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So‘z boshi

Matematika fani yoshlarning mantiqiy fikrlash qobiliyatini o‘stiruvchi vosita sifatida qadimgi Yunoniston maktablarida o‘qita boshlangan. Yangi era boshlarida Xitoyda sonlar nazariyasi, Hindistonda o‘nli sanoq sistemasi, O‘rta Yer dengiz sohillarida trigonometriya yaratila boshlangan. VII-VIII asrlardan boshlab ilm-fan taraqqiyotining markazi O‘rta Osiyoga ko‘chdi. O‘z ilmiy ishlari bilan butun dunyoga tanilgan Muhammad Muso al-Xorazmiy, Ahmad Farg’oniy, Abu Rayhon Beruniy, Abu Ali ibn Sino, Abu Nasr Forobiy, Ismoil Buxoriy, Umar Hayyom, Ulug’bek va boshqalar O‘rta Osiyoda yashab ijod qilganlar.

Ijobiy taffakkur sohibini, ya’ni o‘sayotgan, rivojlanayotgan va taraqqiyot sari yuz tutayotgan mamlakatimiz uchun zarur bo‘lgan ijodkor va mustaqil fikrlay oladigan shaxsni tarbiyalab, voyaga yetkazish oldimizda turgan eng muhim va dolzarb vazifa hisoblanadi.

Bir necha nostandart mulohazalar yordamida yechiladigan tenglama va tengsizliklar murakkab masalalar hisoblanadi. Bunday masalalarga modul va logarifm qatnashgan hamda Nyuton binomiga doir masalalarni aytish mumkin.

Matematikaning turli bo‘limlariga taaluqli murakkab masalalarni yechishda ko‘pincha matematik induksiya usulidan ham foydalaniladi. Shuningdek, masalalar yechishning nostandart usullari umumta’lim maktablar dasturiga kiritilmaganligi sababli umumta’lim maktablarda klassik tengsizliklardan foydalanib yechiladigan masalalarga kam e’tibor beriladi. Shuning uchun klassik tengsizliklar, n-darajali Nyuton binomi, modul hamda logarifmga taaluqli ba’zi formulalar va ular yordamida yechiladigan 500 tadan ortiq masalani (yechimlari bilan) keltirishni lozim topdik.

Matematikaning nostandart misol va masalalari nomli o‘quv qo‘llanma Samarqand Davlat universiteti 5130100 – matematika va 5140300 – mexanika ta’lim yo‘nalishlari talabalariga qo‘shimcha fanlar blokidagi “Matematikani chuqurlashtirish va murakkab tadbiqiy masalalar” fanidan amaliy mashg’ulotlar uchun mo‘ljallangan. Bundan

tashqari bu o‘quv qo‘llanmadan akademik litsey, o‘rta maktab o‘quvchilari va o‘qituvchilari foydalanishlari mumkin.

Bu o‘quv qo‘llanma matematikani chuqurlashtirish va murakkab tadbiqiy masalalarni yechishga katta yordam beradi.

Mazkur o‘quv qo‘llanmaning yaratilishida beqiyos yordamini ayamagan barchaga o‘zimizning chuqur minnatdorchiligimizni izhor etamiz.

Ushbu o‘quv qo‘llanma Samarqand davlat iqtisodiyot va servis institutida uzoq yillar davomida talabalarga ilm ziyosini taratgan, matematikani rivojlanishiga va yosh kadrlarni tayyorlashda o‘z hissasini qo‘shgan, mehri daryo, so‘zi dur zahmatkash inson, iqtisodiyot fanlari nomzodi, katta o‘qituvchi Do‘stov Abdula Lutfilayevichning yorqin xotirasiga bag’ishlanadi.

4-bob. Ko'rsatkichli, logarifmik, trigonometrik tenglamalar va tengsizliklar

4.1. Ko'rsatkichli va logarifmik tenglamalar

4.1.1. $6^x - 2^x = 32$ tenglamani yeching.

Yechish. Tenglamani oddiy usulda yechish ko'zlangan maqsadni bermaydi. Tanlash yo'li bilan $x_1 = 2$ ni osongina topish mumkin. Berilgan tenglamaning ikkala tomonini 2^x ga bo'lib, $3^x - 1 = \frac{32}{2^x}$ tenglamani hosil qilish mumkin. Uning chap tomoni o'suvchi va o'ng tomoni kamayuvchi bo'lib, bu tenglama yagona ildizga ega bo'ladi.

4.1.2. $9^x + 6^x = 2^{2x+1}$ tenglamani yeching.

Yechish. Berilgan tenglamani unga teng kuchli bo'lgan: $(3^x)^2 + (2^x)(3^x) - 2(2^x)^2 = 0$ ko'rinishda yozish mumkin. Uning ikkala tomonini $(2^x)^2$ ga bo'lib, va $\left(\frac{3}{2}\right)^x = u$ almashtirish yordamida ildizlari $u_1 = 1, u_2 = -2$ bo'lgan $u^2 + u - 2 = 0$ kvadrat tenglamani hosil qilish mumkin.$u > 0$ bo'lib, $\left(\frac{3}{2}\right)^x = 1$ va $x_1 = 0$.

4.1.3. $x^{x+1} + x^x = 1$ tenglamani yeching.

Yechish. Tenglama ikki tomonini logarifmlab unga teng kuchli: f(x)=xln x+ln$(x+1)$=0 tenglamani qaraymiz. 0<x<$\frac{1}{e}$ bo'lganda,

$$\ln(x+1) < x,$$

$$x\cdot \ln x + \ln(x+1) < x\cdot \ln x + x = x\cdot \ln \frac{x}{e}.$$

x>$\frac{1}{e}$ bo'lsa, u holda f′(x)=ln x+1+$\frac{1}{1+x}$=ln(xe)+$\frac{1}{1+x}$>0. Demak, f(x) funksiya x>0 da monoton o'suvchi, ya'ni tenglama bittadan ko'p bo'lmagan ildizga ega va bu ildizning taqribiy qiymati 0,43605.

4.1.4. $\left(\sqrt{2+\sqrt{3}}\right)^x + \left(\sqrt{2-\sqrt{3}}\right)^x = 4$ tenglamani yeching.

Yechish. $\left(\sqrt{2+\sqrt{3}}\right)^x \cdot \left(\sqrt{2-\sqrt{3}}\right)^x = 1$ bo'lib, birinchi qo'shiluvchi y bo'lsin, u holda ikkinchi qo'shiluvchi $\frac{1}{y}$ bo'lib, berilgan tenglama $y + \frac{1}{y} = 4$ ko'rinishni oladi va natijada, ildizlari $y_1 = 2 + \sqrt{3}$ hamda $y_2 = 2 - \sqrt{3}$ bo'lgan $y^2 + 4y + 1 = 0$ tenglama hosil bo'ladi.

$\left(\sqrt{2+\sqrt{3}}\right)^x = y$ tenglikdan $\left(\sqrt{2+\sqrt{3}}\right)^x = 2+\sqrt{3}$ va $\left(\sqrt{2+\sqrt{3}}\right)^x = 2-\sqrt{3} = (2+\sqrt{3})^{-1}$ tenglamalarni hosil qilish mumkin.

Birinchi tenglamadan $x_1 = 2$, ikkinchisidan esa $x_2 = -2$ bo'ladi.

4.1.5. $5^x \cdot 8^{\frac{x-1}{x}} = 500$ tenglamani yeching.

Yechish. Berilgan tenglamaning ikkala tomonini 5 asosga ko'ra logarifmlab:

$$x + 3\frac{x-1}{x}\log_5 2 = 3 + 2\log_5 2,$$

$$x^2 + x(\log_5 2 - 3) - 3\log_5 2 = 0,$$

$$x_1 = 3; x_2 = \log_5 2$$

4.1.6. Quyidagi tenglamani yeching.

$(26+15\sqrt{3})^x - 5(7+4\sqrt{3})^x + 6(2+\sqrt{3})^x + (2-\sqrt{3})^x = 5.$

Yechish. $(2+\sqrt{3})^x = y$ belgilash yordamida: $(26+15\sqrt{3})^x = y^3$, $(7+4\sqrt{3})^x = y^2$, $(2-\sqrt{3})^x = \frac{1}{y}$ tengliklarni hosil qilish mumkin va berilgan tenglama $y^4 - 5y^3 + 6y^2 - 5y + 1 = 0$ (*) ko'rinishni oladi. $y \neq 0$ bo'lib, tenglamaning ikkala tomonini y^2 ga bo'lib, hamda $y + \frac{1}{y} = z$ almashtirish yordamida:

$$y^2 - 5y + 6 - \frac{5}{y} + \frac{1}{y^2} = 0, \qquad \left(y^2 + \frac{1}{y^2}\right) - 5\left(y + \frac{1}{y}\right) + 6 = 0,$$

$(z^2-2) - 5z + 6 = 6$ yoki $z^2 - 5z + 4 = 0$ tenglamani hosil qilish mumkin, $z_1 = 1$, $z_2 = 4$, $z = y + \frac{1}{y}$ va $y > 0$ .U holda Koshi (3) tengsizligiga ko'ra: $z \geq 2$

Demak, $z = 4, y + \frac{1}{y} = 4$ yoki $y^2 - 4y + 1 = 0$ bo'lib,

$y_1 = 2 + \sqrt{3}$ va $y_2 = 2 - \sqrt{3} = \frac{1}{2+\sqrt{3}}$.

Demak, $x_1 = 1$ va $x_2 = -1$.

4.1.7. $4^x - (7-x)2^x + 12 - 4x = 0$ tenglamani yeching.

