpha$.

3. f(A) ni hisoblang:

3.1. $f(x) = x^3 + 4x$; $\quad A=\begin{pmatrix} -1 & 9 \\ 5 & 2 \end{pmatrix}.$

3.2. $f(x) = x^2 + 24x + 7$; $\quad A=\begin{pmatrix} -2 & 20 & 1 \\ 6 & -4 & 3 \\ 7 & 5 & 0 \end{pmatrix}.$

3.3. $f(x) = -3x^3 + 15x^2 - 2x + 1$; $\quad A=\begin{pmatrix} -2 & 3 \\ -1 & 0 \end{pmatrix}.$

3.4. $f(x) = -4x^3 + 25x + 9$; $\quad A=\begin{pmatrix} 7 & -1 & 1 \\ 2 & 6 & 3 \\ -1 & 2 & 5 \end{pmatrix}.$

4. Matrisalarni ko'paytirish amalining quyidagi xossalarini isbotlang:

1. $\exists A \bullet B \in F^{mxk} \wedge \exists B \bullet C \in F^{kxs} \Rightarrow (A \bullet B) \bullet C = A \bullet (B \bullet C)$ (assosiativlik).

2. $A \in F^{mxn} \wedge \forall B,C \in F^{nxk} \Rightarrow A \bullet (B+C) = A \bullet B + A \bullet C$ (yig'indini chapdan ko'paytirish);

3. $\forall A,B \in F^{mxn} \wedge \forall C \in F^{nxk} \Rightarrow (A+B) \bullet C = A \bullet C + B \bullet C$ (yig'indini o'ngdan ko'paytirish);

4. $\forall \alpha \in F, \forall A \in F^{mxn}, \forall B \in F^{nxk} \Rightarrow \alpha$™$(A$™$B) = (\alpha$™$A)$™$B$.

5. Quyidagi matrisalar ko'paytmasini toping:

5.1. $A=(0,1,0,0), B=\begin{pmatrix} 5 & 2 & -7 & 3 \\ 0 & 1 & 2 & -1 \\ 9 & -3 & 1 & 5 \\ 2 & -1 & 3 & 4 \end{pmatrix}, AB=?$

5.2. $A=\begin{pmatrix} 21 & 22 & 23 & 24 \\ 25 & 26 & 27 & 28 \\ 29 & 30 & 31 & 32 \\ 33 & 34 & 35 & 37 \end{pmatrix}, B=\begin{pmatrix} 1 \\ -1 \\ -1 \\ 1 \end{pmatrix}, AB=?$

5.3. $A=\begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}, B=\begin{pmatrix} \cos\beta & -\cos\beta \\ \sin\beta & \cos\beta \end{pmatrix}, AB=?, BA=?$

5.4. $A=\begin{pmatrix} -2 & 3 & 0 & 1 \\ 1 & 1 & 2 & -1 \end{pmatrix}, B=\begin{pmatrix} 2 & 0 \\ 1 & -1 \\ -1 & 2 \\ 1 & 3 \end{pmatrix}, AB=?, BA=?$

5.5. $A=\begin{pmatrix} 2 & 3 & 0 & -1 \\ 0 & 1 & -1 & 3 \\ 9 & -11 & -3 & 0 \end{pmatrix}, B=\begin{pmatrix} 12 & -6 & 2 \\ 18 & -9 & 3 \\ 24 & -12 & 4 \end{pmatrix}, C=\begin{pmatrix} 11 \\ 12 \\ -30 \end{pmatrix}, (AB)C=?$

5.6. $A=\begin{pmatrix} 1 \\ 6 \\ -2 \end{pmatrix}, B=(53 \quad 22 \quad -35), C=\begin{pmatrix} 3 \\ -4 \\ 2 \end{pmatrix}, D=(2 \quad -3 \quad -1), ABCD=?$

6. Hisoblang:

1. $\begin{pmatrix} 2 & -1 \\ 3 & -2 \end{pmatrix}^n.$ $\qquad$ 6.2. $\begin{pmatrix} 2 & 1 \\ 0 & 2 \end{pmatrix}^n.$

6.3. $\begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}^n$. 6.4. $\begin{pmatrix} a & 1 \\ 0 & a \end{pmatrix}^n$.

6.5. $\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 4 \end{pmatrix}^n$. 6.6. $\begin{pmatrix} 0 & 0 & 0 & \sin\alpha \\ 0 & 0 & \sin\alpha & 0 \\ 0 & \cos\alpha & 0 & 0 \\ \cos\alpha & 0 & 0 & 0 \end{pmatrix}^{2n}$.

7. Berilgan matrisa bilan kommutativ bo'lgan matrisalarni aniqlang:

7.1. $\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$. 7.2. $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$.

7.3. $\begin{pmatrix} 7 & -3 \\ 5 & -2 \end{pmatrix}$. 7.4. $\begin{pmatrix} 3 & 1 & 0 \\ 0 & 3 & 1 \\ 0 & 0 & 3 \end{pmatrix}$.

7.5. $\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$. 7.6. $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$.

7.7. $\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$. 7.8. $\begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$.

8. R maydonda $\begin{pmatrix} 1 & a \\ 0 & 1 \end{pmatrix}$ ko'rinishdagi matrisalar to'plami mul'tiplikativ

9. gruppa tashkil etishini va uning haqiqiy sonlar additiv gruppasiga izomorfligini isbotlang.

10.R maydonda $\begin{pmatrix} a+bi & c+di \\ c-di & a-bi \end{pmatrix}$ ko'rinishdagi matrisalar to'plami nolning bo'luvchilariga ega bo'lmagan halqa tashkil etishini isbotlang.

11.Q maydonda $\begin{pmatrix} a & b \\ -b & a-b \end{pmatrix}$ ko'rinishdagi matrisalar to'plami kommutativ halqa tashkil etishini isbotlang.

12. Teskari matrisalarning quyidagi xossalarini isbotlang:

1) $(A^{-1})^{-1} = A$;

2) $(AB)^{-1} = B^{-1}A^{-1}$;

3) $(A^T)^{-1} = (A^{-1})^T$.

12. Quyidagi matrisalarning teskarisini toping:

12.1. $\begin{pmatrix} 4 & 3 \\ 3 & 2 \end{pmatrix}$. 12.2. $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$.

12.3. $\begin{pmatrix} -1 & -1 & 1 \\ 2 & 0 & -3 \\ 1 & 1 & 0 \end{pmatrix}$. 12.4. $\begin{pmatrix} 3 & -4 & 5 \\ 2 & -3 & 1 \\ 3 & -5 & -1 \end{pmatrix}$.

12.5. $\begin{pmatrix} 2 & 0 & 0 & 0 \\ 3 & 2 & 0 & 0 \\ 1 & 1 & 3 & 4 \\ 2 & -1 & 2 & 3 \end{pmatrix}$. 12.6. $\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{pmatrix}$.

12.7. $\begin{pmatrix} 1 & 1 & 1 & \dots & 1 \\ 0 & 1 & 1 & \dots & 1 \\ 0 & 0 & 1 & \dots & 1 \\ . & . & . & . & . \\ 0 & 0 & 0 & \dots & 1 \end{pmatrix}$. 12.8. $\begin{pmatrix} 1 & a & a^2 & a^3 & \dots & a^n \\ 0 & 1 & a & a^2 & \dots & a^{n-1} \\ 0 & 0 & 1 & a & \dots & a^{n-2} \\ . & . & . & . & . & . \\ 0 & 0 & 0 & 0 & \dots & 1 \end{pmatrix}$.

12.9. $\begin{pmatrix} 2 & -1 & 0 & 0 & \dots & 0 \\ -1 & 2 & -1 & 0 & \dots & 0 \\ 0 & -1 & 2 & -1 & \dots & 0 \\ . & . & . & . & . & . \\ 0 & 0 & 0 & 0 & \dots & 2 \end{pmatrix}$.

12.10. $\begin{pmatrix} 1+a_1 & 1 & 1 & \dots & 1 \\ 0 & 1+a_2 & 1 & \dots & 1 \\ 0 & 0 & 1+a_3 & \dots & 1 \\ . & . & . & . & . \\ 0 & 0 & 0 & \dots & 1+a_n \end{pmatrix}$.

13. Ixtiyoriy A kvadrat matrisa uchun quyidagi shartlar teng kuchli ekanligini isbotlang:

1) A matrisa teskarilanuvchi.

2) A matrisaning satrlari (ustunlari) chiziqli erkli.

14. Quyidagi matrisali tenglamalarni yeching:

14.1. $\begin{pmatrix} 3 & 5 \\ 2 & 4 \end{pmatrix} X = \begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix}.$

14.2. $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} X = \begin{pmatrix} 3 & 5 \\ 3 & 9 \end{pmatrix}.$

14.3. $X \begin{pmatrix} 1 & -3 \\ 2 & -6 \end{pmatrix} = \begin{pmatrix} 3 & 2 \\ 1 & 4 \end{pmatrix}.$

14.4. $X \begin{pmatrix} 3 & 2 \\ 6 & 4 \end{pmatrix} = \begin{pmatrix} -3 & -2 \\ -9 & -6 \end{pmatrix}.$

14.5. $\begin{pmatrix} 2 & -1 \\ 3 & -2 \end{pmatrix} X \begin{pmatrix} -5 & 1 \\ 2 & 3 \end{pmatrix} = \begin{pmatrix} 1 & -1 \\ 2 & -3 \end{pmatrix}.$

14.6. $\begin{pmatrix} 3 & -1 \\ 5 & -2 \end{pmatrix} X \begin{pmatrix} 5 & 6 \\ 7 & 8 \end{pmatrix} = \begin{pmatrix} 14 & 16 \\ 9 & 10 \end{pmatrix}.$

14.7. $\begin{pmatrix} 1 & 2 & -3 \\ 3 & 2 & -4 \\ 2 & -1 & 0 \end{pmatrix} X = \begin{pmatrix} 1 & -3 & 0 \\ 10 & 2 & 7 \\ 10 & 7 & 8 \end{pmatrix}$

14.8. $\begin{pmatrix} 0 & 2 & 1 \\ 1 & 3 & 0 \\ -3 & 0 & 4 \end{pmatrix} X = \begin{pmatrix} 2 & 0 & 1 \\ -3 & 0 & 4 \\ 1 & 1 & 2 \end{pmatrix}^2.$

14.9. $X \begin{pmatrix} 5 & 3 & 1 \\ 1 & -3 & -2 \\ -5 & 2 & 1 \end{pmatrix} = \begin{pmatrix} -8 & 3 & 0 \\ -5 & 9 & 0 \\ -2 & 15 & 0 \end{pmatrix}.$

14.10. $X \begin{pmatrix} 4 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 2 & 2 & 2 \end{pmatrix}.$

14.11. $\begin{pmatrix} 2 & -3 & 1 \\ 4 & -5 & 2 \\ 5 & -7 & 3 \end{pmatrix} X \begin{pmatrix} 9 & 7 & 6 \\ 1 & 1 & 2 \\ 1 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 2 & 0 & -2 \\ 18 & 12 & 9 \\ 23 & 15 & 11 \end{pmatrix}.$

14.12. $\begin{pmatrix} 1 & 3 & 2 & -5 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{pmatrix} X \begin{pmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 3 & 2 & 1 & -1 \\ 1 & 2 & 0 & 3 \\ 0 & -2 & 1 & 0 \\ 1 & -1 & 2 & -2 \end{pmatrix}.$

15. Quyidagi tenglamalar sistemasini matrisali tenglamaga keltirib yeching:

15.1. $\begin{cases} x_1 - x_2 + x_3 = 6, \\ 2x_1 + x_2 + x_3 = 3, \\ x_1 + x_2 + 2x_3 = 5. \end{cases}$

15.2. $\begin{cases} 5x_1 - 4x_2 + 2x_3 = -9, \\ 3x_1 - 2x_2 + x_3 = 3, \\ 10x_1 - 9x_2 + 2x_3 = 7. \end{cases}$

15.3. $\begin{cases} 2x_1 + 5x_2 + 4x_3 + x_4 = 20, \\ x_1 + 3x_2 + 2x_3 + x_4 = 11, \\ 2x_1 + 10x_2 + 9x_3 + 7x_4 = 40, \\ 3x_1 + 8x_2 + 9x_3 + 2x_4 = 37. \end{cases}$

15.4. $\begin{cases} 3x_1 + 5x_2 - 3x_3 + 2x_4 = 12, \\ 4x_1 - 2x_2 + 5x_3 + 3x_4 = 27, \\ 7x_1 + 8x_2 - x_3 + 5x_4 = 40, \\ 6x_1 + 4x_2 + 5x_3 + 3x_4 = 41. \end{cases}$

15.5. $\begin{cases} 2x_1 + 2x_2 - x_3 + x_4 = 4, \\ 3x_1 + 4x_2 - x_3 + 2x_4 = 6, \\ 5x_1 + 8x_2 - 3x_3 + 4x_4 = 12, \\ 3x_1 + 3x_2 - 2x_3 + 2x_4 = 6. \end{cases}$

15.6. $\begin{cases} 6x_1 + x_2 + 4x_3 - x_4 = 6, \\ 7x_1 + x_2 + 3x_3 + x_4 = -3, \\ 5x_1 + x_2 + 2x_3 - x_4 = 0, \\ 9x_1 + 2x_2 + 5x_3 - 2x_4 = 0. \end{cases}$

15.7. $\begin{cases} 3x_1 - x_2 + x_3 - 4x_4 = 3, \\ 3x_1 - x_2 + 2x_3 - 2x_4 = 3, \\ 5x_1 - x_2 + x_3 - 4x_4 = -1, \\ 11x_1 - 3x_2 + 2x_3 - 5x_4 = -2. \end{cases}$

$$15.8.\quad \begin{cases} 5x_1+2x_2-x_3+3x_4+2x_5=0, \\ 4x_1-7x_3=0, \\ 2x_1+3x_2-7x_3+5x_4+3x_5=2, \\ 2x_1+3x_2-6x_3+4x_4+5x_5=0, \\ 3x_1-4x_3=0. \end{cases}$$

16. Quyidagi matrisali tenglamalar sistemasini yeching:

16.1. $X+Y=\begin{pmatrix}1 & 1\\ 0 & 1\end{pmatrix},\quad 2X+3Y=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}.$

16.2. $2X-Y=\begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix},\quad -4X+2Y=\begin{pmatrix}0 & -2\\ 2 & 0\end{pmatrix}.$

Takrorlash uchun savollar

1. Kvadrat matrisa va uning turlari.
2. Matrisalarni qo'shish va uning xossalari.
3. Skalyarni matrisaga ko'paytirish va uning xossalari.
4. Matrisalarni ko'paytirish va uning xossalari.
5. Teskarilanuvchi matrisa deb qanday matrisaga aytiladi?
6. Elementar matrisalar xossalarini ayting.
7. Teskari matrisani ta'rif asosida topish jarayonini bayon qiling.
8. Matrisaning teskarilanish shartlarini ayting.
9. Teskari matrisani elementar matrisalardan foydalanib topish jarayonini tushuntiring.
10. ChTSning matrisali ifodasi qanday hosil qilinadi?
11. Matrisali tenglamalarning qanday ko'rinishlarini bilasiz?
12. Matrisali tenglamani yechish jarayonini bayon qiling.