Yechish. Berilgan tenglama 2^x ga nisbatan kvadrat tenglama bo'lib uning ildizlari:

$$(2^x)_{1,2} = \frac{7-x \pm \sqrt{(7-x)^2 - 4(12-4x)}}{2} = \frac{7-x \pm \sqrt{x^2+2x+1}}{2}$$

$= \frac{7-x\pm(x+1)}{2}$ bo'ladi.

Bundan, $2^x = \frac{7-x+(x+1)}{2} = 4$ va $2^x = \frac{7-x-(x+1)}{2} = 3-x$ tenglamalarni hosil qilib, birinchi tenglamadan $x_1 = 2$, ikkinch tenglama esa bittadan ko'p ildizga ega emas. Chunki uning chap tomoni uzluksiz o'suvchi o'ng tomoni esa uzluksiz kamayuvchi funksiya. Tanlash usuli bilan $x_2 = 1$ ni topish mumkin.

4.1.8. $\left(\frac{1}{2}\right)^{x+1} + \left(\frac{1}{6}\right)^x - \sqrt{2}\cdot\left(\frac{\sqrt{2}}{6}\right)^x = 1$ tenglamani yeching.

Yechish. Berilgan tenglamani quyidagicha shakl almashtirish mumkin:

$$\frac{1}{2}\cdot\left(\frac{1}{2}\right)^x + \left(\frac{1}{2}\right)^x\left(\frac{1}{3}\right)^x - \sqrt{2}\cdot\left(\frac{1}{\sqrt{2}}\right)^x\left(\frac{1}{3}\right)^x - 1 = 0,$$

$$\left(\frac{1}{2}\cdot\left(\frac{1}{2}\right)^x - 1\right) + \left(\frac{1}{3}\right)^x\cdot\left(\left(\frac{1}{2}\right)^x - \sqrt{2}\cdot\left(\frac{1}{\sqrt{2}}\right)^x\right) = 0,$$

$$\left(\frac{1}{\sqrt{2}}\cdot\left(\frac{1}{\sqrt{2}}\right)^x - 1\right)\left(\frac{1}{\sqrt{2}}\cdot\left(\frac{1}{\sqrt{2}}\right)^x + 1\right) + \sqrt{2}$$

$$\cdot\left(\frac{1}{\sqrt{2}}\right)^x\left(\frac{1}{3}\right)^x\left(\frac{1}{\sqrt{2}}\cdot\left(\frac{1}{\sqrt{2}}\right)^x - 1\right) = 0.$$

$$\left(\frac{1}{\sqrt{2}}\cdot\left(\frac{1}{\sqrt{2}}\right)^x - 1\right)\left(\frac{1}{\sqrt{2}}\cdot\left(\frac{1}{\sqrt{2}}\right)^x + 1 + \sqrt{2}\cdot\left(\frac{1}{\sqrt{2}}\right)^x\left(\frac{1}{3}\right)^x\right) = 0, (*)$$

$$\frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}\right)^x + 1 + \sqrt{2}\cdot\left(\frac{1}{\sqrt{2}}\right)^x\left(\frac{1}{3}\right)^x > 0$$

bo'lib, $(*)$ tenglamadan $\frac{1}{\sqrt{2}}\cdot\left(\frac{1}{\sqrt{2}}\right)^x - 1 = 0$ yoki $\left(\frac{1}{\sqrt{2}}\right)^x = \left(\frac{1}{\sqrt{2}}\right)^{-1}$. Bundan $x_1 = -1$.

4.1.9. $x\cdot 2^{\frac{1}{x}} + \frac{1}{x}\cdot 2^x = 4$ tenglamani yeching.

Yechish. Tenglamaning o'ng tomoni musbat, u holda $x > 0$ bo'lib, Koshi (2) va (3) tengsizliklariga ko'ra:

$$x\cdot 2^{\frac{1}{x}} + \frac{1}{x}2^x \geq 2\sqrt{\left(x2^{\frac{1}{x}}\right)\cdot\left(\frac{1}{x}2^x\right)} = 2\sqrt{2^{x+\frac{1}{x}}} \geq 2\sqrt{2^2} = 4.$$

Bundan va berilgan tenglamadan hamda qo'llanilgan Koshi

tengsizliklariga asosan: $\begin{cases} x \cdot 2^{\frac{1}{x}} = \frac{1}{x} 2^{\frac{1}{x}}, \\ x + \frac{1}{x} = 2 \end{cases}$ bo‘lib, u yagona: $x_1 = 1$ ildizga ega. $x_1 = 1$ ni berilgan tenglamaga qo‘yib tekshirib ko‘rib uning berilgan tenglama ildizi ekanligiga ishonch hosil qilish mumkin.

4.1.10. $2^{1-|x|} - 1 = x^2 + \frac{1}{x^2+1}$ tenglamani yeching.

Yechish. Berilgan tenglamani $2^{1-|x|} = x^2 + 1 + \frac{1}{x^2+1}$ (*) ko‘rinishda yozib $x \geq 0$ bo‘lganligidan (*) tenglama chap tomoni uchun $2^{1-|x|} \leq 2$ bo‘lib, o‘ng tomoni uchun Koshi (3) tengsizligiga ko‘ra: $x^2 + 1 + \frac{1}{x^2+1} \geq 2$.

Demak, (*) tengsizlizligidagi tenglik uning ikkala tomoni ham 2 ga yoki $x_1 = 0$ bo‘lganda o‘rinli bo‘ladi.

4.1.11. $\frac{x}{14} = \left(\frac{2}{7}\right)^{\log_x 4}$ tenglamani yeching.

Yechish. Berilgan tenglamaning aniqlanish sohasi: $x > 0$ va $x \neq 1$

Tenglamaning ikkala qismini x asosga ko‘ra logarifmlab: $\log_x x - \log_x 14 = \log_x 4 \cdot \log_x \frac{2}{7}$ yoki,

$$1 - \log_x 2 - \log_x 7 = 2 \log_x 2 \cdot (\log_x 2 - \log_x 7) \qquad (*)$$

tenglamani hosil qilish mumkin.

$\log_x 2 = a$ va $\log_x 7 = b$ belgilash yordamida (*) tenglamani $1 - a - b = 2a(a - b)$ yoki $2a^2 - a(2b - 1) + b - 1 = 0$ ko‘rinishida yozib hamda uni a o‘zgaruvchiga nisbatan yechib,

$$a_{1,2} = \frac{2b - 1 \pm \sqrt{(2b-1)^2 - 8(b-1)}}{4} = \frac{2b - 1 \pm (2b - 3)}{4}$$

tengliklarga ega bo‘lish mumkin.

Bunda ikki hol bo‘ladi:

1. Agar $a = \frac{2b-1+2b-3}{4} = b - 1$ bo‘lsa, $\log_x 2 = \log_x 7 - 1$ bo‘lib, $\log_x \frac{7}{2} = 1$ yoki $x_1 = \frac{7}{2}$ bo‘ladi.

2. Agar $a = \frac{2b-1+2b+3}{4} = \frac{1}{2}$ bo‘lsa, $\log_x 2 = \frac{1}{2}$, $\sqrt{x} = 2$ yoki $x_2 = 4$ bo‘ladi.

4.1.12. $x^{\sqrt{\log_x 5}} + x^{\sqrt{\log_5 x}} = 2\sqrt{5}$ tenglamani yeching.

Yechish. Tenglamaning aniqlanish sohasi $x > 1$. Logarifmning xossasidan foydalanib:

$$x^{\sqrt{\log_x 5}} = x^{\frac{\log_x 5}{\sqrt{\log_x 5}}} = x^{\log_x 5 \cdot \sqrt{\log_5 x}} = \left(x^{\log_x 5}\right)^{\sqrt{\log_5 x}} = 5^{\sqrt{\log_5 x}}$$

tenglikni hosil qilish mumkin.

Demak, berilgan tenglama $5^{\sqrt{\log_5 x}} = \sqrt{5}$ tenglamaga teng kuchl bo'lib, $\sqrt{\log_5 x} = \frac{1}{2}, \log_5 x = \frac{1}{4}$ va $x_1 = \sqrt[4]{5}$.

Eslatma. Ushbu tenglamani yechishda logarifmning yana bi yuqori sinf o'quvchilari uchun foydali bo'lgan xossasidan foydalanildi.

Agar $a > 1, b > 1$ bo'lsa, u holda $a^{\sqrt{\log_a b}} = b^{\sqrt{\log_b a}}$ tenglil o'rinli bo'ladi. Ushbu tenglikni isbotlash uchun tenglikning ikkal tomonini b asosga ko'ra logarifmlash yetarli $\sqrt{\log_a b} \cdot \log_b a = \sqrt{\log_b a}$ bo'lib, $\sqrt{\log_a b} = \frac{1}{\sqrt{\log_b a}}$, u holda:

$$\sqrt{\log_a b} \cdot \log_b a = \frac{1}{\sqrt{\log_b a}} \cdot \log_b a = \sqrt{\log_b a}.$$

4.1.13. $4^{\lg x} - 32 + x^{lg4} = 0$ tenglamani yeching.

Yechish. Tenglamaning aniqlanish sohasi $x > 0$ va $x \neq 1$ bo'lib (17) formulaga ko'ra berilgan tenglama $4^{\lg x} - 32 + 4^{lgx} = 0$ yok $4^{lgx} = 16$ ko'rinishni oladi va $lgx = 2$ yoki $x_1 = 100$ bo'ladi.

4.1.14. $4^{\log_3(1-x)} = (2x^2 + 2x + 5)^{\log_3 2}$ tenglamani yeching.

Yechish. Tenglamaning aniqlanish sohasi $x < 1$.