VII MODUL. DETERMINANTLAR

17-§. O'rniga qo'yushlar

Asosiy tushunchalar: n-darajali o'rniga qo'yish, n-darajali simmetrik gruppa, inversiya, juft o'rniga qo'yish, toq o'rniga qo'yish, transpozisiya, o'rniga qo'yishning ishorasi.

A={1,2,3,...,n} to'plamni o'ziga biektiv akslantirishga n-darajali o'rniga qo'yish deyiladi.

A to'plamda aniqlangan φ o'rniga qo'yishni

$$\varphi=\begin{pmatrix}1 & 2 & \dots & n\\ \varphi(1) & \varphi(2) & \dots & \varphi(n)\end{pmatrix}$$

ko'rinishda belgilanadi. Agar φ va ψ o'rniga qo'yishlarda $i_k=j_k(k=\overline{1,n})$ bo'lsa, u holda φ va ψ o'rniga qo'yishlar o'zaro teng deyiladi.

φ va ψ o'rniga qo'yishlar ko'paytmasi deb φ va ψ akslantirishlar kompozisiyasi $\varphi\psi(i)=\varphi(\psi(i)), i=1,\dots,n$ ga aytiladi, ya'ni

$$\varphi\cdot\psi=\varphi\cdot\begin{pmatrix}1 & 2 & \dots & n\\ \psi(1) & \psi(2) & \dots & \psi(n)\end{pmatrix}=\begin{pmatrix}\psi(1) & \psi(2) & \dots & \psi(n)\\ \varphi(\psi(1)) & \varphi(\psi(2)) & \dots & \varphi(\psi(n))\end{pmatrix}.$$

A to'plamdan olingan φ o'rniga qo'yishga teskari o'rniga qo'yish deb $\varphi^{-1}=\begin{pmatrix}\varphi(1) & \varphi(2) & \cdots & \varphi(n)\\ 1 & 2 & \cdots & n\end{pmatrix}=\begin{pmatrix}1 & 2 & \dots & n\\ \varphi^{-1}(1) & \varphi^{-1}(2) & \dots & \varphi^{-1}(n)\end{pmatrix}$ o'rniga qo'yishga aytiladi.

A to'plamning xar bir elementini shu elementning o'ziga o'tkazuvchi ε akslantirishga ayniy o'rniga qo'yish deyiladi va u $\varepsilon=\begin{pmatrix}1 & 2 & \cdots & i & \cdots & n\\ 1 & 2 & \cdots & i & \cdots & n\end{pmatrix}$ ko'rinishda belgilanadi.

$<S_n; \cdot, ^{-1}>$ gruppaga n-darajali simmetrik gruppa deyiladi va u S_n orqali belgilanadi.

$\varphi = \begin{pmatrix} 1 & 2 & \dots & n \\ \varphi(1) & \varphi(2) & \dots & \varphi(n) \end{pmatrix}$ o'rniga qo'yishda A={1,2,3,...,n} to'plamning ixtiyoriy i, j elementlaridan tuzilgan juftlik uchun $i - j$ va $\varphi(i) - \varphi(j)$ ayirmalar bir hil ishoraga ega bo'lsa, bu juftlik to'g'ri, bir hil ishoraga ega bo'lmasa to'g'ri emas yoki inversiya tashkil etadi deyiladi. o'rniga qo'yishda inversiyalar soni juft (toq) bo'lsa, o'rniga qo'yish juft (toq) o'rniga qo'yish deyiladi.

O'rniga qo'yishda shunday i, j elementlar mavjud bo'lib, ular uchun $\varphi(i) = j, \varphi(j) = i, \varphi(s) = s, s \in A \setminus \{i, j\}$ shartlar bajarilsa, bunday o'rniga qo'yish transpozisiya deyiladi.

$\varphi = \begin{pmatrix} 1 & 2 & \dots & n \\ \varphi(1) & \varphi(2) & \dots & \varphi(n) \end{pmatrix}$ o'rniga qo'yishning ishorasi deb

$\operatorname{sgn} \varphi = \begin{cases} 1, agar \varphi - juft, \\ -1, agar \quad \varphi - toq \end{cases}$ qiymatga aytiladi.

Misol. Berilgan o'rniga qo'yishlar va ular kompozisiyasining juft-toqligi, ishorasi, inversiyalar sonini aniqlang:

$$\varphi_1 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 1 & 6 & 5 & 4 & 3 \end{pmatrix}; \quad \varphi_2 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 1 & 2 & 5 & 6 & 4 & 3 \end{pmatrix}$$

Yechish:

$$\varphi_1 \circ \varphi_2 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 1 & 4 & 3 & 5 & 6 \end{pmatrix};$$

$$\varphi_2 \circ \varphi_1 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 1 & 3 & 4 & 6 & 5 \end{pmatrix};$$

φ_1 dagi inversiyalar sonini aniqlaymiz:

$\varphi_1 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 1 & 6 & 5 & 4 & 3 \end{pmatrix}$ o'rniga qo'yishda birinchi qatordagi $i - j$ ayirmalar manfiy. 2-qatordagi ularga mos ayirmalardan $\varphi_1(1) - \varphi_1(2) = 2 - 1 = 1$

musbat, qolganlari manfiy, demak φ_1 dagi inversiyalar soni 1 ta. Shuning uchun ishorasi $sqn\varphi_1 = -1$; φ_1 – toq o'rniga qo'yish;

$\varphi_2 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 1 & 2 & 5 & 6 & 4 & 3 \end{pmatrix}$ dagi inversiyalar soni 3 ta; φ_2 – toq o'rniga qo'yish va $sqn\varphi_2 = -1$;

$\varphi_1 \circ \varphi_2 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 1 & 4 & 3 & 5 & 6 \end{pmatrix}$ dagi inversiyalar soni 2 ta, u juft o'rniga qo'yish va $sqn(\varphi_1 \circ \varphi_2) = 1$

$\varphi_2 \circ \varphi_1 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 1 & 3 & 4 & 6 & 5 \end{pmatrix}$ o'rniga quyishda inversiyalar soni 2 ta, $\varphi_2 \circ \varphi_1$ juft o'rniga qo'yish va $sqn(\varphi_2 \circ \varphi_1) = 1$.

Misol. $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 1 & 3 & 4 & 6 & 5 \end{pmatrix}$ o'rniga qo'yishni siklli o'rniga qo'yishlar kompozisiyasi ko'rinishida ifodalang.

Yechish: $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 1 & 3 & 4 & 6 & 5 \end{pmatrix} = (12)\ (3)\ (4)\ (56)$.

Misol va mashqlar

1. O'rniga qo'yishlarni ko'paytiring:

1.1. $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 4 & 1 & 5 & 2 \end{pmatrix}\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 5 & 3 & 1 & 2 & 4 \end{pmatrix}$.

1.2. $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 6 & 4 & 5 & 2 & 1 \end{pmatrix}\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 4 & 1 & 5 & 6 & 3 \end{pmatrix}$.

1.3. $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 4 & 3 & 1 & 2 & 7 & 5 & 6 \end{pmatrix}\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 5 & 3 & 1 & 4 & 2 & 7 & 6 \end{pmatrix}$.

1.4. $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 7 & 5 & 3 & 9 & 1 & 2 & 4 & 6 & 8 \end{pmatrix}\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 9 & 5 & 7 & 2 & 1 & 3 & 4 & 6 & 8 \end{pmatrix}$.

2. O'rniga qo'yishni sikllarga yoying:

2.1. $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 5 & 4 & 1 & 7 & 3 & 6 & 2 \end{pmatrix}$.

2.2. $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 6 & 3 & 2 & 7 & 4 & 5 & 8 & 1 \end{pmatrix}$.

2.3. $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 2 & 4 & 6 & 8 & 1 & 3 & 5 & 7 & 9 \end{pmatrix}$.

2.4. $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ 10 & 5 & 4 & 7 & 6 & 9 & 8 & 3 & 2 & 1 \end{pmatrix}$.

2.5. $\begin{pmatrix} 1 & 2 & 3 & 4 & \dots & 2n-1 & 2n \\ 2 & 1 & 4 & 3 & \dots & 2n & 2n-1 \end{pmatrix}$.

2.6. $\begin{pmatrix} 1 & 2 & \dots & n & n+1 & n+2 & \dots & 2n \\ n+1 & n+2 & \dots & 2n & 1 & 2 & \dots & n \end{pmatrix}$.

3. S_4 dagi barcha toq o'rniga qo'yishlarni aniqlang.

4. Quyidagi shartlar asosida α, β larni toping:

4.1. $(\alpha \ \ 6 \ \ 7 \ \ 1 \ \ \beta \ \ 5 \ \ 3)$-toq.

4.2. $(11 \ \ 5 \ \ 7 \ \ \alpha \ \ 1 \ \ 2 \ \ 9 \ \ 8 \ \ 4 \ \ 3 \ \ \beta)$-juft.

5. Qaysi 10 ta sondan iborat o'rniga qo'yish eng ko'p inversiyaga ega? Inversiyalar sonini aniqlang.

6. S_5 dan quyidagi shartlar bo'yicha o'rniga qo'yishlarni aniqlang:

6.1. 4 ta inversiyaga ega.

6.2. 7 ta inversiyaga ega.

6.3. 9 ta inversiyaga ega.

6.4. 11 ta inversiyaga ega.

7. O'rniga qo'yishlar ishorasining quyidagi xossalarini isbotlang:

1) sgn funksiya mul'tiplikativ, ya'ni har qanday $\varphi, \psi \in S_n$ lar uchun $\operatorname{sgn}(\varphi\psi) = \operatorname{sgn}\varphi \cdot \operatorname{sgn}\psi$ o'rinli;

2) transpozisiya ishorasi (-1) ga teng;

3) o'zaro teskari o'rniga qo'yishlar ishorasi bir hil;

4) agar τ-transpozisiya va φ ixtiyoriy o'rniga qo'yish bo'lsa, u holda $\operatorname{sgn}(\tau\varphi) = \operatorname{sgn}(\varphi\tau) = -\operatorname{sgn}\varphi$ bo'ladi.

8. O'rniga o'yishlardagi inversiyalar sonini aniqlang:

8.1. $(1 \ \ 4 \ \ 7 \ \ \dots \ \ 3n-2 \ \ 2 \ \ 5 \ \ 8 \ \ \dots \ \ 3n-1 \ \ 3 \ \ 6 \ \ 9 \ \ \dots \ \ 3n)$.

8.2. $(3 \ \ 6 \ \ 9 \ \ \dots \ \ 3n \ \ 2 \ \ 5 \ \ 8 \ \ \dots \ \ 3n-1 \ \ 1 \ \ 4 \ \ 7 \ \ \dots \ \ 3n-2)$.

8.3. $(2 \ \ 5 \ \ 8 \ \ \dots \ \ 3n-1 \ \ 3 \ \ 6 \ \ 9 \ \ \dots \ \ 3n \ \ 1 \ \ 4 \ \ 7 \ \ \dots \ \ 3n-2)$.

8.4. $(2 \ \ 5 \ \ 8 \ \ \dots \ \ 3n-1 \ \ 1 \ \ 4 \ \ 7 \ \ \dots \ \ 3n-2 \ \ 3 \ \ 6 \ \ 9 \ \ \dots \ \ 3n)$.

Takrorlash uchun savollar

1. n-darajali o'rniga qo'yishga ta'rif bering.
2. O'rniga qo'yishlar gruppa tashkil etishini tekshiring.
3. n-darajali simmetrik gruppaga misol keltiring.
4. Inversiyaga ta'rif bering.
5. Juft, toq o'rniga qo'yishlarni ta'riflang.
6. Transpozisiya nima?
7. O'rniga qo'yishning ishorasi qanday aniqlanadi?

18-§. Determinantlar

Asosiy tushunchalar: determinant, matrisaosti (qismmatrisa), n-tartibli minor, algebraik to'ldiruvchi, Laplas teoremasi, algebraik to'ldiruvchi, Kramer formulalari.

Kvadrat matrisaning har bir satr va har bir ustunidan bittadan elementlar olib tuzilgan ko'paytmalarning algebraik yig'indisiga berilgan kvadrat matrisaning determinanti deyiladi.

n-tartibli kvadrat matrisa $A=\begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{pmatrix}$ ning determinanti deb

$|A| = \sum_{\tau\in S_n} \mathrm{sgn}(\tau) a_{1\tau(1)} \cdot \dots \cdot a_{n\tau(n)}$ ($n!$ qo'shiluvchilardan iborat) yig'indiga aytiladi.

$F=<F;+,\cdot,-,^{-1},0,1>$ maydon va maydon ustida $F^{m\times n}$ matrisalar to'plami berilgan bo'lsin.

A matrisaning matrisaosti deb, uning qandaydir satr va ustunlarini o'chirishdan hosil bo'lgan matrisaga aytiladi.

k-tartibli matrisaosti determinanti A matrisaning k-tartibli minori deyiladi.

Kvadrat matrisaning i- qatori j-ustunini o'chirishdan hosil bo'lgan matrisaosti determinanti a_{ij} elementning minori deyiladi va M_{ij} ko'rinishda belgilanadi.

$A_{ij} = (-1)^{i+j} \cdot M_{ij}$ ko'paytmaga a_{ij} elementning algebraik to'ldiruvchisi deyiladi.

Laplas teoremasi. A=$\begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{pmatrix}$ kvadrat matrisaning determinanti biror-bir satr (ustun) elementlari bilan ularning algebraik to'ldiruvchilari ko'paytmalarining yig'indisiga, ya'ni

$|A| = a_{1j}A_{1j} + \dots + a_{nj}A_{nj}$ $(|A| = a_{i1}A_{i1} + \dots + a_{in}A_{in}), i, j \in \{1,\dots,n\}$ ga teng.

A matrisaning a_{ij} elementining A_{ij} $(i, j \in \{1,\dots,n\}$ algebraik to'ldiruvchilaridan iborat $A^* = \begin{pmatrix} A_{11} & A_{21} & \dots & A_{n1} \\ A_{12} & A_{22} & \dots & A_{n2} \\ . & . & \dots & . \\ A_{1n} & A_{2n} & \dots & A_{nn} \end{pmatrix}$ matrisaga A matrisaga biriktirilgan matrisa deyiladi.

Agar $|A| \neq 0$ bo'lsa, u holda A matrisa teskarilanuvchi va $A^{-1} = |A|^{-1} \cdot A^*$.

$F=<F;+,\cdot,-,^{-1},0,1>$ maydon ustida quyidagi

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = b_1, \\ a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n = b_2, \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots \\ a_{n1}x_1 + a_{n2}x_2 + \dots + a_{nn}x_n = b_n. \end{cases}$$

ChTS berilgan va uning asosiy matrisasi A bo'lsin.

Agar $|A| \neq 0$ bo'lsa, u holda ChTS yagona yechimga ega va u quyidagi formulalar orqali ifodalanadi: $x_1 = \frac{|A(1)|}{|A|}, \dots, x_n = \frac{|A(n)|}{|A|}$

Misol. Дeterminantni elementar almashtirishlar yordamida hisoblang:

Yechish:

$$\begin{vmatrix} 1 & -1 & 0 & 1 \\ 2 & 1 & 3 & 1 \\ -1 & 4 & 1 & 2 \\ 3 & 1 & 0 & 1 \end{vmatrix} = \begin{vmatrix} 1 & -1 & 0 & 1 \\ 0 & 3 & 3 & -1 \\ 0 & 3 & 1 & 3 \\ 0 & 4 & 0 & -2 \end{vmatrix} = \begin{vmatrix} 1 & -1 & 0 & 1 \\ 0 & 3 & 3 & -1 \\ 0 & 0 & -2 & 4 \\ 0 & 0 & -12 & -2 \end{vmatrix} = \begin{vmatrix} 1 & -1 & 0 & 1 \\ 0 & 3 & 3 & -1 \\ 0 & 0 & -2 & 4 \\ 0 & 0 & 0 & -26 \end{vmatrix} =$$

$= 1 \cdot 3 \cdot (-2) \cdot (-26) = 156$

Misol. Дeterminantni 1-qator hamda 2-ustun elementlari yoyilmasi orqali hisoblang.

Yechish: 1-qator bo'yicha yoyamiz:

$$\begin{vmatrix} 1 & -1 & 0 & 1 \\ 2 & 1 & 3 & 1 \\ -1 & 4 & 1 & 2 \\ 3 & 1 & 0 & 1 \end{vmatrix} = 1 \cdot (-1)^{1+1} \begin{vmatrix} 1 & 3 & 1 \\ 4 & 1 & 2 \\ 1 & 0 & 1 \end{vmatrix} + (-1)(-1)^{1+2} \begin{vmatrix} 2 & 3 & 1 \\ -1 & 1 & 2 \\ 3 & 0 & 1 \end{vmatrix} +$$

$$+0 \cdot (-1)^{1+3} \begin{vmatrix} 2 & 1 & 1 \\ -1 & 4 & 2 \\ 3 & 1 & 1 \end{vmatrix} + 1 \cdot (-1)^{1+4} \begin{vmatrix} 2 & 1 & 3 \\ -1 & 4 & 1 \\ 3 & 1 & 0 \end{vmatrix} = (1+6-1-12)+(2+18-$$

-3+3)+0+(-1)(-3+3-36-2)=-6+20+38=52.

2-ustun bo'yicha hisoblaymiz:

$$\begin{vmatrix} 1 & -1 & 0 & 1 \\ 2 & 1 & 3 & 1 \\ -1 & 4 & 1 & 2 \\ 3 & 1 & 0 & 1 \end{vmatrix} = (-1)\cdot(-1)^{1+2}\begin{vmatrix} 2 & 3 & 1 \\ -1 & 1 & 2 \\ 3 & 0 & 1 \end{vmatrix} + 1\cdot(-1)^{2+2}\begin{vmatrix} 1 & 0 & 1 \\ -1 & 1 & 2 \\ 3 & 0 & 1 \end{vmatrix} +$$

$$+4\cdot(-1)^{3+2}\begin{vmatrix} 1 & 0 & 1 \\ 2 & 3 & 1 \\ 3 & 0 & 1 \end{vmatrix} + 1\cdot(-1)^{4+2}\begin{vmatrix} 1 & 0 & 1 \\ 2 & 3 & 1 \\ -1 & 1 & 2 \end{vmatrix} = (2+18-3+3)+4(3-9)+$$

$+(6+2+3-1)=20-2-4(-6)+10=18+24+10=52$

Misol. $\begin{cases} x_1 - 2x_2 + 3x_3 = 1 \\ -x_1 + x_2 - x_3 = 2 \\ 2x_1 + 4x_2 + x_3 = 3 \end{cases}$ tenglamalar sistemasini Kramer formulalari yordamida yeching.

Yechish: Kramer formulalari:

$$x_1 = \frac{\Delta(1)}{\Delta}; \qquad x_2 = \frac{\Delta(2)}{\Delta}; \qquad x_3 = \frac{\Delta(3)}{\Delta};$$

Демак, $\Delta, \Delta(1), \Delta(2), \Delta(3)$ larni hisoblaymiz:

$$\Delta = \begin{vmatrix} 1 & -2 & 3 \\ -1 & 1 & -1 \\ 2 & 4 & 1 \end{vmatrix} = 1-12+4-6+4-2=9-20=-11$$

$$\Delta(1) = \begin{vmatrix} 1 & -2 & 3 \\ 2 & 1 & -1 \\ 3 & 4 & 1 \end{vmatrix} = 1+24+6-9+4+4=30$$

$$\Delta(2) = \begin{vmatrix} 1 & 1 & 3 \\ -1 & 2 & -1 \\ 2 & 3 & 1 \end{vmatrix} = 2-9-2-12+3+1=6-23=-17$$

$$\Delta(3) = \begin{vmatrix} 1 & -2 & 1 \\ -1 & 1 & 2 \\ 2 & 4 & 3 \end{vmatrix} = 3-4-8-2-8-6=-25$$

Bundan, $\quad x_1 = -\dfrac{30}{11}; \qquad x_2 = \dfrac{17}{11}; \qquad x_3 = \dfrac{25}{11};$

Misol va mashqlar

1. Determinantni hisoblang:

1.1. $\begin{vmatrix} -2 & 5 \\ -4 & 7 \end{vmatrix}$. 1.2. $\begin{vmatrix} n+1 & n \\ n & n-1 \end{vmatrix}$.

1.3. $\begin{vmatrix} 1-3i & 2i \\ 4i^3 & 1+3i \end{vmatrix}$. 1.4. $\begin{vmatrix} \log_b a & 1 \\ 1 & \log_a b \end{vmatrix}$.

1.5. $\begin{vmatrix} tg\alpha & \sin\alpha \\ 2 & \cos\alpha \end{vmatrix}$. 1.6. $\begin{vmatrix} \dfrac{1-a^2}{1+a^2} & \dfrac{2a}{1+a^2} \\ \dfrac{-2a}{1+a^2} & \dfrac{1-a^2}{1+a^2} \end{vmatrix}$.

1.7. $\begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & 6 \end{vmatrix}$. 1.8. $\begin{vmatrix} 2 & -2 & 4 \\ -3 & 3 & -6 \\ 5 & 1 & 0 \end{vmatrix}$.

1.9. $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$. 1.10. $\begin{vmatrix} a+x & x & x \\ x & b+x & x \\ x & x & c+x \end{vmatrix}$.

1.10. $\begin{vmatrix} \sin\alpha & \sin 2\alpha & 1 \\ \cos\alpha & 1+\cos\alpha & 1 \\ tg\alpha & 2\sin\alpha & 1 \end{vmatrix}$.

1.11. $\begin{vmatrix} 1 & -1 & \varepsilon \\ \varepsilon & \varepsilon^2 & 1 \\ -1 & \varepsilon & -1 \end{vmatrix}, \varepsilon = \cos\dfrac{\pi}{3} + i\sin\dfrac{\pi}{3}$.

1.12. $\begin{vmatrix} \alpha^2+1 & \alpha\beta & \alpha\gamma \\ \alpha\beta & \beta^2+1 & \beta\gamma \\ \alpha\gamma & \beta\gamma & \gamma^2+1 \end{vmatrix}$.

1.13. $\begin{vmatrix} \alpha & \beta & \gamma \\ \beta & \gamma & \alpha \\ \gamma & \alpha & \beta \end{vmatrix}, \alpha, \beta, \gamma \in \{x \mid x^3 + px + q = 0\}$.

2. Determinantning quyidagi xossalarini isbotlang:

2.1. Nol satr yoki ustunga ega kvadrat matrisaning determinanti nolga teng.

2.2. Diagonal matrisaning determinanti asosiy diagonal elementlari ko'paytmasiga teng.

2.3. Uchburchak matrisaning determinanti asosiy diagonal elementlari ko'paytmasiga teng.

2.4. Kvadrat matrisa va unga transponirlangan matrisalar determinantlari teng.

2.5. Kvadrat matrisaning ikkita satr (ustun)lari o'rnini almashtirish natijasida determinant ishorasi o'zgaradi.

2.6. Ikkita bir hil satr (ustun)ga ega kvadrat matrisa determinanti nolga teng.

2.7. A kvadrat matrisaning biror bir satr (ustun) elementlarini noldan farqli λ skalyarga ko'paytirilsa, u holda A matrisaning determinanti λ skalarga ko'paytiriladi.

2.8. Qandaydir ikkita satr (ustun)lari proporsional bo'lgan kvadrat matrisaning determinanti nolga teng.

2.9. Kvadrat matrisa i- qatori (ustuni)ning har bir elementi m ta qo'shiluvchilardan iborat bo'lsa, bunday kvadrat matrisaning determinanti m ta determinantlar yig'indisidan iborat bo'lib, birinchi determinant i- qatori (ustuni)da birinchi, ikkinchi determinantda ikkinchi qo'shiluvchilar va h.z. boshqa qatorlar A matrisanikidek bo'ladi.

2.10. Kvadrat matrisaning biror-bir satr (ustun)iga noldan farqli skalyarga ko'paytirilgan boshqa satr (ustun)ni qo'shish natijasida determinant o'zgarmaydi.

2.11. Kvadrat matrisaning biror-bir satr (ustun)iga qolgan satr (ustun)lar chiziqli kombinasiyasini qo'shish natijasida determinant o'zgarmaydi.

2.12. Kvadrat matrisaning oxirgisidan boshqa barcha qatorlariga undan keyingi qator qo'shilsa, uning determinanti o'zgarmaydi.

2.13. Kvadrat matrisaning ikkinchi ustunidan boshlab har bir ustuniga undan oldingi ustun qo'shilsa, uning determinanti o'zgarmaydi.

2.14. Kvadrat matrisaning biror-bir satri (ustuni) qolganlarining chiziqli kombinasiyasidan iborat bo'lsa, uning determinanti nolga teng.

2.15. Har qanday elementar matrisaning determinanti noldan farqli.

2.16. Kvadrat matrisalar ko'paytmasining determinanti berilgan matrisalar determinantlari ko'paytmasiga teng.

2.17. Kvadrat matrisaning determinanti nolga teng bo'lishi uchun uning satr (ustun)lari chiziqli bog'langan bo'lishi zarur va etarli.

3. Determinant xossalaridan foydalanib quyidagilarni isbotlang:

3.1. $$\begin{vmatrix} (a+b)^2 & c^2 & c^2 \\ a^2 & (b+c)^2 & a^2 \\ b^2 & b^2 & (c+a)^2 \end{vmatrix} = 2abc(a+b+c)^3.$$

3.2. $$\begin{vmatrix} \frac{1}{a+x} & \frac{1}{a+y} & 1 \\ \frac{1}{b+x} & \frac{1}{b+y} & 1 \\ \frac{1}{c+x} & \frac{1}{c+y} & 1 \end{vmatrix} = \frac{(a-b)(a-c)(b-c)(x-y)}{(a+x)(b+x)(c+x)(a+y)(b+y)(c+y)}.$$

3.3. $$\begin{vmatrix} a & b & c & d \\ -b & a & d & -c \\ -c & -d & a & b \\ -d & c & -b & a \end{vmatrix} = (a^2+b^2+c^2+d^2)^2.$$

3.4. $$\begin{vmatrix} -1 & 1 & 1 & 1 & x \\ 1 & -1 & 1 & 1 & y \\ 1 & 1 & -1 & 1 & z \\ 1 & 1 & 1 & -1 & t \\ x & y & z & t & 0 \end{vmatrix} =$$

$$= -4(x^2+y^2+z^2+t^2-2(xy+xz+xt+yz+yt+zt)).$$

4. Quyidagi determinantlarni hisoblang:

4.1. $$\begin{vmatrix} 1 & 2 & 3 & \dots & n \\ -1 & 0 & 3 & \dots & n \\ -1 & -2 & 0 & \dots & n \\ . & . & . & . & . \\ -1 & -2 & -3 & \dots & 0 \end{vmatrix}.$$

4.2. $\begin{vmatrix} 1 & n & n & \dots & n \\ n & 2 & n & \dots & n \\ n & n & 3 & \dots & n \\ . & . & . & . & . \\ n & n & n & \dots & n \end{vmatrix}$.

4.3. $\begin{vmatrix} 1+x_1 & 1 & 1 & \dots & 1 \\ 1 & 1+x_2 & 1 & \dots & 1 \\ 1 & 1 & 1+x_3 & \dots & 1 \\ . & . & . & . & . \\ 1 & 1 & 1 & \dots & 1+x_n \end{vmatrix}$.

4.4. $\begin{vmatrix} x_1 & a_{12} & a_{13} & \dots & a_{1n} \\ x_1 & x_2 & a_{23} & \dots & a_{2n} \\ x_1 & x_2 & x_3 & \dots & a_{3n} \\ . & . & . & . & . \\ x_1 & x_2 & x_3 & \dots & x_n \end{vmatrix}$.

4.5. $\begin{vmatrix} 1 & 2 & 3 & \dots & n \\ 1 & x+1 & 3 & \dots & n \\ 1 & 2 & x+1 & \dots & n \\ . & . & . & . & . \\ 1 & 2 & 3 & \dots & x+1 \end{vmatrix}$.

4.6. $\begin{vmatrix} 1 & 1 & 1 & \dots & 1 \\ 1 & 2-a & 1 & \dots & 1 \\ 1 & 1 & 3-a & \dots & 1 \\ . & . & . & . & . \\ 1 & 1 & 1 & \dots & n+1-a \end{vmatrix}$.

4.7. $\begin{vmatrix} 2 & 1 & 0 & \dots & 0 \\ 1 & 2 & 1 & \dots & 0 \\ 0 & 1 & 2 & \dots & 0 \\ . & . & . & . & . \\ 0 & 0 & 0 & \dots & 2 \end{vmatrix}$.

4.8. $\begin{vmatrix} 3 & 2 & 0 & \dots & 0 \\ 1 & 3 & 2 & \dots & 0 \\ 0 & 1 & 3 & \dots & 0 \\ . & . & . & . & . \\ 0 & 0 & 0 & \dots & 3 \end{vmatrix}$.

4.9. $\begin{vmatrix} x+a_1 & a_2 & a_3 & \dots & a_n \\ a_1 & x+a_2 & a_3 & \dots & a_n \\ a_1 & a_2 & x+a_3 & \dots & a_n \\ . & . & . & . & . \\ a_1 & a_2 & a_3 & \dots & x+a_n \end{vmatrix}$.

4.10. $\begin{vmatrix} 1 & x & x^2 & \dots & x^n \\ a_{11} & 1 & x & \dots & x^{n-1} \\ a_{21} & a_{22} & 1 & \dots & x^{n-2} \\ . & . & . & . & . \\ a_{n1} & a_{n2} & a_{n3} & \dots & 1 \end{vmatrix}$.