Logarifmning xossalariga ko'ra: $4^{\log_3(1-x)} = 2^{\log_3(2x^2+2x+5)}$ yoki $2^{2\log_3(1-x)} = 2^{\log_3(2x^2+2x+5)}$.

Bundan quyidagilarni hosil qilish mumkin:

$$2\log_3(1-x) = \log_3(2x^2 + 2x + 5),$$
$$(1-x)^2 = 2x^2 + 2x + 5,$$
$$(1-x)^2 = 2x^2 + 2x + 5,$$
$$x^2 - 2x + 1 = 2x^2 + 2x + 5,$$
$$x^2 + 4x + 4 = 0 \quad \text{yoki} \quad (x+2)^2 = 0$$

Demak, berilgan tenglama $x_1 = -2$ ildizga ega.

4.1.15. $3\lg x^2 - lg^2(-x) = 9$ tenglamani yeching.

Yechish. Tenglamaning aniqlanish sohasi $x < 0$ tengsizlik bilan aniqlanadi.

Logarifmlash qoidalariga ko'ra: $6lg|x| - \lg^2(-x) = 9$ bo'lib,

$x < 0$ bo'lganligi uchun

$$6\lg(-x) - lg^2(-x) = 9$$

$$(\lg(-x) - 3)^2 = 0, \lg(-x) = 3 \text{ yoki } x_1 = -1000.$$

4.1.16. $\lg(x+10) + \frac{1}{2}lgx^2 = 2 - lg4$ tenglamani yeching.

Yechish. Tenglamaning aniqlanish sohasi $x \neq 0$ va $x > -10$ shartni qanoatlantiradi $\frac{lgx^2}{2} = lg|x|$ tenglikdan foydalanib berilgan tenglama $\lg(x+10) + lg|x| + lg4 = 2$ ko'rinishga kelib logarifmning xossalariga asosan:

$$lg\big(|x| \cdot (x+10)\big) = 2 \text{ yoki } 4|x| \cdot (x+10) = 100) \qquad (*)$$

bo'lib, bunda ikki hol bo'lishi mumkin:

$x > 0$ bo'lsin, u holda $|x| = x$ bo'lib, $(*)$ tenglama

$$4x(x+10) = 100 \text{ yoki } x^2 + 10x - 25 = 0$$

ko'rinishni oladi va ildizlari $x_{1,2} = -5 \pm 5\sqrt{2}$.

Demak, $x > 0$ bo'lganda berilgan tenglamaning ildizi

$$x_1 = 5\sqrt{2} - 5.$$

$-10 < x < 0$ bo'lsin, u holda $|x| = -x$ bo'lib, $(*)$ tenglama

$-4x(x+10) = 100$ yoki $x^2 + 10x + 25 = 0$, $(x+5)^2 = 0$

ko'rinishga keladi va $x_2 = -5$.

4.1.17. $\log_2\left(1 + \sqrt{x^4 + x^2}\right) + \log_2(1 + 2x^2) = 0$ tenglamani yeching.

Yechish. $\sqrt{x^4 + x^2} \geq 0$ va $x^2 \geq 0$ bo'lganligidan:

$$1 + \sqrt{x^4 + x^2} \geq 1 \text{ va } 1 + 2x^2 \geq 1$$

tenglik doim o'rinli bo'ladi. U holda: $\log_2\left(1 + \sqrt{x^4 + x^2}\right) \geq 0$ va $\log_2(1 + 2x^2) \geq 0$ natijada, quyidagi tengsizlik o'rinli bo'ladi

$$\log_2\left(1 + \sqrt{x^4 + x^2}\right) + \log_2(1 + 2x^2) \geq 0.$$

Bundan va berilgan tenglamadan: $\begin{cases} \log_2\left(1 + \sqrt{x^4 + x^2}\right) = 0 \\ \log_2(1 + 2x^2) = 0 \end{cases}$

bo'lib, berilgan tenglama yagona $x_1 = 0$ ildizga ega.

4.1.18. Quyidagi tenglamani yeching.

$$|\log_2(2x+7)| = \log_2(1 + |x+3|) + \log_2(1 - |x+3|)$$

Yechish. Tenglamaning aniqlanish sohasi $\begin{cases} 2x + 7 > 0, \\ 1 - |x+3| > 0 \end{cases}$ tengsizliklar sistemasi yordamida aniqlanadi, ya'ni $-3{,}5 < x < -2$.

Berilgan tenglamaning chap tomoni nomanfiy bo'lib, o'n tomonini quyidagicha baholash mumkin:

$$\log_2(1+|x+3|)+\log_2(1-|x+3|)=$$
$$=\log_2(1-(x+3)^2)\le\log_2 1=0.$$

Natijada, berilgan tenglama uning ikkala tomoni ham 0 ga ten bo'lgandagina o'rinli bo'ladi.

$$\begin{cases}\log_2(2x+7)=0,\\ \log_2(1-(x+3)^2)=0.\end{cases}$$

Demak, $x_1=-3$ bo'lib, o'zgaruvchining bu qiymatini berilga tenglamaga qo'yib $x_1=-3$ haqiqatan berilgan tenglamaning ildiz ekanligiga ishonch hosil qilish mumkin.

4.1.19. $\log_{\sqrt{x}}(x+|x-2|)=\log_x(5x-6+5|x-2|$ tenglamani yeching.

Yechish. Berilgan tenglamada $x>0, x\neq 1$

$u=x+|x+2|$ belgilash yordamida $\log_{\sqrt{x}}u=\log_x(5u-6$ yoki $\log_x u^2=\log_x(5u-6)$ (bu yerda $u>\frac{6}{5}$) va undan ildizlari $u_1=2, u=3$ bo'lgan $u^2-5x+6=0$ tenglamani hosil qilish mumkin.

$u=x+|x-2|$ bo'lganligi uchun: $x+|x-2|=2$ bo'lsin Agar $0<x<1$ yoki $1<x\le 2$ bo'lsa, u holda $|x-2|=-x+2$ bo'lib, bu tenglik doim to'g'ri, ya'ni $x+(-x+2)=2$.

$x+|x-2|=3$ bo'lsin. Bu tenglamaning yagona $x_2=\frac{5}{2}$ ildizini osongina topish mumkin.

Demak, berilgan tenglama $0<x<1$ va $1<x\le 2$ oraliqla hamda $x_2=\frac{5}{2}$ da o'rinli ekan.

4.1.20. $log_2^2 x+(x-1)\cdot\log_2 x+2x-6=0$ tenglaman yeching.

Yechish. Ushbu tenglama $\log_2 x$ ga nisbatan kvadrat tenglama bo'lib,

$$(\log_2 x)_{1,2}=\frac{-x+1\pm\sqrt{(x-1)^2-4(2x-6)}}{2}$$
$$=\frac{-x+1\pm\sqrt{x^2-10x+25}}{2}=\frac{-x+1\pm(x-5)}{2}$$

Agar $\log_2 x = \frac{-x+1+x-5}{2} = -2$ bo'lsa, $x_1 = \frac{1}{4}$ bo'ladi.

Agar $\log_2 x = \frac{-x+1-x+5}{2} = 3 - x$ bo'lsa, u holda $\log_2 x = 3 - x$ $(*)$ tenglama hosil bo'ladi.

$u = \log_2 x$ funksiya o'zining aniqlanish sohasida uzluksiz va o'suvchi bo'lib, $v = 3 - x$ funksiya esa uzluksiz va kamayuvchidir.

Demak, $(*)$ tenglama bittadan ko'p bo'lmagan ildizga ega. Tanlash yo'li bilan bu ildiz $x_2 = 2$ ekanligini osongina topish tmumkin.

Javob: $x_1 = \frac{1}{4}$ va $x_2 = 2$.

4.1.21. $\log_{x^2}(x^2 - 12) - \log_{-x}\sqrt{2-x} = 1$ tenglamani yeching.

Yechish. Tenglamaning aniqlanish sohasi $x < 0$ va $x \neq -1$.

$x < 0$ shartdan va logarifm xossalaridan foydalanib:

$$\log_{|x|}(x^2 + 12) - \log_{-x}(2 - x) = 2,$$

$$\log_{-x}(x^2 + 12) - \log_{-x}(2 - x) = 2, \log_{-x}\frac{x^2+12}{2-x} = 2,$$

$$\frac{x^2+12}{2-x} = x^2 \text{ yoki } x^3 - x^2 + 12 = 0,$$

$$(x + 2)(x^2 - 3x + 6) = 0 \text{ va } x^2 - 3x + 6 > 0$$

bo'lganligi uchun berilgan tenglama yagona $x_1 = -2$ ildizga ega.

4.1.22. $\frac{2-4\cdot\log_{12}2}{\log_{12}(x+2)} - 1 = \frac{\log_6(8-x)}{\log_6(x+2)}$ tenglamani yeching.

Yechish. Tenglamaning aniqlanish sohasi $-2 < x < 8$ va $x \neq -1$.

$\frac{\log_a f(x)}{\log_a g(x)} = \frac{\log_b f(x)}{\log_b g(x)}$ tenglik ixtiyoriy 1 dan farqli musbat a va b sonlar uchun o'rinli bo'ladi.

Bu tenglikdan foydalanib berilgan tenglamani

$$\frac{\log_{12}9}{\log_{12}(x+2)} - 1 = \frac{\log_{12}(8-x)}{\log_{12}(x+2)},$$

$$\log_{12}9 - \log_{12}(x + 2) = \log_{12}(8 - x),$$

$$(x + 2)(8 - x) = 9 \text{ yoki } x^2 + 6x - 7 = 0$$

ko'rinishda yozish mumkin.

Bu tenglamaing ildizlari $x_1 = 7$ va $x_2 = -1$. Ammo $2 < x < 8$ va $x \neq -1$ bo'lganligidan berilgan tenglama yagona $x_1 = 7$ ildizga ega.

4.1.23. $x = \lg(9x + 1)$ tenglamani yeching.

Yechish. Tenglama manfiy ildizlarga ega emasligi ko'rinib turibdi, $x_1 = 0$ esa tenglamaning ildizi bo'ladi.

Berilgan tenglamani $10^x = 1 + 9x$ yoki $(1 + 9)^x = 1 + 9$ ko'rinishda yozib olib, Bernulli (6) tengsizligiga ko'ra, ixtiyoriy r natural soni uchun $(1 + x)^n \geq 1 + nx$ tengsizlik o'rinli bo'lib, tengli $n = 1$ bo'lganda bajariladi, ya'ni $(1 + 9)^x = 1 + 9x$ tenglama yagon musbat $x_2 = 1$ butun ildizga ega.

Demak, $x_1 = 0$ va $x_2 = 1$.

Mustaqil yechish uchun misollar

Tenglamalarni yeching.

1. $\sqrt{2^{x^2-2x-10}}=\sqrt{33+\sqrt{128}}-1$, $\{-3;5\}$;
2. $3^{2x^2}-2\cdot 3^{x^2+x+6}+3^{2x+12}=0$, $\{-2;3\}$;
3. $2\sqrt{x}\cdot 4^x+5\cdot 2^{x+1}+2\sqrt{x}=2^{2x+2}+5\sqrt{x}2^x+4$, $\{2;4\}$;
4. $\sqrt{1+\log_x\sqrt{27}}\cdot\log_3 x+1=0$, $x=\frac{1}{9}$;
5. $\log_2(x+1)^2+\log_2(x+1)=6$, $\{-5;3\}$;
6. $\frac{1}{\log_6(3+x)}+\frac{2\log_{1/4}(4-x)}{\log_2(3+x)}=1$, x=3;
7. $(\log_a c)^{\log_b \log_a{}^2 x}+\log_a c=\log_a ac\,((\log_a x)^{\log_b \log_a c}$)(1 $a\neq c, 0<a\neq 1, 0<b\neq 1, 0<c\neq 1$ bo'lsa, u holda $x_1=a, x_2=a^b$;2) $0<a=c\neq 1, 0<b\neq 1$ bo'lsa, u holda $0<x\neq 1$,ya'ni 1 dan boshqa barcha musbat sonlar)
8. $\sqrt[3]{1+lgtgx}+\sqrt[3]{1-lgtgx}=2$, $x=\frac{\pi}{4}+\pi k, k\in Z$;
9. $(\frac{1-a^2}{2a})^x+1=(\frac{1+a^2}{2a})^x$, x=2;
10. $a^{\frac{2}{x}}+b^{\frac{2}{x}}=m(ab)^{\frac{1}{x}}, a\geq 0, b\geq 0$ 1) a=b=0,x> 0; 2) a=0,b> 0 b=0,a> 0,Ø; 3) $a\neq 0, b=0, a>0, b>0, m>2$

$x=\frac{lga-lgb}{\lg(m\pm\sqrt{m^2-m})-lg2}$; 4) a=b> $0, m=2$ $x\neq 0$);

11. $\frac{1}{4^{\frac{1}{x}}}+\frac{1}{6^{\frac{1}{x}}}=\frac{1}{9^{\frac{1}{x}}}$, $x=\frac{1}{\log_{3/2}\frac{-1+\sqrt{5}}{2}}$;
12. $2^x-3^x=\sqrt{6^x-9^x}x_1=0$, $x_2=\frac{1}{2-\log_2 3}$;
13. $16^{\frac{x-1}{x}}5^x=100$, $\{-2\log_5 2;2\}$;

14. $\frac{\log_2 x}{\log_4 2x} = \frac{\log_8 4x}{\log_{16} 8x}$, $\left\{\frac{1}{16}; 2\right\}$;

15.

$x^2 \log_2 \frac{3+x}{10} - x^2 \log_{1/2}(2+3x) = x^2 - 4 + 2\log_{\sqrt{2}} \frac{3x^2+11x+6}{10}$, $\{1; 2\}$;

16. $10^{\sqrt{lg\, x}} + x^{\sqrt{\log_x 10}} = 200$, $x=10^4$;

4.2. Ko'rsatkichli va logarifmik tengsizliklar

4.2.1. $1 + 9^x + 16^x \leq 3^x + 4^x + 12^x$ tengsizlikni yeching.

Yechish. $3^x = a$ va $4^x = b$ almashtirishlar yordamida berilgan tengsizlikni $1 + a^2 + b^2 \leq a + b + ab$, $a^2 + b^2 - ab - a - b + 1 \leq 0$ ko'rinishda yozish mumkin.

Oxirgi tengsizlikni 2 ga ko'paytirib, so'ngra chap tomonida to'la kvadratlar ajratish mumkin .U holda:

$$2a^2 + 2b^2 - 2ab - 2a - 2b + 2 \leq 0,$$

$$(a^2 - 2ab + b^2) + (a^2 - 2a + 1) + (b^2 - 2b + 1) \leq 0$$

yoki

$(a-b)^2 + (a-1)^2 + (b-1)^2 \leq 0$ $(*)$ bo'lib, $(a^2 + b^2) \geq 0$, $(a-1)^2 \geq 0$ va $(b-1)^2 \geq 0$ tengsizliklardan $(*)$ tengsizlik $a - b = 0$, $a - 1 = 0$ va $b - 1 = 0$ yoki $a = 1$ va $b = 1$ bo'lib, $3^x = 4^x$ ya'ni, $x = 0$ bo'ladi.

Eslatma. Tengsizlikni yechish jarayonida ixtiyoriy x va y o'zgaruvchilar uchun $x^2 + y^2 - xy - x - y + 1 \geq 0$ tengsizlik o'rinli bo'lishi ham isbotlandi.

4.2.2. $\log_{\frac{1}{2}} x \geq 16^x$ tengsizlikni yeching.

Yechish. Quyidagi qo'shimcha tenglamani qaraymiz $\log_{\frac{1}{2}} x = 16^x$ $(*)$ bunda, $0< x < 1$.

$f(x) = \log_{\frac{1}{2}} x$ va $g(x) = 16^x$ bo'lsin. $u = f(x)$ o'zining aniqlanish sohasida uzluksiz va kamayuvchi $v = g(x)$ esa uzluksiz o'suvchi bo'lib $(*)$ tenglama bittadan ko'p ildizga ega bo'la olmaydi.Tanlash yo'li bilan bu ildiz $x_1 = \frac{1}{4}$ ekanligini ko'rish mumkin.

Yuqoridagilardan $\left(0; \frac{1}{4}\right]$ oraliqda $f(x) \geq f\left(\frac{1}{4}\right) = g\left(\frac{1}{4}\right) \geq g(x)$

tengsizlikni yozish mumkin.

Demak, $\log_{\frac{1}{2}} x \geq 16^x$ tengsizlik $0 < x \leq \frac{1}{4}$ bo'lganda o'rin bo'ladi.

4.2.3. $|x+1|^{x^2-\frac{5x}{2}+\frac{3}{2}} \leq 1$ tengsizlikni yeching.

Yechish. Ma'lumki, $|x+1| \neq 1$. Agar $0<|x+1|<1$, ya'ni $-2 < x < -1$ yoki $-1 < x < 0$ bo'lsa, u holda:

$$x^2 - \frac{5x}{2} + \frac{3}{2} > 0 \text{ va } x > 1{,}5 \text{ yoki } x < 1$$

bo'ladi.

Demak, berilgan tengsizlik qaralayotgan oraliqda doim bajariladi.

Agar $|x+1|>1$, ya'ni $x>0$ yoki $x \leq -2$ bo'lsa, u holda:

$x^2 - \frac{5x}{2} + \frac{3}{2} < 0$ yoki $1 < x < \frac{3}{2}$ bo'lib, bu holda berilgan tengsizlik $1 < x < \frac{3}{2}$ shartda bajariladi.

Javob. $-2 < x < -1$; $-1 < x < 0$; $1 < x < 1{,}5$

4.2.4. $\log_{\frac{1}{2}}(\log_2(\log_{x-1} 9)) > 0$ tengsizlikni yeching.

Yechish. Tengsizlikda x-o'zgaruvchining qabul qilishi mumkin bo'lgan qiymatlari $1 < x < 2$ va $x > 2$.

Berilan tengsizlikni unga teng kuchli bo'lgan tengsizliklarga quyidagicha almashtirish mumkin:

$\log_{\frac{1}{2}}(\log_2(\log_{x-1} 9)) > 0$,

$0 < \log_2(\log_{x-1} 9) < 1$,

$1 < \log_{x-1} 9 < 2$,

$\frac{1}{2} < \log_{x-1} 3 < 1$,

$\frac{1}{2} < \frac{1}{\log_3(x-1)} < 1$,

Oxirgi tengsizlikdan $1 < \log_3(x-1) < 2$, $3 < x-1 < 9$ va $4 < x < 10$.

Tengsizlikning aniqlanish sohasini inobatga olib (4;10) berilgan tengsizlikning yechimi bo'lishini ko'rish mumkin.

4.2.5. $x^{2\lg 2} \cdot 2^{-\lg x} \leq 2$ tengsizlikni yeching.

Yechish. Tengsizlikning aniqlanish sohasi $x > 0$. (17) formulaga ko'ra: $x^{lg2} = 2^{\lg x}$ bo'lib, u holda $(2^{lgx})^2 \cdot 2^{-\lg x} \leq 2$, $2^{lgx} \leq 2$, $lgx \leq 1$, $0 < x \leq 10$.