5. ChTSni Kramer formulalari yordamida yeching:

5.1. $\begin{cases} x_1 + x_2 - x_3 = 2, \\ -2x_1 + x_2 + x_3 = 3, \\ x_1 + x_2 + x_3 = 6. \end{cases}$

5.2. $\begin{cases} 3x_1 - x_2 = 5, \\ -2x_1 + x_2 + x_3 = 0, \\ 2x_1 - x_2 + 4x_3 = 15. \end{cases}$

5.3. $\begin{cases} 2x_1 + 2x_2 - x_3 = 4, \\ 3x_1 + x_2 - x_3 = 7, \\ x_1 + x_2 - 2x_3 = 3. \end{cases}$

5.4. $\begin{cases} 5x_1 + 4x_3 = 1, \\ x_1 - x_2 + 2x_3 = 0, \\ 4x_1 + x_2 + 2x_3 = 1. \end{cases}$

5.5. $\begin{cases} x_1+x_2+2x_3+3x_4=1, \\ 3x_1-x_2-x_3-2x_4=-4, \\ 2x_1+3x_2-x_3-x_4=-6, \\ x_1+2x_2+3x_3-x_4=-4. \end{cases}$

5.6. $\begin{cases} 2x_1+x_2+x_3-x_4=1, \\ 3x_1-x_2+x_3+x_4=2, \\ 4x_1+2x_2+x_3-x_4=1, \\ x_4=2. \end{cases}$

5.7. $\begin{cases} 2x_1-x_2-6x_3+3x_4=-1, \\ 7x_1-4x_2+2x_3-15x_4=-32, \\ x_1-2x_2-4x_3+9x_4=5, \\ x_1-x_2+2x_3-6x_4=-8. \end{cases}$

5.8. $\begin{cases} 2x_1-3x_2+5x_3+x_4=-7, \\ -x_1+x_2+4x_3-2x_4=1, \\ 5x_1+2x_2-3x_3+6x_4=5, \\ -2x_1+5x_2+x_3-7x_4=5. \end{cases}$

5.9. $\begin{cases} x_1+2x_2-3x_3-x_4=-8, \\ 2x_1-x_2+2x_3-x_4=2, \\ 4x_1+3x_2-x_3-x_4=3, \\ x_1+2x_2+x_3+x_4=12. \end{cases}$

5.10. $\begin{cases} 5x_1+2x_2+x_3+4x_4=4, \\ 4x_1+6x_2+3x_3+7x_4=1, \\ 3x_1+x_2+2x_3+4x_4=1, \\ 2x_1+3x_2+4x_3+5x_4=-2. \end{cases}$

6. Bir jinsli n noma'lumli n ta chiziqli tenglamalar sistemasining nolmas yechimga ega bo'lishi uchun, uning determinanti nolga teng bo'lishi zarur va etarli ekanligini isbotlang.

7. Har qanday kvadrat matrisa uchun quyidagi shartlar teng kuchli ekanligini isbotlang:

7.1. $|A| \neq 0$.

7.2. Matrisaning satr (ustun)lari chiziqli erkli.

7.3. A matrisa teskarilanuvchi.

7.4. A matrisa elementar matrisalar yordamida ifodalanadi.

8. Matrisa rangini minorlar yordamida toping:

8.1. $\begin{pmatrix} 2 & -1 & 3 & -2 & 4 \\ 4 & -2 & 5 & 1 & 7 \\ 2 & -1 & 1 & 8 & 2 \end{pmatrix}$. 8.2. $\begin{pmatrix} 8 & 2 & 2 & -1 & 1 \\ 1 & 7 & 4 & -2 & 5 \\ -2 & 4 & 2 & -1 & 3 \end{pmatrix}$.

8.3. $\begin{pmatrix} 1 & 7 & 7 & 9 \\ 7 & 5 & 1 & -1 \\ 4 & 2 & -1 & -3 \\ -1 & 1 & 3 & 5 \end{pmatrix}$. 8.4. $\begin{pmatrix} 1 & 3 & 5 & -1 \\ 2 & -1 & -3 & 4 \\ 5 & 1 & -1 & 7 \\ 7 & 7 & 9 & 1 \end{pmatrix}$.

8.5. $\begin{pmatrix} 4 & 1 & 7 & -5 & 1 \\ 0 & -7 & 1 & -3 & -5 \\ 3 & 4 & 5 & -3 & 2 \\ 2 & 5 & 3 & -1 & 3 \end{pmatrix}$. 8.6. $\begin{pmatrix} -6 & 4 & 8 & -1 & 6 \\ -5 & 2 & 4 & 1 & 3 \\ 7 & 2 & 4 & 1 & 3 \\ 2 & 4 & 8 & -7 & 6 \\ 3 & 2 & 4 & -5 & 3 \end{pmatrix}$.

8.7. $\begin{pmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$. 8.8. $\begin{pmatrix} 1 & 1 & 0 & 0 & \dots & 0 & 0 \\ 0 & 1 & 1 & 0 & \dots & 0 & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & 0 & 0 & \dots & 1 & 1 \\ 1 & 0 & 0 & 0 & \dots & 0 & 1 \end{pmatrix}$.

9. Matrisaga teskari matrisani biriktirilgan matrisa yordamida toping:

9.1. $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$. 9.2. $\begin{pmatrix} 3 & 4 \\ 5 & 7 \end{pmatrix}$.

9.3. $\begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}$. 9.4. $\begin{pmatrix} 2-3i & 1-i \\ 3+i & 5+4i \end{pmatrix}$.

9.5. $\begin{pmatrix} 2 & 5 & 7 \\ 6 & 3 & 4 \\ 5 & -2 & -3 \end{pmatrix}$. 9.6. $\begin{pmatrix} 7 & 9 & 2 \\ 2 & -2 & 6 \\ 5 & 6 & 3 \end{pmatrix}$.

9.7. $\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{pmatrix}$. 9.8. $\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & i & -1 & -i \\ 1 & -1 & 1 & -1 \\ 1 & -i & -1 & i \end{pmatrix}$.

10. Berilgan matrisalarni elementar matrisalar ko'paytmasi ko'rinishida ifodalang:

10.1. $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$. 10.2. $\begin{pmatrix} 3 & 4 \\ 5 & 7 \end{pmatrix}$.

10.3. $\begin{pmatrix} 7 & -4 \\ -5 & 3 \end{pmatrix}$. 10.4. $\begin{pmatrix} -1 & -1 \\ 2 & 3 \end{pmatrix}$.

10.5. $\begin{pmatrix} -3 & -5 \\ 2 & 4 \end{pmatrix}$. 10.6. $\begin{pmatrix} 3 & -2 \\ 5 & -4 \end{pmatrix}$.

Takrorlash uchun savollar

1. Determinantning asosiy xossalarini ayting.
2. n-tartibli minor deb nimaga aytiladi?.
3. Determinantni algebraik to'ldiruvchi yordamida aniqlash jarayonini tushuntiring.
4. Determinant nolga teng bo'lishining zarur va etarli shartini ayting.
5. Algebraik to'ldiruvchilar yordamida teskari matrisani topish jarayonini tushuntiring.
6. ChTSni Kramer qoidasi bilan yechish usulini tushuntiring.
7. n noma'lumli n ta tenglamadan iborat bir jinsli chiziqli tenglamalar sistemasi qachon yagona yechimga ega?

VIII MODUL. VEKTOR FAZOLAR

19-§. Vektor fazo. Fazoostilar kesishmasi, yig'indisi.

Asosiy tushunchalar: vektor fazo, fazoosti, fazoostilar kesishmasi, fazoostilar yig'indisi, fazoostilar to'g'ri yig'indisi, vektor fazo bazisi, vektor fazo o'lchovi.

Bo'sh bo'lmagan $V=\{\bar{x},\bar{y},\bar{z},...\}$ to'plam va $\mathcal{F}=\{\alpha,\beta,\gamma,...\}$ maydon berilgan bo'lib, quyidagi aksiomalar bajarilsa, u holda V to'plam $\mathcal{F}$ sonlar maydoni ustiga qurilgan vektor fazo deyiladi:

V – additiv abel gruppa;

$(\alpha\cdot\beta)\bar{x}=\alpha(\beta\bar{x})\ (\forall\bar{x}\in V,\forall\alpha,\beta\in F)$;

$\alpha(\bar{x}+\bar{y})=\alpha\bar{x}+\alpha\bar{y}\quad(\forall\bar{x},\bar{y}\in V,\ \ \forall\alpha\in F)$;

$(\alpha+\beta)\bar{x}=\alpha\bar{x}+\beta\bar{x}\ (\forall\bar{x}\in V,\forall\alpha,\beta\in F)$;

$1\cdot\bar{x}=\bar{x}\qquad(\forall\bar{x}\in V,\ \ 1\in F)$.

$\mathcal{F}$ maydon ustida aniqlangan V vektor fazoning biror L to'plamostisi V da aniqlangan algebraik amallarga nisbatan vektor fazosini tashkil etsa, u holda L ga V fazoning fazoosti deyiladi.

V vektor fazoning biror L to'plamostisi shu vektor fazoning fazoostisi bo'lishi uchun quyidagi ikkita shartning bajarilishi zarur va etarli:

$a)\ (\forall\bar{x},\bar{y}\in L)\ (\bar{x}-\bar{y})\in L$;

$b)\ (\forall\bar{x}\in L,\ \ \forall\alpha\in F)\ \ \alpha\bar{x}\in L$.

Agar U_1 ,..., U_n lar V vektor fazoning fazoostilari bo'lsa, u holda $U=U_1\cap U_2\cap...\cap U_n$ ga U_1,..., U_n fazoostilarning kesishmasi deyiladi.

$\bar{x}_1 \in U_1$, $\bar{x}_2 \in U_2$, ..., $\bar{x}_n \in U_n$ bo'lganda $\bar{x}_1 + \bar{x}_2 + ... + \bar{x}_n$ ko'rinishdagi barcha yig'indilar to'plamiga U_1 ,..., U_n fazoostilar yig'indisi deyiladi va u $U_1 + U_2 + .. . + U_n$ ko'rinishda belgilanadi.

Agar V vektor fazoning chiziqli bog'lanmagan $\bar{x}_1, \bar{x}_2, ..., \bar{x}_n$ (1) vektorlar sistemasi mavjud bo'lsaki, V ning qolgan barcha vektorlari (1) sistema orqali chiziqli ifodalansa, u holda (1) vektorlar sistemasi V vektor fazoning bazisi deyiladi.

V vektor fazoning bazislaridagi vektorlar soni V vektor fazoning o'lchovi deyiladi.

V fazoning o'lchovi dimV orqali belgilanadi.

Misol. V= $\left\{\begin{pmatrix} a & b \\ c & d \end{pmatrix} \middle| a,b,c,d \in R\right.$ to'plmni R maydon ustida chiziqli fazo tashkil etishini va uni bazisi, o'lchovini aniqlang.

Yechish. Berilgan $V \neq \varnothing$ to'plamda qo'shish va skalyarni to'plam elementiga ko'paytirish amallarini aniqlaymiz:

1). $\forall A_1 A_2 \in V$, $A_1 = \begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix}$, $A_2 = \begin{pmatrix} a_2 & b_2 \\ c_2 & d_2 \end{pmatrix}$ larga yagona $A_1 + A_2 = \begin{pmatrix} a_1 + a_2 & b_1 + b_2 \\ c_1 + c_2 & d_1 + d_2 \end{pmatrix}$ ni mos qo'yamiz. Bu erda $a_1, a_2, b_1, b_2, c_1, c_2, d_1, d_2 \in R$ va haqiqiy sonlar to'plamida qo'shish amali aniqlanganligi uchun $a_1 + a_2, b_1 + b_2, c_1 + c_2, d_1 + d_2 \in R$, ya'ni A_1+A_2 matrisa V ning elementi. Bundan tashqari ikkita berilgan haqiqiy sonlar yig'indisi yagona uchinchi haqiqiy son ekanligidan berilgan ikkita kvadrat matrisalar yig'indisi bo'lgan uchinchi kvadrat matrisaning yagonaligi kelib chiqadi.

2). V to'plamning ixtiyoriy $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ elementi va ixtiyoriy $\alpha \in R$ uchun $\omega_\alpha(A) = \alpha A = \alpha\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} \alpha a & \alpha b \\ \alpha c & \alpha d \end{pmatrix}$ matrisani hosil qilamiz. Haqiqiy sonlar to'plamida ko'paytirish amali aniqlanganligi uchun $\alpha a, \alpha b, \alpha c, \alpha d \in R$, ya'ni

$\alpha A \in V$. Hosil qilingan $V = < V; +, \{\omega_\alpha | \alpha \in R\} >$ algebra chiziqli vektor fazo tashkil etishini isbotlaymiz. Buning uchun qo'shish, skalyarni matrisaga ko'paytirish amallarining quyidagi xossalari bajarilishini isbotlaymiz:

1) $\forall A_1, A_2 \in V$, $\quad A_1 + A_2 = A_2 + A_1$;
2) $\forall A_1, A_2, A_3 \in V$, $\quad A_1 + (A_2 + A_3) = (A_1 + A_2) + A_3$;
3) $\forall A \in V \wedge \exists O \in V$, $\quad A + O = A$;
4) $\forall A \in V \wedge \exists A' \in V$, $\quad A + A' = O$;
5) $\forall \alpha, \beta \in R \wedge \forall A \in V$, $\quad (\alpha + \beta)A = \alpha A + \beta A$;
6) $\forall \alpha, \beta \in R \wedge \forall A \in V$, $\quad (\alpha\beta)A = \alpha(\beta A)$;
7) $\forall \alpha \in R \wedge \forall A_1, A_2 \in V$, $\quad \alpha(A_1 + A_2) = \alpha A_1 + \alpha A_2$;
8) $\forall A \in V$, $\quad 1 \cdot A = A$..

Isbot. 1.V to'plamning ixtiyoriy $A_1 = \begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix}$, $A_2 = \begin{pmatrix} a_2 & b_2 \\ c_2 & d_2 \end{pmatrix}$ elementlari uchun A_1+A_2= |matrisalarni qo'shish amali ta'rifiga ko'ra|=

$$= \begin{pmatrix} a_1 + a_2 & b_1 + b_2 \\ c_1 + c_2 & d_1 + d_2 \end{pmatrix} = \left| \begin{matrix} R\, da \;\; qo'shish \;\; amali \\ kommutativ \;\; ekanligidan \end{matrix} \right| = \begin{pmatrix} a_2 + a_1 & b_2 + b_1 \\ c_2 + c_1 & d_2 + d_1 \end{pmatrix} =$$

$$= \left| \begin{matrix} V da \;\; qo'shish \;\; amali \\ ta'rifiga \;\; ko'ra \end{matrix} \right| = \begin{pmatrix} a_2 & b_2 \\ c_2 & d_2 \end{pmatrix} + \begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix} = B + A.$$

Demak,V to'plamda aniqlangan qo'shish amali kommutativ va <V ;+ > additiv abel gruppoid .