Javob. $0 < x \leq 10$ bo‘ladi.

4.2.6. $\log_{\frac{25-x^2}{16}}\left(\frac{24-2x-x^2}{14}\right) \geq 1$ tengsizlikni yeching.

Yechish. Berilgan tengsizlikning aniqlanish sohasi:

$$\begin{cases} \frac{25-x^2}{16} > 1, \\ 24-2x-x^2 > 0 \end{cases} \text{ yoki } \begin{cases} 0 < \frac{25-x^2}{16} < 1, \\ 24-2x-x^2 > 0 \end{cases}$$

tengsizliklar sistemalari bilan aniqlanadi.

Birinchi sistemadan $\begin{cases} -3 < x < 3, \\ -6 < x < 4. \end{cases}$ bo‘lib, $-3 < x < 3$ bo‘ladi.

Ikkinchi sistemadan esa: $\begin{cases} -5 < x < 5, \\ x \leq -3, \\ -6 < x < 4. \end{cases}$ yoki $\begin{cases} -5 < x < 5, \\ x > 3, \\ -6 < x < 4. \end{cases}$ bo‘lib

$-5 < x < -3$ yoki $3 < x < 4$ bo‘ladi.

Shunday qilib, berilgan tengsizlikning aniqlanish sohasi quyidagi uchta oraliqdan iborat:

$$-5 < x < -3;\ -3 < x < 3;\ 3 < x < 4.$$

Agar $-3 < x < 3$ bo‘lsin, u holda:

$$\frac{24-2x-x^2}{14} \geq \frac{25-x^2}{16},$$
$$-17 \leq x \leq 1.$$

bo‘lib, tengsizlikning aniqlanish sohasini hisobga olsak, $-3 < x \leq 1$ bo‘ladi.

Agar $-5 < x < 3$ yoki $3 < x < 4$ bo‘lsa, u holda hosil bo‘lgan $(x+17)(x-1) \geq 0$ tengsizlikdan $x \geq 1$ yoki $x \leq -17$ bo‘lib, aniqlanish sohasini hisobga olsak, $3 < x < 4$ bo‘ladi.

Javob. $-3 < x \leq 1, 3 < x < 4$.

4.2.7. $\log_{\sqrt{2}+\sqrt{3}}(2-|x-1|) > \log_{\sqrt{10}}(2x-x^2)$ tengsizlikni yeching.

Yechish. Tengsizlikning aniqlanish sohasi $\begin{cases} 2-|x-1| > 0 \\ 2x-x^2 > 0 \end{cases}$ sistemadan topiladi. Bundan $0 < x < 2$.

Demak, $-1 < x-1 < 1$ *yoki* $|x-1| < 1$.

$2-|x-1| > 1$ tengsizlik berilgan tengsizlikning aniqlanish sohasida to‘g‘ri bo‘lib, uni $\sqrt{2}+\sqrt{3}$ asosga ko‘ra logarifmlab:

$\log_{\sqrt{2}+\sqrt{3}}(2-|x-1|) > 0$ tengsizlikni hosil qilish mumkin.

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Bundan: $0 < 2x - x^2 = 1 - (x-1)^2 \le 1$ bo'lib, $\sqrt{10} > 1$ bo'lganligi uchun $\log_{\sqrt{10}}(2x - x^2) \le 0$ bo'ladi.

Shunday qilib,

$$\log_{\sqrt{2}+\sqrt{3}}(2 - |x-1|) > 0 \text{ va } \log_{\sqrt{10}}(2x - x^2) \le 0$$

tengsizliklar berilgan tengsizlik aniqlanish sohasiga tegishli barcha x lar uchun o'rinli bo'ladi, ya'ni berilgan tengsizlikning yechimi $0 < x \le 2$ bo'ladi.

4.2.8. $0 < b < a$ shartni qanoatlantiruvchi a va b sonlari uchun quyidagi tengsizlikni isbotlang

$$ln\frac{a}{b} > 2\frac{a-b}{a+b}.$$

Yechish. $0 < b < a$ shartdan $a = bk$ va $k > 1$ bo'lib, berilgan tengsizlikni quyidagi ko'rinishda yozish mumkin:

$$ln\frac{bk}{b} > 2\frac{bk-b}{bk+b}, \quad lnk > 2\frac{k-1}{k+1} \qquad (*)$$

$x > 1$ bo'lganda $f(x) = \ln x - 2\frac{x-1}{x+1}$ funksiyani tahlil qilamiz. $f(1) = 0$.

$x > 1$ da $f'_x = \frac{1}{x} - \frac{4}{(x+1)^2} = \frac{(x-1)^2}{(x+1)^2} > 0$ bo'lib, bu oraliqda $f(x)$ funksiya o'suvchi bo'ladi.

Demak, $f(x) > f(1) = 0$ bo'lib, $\ln x > 2\frac{x-1}{x+1}$ bo'ladi, ya'ni $(*)$ tengsizlik va berilgan tengsizlik to'g'ri ekan.

4.2.9. $\log_{17} 19 > \log_{19} 20$ tengsizlikni isbotlang.

Yechish. Berilgan tengsizlikni isbotlash uchun $\frac{\log_{19} 20}{\log_{17} 19} < 1$ yoki $\log_{19} 17 \cdot \log_{19} 20 < 1$ tengsizlikni isbotlash kifoya. Koshi (2) tengsizligiga asosan:

$$\log_{19} 17 \log_{19} 20 \le \left(\frac{\log_{19} 20 + \log_{19} 17}{2}\right)^2 = \left(\frac{\log_{19} 340}{2}\right)^2$$

$$< \left(\frac{\log_{19} 361}{2}\right)^2 = 1.$$

Chunki, $u = \log_{19} x$ funksiya $x > 0$ da o'suvchi va $\log_{19} 340 < \log_{19} 361$.

4.2.10. $\lg(n+1) > \frac{lg1+lg2+\ldots+lgn}{n}$ tengsizlikni isbotlang.

Yechish. Logarifmning xossalariga ko'ra: $(n+1)^n > n!$ bo'lib,

bu yerda $n! = 1 \cdot 2 \cdot \ldots \cdot n$ va $n \geq 1$.

Ma'lumki, $n+1 > 1$, $n+1 > 2$, $n+1 > 3$, ….. , $n+1 > n$ bo'lib, ularni ko'paytirish natijasida $(n+1)^n > n!$ tengsizlikni hosil qilish mumkin.

4.12.11. Musbat $x_1, x_2, \cdots, x_n$ sonlari uchun $x_1 x_2 \cdots x_n = 1$ tenglik bajarilsa, quyidagi tengsizlikni isbotlang

$$x_1^{lgx_1} + x_2^{lgx_2} + \ldots + x_n^{lgx_n} \geq n.$$

Yechish. Berilgan tengsizlik chap tomonining o'nli logarifmi uchun Koshi (1) va Koshi-Bunyakovskiy (9) tengsizliklaridan foydalanib:

$$lg\left(x_1^{lgx_1} + x_2^{lgx_2} + \ldots + x_n^{lgx_n}\right) \geq$$

$$lg\left(n \cdot \sqrt[n]{x_1^{lgx_1} \cdot x_2^{lgx_2} \cdot \ldots \cdot x_n^{lgx_n}}\right) =$$

$$= lgn + \frac{1}{n} \cdot (lg^2 x_1 + lg^2 x_2 + \ldots + lg^2 x_n) =$$

$$= lgn + \frac{1}{n^2} \cdot \left(\underbrace{1^2 + 1^2 + \cdots + 1^2}_{n}\right) \cdot (lg^2 x_1 + lg^2 x_2 + \cdots + lg^2 x_n) \geq$$

$$\geq lgn + \frac{1}{n^2}(1 \cdot lgx_1 + 1 \cdot lgx_2 + \ldots + 1 \cdot lgx_n)^2$$

$$= lgn + \frac{1}{n^2} \cdot lg^2(x_1 x_2 \ldots x_n)$$

ni hosil qilish mumkin. Shartga ko'ra, $x_1 x_2 \ldots . x_n = 1$ bo'lib, u holda:

$$lgn + \tfrac{1}{n^2} \cdot lg^2(x_1 x_2 \ldots x_n) = lgn + \tfrac{1}{n^2} lg^2 1 = lgn + 0 = \lg n.$$

Demak, $lg\left(x_1^{lgx_1} + x_2^{lgx_2} + \ldots + x_n^{lgx_n}\right) \geq \lg n$ bo'lib, berilgan tengsizlikning to'g'riligi kelib chiqadi.

Mustaqil yechish uchun misollar.