2. V to'plamning ixtiyoriy $A_1 = \begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix}$, $A_2 = \begin{pmatrix} a_2 & b_2 \\ c_2 & d_2 \end{pmatrix}$, $A_3 = \begin{pmatrix} a_3 & b_3 \\ c_3 & d_3 \end{pmatrix}$ elementlari uchun

$$A_1+(A_2+A_3)=\begin{pmatrix}a_1 & b_1\\ c_1 & d_1\end{pmatrix}+\left(\begin{pmatrix}a_2 & b_2\\ \tilde{n}_2 & d_2\end{pmatrix}+\begin{pmatrix}a_3 & b_3\\ c_3 & d_3\end{pmatrix}\right)=\left|\begin{array}{l}V\ da\ qo'shish\ amali\\ ta'rifiga\ ko'ra\end{array}\right|=$$

$$=\begin{pmatrix}\grave{a}_1 & b_1\\ c_1 & d_1\end{pmatrix}+\begin{pmatrix}a_2+a_3 & b_2+b_3\\ c_2+c_3 & d_2+d_3\end{pmatrix}=\left|\begin{array}{l}V\ da\ qo'shish\ amali\\ ta'rifiga\ ko'ra\end{array}\right|=$$

$$=\begin{pmatrix}a_1+(a_2+a_3) & b_1+(b_2+b_3)\\ c_1+(c_2+c_3) & d_1+(d_2+d_3)\end{pmatrix}=\left|\begin{array}{l}V\ da\ qo'shish\ amalining\\ assosiativligidan\end{array}\right|=$$

$$=\begin{pmatrix}(a_1+a_2)+a_3 & (b_1+b_2)+b_3\\ (c_1+c_2)+c_3 & (d_1+d_2)+d_3)\end{pmatrix}=\left|\begin{array}{l}V\ da\ qo'shish\ amali\\ ta'rifiga\ ko'ra\end{array}\right|=$$

$$=\begin{pmatrix}a_1+a_2 & b_1+b_2\\ c_1+c_2 & d_1+d_2\end{pmatrix}+\begin{pmatrix}\grave{a}_3 & b_3\\ c_3 & d_3\end{pmatrix}=\left|\begin{array}{l}V\ da\ qo'shish\ amali\\ ta'rifiga\ ko'ra\end{array}\right|=(A+B)+C.$$

Demak, <V ;+ > additiv abel yarimgruppa.

3.V to'plamdan olingan ixtiyoriy $A=\begin{pmatrix}a & b\\ c & d\end{pmatrix}$ element uchun shu to'plamda yagona $O=\begin{pmatrix}0 & 0\\ 0 & 0\end{pmatrix}$ element mavjudki, A+O=A+O=O.

Demak, <V ;+,0 > - additiv abel manoid .

4. V to'plamdan olingan har qanday $A=\begin{pmatrix}a & b\\ c & d\end{pmatrix}$ element uchun shunday $A'\in V$ element mavjudki, $A+A'=0$. Bu erda $A'=\begin{pmatrix}-a & -b\\ -c & -d\end{pmatrix}$ ekanligi har qanday haqiqiy son uchun qarama-qarshisi mavjidligidan kelib chiqadi. Yoki bo'lmasa, (-1)A=-A ni A ga qarama-qarshi element sifatida hosil qilish mumkin.

Demak, <V ;+,-,0 > - additiv abel gruppa ekan.

5.$\forall\alpha,\beta\in R\ va\ \forall A\in V$ uchun

$$(\alpha+\beta)A=(\alpha+\beta)\begin{pmatrix}a & b\\ c & d\end{pmatrix}=\left|\begin{array}{l}Vda\ skalyarni\ matrisaga\ ko'paytirish\\ amali\ ta'rifiga\ ko'ra\end{array}\right|=$$

$$=\begin{pmatrix}(\alpha+\beta)a & (\alpha+\beta)b\\ (\alpha+\beta)c & (\alpha+\beta)d)\end{pmatrix}=\left|\begin{array}{l}R\,da\ ko'paytirishning\ qo'shishga\\ nisba\text{ tan }distributivligidan\end{array}\right|=$$

$$=\begin{pmatrix}\alpha a+\beta a & \alpha b+\beta b\\ \alpha c+\beta c & \alpha d+\beta d\end{pmatrix}=\left|\begin{array}{l}Vda\ qo'shish\\ amali\ ta'rifiga\ ro'ra\end{array}\right|=$$

$$=\begin{pmatrix}\alpha a & \alpha b\\ \alpha c & \alpha d)\end{pmatrix}+\begin{pmatrix}\beta a & \beta b\\ \beta c & \beta d)\end{pmatrix}=\left|\begin{array}{l}Vda\ skalyarni\ matrisaga\ ko'paytirish\\ amali\ ta'rifiga\ ko'ra\end{array}\right|=$$

$$=\alpha\begin{pmatrix}a & b\\ c & d\end{pmatrix}+\beta\begin{pmatrix}a & b\\ c & d\end{pmatrix}=\alpha A+\beta A.$$

6. $\forall\alpha,\beta\in R\ va\forall A\in V\ lar\ uchun$

$$(\alpha\beta)A=(\alpha\,\beta)\begin{pmatrix}a & b\\ c & d\end{pmatrix}=\left|\begin{array}{l}Vda\ skalyarni\ matrisaga\ ko'paytirish\\ amali\ ta'rifiga\ ko'ra\end{array}\right|=$$

$$=\begin{pmatrix}(\alpha\,\beta)a & (\alpha\,\beta)b\\ (\alpha\,\beta)c & (\alpha\,\beta)d\end{pmatrix}=\left|\begin{array}{l}R\,da\ ko'paytirish\\ amali\ assosiativligidan\end{array}\right|=\begin{pmatrix}\alpha(\beta\,a) & \alpha(\beta\,b)\\ \alpha(\beta\,c) & \alpha(\beta\,d)\end{pmatrix}=$$

$$=\left|\begin{array}{l}Vda\ skalyarni\ matrisaga\ ko'paytirish\\ amali\ ta'rifiga\ ko'ra\end{array}\right|=\alpha\begin{pmatrix}\beta a & \beta b\\ \beta c & \beta d\end{pmatrix}=$$

$$=\left|\begin{array}{l}Vda\ skalyarni\ matrisaga\ ko'paytirish\\ amali\ ta'rifiga\ ko'ra\end{array}\right|=\alpha\left(\beta\begin{pmatrix}a & b\\ c & d\end{pmatrix}\right)=\alpha(\beta A).$$

7. $\forall\alpha\in R\wedge\forall A_1,A_2\in V$ lar uchun $\alpha(A_1+A_2)=$

$$=\alpha\left(\begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix}+\begin{pmatrix} a_2 & b_2 \\ c_2 & d_2 \end{pmatrix}\right)=\left|\begin{matrix} V \text{ dà qo'shish va skalyarni} \\ \text{ko' paytirish amallarining} \\ \text{ta' rifiga ko'ra} \end{matrix}\right|=$$

$$=\begin{pmatrix} \alpha(a_1+a_2) & \alpha(b_1+b_2) \\ \alpha(c_1+c_2) & \alpha(d_1+d_2) \end{pmatrix}=\left|\begin{matrix} R \text{ dà ko' paytirishning qo' shishga} \\ \text{nisba tan distributivligidan} \end{matrix}\right|=$$

$$=\begin{pmatrix} \alpha a_1+\alpha a_2 & \alpha b_1+\alpha b_2 \\ \alpha c_1+\alpha c_2 & \alpha d_1+\alpha d_2 \end{pmatrix}=\left|\begin{matrix} V \text{ dà qo' shish amalining} \\ \text{ta' rifiga ko'ra} \end{matrix}\right|=$$

$$=\begin{pmatrix} \alpha a_1 & \alpha b_1 \\ \alpha c_1 & \alpha d_1 \end{pmatrix}+\begin{pmatrix} \alpha a_2 & \alpha b_2 \\ \alpha c_2 & \alpha d_2 \end{pmatrix}=\left|\begin{matrix} V\text{da skalyarni matrisaga ko' paytirish} \\ \text{amali ta' rifiga ko'ra} \end{matrix}\right|=$$

$$=\alpha \grave{A}_1+\alpha \grave{A}_2 .$$

8.V to'plamning har qanday $A=\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ elementi va $1\in R$ skalyarlar uchun

1·A=A ekaligi V da skalyarni ko'paytirish amali ta'rifi va R to'plamda

$1\cdot a=a$ $(\forall a\in R)$ ekaligidan kelib chiqadi.

Demak, V = $<V;+,\{\omega_\lambda|\lambda\in R\}>$ -chiziqli vektor fazo ekan.

9. Bu fazoning bazisini aniqlaymiz. Chiziqli vektor fazo bazisi ta'rifiga ko'ra V to'plamning ixtiyoriy $A=\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ elementini chiziqli ifodalovchi chiziqli erkli sistemani topamiz. Bunday sistema sifatida $A_a=\begin{pmatrix} a & 0 \\ 0 & 0 \end{pmatrix}$, $A_b=\begin{pmatrix} 0 & b \\ 0 & 0 \end{pmatrix}$, $A_c=\begin{pmatrix} 0 & 0 \\ c & 0 \end{pmatrix}$, $A_d=\begin{pmatrix} 0 & 0 \\ 0 & d \end{pmatrix}$ sistemani olsak, u holda $\begin{pmatrix} a & b \\ c & d \end{pmatrix}=A_a+A_b+A_c+A_d$ ekanligi ravshan. Bazis vektor sifatida A_a, A_b, A_c, A_d bazisdagi a,b,c,d haqiqiy sonlar o'rniga 0 dan farqli ixtiyoriy haqiqiy sonni qo'yish natijasida V fazoning boshqa bazislarini hosil qilish mumkin.

Demak, V = $<V;+,\{\omega_\lambda|\lambda\in R\}>$ - vektor fazoning bazisi cheksiz ko'p.

10. Chiziqli vektor fazoning o'lchovi ta'rifiga ko'ra

V = $<V;+,\{\omega_\lambda|\lambda\in R\}>$- fazoning ixtiyoriy bazisidagi vektorlar soni uning o'lchovidir, ya'ni $\dim V=4$ ga teng.

Misol. $(a):\vec{a}_1=(1,1,2)$, $\vec{a}_2=(0,1,1)$; $(b):\vec{b}_1=(2,1,3)$, $\vec{b}_2=(1,1,3)$, $\vec{b}_3=(0,1,1)$ vektorlar sistemalari tashkil etgan chiziqli fazolar, ularning yig'indisi, kesishmasining bazisi va o'lchovini aniqlang.

Yechish . 1)$(a):\vec{a}_1=(1,1,2)$, $\vec{a}_2=(0,1,1)$ vektorlar sistemasi chiziqli erkli, rang(a)=2. Shuning uchun $L((a))$ chiziqli fazoning o'lchovi $dimL((a))$=2.

2) $(b):\vec{b}_1=(2,1,3)$, $\vec{b}_2=(1,1,3)$, $\vec{b}_3=(0,1,1)$ sistemani tekshiramiz:

$$\begin{array}{c} \vec{b}_1 \\ \vec{b}_2 \\ \vec{b}_3 \end{array}\begin{pmatrix} 2 & 1 & 3 \\ 1 & 1 & 3 \\ 0 & 1 & 1 \end{pmatrix}\sim\begin{array}{c} \vec{b}_2 \\ \vec{b}_3 \\ \vec{b}_1-2\vec{b}_2 \end{array}\begin{pmatrix} 1 & 1 & 3 \\ 0 & 1 & 1 \\ 0 & -1 & -3 \end{pmatrix}\sim\begin{array}{c} \vec{b}_2 \\ \vec{b}_3 \\ \vec{b}_3+\vec{b}_1-2\vec{b}_2 \end{array}\begin{pmatrix} 1 & 1 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & -2 \end{pmatrix}.$$

Demak ,(b) sistema chiziqli erkli. Bundan dim $L((b))$=3.

3) $L((a))$, $L((b))$ qism fazolar yig'indisining bazisini topish uchun ularning bazis vektorlaridan

$\vec{a}_1=(1,1,2)$, $\vec{a}_2=(0,1,1)$, $\vec{b}_1=(2,1,3)$, $\vec{b}_2=(1,1,3)$, $\vec{b}_3=(0,1,1)$ sistemani hosil qilamiz va bu sistemaning bazis vektorlarini topamiz. Qaralayotgan misolda $L((b))=R^3$ bo'lganligi uchun $L((a))$, $L((b))$ qism fazolar yig'indisi ham R^3 dan iborat bo'ladi. U holda $dim(L((a))+L((b)))=3$. Agar

$\dim(L((a))+L((b)))+dim(L((a))\cap L((b)))=dimL((a))+dimL((b))$

tenglikni e'tiborga olsak, u holda $dimL((a))\cap dimL((b))$ =2 ekanligi kelib chiqadi .

Demak, $dim\ L((a))=2;\ dim\ L((b))=3,\ dim\ L((a))+dim\ L((b))=3,$

$dim(L((a))\cap L((b)))=2.$

Misol va mashqlar

1.Berilgan to'plamlarni chiziqli fazo tashkil etishini isbotlang va uning bazisi, o'lchovini aniqlang:

1.1. $V = \left\{ \begin{pmatrix} a & 0 \\ 0 & 0 \end{pmatrix} \mid a \in R \right\}$.

1.2. $V = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \mid a,b,c,d \in R \right\}$.

1.3. $V = \left\{ \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \mid a_{ij} \in R \wedge \ i = \overline{1,3} \wedge j = \overline{1,3} \right\}$.

1.4. $V = \left\{ \begin{pmatrix} a_{11} & 0 & 0 \\ a_{21} & a_{22} & 0 \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \mid a_{ij} \in R \wedge \ i = \overline{1,3} \wedge j = \overline{1,3} \right\}$.

1.5. $V = \left\{ \begin{pmatrix} a_{11} & 0 & a_{13} \\ a_{21} & 0 & a_{23} \\ a_{31} & 0 & a_{33} \end{pmatrix} \mid a_{ij} \in R \wedge \ i = \overline{1,3} \wedge j = 1,3 \right\}$.

1.6. $V = \left\{ \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ 0 & 0 & 0 \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \mid a_{ij} \in R \wedge \ i = 1,3 \wedge j = \overline{1,3} \right\}$.

1.7. $V = \left\{ \begin{pmatrix} 0 & a_{12} & a_{13} \\ a_{21} & 0 & a_{23} \\ a_{31} & a_{32} & 0 \end{pmatrix} \mid a_{ij} \in R \wedge \ i = \overline{1,3} \wedge j = \overline{1,3} \right\}$.