1. $4x^2 + 3^{\sqrt{x}+1} + x \cdot 3^{\sqrt{x}} < 2x^2 \cdot 3^{\sqrt{x}} + 2x + 6$ $0 \leq x < log_3{}^2 2$, $x > \frac{3}{2}$;

2. $(\frac{1}{2})^{(x^6 - 2x^3 + 1)^{1/2}} < 2^{x-1}$, $x < -1, 0 < x < 1, x > 1$;

3. $4^{1+\sqrt{x}} + 3 \cdot 2^{x+\sqrt{x}} \leq 2^{2x}$ $x \geq 4$;

4. $(4^x - 1)^2 + 2^{x+1}(4^x - 1) < 8 \cdot 4^x$, $\log_2(\sqrt{5} - 2) < x <$

$\log_2(1+\sqrt{2})$;

5. $(\sqrt{5}+2)^{x-1} \geq (\sqrt{5}-2)^{\frac{x-1}{x+1}}$, $-2 \leq x-1,\ x \geq 1$;

6. $\log_{2-x}(x+2)\log_{x+3}(3-x) \leq 0$, $-2 < x \leq -1;\ 1 < x < 2$;

7. $\frac{\log_{x^2+3}(x+1)-\log_{x^2+3}(x^2+1)}{\log_{x^2+3}(x+2)-\log_{x^2+3}(x^2+2)} \leq 2$, $0< x < 1; x > 1$;

8. $\frac{1}{\log_{x^2-16}(x^2-8x+16)} + \frac{1}{\log_{x^2-16}(x^2+8x+16)} < \frac{9}{4}$

$x< -\frac{9+\sqrt{33}}{2}; -5 < x < -\sqrt{17}; -\sqrt{17} < x < -4; 4 < x < \sqrt{17}; \sqrt{17} < x < 5$;

9. $\log_2 \log_3 \frac{x+1}{x-1} < \log_{\frac{1}{2}} \log_{\frac{1}{3}} \frac{x-1}{x+1}$, $x> 2$;

10. $(lgx+1)^2 < \sqrt{lgx}(\sqrt{lgx}+1)^2$, $\emptyset$;

4.3. Ko‘rsatkichli va logarifmik tenglama va tengsizliklar sistemalari

4.3.1. Quyidagi sistemani yeching:

$$\begin{cases} \log_2 x \log_x(x-3y) = 2, \\ x \cdot y^{\log_x y} = y^{5/2}. \end{cases}$$

Yechish. $\log_2 x = \frac{1}{\log_x 2}$ bo‘lganligidan sistemaning birinchi tenglamasi $\frac{\log_x(x-3y)}{\log_x 2} = 2$ yoki

$\log_x(x-3y) = 2\log_x 2$ bo‘lib, $x-3y=4$ bo‘ladi.

Sistema ikkinchi tenglamasining ikkala tomonini x asosga ko‘ra logarifmlab:

$$\log_x(x \cdot y^{\log_x y}) = \log_x y^{5/2},$$

$$1 + log_x{}^2 y = \frac{5}{2}\log_x y,$$

$$2log_x{}^2 y - 5\log_x y + 2 = 0.$$

$\log_x y = 2$ va $\log_x y = \frac{1}{2}$ yoki $y=x^2$ va $x=y^2$ mos ravishda $3x^2 - x + 4 = 0$ va $y^2 - 3y - 4 = 0$ tenglamalarni hosil qilish mumkin. Birinchi tenglama haqiqiy ildizlarga ega emas, ikkinchi tenglamaning musbat ildizi y=4 bo‘lib, u holda: x=2 bo‘ladi.

4.3.2. Quyidagi sistemani yeching:

$$\begin{cases}(xy)^y \cdot x^{6x} = y^x \\ x^2y = 1\end{cases}$$

Yechish. $x_1 = 1$ va $y = 1$ berilgan tenglamalar sistemasining ildizlari ekanligini osongina tekshirib ko'rish mumkin.$x \neq 1$ bo'lsin, u holda sistemaning birinchi tenglamasini x asosga ko'ra logarifmlab:

$$y \cdot \log_x(xy) + 6x \log_x x = x \log_x y$$

yoki

$$y(1 + \log_x y) + 6x = x \log_x y \qquad (*)$$

tenglikni hosil qilish mumkin.

Ikkinchi tenglamani ham x asosga ko'ra logarifmlab:

$2 + \log_x y = 0$ yoki $\log_x y = -2$ tenglikni hosil qilib va $(*)$ tenglikdan: $y = 8x$ bo'ladi.

$\log_x y = -2$ tenglikdan $y = \frac{1}{x^2}$, $8x = \frac{1}{x^2}$ va $x_2 = \frac{1}{2}$.

Shunday qilib, berilgan sistema yechimlari (1;1) va $\left(\frac{1}{2}; 4\right)$.

4.3.3. Quyidagi sistemani yeching:

$$\begin{cases}3\left(2\log_{y^2} x - \log_{\frac{1}{x}} y\right) = 10, \\ xy = 81.\end{cases}$$

Yechish. Sistemaning birinchi tenglamasida x asosga ko'ra logarifmga o'tib, quyidagi tenglamani hosil qilish mumkin:

$$3\left(\log_x y + \frac{1}{\log_x y}\right) = 10.$$

$\log_x y = a$ almashtirish yordamida ildizlari $a_1 = 3$ va $a_2 = \frac{1}{3}$ bo'lgan

$$3a^2 - 10a + 3 = 0.$$

tenglamani hosil qilish mumkin. Misolning shartiga ko'ra, mos ravishda (3; 27) va (27; 3) yechimlarni hosil qilish mumkin

4.3.4. Quyidagi sistemani yeching:

$$\begin{cases}y^{\log_5 x} = 64, \\ xy = 500.\end{cases}$$

Yechish. Shartga ko'ra: $x > 0$, $x \neq 1$ va $y > 0$.

Har ikkala tenglamani 2 asosga ko'ra logarifmlab:

$$\begin{cases} \log_5 x \log_2 y = 6, \\ \log_2 x + \log_2 y = 2 + 3\log_2 5. \end{cases}$$

Bundan:

$$\log_2 x + \frac{6}{\log_5 x} = 2 + 3\log_2 5$$

yoki

$$\log_2 x + \frac{6\log_2 5}{\log_2 x} = 2 + 3\log_2 5 \qquad (*)$$

tenglamani hosil qilish mumkin.

Yangi $\log_2 x = z$ va $\log_2 5 = a$, z va a o'zgaruvchilar yordamida $(*)$ tenglamadan $z + \frac{6a}{z} = 2 + 3a$ yoki $z^2 - z(2 + 3a) + 6a = 0$ tenglamani hosil qilish mumkin va bu kvadrat tenglamaning ildizlari:

$$z_{1,2} = \frac{2 + 3a \pm \sqrt{(2 + 3a)^2 - 24a}}{2} = \frac{2 + 3a \pm (2 - 3a)}{2}$$

bo'ladi.

Agar $z_1 = 2$ bo'lsa, $\log_2 x_1 = 2$ va $x_1 = 4$ hamda $y = \frac{500}{x}$ tenglikdan $y_1 = 125$.

Agar $z_2 = 3a$ bo'lsa, $\log_2 x_2 = 3\log_2 5$ va $x_2 = 125, y = \frac{500}{x}$ tenlikdan $y_2 = 4$.

4.3.5. Quyidagi sistemani yeching:

$$\begin{cases} 2\log_4 x^2 + \log_{0,2} y^3 = -1, \\ 2\log_4 x^4 - \log_{0,2} y = 5. \end{cases}$$

Yechish. Tenglamalar sistemasining aniqlanish sohasi $x \neq 0$ va $y > 0$. Logarifmlash qoidalariga ko'ra: $\begin{cases} 4\log_4 |x| + 3\log_{0,2} y = -1, \\ 8\log_4 |x| - \log_{0,2} y = 5 \end{cases}$ $(*)$ bo'lib, $(*)$ tenglamalar sistemasining yechimi $\log_4 |x| = \frac{1}{2}$ va $\log_{0,2} y = -1$ bo'lib bundan: $x_1 = 2$, $y_1 = 5$ va $x_2 = -2$, $y_2 = 5$ lar berilgan tenglamalar sistemasining yechimi bo'ladi.

4.3.6. Quyidagi tenglama b va c ning qanday qiymatlarida yechimga ega bo'ladi

$$\begin{cases} a^{2x} + a^{2y} = 2b, \\ a^{x+y} = c, \\ a > 0. \end{cases}$$

Yechish. $a^x = u, a^y = v$ almashtirishlar yordamida quyidagi tenglamalar sistemasi hosil bo'ladi

$$\begin{cases} u^2 + v^2 = 2b, \\ uv = c. \end{cases}$$

u va v faqat musbat qiymatlar qabul qilishligini hisobga olgan holda quyidagi tengliklarni hosil qilamiz:

$(u+v)^2 = 2(b+c), (u-v)^2 = 2(b-c)$.

Birinchi tenglamadan:

$$u + v = \sqrt{2(b+c)}\,. \qquad (*)$$

Ikkinchi tenglamada $b \geq c$ tengsizlik bajarilishini hisobga olib:

$$u - v = \pm\sqrt{2(b-c)} \qquad (**)$$

tenglikni hosil qilish mumkin.

$b \geq c$ bo'lganda $(*)$ va $(**)$ tengliklardan berilgan sistema yechimlarini topish mumkin:

$$u_{1,2} = \frac{\sqrt{2}}{2}\left(\sqrt{b+c} \pm \sqrt{b-c}\right), x_{1,2} = \log_a\left(\frac{\sqrt{2}}{2}\left(\sqrt{b+c} \pm \sqrt{b-c}\right)\right).$$

$$v_{1,2} = \frac{\sqrt{2}}{2}\left(\sqrt{b+c} \mp \sqrt{b-c}\right), y_{1,2} = \log_a\left(\frac{\sqrt{2}}{2}\left(\sqrt{b+c} \mp \sqrt{b-c}\right)\right).$$

4.3.7. Quyidagi sistemani yeching:

$$\begin{cases} x^{\log_8 y} + y^{\log_8 x} = 4, \\ \log_4 x - \log_4 y = 1. \end{cases}$$

Yechish. Logarifmning xossalariga ko'ra: $x^{\log_g y} = y^{\log_g x}$ tenglikdan sistemaning birinchi tenglamasini

$x^{\log_8 y} + y^{\log_8 x} = 4$, $\quad x^{\log_8 y} = 2$ $\quad$ yoki $\quad \log_2 x \cdot \log_2 y = 3$

ko'rinishida yozish mumkin.

Sistemaning ikkinchi tenglamasi $\log_2 x - \log_2 y = 2$ tenglamaga teng kuchli bo'lib, berilgan tenglamalar sistemasi $\begin{cases} \log_2 x \cdot \log_2 y = 3, \\ \log_2 x - \log_2 y = 2 \end{cases}$ tenglamalar sistemasiga teng kuchli bo'ladi va undan $\log_2 x = 3$, $\log_2 y = 1$ va $\log_2 x = -1, \log_2 y = -3$.

Shunday qilib, berilgan tenglamalar sistemasining yechimlari

$x_1 = 8, y_1 = 2$ va $x_2 = \frac{1}{2}, y_2 = \frac{1}{8}$.

4.3.8. Quyidagi sistemani yeching:

$$\begin{cases} 4^{x+y-1} + 3 \cdot 4^{2y-1} \le 2, \\ x + 3y \ge 2 - \log_4 3. \end{cases}$$

Yechish. Koshi (2) tengsizligiga ko'ra tengsizlikning chap qismi uchun quyidagilarni yozish mumkin:

$$4^{x+y-1} + 3 \cdot 4^{2y-1} \ge 2\sqrt{4^{x+y-1} \cdot 3 \cdot 4^{2y-1}} = 2\sqrt{3 \cdot 4^{x+3y-2}} \ge$$

$$\ge 2\sqrt{3 \cdot 4^{2-\log_4 3-2}} = 2.$$

Demak,

$$\begin{cases} 4^{x+y-1} = 3 \cdot 4^{2y-1} \\ x + 3y = 2 - \log_4 3 \end{cases} \qquad (*)$$

sistema birinchi tenglamasining ikkala tomonini 4 asosga ko'ra logarifmlab: $(*)$ sistemaning ikkinchi tenglamasidan:

$x + 3(x - \log_4 3) = 2 - \log_4 3, 4x = 2 + 2\log_4 3$ yoki

$$x_1 = \frac{1}{2} + \frac{1}{2}\log_4 3 = \log_4(2\sqrt{3}).$$

$y = x - \log_4 3$ tenglikdan esa $y_1 = \frac{1}{2} - \frac{1}{2}\log_4 3 = \log_4 \frac{2}{\sqrt{3}}$ bo'ladi.

Mustaqil yechish uchun misollar

1. $\begin{cases} \log_2(x+y) - \log_3(x-y) = 1, \\ x^2 - y^2 = 2 \end{cases}$ $(\frac{3}{2}; \frac{1}{2})$;

2. $\begin{cases} x^a = y^b, \\ \log_c \frac{x}{y} = \frac{\log_c x}{\log_c y}, \\ a \ne b, ab \ne 0. \end{cases}$ $x = c^{\frac{b^2}{a(b-a)}}$; $y = c^{\frac{b}{b-a}}$.

3. $\begin{cases} \log_5 x + 3^{\log_3 y} = 7, \\ x^y = 5^{12}. \end{cases}$ (125;4) ,(625;3)

4. $\begin{cases} yx^{\log_y x} = x^{5/2}, \\ \log_4 y \log_y(y - 3x) = 1. \end{cases}$ (4;16);

5. $\begin{cases} a^x b^y = ab, \\ 2\log_a x = \log_{1/b} y \log_{\sqrt{a}} b. \end{cases}$ $(\log_a b ; \log_b a), (1; 1)$;

6. $\begin{cases} \log_{12} x \left(\frac{1}{\log_x 2} + \log_2 y\right) = \log_2 x, \\ \log_2 x \log_3(x+y) = 3\log_3 x. \end{cases}$ (6;2),(2;6)

7. $\begin{cases} x\log_2 y \log_{1/x} 2 = y\sqrt{y}(1-\log_x 2), \\ \log_{y^3} 2 \log_{\sqrt{2}} x = 1. \end{cases}$ $(2^{3}/_{5}; 2^{2}/_{5})$

8. $\begin{cases} \log_2 x + \log_4 y + \log_4 z = 2, \\ \log_3 y + \log_9 z + \log_9 x = 2, \\ \log_4 z + \log_{16} x + \log_{16} y = 2. \end{cases}$ $(\frac{2}{3}; \frac{27}{8}; \frac{32}{3})$

9. $\begin{cases} \log_{0,5}(y-x) + \log_2 \frac{1}{y} = -2, \\ x^2 + y^2 = 25. \end{cases}$ $(-\frac{7}{\sqrt{2}}; \frac{1}{\sqrt{2}}), (3; 4)$

10. $\begin{cases} x^{x+y} = y^{x-y}, \\ x^2 y = 1, \\ x > 0, y > 0. \end{cases}$ $(1;1), (\frac{1}{\sqrt[3]{3}}; \sqrt[3]{9})$

11. $\begin{cases} x^{x+y} = y^n, \\ y^{x+y} = x^{2n} y^n, \\ x > 0, y > 0, n > 0. \end{cases}$ $(\frac{\sqrt{8n+1}-1}{2}; \frac{(\sqrt{8n1}-1)^2}{4}), (1;1)$

12. $\begin{cases} (3x+y)^{x-y} = 9, \\ \sqrt[x-y]{324} = 18x^2 + 12xy + 2y^2. \end{cases}$ $(\frac{5}{4}; -\frac{3}{4}), (-\frac{1}{4}; -\frac{9}{4})$

13. $\begin{cases} x^y = y^x, \\ x^p = y^q, \\ x > 0, y > 0, pq > 0. \end{cases}$ $(1;1), ((\frac{q}{p})^{\frac{q}{q-p}}; (\frac{q}{p})^{\frac{p}{q-p}})$

14. $\begin{cases} x^y = y^x, \\ p^x = q^y, \\ x > 0, y > 0, p > 0, q > 0. \end{cases}$ $(\frac{\log_p q}{\log_p q - 1}; \frac{1}{\log_p q - 1})$

4.4. Trigonometrik tenglamalar va ulalarning sistemasi

4.4.1. $\sin x \cdot (1 + \cos x) = 1 + \cos x + \cos^2 x$ tenglamani yeching.

Yechish. Agar $\cos x = 1$ bo'lsa, u holda $\sin x = \frac{3}{2} > 1$ bo'lib, bunday bo'lishi mumkin emas.

Demak, $\cos x \neq 1$ yoki $1 - \cos x \neq 0$ U holda, berilgan tenglama ikkala tomonini $(1 - \cos x)$ ga ko'paytirib

$\sin x \cdot (1 - \cos^2 x) = 1 - \cos^3 x$ yoki $\sin^3 x + \cos^3 x = 1$ $(*)$ tenglikni hosil qilish mumkin.

$\sin^3 x \leq \sin^2 x$, $\cos^3 x \leq \cos^2 x$ va $\sin^2 x + \cos^2 x = 1$

munosabatlardan $\sin^3 x + \cos^2 x \leq 1$ bo'lib, $(*)$ tenglama $\begin{cases}\sin^3 x = \sin^2 x,\\ \cos^3 x = \cos^2 x\end{cases}$ tenglamalar sistemasiga teng kuchli bo'ladi, bundan:

$$\begin{cases}\sin x \cdot (\sin x - 1) = 0,\\ \cos x \cdot (\cos x - 1) = 0\end{cases} \qquad (**)$$

bo'lib, $\cos x \neq 1$ shartga ko'ra, $(**)$ tenglamalar sistemasining ikkinchi tenglamasidan $\cos x = 0$ bo'lib, u holda $\sin x = \pm 1$ bo'ladi. Ammo, birinchi tenglamadan $\sin x = 1$, ya'ni $x_1 = \frac{\pi}{2} + 2\pi n$ bu yerda n-butun son.

4.4.2. a, b, c, d-musbat sonlar biror arifmetik progressiyaning ketma-ket hadlari bo'lsa, quyidagi tenglamani yeching

$$\sin ax \sin bx = \sin cx \sin dx.$$

Yechish. a, b, c, d sonlari arifmetik progressiyaning ketma-ket hadlari bo'lganligidan (r-arifmetik progressiyaning ayirmasi)

$$b=a+r, c = a + 2r, d = a + 3r,$$

$$\cos(2a + r)x - \cos(2a + 5r)x = 0,$$

$$\sin(2a + 3r)x \sin 2rx = 0.$$

Bu tenglamaning yechimlari quyidagicha bo'ladi:

$$x_1 = \frac{\pi k}{2a+3r}, x_2 = \frac{\pi k}{2r}, k \epsilon Z.$$

Oxirgi tengliklar ma'noga ega, chunki

$$2a + 3r = b + c, r \neq 0.$$

Javob. $x_1 = \frac{\pi k}{b+c}, x_2 = \frac{\pi k}{2(b-a)}, k \epsilon Z.$

4.4.3. $1 + \cos 2x \cdot \cos 3x = \frac{1}{2}\sin^2 3x$ tenglamani yeching.

Yechish. Tenglamaning ikkala tomonini 2 ga ko'paytirib va $\sin^2 3x = 1 - \cos^2 3x$ ayniyatdan foydalanib

$\cos^2 3x + 2\cos 2x \cos 3x = -1$ tenglamani hosil qilish mumkin va bu tenglama ikkala tomoniga $\cos^2 2x$ qo'shilsa,

$(\cos 3x + \cos 2x)^2 = -\sin^2 2x$ tenglama hosil bo'ladi va bu tenglama uning ikkala qismi ham 0 ga teng bo'lgandagina o'rinli bo'ladi, ya'ni

$$\begin{cases}\cos 3x + \cos 2x = 0,\\ \sin 2x = 0\end{cases} \qquad (*)$$

$cos2x = 2\cos^2 x - 1$ va $\cos 3x = 4\cos^3 x - 3\cos x$ ayniyatlar yordamida sistemaning birinchi tenglamasi $\cos x$ ga nisbatan uchinchi

darajali:

$$4\cos^3 x + 2\cos^2 x - 3\cos x - 1 = 0 \qquad (**)$$

tenglamaga teng kuchli bo‘ladi.