2. Kompleks sonlar maydoni tashkil etgan vektor fazoning bazisi va o'lchovini aniqlang.

3. V_n arifmetik vektor fazoning quyidagi vektorlar sistemasi tashkil etgan fazoostilari bazisi va o'lchovini aniqlang:

3.1. Birinchi va keyingi koordinatalari teng bo'lgan barcha vektorlar.

3.2. Koordinatalari yig'indisi nolga teng bo'lgan barcha vetorlar.

3.3. Juft o'rindagi koordinatalari nolga teng bo'lgan barcha vektorlar.

3.4. Juft o'rindagi koordinatalari teng bo'lgan barcha vektorlar.

3.5. Koordinatalari teng bo'lgan barcha vektorlar.

3.6. Har bir koordinatasi o'zidan oldingi koordinataning qarama-qarshisiga teng bo'lgan barcha vektorlar.

4. Quyidagi to'plamlarning qaysilari vektor fazo tashkil etadi:

4.1. R_n vektor fazoning butun koeffitsientli vektorlar to'plami.

4.2. Tekislikning koordinatalar o'qlaridan birida joylashgan barcha vektorlar to'plami.

4.3. Tekislikning boshi koordinatalar boshida oxiri berilgan to'g'ri chiziqda yotuvchi barcha vektorlari to'plami.

4.4. Tekislikning boshi va oxiri bir to'g'ri chiziqda yotuvchi vektorlari to'plami.

4.5. R_n vektor fazoning barcha koordinatalari yig'indisi 1 ga teng bo'lgan barcha vektorlar to'plami.

4.6. R_n vektor fazoning barcha koordinatalari yig'indisi 0 ga teng bo'lgan barcha vektorlar to'plami.

5. Fazoostining quyidagi xossalarini isbotlang:

5.1. Agar V fazo $\mathcal{F}$ maydon ustida vektor fazo bo'lsa, u holda uning ixtiyoriy fazoostisi $\mathcal{F}$ maydon ustidagi vektor fazo bo'ladi.

5.2. Agar U fazo V vektor fazoning fazoosti va V fazo W vektor fazoning fazoosti bo'lsa, u holda U fazo W vektor fazoning fazoosti bo'ladi.

5.3. V vektor fazoning ixtiyoriy fazoostilarining kesishmasi V vektor fazoning qism fazosi bo'ladi.

6. Quyidagi vektorlar sistemasi tashkil etgan fazoostilar bazisi va o'lchovini aniqlang:

6.1. $\vec{a}_1(1,0,0,-1)$, $\vec{a}_2(2,1,1,0)$, $\vec{a}_3(1,1,1,1)$, $\vec{a}_4(0,1,2,3)$.

6.2. $\vec{a}_1(0,-1,0,-1)$, $\vec{a}_2(1,2,1,2)$, $\vec{a}_3(1,1,1,1)$, $\vec{a}_4(2,3,1,0)$, $\vec{a}_5(4,5,3,2)$.

6.3. $\vec{a}_1(1,1,1,1,0)$, $\vec{a}_2(1,1,-1,-1,-1)$, $\vec{a}_3(2,2,0,0,-1)$, $\vec{a}_4(1,1,5,5,2)$,

6.4. $\vec{a}_5(1,-1,-1,0,0)$.

6.5. $\vec{a}_1(1,1,0,0)$, $\vec{a}_2(0,1,1,0)$, $\vec{a}_3(0,0,1,1)$.

6.6. $\vec{a}_1(1,-1,-1)$, $\vec{a}_2(-2,2,2)$, $\vec{a}_3(1,0,1)$, $\vec{a}_4(2,3,1)$, $\vec{a}_5(5,3,2)$.

6.7. $\vec{a}_1(-4,3,-2,1)$, $\vec{a}_2(2,2,2,2)$, $\vec{a}_3(=0,1,-1,1)$, $\vec{a}_4(3,3,1,1)$, $\vec{a}_5(0,1,2,3)$.

7.Fazoostilar yig'indisi va to'g'ri yig'indisining quyidagi xossalarini isbotlang:

7.1. Agar L va U lar V vektor fazoning fazoostilari bo'lsa, u holda $L+U=U+L$ bo'ladi.

7.2. Agar L, U, W lar V vektor fazoning fazoostilari bo'lsa, u holda $L+(U+W)=(L+U)+W$ bo'ladi.

7.3. Agar L fazoosti V vektor fazoning fazoostisi bo'lsa, u holda L+V=V bo'ladi.

7.4. L va U lar V fazoning fazoostilari bo'lsa, u xolda L+U yig'indi to'g'ri yig'indi bo'lishi uchun $L\cap U=\{\bar{0}\}$ bo'lishi zarur va etarli.

8. Bektorlarning (a) va (b) sistemalari tashkil etgan chiziqli fazolar, ularning yig'indisi, kesishmasining bazisi va o'lchovini aniqlang:

8.1. (a): $\vec{a}_1(1,2,3)$, $\vec{a}_2(0,1,1)$;

(b): $\vec{b}_1(1,0,1)$, $\vec{b}_2(2,1,1)$.

8.2. (a): $\vec{a}_1(1,2,3,3)$, $\vec{a}_2(0,1,1,5)$;

(b): $\vec{b}_1(1,0,7,1)$, $\vec{b}_2(2,0,1,1)$.

8.3. (a): $\vec{a}_1(1,2,3)$, $\vec{a}_2(0,1,1)$, $\vec{a}_3(-1,4,3)$;

(b): $\vec{b}_1(1,0,1)$, $\vec{b}_2(2,1,1)$, $\vec{b}_3(-3,-2,-1)$.

8.4. (a): $\vec{a}_1(1,2,3,6)$, $\vec{a}_2(0,1,1,7)$, $\vec{a}_3(-1,4,3,8)$;

(b): $\vec{b}_1(1,0,1,-4)$, $\vec{b}_2(2,1,1,-3)$, $\vec{b}_3(-3,-2,-1,-2)$.

8.5. (a): $\vec{a}_1(1,-2,3)$, $\vec{a}_2(0,1,-1)$, $\vec{a}_3(-1,4,3)$, $\vec{a}_4(-1,0,-3)$;

(b): $\vec{b}_1(9,0,1)$, $\vec{b}_2(-5,1,1)$, $\vec{b}_3(-3,2,-1)$.

8.6. (a): $\vec{a}_1(0,2,3)$, $\vec{a}_2(0,1,0)$, $\vec{a}_3(-1,-4,3)$;

(b): $\vec{b}_1(1,0,1)$, $\vec{b}_2(2,1,1)$, $\vec{b}_3(-3,-2,-1)$, $\vec{b}_4(-5,4,-3)$.

8.7. (a): $\vec{a}_1(-5,1,2,3)$, $\vec{a}_2(-6,0,1,1)$, $\vec{a}_3(-1,-1,4,3)$;

(b): $\vec{b}_1(-3,1,0,1)$, $\vec{b}_2(-4,2,1,1)$, $\vec{b}_3(-2,-3,-2,-1)$.

8.8. (a): $\vec{a}_1(-3,1,2,3)$, $\vec{a}_2(-4,0,1,1)$, $\vec{a}_3(-8,-1,4,3)$;

(b): $\vec{b}_1(-5,1,0,1)$, $\vec{b}_2(-6,2,1,1)$, $\vec{b}_3(-2,-3,-2,-1)$.

Takrorlash uchun savollar

1. Maydon ustida vektor fazo deb nimaga aytiladi?
2. Vektor fazoning asosiy xossalarini bayon eting.
3. Vektor fazoga misollar keltiring.
4. Vektor fazoning fazoostisi deb nimaga aytiladi?
5. Fazoostilar kesishmasi deb nimaga aytiladi?
6. Fazoostilar yig'indisi, to'g'ri yig'indisi ta'rifini ayting.
7. Fazoostilar yig'indisi va to'g'ri yig'indisining qanday xossalarini bilasiz?
8. Vektor fazoning bazisi deb nimaga aytiladi?
9. Vektor fazoning o'lchovi deb nimaga aytiladi?

20-§. Skalyar ko'paytmali vektor fazolar. Evklid vektor fazolar. Vektor fazolar izomorfizmi.

Asosiy tushunchalar: skalyar ko'paytma, xosmas, nol skalyar ko'paytmalar, unitar vektor fazo, ortogonal vektorlar, ortogonal bazis, ortonormal bazis, Evklid vektor fazo.

Agar V fazoning xar bir juft $\bar{x}$ va $\bar{y}$ elementlariga ularning skalyar ko'paytmasi deb ataluvchi yagona $(\bar{x},\bar{y})$ haqiqiy son mos qo'yilib, bu moslik uchun

1) $(\bar{x},\bar{y})=(\bar{y},\bar{x})$;

2) $(\bar{x}+\bar{y},\bar{z})=(\bar{x},\bar{z})+(\bar{y},\bar{z})$;

3) $(\lambda\bar{x},\bar{y})=\lambda(\bar{x},\bar{y})$, $\forall\lambda\in R$;

4) $(\bar{x},\bar{x})\geq 0$

shartlar bajarilsa, u holda V vektorlar fazosiga skalyar ko'paytmali fazo deyiladi.

Agar V fazoning istalgan $\bar{x} \neq \bar{0}$ vektori uchun $(\bar{x},\bar{x}) \neq \bar{0}$ bo'lsa, V fazoda aniqlangan skalyar ko'paytma xosmas skalyar ko'paytma deyiladi.

Agar V fazoning istalgan $\bar{x}$ va $\bar{y}$ vektorlari uchun $(\bar{x},\bar{y}) = 0$ bo'lsa, V fazoda aniqlangan skalyar ko'paytma nol skalyar ko'paytma deyiladi.

Agar V fazoning istalgan $\bar{x} \neq 0$ vektori uchun $(\bar{x},\bar{x}) > 0$ bo'lsa, bunday fazoga unitar fazo deyiladi.

Agar unitar fazoning ikkita $\bar{x}$ va $\bar{y}$ vektorlari uchun $(\bar{x},\bar{y}) = 0$ bo'lsa, u holda $\bar{x}$ va $\bar{y}$ vektorlar ortogonal vektorlar deyiladi.

Agar V fazoning $\bar{a}_1, \bar{a}_2, \ldots, \bar{a}_n$ (1)

vektorlar sistemasining istalgan ikkita elementi o'zaro ortogonal bo'lsa, u xolda (1) sistema ortogonal vektorlar sistemasi deyiladi.

Agar ortogonal vektorlar sistemasi qaralayotgan fazoning bazisi bo'lsa, bunday sistema ortogonal bazis deyiladi.

R maydon ustida aniqlangan V_n fazoning ixtiyoriy $\bar{a}_1, \bar{a}_2, \ldots, \bar{a}_n$ (1)

bazisini $\bar{e}_1, \bar{e}_2, \ldots, \bar{e}_n$ (2)

ortogonal bazisga aylantirish jarayoni bilan tanishamiz. Bu erda (1) dan (2) ni hosil qilish ortogonallash jarayoni deyilib, u quyidagicha amalga oshiriladi: $\bar{e}_1 = \bar{a}_1$ deb olamiz, $\bar{a}_1 \neq \bar{0}$ bo'lgani uchun $\bar{e}_1 \neq \bar{0}$ bo'ladi. Endi $\bar{e}_2$ ni $\bar{e}_2 = \bar{a}_2 + \alpha\bar{a}_1 = \bar{a}_2 + \alpha\bar{e}_1$ shaklda olib, α sonni shunday aniqlaymizki, natijada $(\bar{e}_1, \bar{e}_2) = 0$, ya'ni $(\bar{e}_1, \bar{e}_2) = (\bar{e}_1, \bar{a}_2 + \alpha\bar{e}_1) = (\bar{e}_1, \bar{a}_2) + \alpha(\bar{e}_1, \bar{e}_1) = 0$ (3)

bo'lsin. $\bar{a}_1 = \bar{e}_1 \neq \bar{0}$ va $\bar{a}_2 \neq \bar{0}$ bo'lgani uchun $\bar{e}_2 \neq \bar{0}$ bo'ladi. (3) tenglikdan

$$\alpha = -\frac{(\bar{e}_1, \bar{a}_2)}{(\bar{e}_1, \bar{e}_1)}$$ topiladi.

Endi $\bar{e}_3$ ni $\bar{e}_3 = \bar{a}_3 + \gamma\bar{e}_2 + \beta\bar{e}_1$ shaklda yozib olib, β va γ larni shunday tanlaymizki, natijada $(\bar{e}_1, \bar{e}_3) = 0$ va $(\bar{e}_2, \bar{e}_3) = 0$ bo'lsin, ya'ni

$$(\bar{e}_1, \bar{a}_3 + \gamma\bar{e}_2 + \beta\bar{e}_1) = 0, \quad (4)$$

$$(\bar{e}_2, \bar{a}_3 + \gamma\bar{e}_2 + \beta\bar{e}_1) = 0, \quad (5)$$

tengliklar bajarilsin. (4) va (5) tengliklardan

$$(\bar{e}_1, \bar{a}_3) + \gamma(\bar{e}_1, \bar{e}_2) + \beta(\bar{e}_1, \bar{e}_1) = 0,$$

$$(\bar{e}_2, \bar{a}_3) + \gamma(\bar{e}_2, \bar{e}_2) + \beta(\bar{e}_2, \bar{e}_1) = 0$$

hosil bo'lib, bunda $(\bar{e}_1, \bar{e}_2) = (\bar{e}_2, \bar{e}_1) = 0$ ekanligini e'tiborga olsak,

$$\beta = -\frac{(\bar{e}_1, \bar{a}_3)}{(\bar{e}_1, \bar{e}_1)}$$ va $$\gamma = -\frac{(\bar{e}_2, \bar{a}_3)}{(\bar{e}_2, \bar{e}_2)}$$ lar kelib chikadi.

Bu jarayonni oxirigacha davom ettirib, (2) ortogonal bazisni hosil qilamiz.

Hakikiy sonlar maydoni ustida aniqlangan V unitar fazoga Evklid fazosi deyiladi.

$+\sqrt{(\bar{a},\bar{a})}$ miqdor $\bar{a} \in V$ vektorning normasi (uzunligi) deyiladi va $\|\bar{a}\|$ orqali belgilanadi. Agar $\|\bar{a}\| = 1$ bo'lsa, $\bar{a}$ normallangan vektor deyiladi.

Evklid fazosining xar biri normallangan $\bar{b}_1, \bar{b}_2, \ldots, \bar{b}_n$ (6)

ortogonal vektorlar sistemasiga ortonormallangan vektorlar sistemasi deyiladi.

Agar (6) sistema bazis tashkil etsa, unga Evklid fazoning ortonormallangan bazisi deyiladi.

$\mathcal{F}$ maydonda berilgan V_n va V'_n chiziqli fazolar orasida shunday φ akslantirish mavjud bo'lib, u V_n ning xar bir $\bar{x}$ vektorini V'_n ning yagona bitta $\bar{x}'$ vektoriga o'zaro bir qiymatli akslantirsa va quyidagi shartalar bajarilsa, V_n va V'_n fazolar o'zaro izomorf chiziqli fazolar deyiladi:

1) $\forall \bar{x}, \bar{y} \in V_n (\varphi(\bar{x} + \bar{y}) = \varphi(\bar{x}) + \varphi(\bar{y}))$;

2) $\forall \bar{x} \in V_n \wedge \forall \alpha \in F(\varphi(\alpha\bar{x}) = \alpha\varphi(\bar{x}))$.

Misol. Berilgan $(a): \bar{a}_1 = (1,2,1),\ \bar{a}_2 = (1,1,1)$ vektorlar sistemasini 3 usulda bazisgacha to'ldiring .

Yechish. **1-usul.** Vektorlar sistemasini bazisgacha to'ldiririshning birinchi usuli –sistemani fazoning maksimal sondagi chiziqli erkli sistemasiga aylantirish uchun bitta- bitta vektorlarni sistemaga qo'shish va har gal sistemani chiziqli erkliligini tekshirishdan iborat.

Berilgan vektorlar sistemasini chiziqli bog'liq yoki chiziqli erkli ekanligini tekshirib olamiz :

$$\begin{array}{c} \vec{a}_1 \\ \vec{a}_2 \end{array}\begin{pmatrix} 1 & 2 & 1 \\ 1 & 1 & 1 \end{pmatrix} \sim \begin{array}{c} \vec{a}_1 \\ \vec{a}_2-\vec{a}_1 \end{array}\begin{pmatrix} 1 & 2 & 1 \\ 0 & -1 & 0 \end{pmatrix}$$

Demak, $\vec{a}_1, \vec{a}_2$ vektorlar sistemasi chiziqli erkli.

$\vec{a}_1, \vec{a}_2$ vektorlar R^3 fazo vektorlari bo'lganligi uchun $\vec{a}_1, \vec{a}_2$ sistemani R^3 ning bazisigacha to'ldiramiz, ya'ni shunday $\vec{a}_3$ vektorni R^3 dan topamizki, $\vec{a}_1, \vec{a}_2$ vektorlar bilan $\vec{a}_3$ chiziqli ifodalanmasin. Masalan, $\vec{a}_3$=(0,0,1) bo'lsa, u holda $\vec{a}_1, \vec{a}_2, \vec{a}_3$ sistema chiziqli erkli sistema bo'lishini tekshiramiz :

$$\begin{array}{c} \vec{a}_1 \\ \vec{a}_2 \\ \vec{a}_3 \end{array}\begin{pmatrix} 1 & 2 & 1 \\ 1 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} \sim \begin{array}{c} \vec{a}_1 \\ \vec{a}_2-\vec{a}_1 \\ \vec{a}_3 \end{array}\begin{pmatrix} 1 & 2 & 1 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Demak, $\vec{a}_1, \vec{a}_2, \vec{a}_3$- sistema R^3 uchun bazis .

2-usul . Berilgan vektorlar sistemasiga fazoning biror- bir bazisini qo'shish yordamida vektorlar sistemasini bazisgacha to'ldiramiz. Buning uchun R^3 fazoning $\vec{e}_1(1,0,0)$, $\vec{e}_2(0,1,0)$, $\vec{e}_3(0,0,1)$ bazisini olamiz. U holda hosil bo'lgan $\vec{a}_1, \vec{a}_2, \vec{a}_3, \vec{e}_1, \vec{e}_2, \vec{e}_3$ vektorlar sistemansi bazis xossalariga ko'ra chiziqli bog'liq. Bu sistemadagi $\vec{a}_1, \vec{a}_2$ vektorlarni saqlab qolgan holda uni chiziqli erkli vektorlar sistemasiga keltiramiz :

$$\begin{array}{c} \vec{a}_1 \\ \vec{a}_2 \\ \vec{e}_1 \\ \vec{e}_2 \\ \vec{e}_3 \end{array}\begin{pmatrix} 1 & 2 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & 2 & 1 \\ 0 & -1 & 0 \\ 0 & -2 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{array}{c} \vec{a}_1 \\ \vec{a}_2-\vec{a}_1 \\ \vec{e}_1-\vec{a}_1 \\ \vec{e}_2 \\ \vec{e}_3 \end{array} \sim \begin{pmatrix} 1 & 2 & 1 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{array}{c} \vec{a}_1 \\ \vec{a}_2-\vec{a}_1 \\ \vec{e}_1-\vec{a}_1-2(\vec{a}_2-\vec{a}_1) \\ \vec{e}_2+\vec{a}_2-\vec{a}_1 \\ \vec{e}_3 \end{array} \sim$$

$$\sim \begin{pmatrix} 1 & 2 & 1 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \end{pmatrix}\begin{array}{c} \vec{a}_1 \\ \vec{a}_2-\vec{a}_1 \\ \vec{e}_3 \\ \vec{e}_1-3\vec{a}_1-2\vec{a}_2 \\ \vec{e}_2+\vec{a}_2-\vec{a}_1 \end{array}$$

Demak, $\vec{a}_1, \vec{a}_2, \vec{a}_3, \vec{e}_3$-sistema bazis bo'ladi.

3-usul. Berilgan sistema asosida ortogonal bazis hosil qilish.

Buning uchun berilgan $\vec{a}_1, \vec{a}_2$ sistemani ortogonallash jarayoni asosida ortogonal sistemaga keltiramiz:

$\vec{b}_1 = \vec{a}_1$ va $\vec{b}_2 = \vec{a}_2 - \alpha\vec{b}_1$ belgilashlar kiritsak, u holda $\vec{b}_1, \vec{b}_2$ vektorlar ortogonal bo'lishi uchun $(\vec{b}_1, \vec{b}_2) = 0$ shartni qo'llaymiz, ya'ni

$0 = (\vec{b}_1, \vec{b}_2) = (\vec{b}_1, \vec{a}_2) - \alpha(\vec{b}_1, \vec{b}_1)$. Bu tenglamadan

$$\alpha = \frac{(\vec{b}_1, \vec{a}_2)}{(\vec{b}_1, \vec{b}_1)} \frac{((1,2,1),(1,1,1))}{6} = \frac{4}{6} = \frac{2}{3}$$ ni hosil qilamiz.

U holda $\vec{b}_2 = \vec{a}_2 - \alpha\vec{b}_1 = (1,1,1) - \frac{2}{3}(1,2,1) = \left(\frac{1}{3}, -\frac{1}{3}, \frac{1}{3}\right)$ hosil bo'lgan $\vec{b}_2 = \left(\frac{1}{3}, -\frac{1}{3}, \frac{1}{3}\right)$ vektor $\vec{b}_1$ vektorga ortogonal va $\vec{b}_1, \vec{b}_2$–ortogonal sistema chiziqli erkli.

$\vec{b}_1, \vec{b}_2$– sistemani ortogonal bazisgacha to'ldirish uchun shunday $\vec{b}_3$ vektorni topamizki, u $(\vec{b}_1, \vec{b}_3) = 0 \wedge (\vec{b}_2, \vec{b}_3) = 0$ shartlarni qanoatlantirsin. $\vec{b}_3 = (x, y, z)$ deb olib, $(\vec{b}_1, \vec{b}_3) = 0 \wedge (\vec{b}_2, \vec{b}_3) = 0$ shartlardan $\begin{cases} x+2y+z=0 \\ \frac{x}{3}-\frac{y}{3}+\frac{z}{3}=0 \end{cases}$ tenglamalar sistemasini tuzamiz. Keltirib chiqarilgan chiziqli tenglamalar sistemasini Gauss usulida yechimlarini topamiz :

$$\begin{cases} x+2y+z=0 \\ \frac{x}{3}-\frac{y}{3}+\frac{z}{3}=0 \end{cases} \begin{matrix} (-1) \\ 3 \end{matrix} \Leftrightarrow \begin{cases} x+2y+z=0 \\ -4y=0 \end{cases} \Leftrightarrow \begin{cases} x=-z \\ y=0 \\ z \in R \end{cases}$$

Demak, chiziqli tenglamalar sistemasining cheksiz ko'p yechimlari mavjud. Uning noldan farqli biror-bir yechimini, masalan, (-1,0,1) ni $\vec{b}_3$ vektor sifatida olish natijasida $\vec{b}_1,\vec{b}_2,\vec{b}_3$-ortoganal bazisni hosil qilamiz.

Misol. $\vec{a}_1=(1,2,3)$, $\vec{a}_2=(1,1,1)$ vektorlar berilgan bo'lsa $L(\vec{a}_1,\vec{a}_2)$ qismfazoning ortogonal to'ldiruvchisini toping.

Yechish. Qismfazo ortogonal to'ldiruvchisi ta'rifiga ko'ra $\vec{a}_1,\vec{a}_2$ vektorlarga ortogonal vektorlar tashkil etgan qismfazoni topamiz. Buning uchun $(\vec{a}_1,\vec{x})=0\wedge(\vec{a}_2,\vec{x})=0$ shartlarni qanoatlantiruvchi $\vec{x}$ vektorlarni aniqlaymiz. Berilgan $\vec{a}_1,\vec{a}_2$ vektorlar yordamida va $\vec{x}=(x,y,z)$ belgilashdan quyidagi chiziqli tenglamalar sistemasini tuzamiz. $\begin{cases}x+2y+3z=0\\ x+y+z=0\end{cases}$. Tenglamalar sistemasi cheksiz ko'p yechimga ega, chunki sistema 3 noma'lumli va 2 ta tenglamadan iborat. Tenglamalar sistemasining umumiy yechimini aniqlaymiz:

$$\begin{cases}x+y+z=0\\ x+2y+3z=0\end{cases}\Leftrightarrow\begin{cases}x+y+z=0\\ y+2z=0\end{cases}\Leftrightarrow\begin{cases}x=y\\ y=-2z\\ z\in R\end{cases}$$

Bundan $\vec{a}_1,\vec{a}_2$ sistemaga ortogonal vektorlar $A=\{(z;-2z;z)|z\in R\}$ to'plamdan iborat. Bu to'plam z ning qiymatiga bog'liq bo'lgan vektorlardan iborat. Shuning uchun A to'plam tashkil etgan A=$<A;+,\{\omega_\lambda|\lambda\in R\}>$-chiziqli fazo o'lchovi *dim* A =1 va A chiziqli fazo $L(\vec{a}_1,\vec{a}_2)$ chiziqli fazoning ortogonal to'ldiruvchisi bo'ladi.

Misol. Berilgan to'plamlar tashkil etgan chiziqli fazolar orasida izomorfizm o'rnating $V=\left\{\begin{pmatrix}a&b&c\\0&0&0\\0&0&0\end{pmatrix}\middle| a,b,c\in R\right\}, V'=\left\{\begin{pmatrix}a&0&0\\0&b&0\\0&0&c\end{pmatrix}\middle| a,b,c\in R\right\}$.

Yechish. Berilgan to'plamlar tashkil etgan chiziqli vektor fazolar 3 o'lchovli fazolar. Berilgan V ning bazislaridan biri

$$A=\begin{pmatrix}1&0&0\\0&0&0\\0&0&0\end{pmatrix},B=\begin{pmatrix}0&1&0\\0&0&0\\0&0&0\end{pmatrix},C=\begin{pmatrix}0&0&1\\0&0&0\\0&0&0\end{pmatrix}$$ va V ' ning bazislaridan biri

$$A'=\begin{pmatrix}1&0&0\\0&0&0\\0&0&0\end{pmatrix},B'=\begin{pmatrix}0&1&0\\0&0&0\\0&0&0\end{pmatrix},C'=\begin{pmatrix}0&0&1\\0&0&0\\0&0&0\end{pmatrix}$$ bo'lishi ayon.

Bu fazolar orasida $f(V)=V'$ akslantirishni quyidagicha aniqlaymiz:

$$f\left(\begin{pmatrix}a&b&c\\0&0&0\\0&0&0\end{pmatrix}\right)=\begin{pmatrix}a&0&0\\0&b&0\\0&0&c\end{pmatrix}.$$

f - akslantirish izomorfizm bo'lishini isbotlaymiz :

1) $\forall A_1,A_2\in V,\ A_1=\begin{pmatrix}a_1&b_1&c_1\\0&0&0\\0&0&0\end{pmatrix},A_2=\begin{pmatrix}a_2&b_2&c_2\\0&0&0\\0&0&0\end{pmatrix}$ lar uchun

$$f(A_1+A_2)=f\left(\begin{pmatrix}a_1&b_1&c_1\\0&0&0\\0&0&0\end{pmatrix}+\begin{pmatrix}a_2&b_2&c_2\\0&0&0\\0&0&0\end{pmatrix}\right)=\left|\begin{matrix}\textit{matrisalarni qo'shish}\\ \textit{amali ta'rifidan}\end{matrix}\right|=$$

$$=f\left(\begin{pmatrix}a_1+a_2&b_1+b_2&c_1+c_2\\0&0&0\\0&0&0\end{pmatrix}\right)=\left|\begin{matrix}\textit{akslsntirish}\\ \textit{ta'rifidan}\end{matrix}\right|=\begin{pmatrix}a_1+a_2&0&0\\0&b_1+b_2&0\\0&0&c_1+c_2\end{pmatrix}=$$

$$=\left|\begin{matrix}\textit{matrisalarni qo'shish}\\ \textit{amali ta'rifidan}\end{matrix}\right|=\begin{pmatrix}a_1&0&0\\0&b_1&0\\0&0&c_1\end{pmatrix}+\begin{pmatrix}a_2&0&0\\0&b_2&0\\0&0&c_2\end{pmatrix}=f(A_1)+f(A_2).$$

Demak, f akslantirish qo'shish binar amalini saqlaydi.

2). $\forall A\in V\wedge\forall\alpha\in R$ lar uchun

$$f(\alpha\cdot A)=f\left(\alpha\begin{pmatrix} a & b & c\\ 0 & 0 & 0\\ 0 & 0 & 0\end{pmatrix}\right)=\left|\begin{matrix}\textit{skalyarni matrisaga ko'paytirish}\\ \textit{amali ta'rifidan}\end{matrix}\right|=$$

$$=f\left(\begin{pmatrix} \alpha a & \alpha b & \alpha c\\ 0 & 0 & 0\\ 0 & 0 & 0\end{pmatrix}\right)=\left|\begin{matrix}\textit{akslsntirish}\\ \textit{ta' rifidan}\end{matrix}\right|=\left(\begin{pmatrix} \alpha a & 0 & 0\\ 0 & \alpha b & 0\\ 0 & 0 & \alpha c\end{pmatrix}\right)=$$

$$=\left|\begin{matrix}\textit{skalyarni matrisaga ko' paytirish}\\ \textit{amali ta' rifidan}\end{matrix}\right|=\alpha\begin{pmatrix} a & 0 & 0\\ 0 & b & 0\\ 0 & 0 & c\end{pmatrix}=\alpha\cdot f(A)$$

Demak, f akslantirish skalyarni matrisaga ko'paytirish amalini saqlaydi.

V ,V ' chiziqli fazolar orasida aniqlangan f -akslantirish gomomorfizm ekan.

3) V ' dan olingan ixtiyoriy $f(A_1)$, $f(A_2)$ lar uchun $f(A_1)\neq f(A_2)$ bo'lsin.

U holda $f(A_1)=\begin{pmatrix} a_1 & 0 & 0\\ 0 & b_1 & 0\\ 0 & 0 & c_1\end{pmatrix}$ ning asli $A_1=\begin{pmatrix} a_1 & b_1 & c_1\\ 0 & 0 & 0\\ 0 & 0 & 0\end{pmatrix}$ va

$f(A_2)=\begin{pmatrix} a_2 & 0 & 0\\ 0 & b_2 & 0\\ 0 & 0 & c_2\end{pmatrix}$ ning asli $A_2=\begin{pmatrix} a_2 & b_2 & c_2\\ 0 & 0 & 0\\ 0 & 0 & 0\end{pmatrix}$.

Agar $\begin{pmatrix} a_1 & 0 & 0\\ 0 & b_1 & 0\\ 0 & 0 & c_1\end{pmatrix}\neq\begin{pmatrix} a_2 & 0 & 0\\ 0 & b_2 & 0\\ 0 & 0 & c_2\end{pmatrix}$ bo'lsa, u holda

$a_1\neq a_2 \vee b_1\neq b_2 \vee c_1\neq c_2$ bo'ladi. Bundan $\begin{pmatrix} a_1 & b_1 & c_1\\ 0 & 0 & 0\\ 0 & 0 & 0\end{pmatrix}\neq\begin{pmatrix} a_2 & b_2 & c_2\\ 0 & 0 & 0\\ 0 & 0 & 0\end{pmatrix}$

ekanligi kelib chiqadi.

Ya'ni $f(V)=V'$ in'ektiv akslantirish.

4) V ' to'plamning ixtiyoriy $f(A)=\begin{pmatrix} a & 0 & 0\\ 0 & b & 0\\ 0 & 0 & c\end{pmatrix}$ elementi uchun V to'plamda $A=\begin{pmatrix} a & b & c\\ 0 & 0 & 0\\ 0 & 0 & 0\end{pmatrix}$ element mavjud. Ya'ni f - syur'ektiv akslantirish.

Demak, $f(V)=V'$ akslantirish izomorfizm ekan.

⟹ Misol va mashqlar

1. Skalyar ko'paytmaning quyidagi xossalarini isbotlang:

1.1. $(\bar{x},\bar{y}+\bar{z})=(\bar{y}+\bar{z},\bar{x})=(\bar{y},\bar{x})+(\bar{z},\bar{x})=(\bar{x},\bar{y})+(\bar{x},\bar{z})$;

1.2. $(\bar{x},\lambda\bar{y})=(\lambda\bar{y},\bar{x})=\lambda(\bar{y},\bar{x})=\lambda(\bar{x},\bar{y})$.

2. Agar V skalyar ko'paytmaga ega bo'lgan fazo bo'lsa, u holda $\forall\bar{x}\in V$ uchun $(\bar{x},\bar{0})=0$ bo'lishini isbotlang.

3. Biror V fazo Evklid fazosi bo'lishi uchun uning elementlari ustida quyidagi shartlar bajarilishi lozimligini isbotlang:

3.1. $(\bar{x},\bar{y})=(\bar{y},\bar{x}) \quad (\forall\bar{x},\bar{y}\in V)$;

3.2. $(\bar{x},\bar{y}+\bar{z})=(\bar{x},\bar{y})+(\bar{x},\bar{z}) \quad (\forall\bar{x},\bar{y},\bar{z}\in V)$;

3.3. $(\lambda\bar{x},\bar{y})=\lambda(\bar{x},\bar{y}) \quad (\forall\bar{x},\bar{y}\in V,\ \forall\lambda\in R)$;

3.4. $(\bar{x},\bar{x})>0 \quad (\forall\bar{x}\in V,\ \bar{x}\neq\bar{0}), \quad (\bar{x},\bar{x})=0 \quad (\bar{x}\in V,\ \bar{x}=\bar{0})$.

4. $\bar{a}$, $\bar{b}$ - Evklid fazosining ixtiyoriy vektorlari va $\lambda\in R$ uchun quyidagi xossalar o'rinli ekanligini isbotlang:

4.1. $\|\bar{a}\|\geq 0 \quad (\|\bar{a}\|=0 <=> \bar{a}=\bar{0})$;

4.2. $\|\lambda\bar{a}\|=|\lambda|\ \|\bar{a}\|$;

4.3. $|(\bar{a},\bar{b})|\leq\|\bar{a}\|\cdot\|\bar{b}\|$ (Koshi – Bunyakovskiy tengsizligi);

4.4. $\|\bar{a}+\bar{b}\| \le \|\bar{a}\| + \|\bar{b}\|$ (uchburchak tengsizligi).

5. Quyidagi vektorlar sistemasini ortogonallang:

5.1. $\bar{a}_1(-1,1,1)$, $\bar{a}_2(0,1,2)$, $\bar{a}_3(1,2,3)$.

5.2. $\bar{a}_1(1,2,1,0)$, $\bar{a}_2(-1,1,2,0)$, $\bar{a}_3(1,1,1,1)$, $\bar{a}_4(1,4,4,1)$.

5.3. $\bar{a}_1(-1,3,2,1)$, $\bar{a}_2(0,0,5,5)$, $\bar{a}_3(2,1,-1,1)$.

5.4. $\bar{a}_1(1,0,1)$, $\bar{a}_2(-1,1,3)$, $\bar{a}_3(13,34,5)$.

5.5. $\bar{a}_1(1,0,1,1)$, $\bar{a}_2(1,1,1,1)$, $\bar{a}_3(0,0,1,2)$, $\bar{a}_4(-1,0,3,1)$.

5.6. $\bar{a}_1(1,0,1,1)$, $\bar{a}_2(3,1,-3,0)$, $\bar{a}_3(5,7,1,1)$.

6. Chekli o'lchovli Evklid fazosining istalgan bazisini ortonormallash mumkinligini isbotlang.

7. Agar V xosmas skalyar ko'paytmali vektor fazo bo'lsa, u holda V fazoning nolmas vektorlaridan tuzilgan ortogonal vektorlar sistemasi chiziqli erkli bo'lishini isbotlang.

8.Berilgan vektorlar sistemasini 3 usulda bazisgacha to'ldiring:

8.1. $\bar{a}_1(1,0,3)$, $\bar{a}_2(0,-4,-1)$.

8.2. $\bar{a}_1(-1,1,0,3)$, $\bar{a}_2(2,0,-4,-1)$, $\bar{a}_3(3,-1,0,-3)$.

8.3. $\bar{a}_1(0,2,3)$, $\bar{a}_2(3,1,2)$, $\bar{a}_3(6,2,4)$.

8.4. $\bar{a}_1(1,-2,1,0,1)$, $\bar{a}_2(2,0,-5,1,1)$, $\bar{a}_3(3,2,-3,-2,-1)$.

8.5. $\bar{a}_1(-1,1,0,3)$, $\bar{a}_2(2,0,-4,-1)$, $\bar{a}_3(1,1,-4,2)$.

8.6. $\bar{a}_1(5,-1,-4,-4)$, $\bar{a}_2(2,0,-4,-1)$.

8.7. $\bar{a}_1(1,2,1,2,3)$, $\bar{a}_2(3,4,0,1,1)$.

8.8. $\bar{a}_1(7,0,5,1)$, $\bar{a}_2(3,5,5,0)$, $\bar{a}_3(-3,2,-7,4)$.

9. Evklid fazo fazoostisi ortogonal to'ldiruvchisining quyidagi xossalarini isbotlang:

9.1. $(U^{\perp})^{\perp} = U$.

9.2. $(U \cap V)^{\perp} = U^{\perp} + V^{\perp}$.

9.3. $(E)^{\perp} = \{\bar{0}\}$.

9.4. $\{\bar{0}\}^{\perp} = E$.

9.5. $(U^{\perp} \cap V^{\perp})^{\perp} = U + V$.

9.6. $(U^{\perp} + V^{\perp})^{\perp} = U \cap V$.

9.7. $(U \cap V \cap T)^{\perp} = U^{\perp} + V^{\perp} + T^{\perp}$.

9.8. $(U + V + T)^{\perp} = U^{\perp} \cap V^{\perp} \cap T^{\perp}$.

10. Berilgan L($\bar{a}_1, \dots, \bar{a}_n$) fazoostining ortogonal to'ldiruvchisini toping:

10.1. $\bar{a}_1(3,-2)$.

10.2. $\bar{a}_1(1,-2,0)$.

10.3. $\bar{a}_1(1,-2,0,1)$.

10.4. $\bar{a}_1(-1,2,3)$, $\bar{a}_2(3,-4,1)$.

10.5. $\bar{a}_1(1,1,1,0)$, $\bar{a}_2(1,2,-4,0)$.

10.6. $\bar{a}_1(1,-2,0,1)$, $\bar{a}_2(2,-4,0,2)$, $\bar{a}_3(1,1,1,1)$.

10.7. $\bar{a}_1(4,1,0,1,1)$, $\bar{a}_2(3,1,0,1,0)$, $\bar{a}_3(1,0,0,0,2)$.

10.8. $\bar{a}_1(2,1,2,1,3)$, $\bar{a}_2(1,2,1,2,1)$, $\bar{a}_3(3,3,3,3,3)$, $\bar{a}_4(-1,1,-1,1,0)$.

11. Berilgan L($\bar{a}_1, \dots, \bar{a}_n$) fazoostining ortogonal to'ldiruvchisi bazisini toping:

11.1. $\bar{a}_1(1,-2,3)$.

11.2. $\bar{a}_1(1,-1,2)$, $\bar{a}_2(1,0,1)$, $\bar{a}_3(2,-1,4)$.

11.3. $\bar{a}_1(1,1,0,-2)$, $\bar{a}_2(2,1,-1,1)$, $\bar{a}_3(3,2,-1,-1)$.

11.4. $\bar{a}_1(1,-1,2,1)$, $\bar{a}_2(1,0,1,-1)$, $\bar{a}_3(2,-1,3,0)$.

11.5. $\bar{a}_1(1,1,-1,2)$, $\bar{a}_2(2,0,1,3)$, $\bar{a}_3(4,2,-1,7)$, $\bar{a}_4(3,1,0,5)$.

11.6. $\bar{a}_1(2,-1,1,-3,1,1)$, $\bar{a}_2(1,-1,1,-1,1)$, $\bar{a}_3(0,2,1,2,0)$, $\bar{a}_4(-1,1,-1,1,-1)$.

12. Quyidagi tenglamalar sistemasi yechimlar to'plami tashkil etgan fazoostining ortogonal to'ldiruvchisini toping:

12.1. $\begin{cases} x_1 + x_2 + x_3 + x_4 = 0, \\ -x_1 + x_2 - x_3 + x_4 = 0. \end{cases}$

12.2. $\begin{cases} x_1 - 2x_2 + x_3 = 0, \\ 2x_1 - x_2 - x_3 = 0, \\ x_1 + x_2 - 2x_3 = 0. \end{cases}$

12.3. $\begin{cases} 2x_1 + x_2 + 3x_3 - x_4 = 0, \\ 3x_1 + 2x_2 - 2x_4 = 0, \\ 3x_1 + x_2 + 9x_3 - x_4 = 0. \end{cases}$

12.4. $\begin{cases} x_1 + x_2 + x_3 + x_4 = 0, \\ x_1 - x_2 - x_3 - 3x_4 = 0, \\ 3x_1 + x_2 + x_3 - x_4 = 0. \end{cases}$

13. $\mathcal{F}$ maydon ustidagi n o'lchovli har qanday ikkita V_n va V_n' chiziqli fazolar izomorfligini isbotlang.

14. Berilgan to'plamlar tashkil etgan chiziqli fazolar orasida izomorfizm o'rnating:

14.1. $V = \left\{ \begin{pmatrix} a & 0 \\ 0 & 0 \end{pmatrix} \mid a \in R \right\}$ va $V = \left\{ \begin{pmatrix} 0 & a \\ a & 0 \end{pmatrix} \mid a \in R \right\}$.

14.2. $V = \left\{ \begin{pmatrix} a & b \\ 0 & 0 \end{pmatrix} \mid a,b \in R \right\}$ va $V = \left\{ \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix} \mid a,b \in R \right\}$.

14.3. $V = \left\{ \begin{pmatrix} a & b \\ c & 0 \end{pmatrix} \mid a,b,c \in R \right\}$ va $V = \left\{ \begin{pmatrix} 0 & c \\ b & a \end{pmatrix} \mid a \in R \right\}$.

14.4. $V = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \mid a,b,c,d \in R \right\}$ va $V = \left\{ \begin{pmatrix} a & 2b \\ 3c & 4d \end{pmatrix} \mid a,b,c,d \in R \right\}$.

14.5. $V = \left\{ \begin{pmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{pmatrix} \mid a,b,c \in R \right\}$ va $V = \left\{ \begin{pmatrix} 0 & -a & -b \\ a & 0 & -c \\ b & c & 0 \end{pmatrix} \mid a,b,c \in R \right\}$.

14.6. $V = \left\{ \begin{pmatrix} a & b & 0 \\ c & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \mid a,b,c \in R \right\}$ va $V = \left\{ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & c \\ 0 & b & a \end{pmatrix} \mid a,b,c \in R \right\}$.

14.7. $V = \left\{ \begin{pmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{pmatrix} \mid a,b,c \in R \right\}$ va $V = \left\{ \begin{pmatrix} 0 & 0 & c \\ 0 & b & 0 \\ a & 0 & 0 \end{pmatrix} \mid a,b,c \in R \right\}$.

14.8. $V = \left\{ \begin{pmatrix} a & 0 & 0 \\ 0 & b & 0 \\ c & 0 & 0 \end{pmatrix} \mid a,b,c \in R \right\}$ va $V = \left\{ \begin{pmatrix} 0 & 0 & a \\ 0 & b & 0 \\ 0 & 0 & c \end{pmatrix} \mid a,b,c \in R \right\}$.

14.9. $V = \left\{ \begin{pmatrix} a & 1 & 0 \\ b & 2 & 0 \\ c & 3 & 0 \end{pmatrix} \mid a,b,c \in R \right\}$ va

$V = \left\{ \begin{pmatrix} a & b & c \\ \alpha & \beta & \gamma \\ 0 & 0 & 0 \end{pmatrix} \mid a,b,c,\alpha,\beta,\gamma \in R \right\}$.

14.10. $V = \{a + bi \mid a, b \in \mathrm{R}\}$ va $V = \{a - bi \mid a, b \in \mathrm{R}\}$.

14.11. $V = \{a + bi \mid a, b \in \mathrm{R}\}$ va $V = \mathrm{R}^2$.

14.12. $V=\{a+b\sqrt{q} \mid a,b \in \mathrm{Q} \wedge q$-tub son$\}$ va $V=\{a+b\sqrt{p} \mid a,b \in Q \wedge p$-tub son$\}$.

Takrorlash uchun savollar

1. Vektor fazolar izomorfizmini tushuntiring.
2. Skalyar ko'paytmaning xossalarini bayon eting.
3. Skalyar ko'paytmali vektor fazo deb nimaga aytiladi?
4. Xosmas skalyar ko'paytma ta'rifini ayting.
5. Unitar fazo deb nimaga aytiladi?
6. Ortogonal vektorlar deb nimaga aytiladi?
7. Ortogonal vektorlar sistemasi deb nimaga aytiladi?
8. Ortogonal bazis deb nimaga aytiladi?
9. Ortogonallash jarayonini bayon qiling.
10. Evklid fazo deb nimaga aytiladi?
11. Vektorning normasi deb nimaga aytiladi?
12. Vektor normasining xossalarini bayon qiling.
13. Normallangan vektor deb nimaga aytiladi?
14. Ortonormallangan bazis deb nimaga aytiladi?

MUNDARIJA

D.YUNUSOVA, A.YUNUSOV

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