$(*)$ sistemaning ikkinchi tenglamasidan $\cos x = 0$ yoki $\sin x = 0$ bo‘ladi. Agar $\sin x = 0$ bo‘lsa, $\cos x = \pm 1$ bo‘lib, $\cos x = -1$ $(**)$ sistemani qanoatlantiradi. $\cos x = -1$ $(*)$ tenglamaning yoki berilgan tenglamaning ildizlari

$x_1 = \pi(2n + 1)$ bu yerda n – butun son.

4.4.4. $\cos 6x + \sin\frac{5}{2}x = 2$ tenglamani yeching.

Yechish. $\cos 6x \le 1$ va $\sin\frac{5}{2}x \le 1$. U holda:

$$\cos 6x + \sin\frac{5}{2}x \le 2$$

bo‘lib, berilgan tenglamadan quyidagi tenglamalar sistemasi hosil bo‘ladi:

$$\begin{cases}\cos 6x = 1,\\ \sin\frac{5}{2}x = 1.\end{cases} \qquad (*)$$

Ushbu sistemaning yechimi undagi tenglamalar ildizlarining kesishmasidan iborat bo‘lib, bu kesishma bo‘sh to‘plamdan iborat bo‘lsa berilgan tenglama ildizlari mavjud emas. $(*)$ sistema tenglamalari ildizlari $\begin{cases}6x = 2\pi n,\\ \frac{5}{2}x = \frac{\pi}{2} + 2\pi m\end{cases}$ sistemadan iborat, bu yerda m va n butun sonlar.

Bundan $\begin{cases}x = \frac{\pi n}{3}\\ x = \frac{\pi}{5}(4m + 1)\end{cases}$ bo‘lib $(**)$ ularning kesishmasini topish uchun $\frac{\pi n}{3} = \frac{\pi}{5}(4m + 1)$ qaraymiz. Demak, $5n = 12n + 3$ bo‘lib, n 3 ga karrali ya’ni ixtiyoriy butun k son uchun $n = 3k$. Natijada,

$15k = 12m + 3$ yoki $5k = 4m + 1$ bu son toq son bo‘lib, undan k ham toq son bo‘ladi ya’ni u-butun son uchun: $k = 2u + 1$ u holda,

$10u + 5 = 4m + 1$ yoki $5u = 2m - 2$. Ko‘rinib turibdiki, u juft son, ya’ni butun t lar uchun $u = 2t$ bo‘lib, $10t = 2m - 2$ yoki $m = 5t + 1$ u holda $(**)$ sistemaning ikkinchi tenglamasidan butun t sonlar uchun:

$$x_1 = \frac{\pi}{5}(4m + 1) = \frac{\pi}{5}(20t + 5) = \pi(4t + 1)$$

bo‘ladi.

4.4.5. $\frac{1}{\sin x}-\frac{1}{\sin 2x}=\frac{1}{\sin 4x}$ tenglamani yeching.

Yechish. Tenglamaning aniqlanish sohasi: $\begin{cases}\sin x\neq 0,\\ \sin 2x\neq 0,\\ \sin 4x\neq 0\end{cases}$ tengsizliklar sistemasi bilan aniqlanib, butun k- sonlari uchun $x\neq\frac{\pi}{4}k$ bo‘ladi.

Demak,

$$\frac{1}{\sin x}=\frac{1}{\sin 2x}+\frac{1}{\sin 4x},\ \frac{1}{\sin x}=\frac{\sin 2x+\sin 4x}{\sin 2x\cdot\sin 4x},$$
$$\frac{1}{\sin x}=\frac{2\sin 3x\cdot\cos x}{2\sin x\cdot\cos x\cdot\sin 4x} \qquad (*)$$

bo‘lib, $x\neq\frac{\pi}{4}k$ shartdan $\sin x\neq 0$, $\cos x\neq 0$ va $(*)$ tenglamadan $\sin 4x-\sin 3x=0$ yoki $2\sin\frac{x}{2}\cos\frac{7x}{2}=0$, $\sin\frac{x}{2}\neq 0$ chunki, $x\neq\frac{\pi}{4}k$ (k-butun son), u holda $\cos\frac{7x}{2}=0$ tenglama ildizlari $x_1=\frac{\pi}{7}(2n+1)$ $(n-butun\ son)$ bo‘ladi. Ammo, x ning topilgan barcha qiymatlari berilgan tenglama ildizlari bo‘lmasligi mumkin. Argument x ning bu qiymatlaridan tenglamaning aniqlanish sohasiga kirmaydigan qiymatlarini chiqarib tashlashga to‘g‘ri keladi.

Shunday qilib, x ning yechim bo‘lmaydigan qiymatlarini $\frac{\pi}{7}(2n+1)=\frac{\pi}{4}k$ tenglik yordamida aniqlaymiz. Bundan $8n+4=7k$, va $k=4m$, m-butun son. Demak, $8n+4=28m$ yoki $2n+1=7m$ bo‘lib bunda m toq son ya’ni t-butun son uchun $m=2t+1$ bo‘lib, $2n+1=14t+7$ yoki $n=7t+3$.

Natijada, berilgan tenglama ildizlari $x_1=\frac{\pi}{7}(2n+1)$ bo‘lib, bu yerda $n\neq 7t+3$ va n, t-butun sonlar.

4.4.6 $sin^8x+cos^8x=\frac{17}{32}$ tenglamani yeching.

Yechish. Trigonometrik va qisqa ko‘paytirish ayniyatlaridan foydalanib berilgan tenglamada quyidagi shakl almashtirishlarni bajarish mumkin:

$$(sin^4x+cos^4x)^2-2sin^4xcos^4x=\frac{17}{32},$$
$$((sin^2x+cos^2x)^2-\frac{1}{2}sin^22x)^2-\frac{1}{8}sin^42x=\frac{17}{32},$$

$$1 - sin^2 2x + \frac{1}{8} sin^4 2x = \frac{17}{32},$$

$$sin^4 2x - 8sin^2 2x + \frac{15}{4} = 0.$$

Oxirgi tenglamadan

$$sin^2 2x = 4 \pm \frac{7}{2} \qquad yoki \quad sin^2 2x = \frac{1}{2} \quad bo'lib,$$

$$2x = \frac{\pi}{4} + \frac{\pi k}{2} \quad yoki \quad x = \frac{(2k+1)\pi}{8}. (k \epsilon Z)$$

4.4.7. $\cos^2 x + \frac{1}{\cos^2 x} = \cos 2x + \cos 4x$ tenglamani yeching.

Yechish. Koshi (3) tengsizligiga ko'ra: $\cos^2 x + \frac{1}{\cos^2 x} \geq 2$ bo'lib, ikkinchi tomondan $\cos 2x \leq 1$ va $cos4x \leq 1$, u holda:

$\cos 2x + \cos 4x \leq 2.$

Demak, $\begin{cases} \cos^2 x + \frac{1}{\cos^2 x} = 2, \\ \cos 2x + \cos 4x = 2 \end{cases}$ bo'lib, $\begin{cases} \cos^2 x = 1, \\ \cos 2x = 1, \\ \cos 4x = 1 \end{cases}$ tengliklar bajarilishi kerak bo'ladi.

Bulardan $x = \pi k$, k-butun son .

4.4.8. $3 \sin x + 4 \cos 3x \cdot \cos x + 2 \sin 5x = 7$ tenglamani yeching.

Yechish. Koshi-Bunyakovskiy (9) tengsizligidan foydalanib quyidagi tengsizlikni yozish mumkin:

$(3 \sin x + 4 \cos 3x \cdot \cos x)^2 \leq (9 + 16 \cos^2 3x) \cdot (\sin^2 x + \cos^2 x) \leq$

≤ 25

Bundan $3 \sin x + 4 \cos 3x \cdot \cos x \leq 5$ bo'ladi. $2 \sin 5x \leq 2$ bo'lganligidan,

$$3 \sin x + 4 \cos 3x \cdot \cos x + 2 \sin 5x \leq 7$$

tengsizlikda tenglik bajarilishi uchun bir vaqtda $\frac{3}{4 \cos 3x} = \frac{\sin x}{\cos x}$, $\cos^2 3x = 1$ va $\sin 5x = 1$ shartlar bajarilishi lozim.

Oxirgi ikkita tenglik bir vaqida bajarilishi uchun $\begin{cases} \cos 3x = \pm 1, \\ \sin 5x = 1 \end{cases}$

yoki $\begin{cases} x = \frac{\pi}{3} n, \\ x = \frac{\pi}{10}(4m+1) \end{cases}$ bo'lishi kerak bu yerda n, m-butur sonlar

$\frac{\pi}{3}n = \frac{\pi}{10}(4m+1)$ u holda, $10m = 12m + 3$ bo'lib, bu tenglikning chap tomoni juft o'ng tomoni esa doim toq bo'ladi. Demak, berilgan tenglama yechimga ega emas.

4.4.9. $sin^{10}x + cos^{10}x = \frac{29}{16}cos^4 2x$ tenglamani yeching.

Yechish. Darajani pasaytirish formulalaridan foydalarib berilgan tenglamada quyidagi almashtirishlarni bajarish mumkin:

$$\left(\frac{1-\cos 2x}{2}\right)^5 + \left(\frac{1+\cos 2x}{2}\right)^5 = \frac{29}{16}cos^4 2x,$$

$$(1-\cos 2x)^5 + (1+\cos 2x)^5 = 58cos^4 2x,$$

$\cos 2x = y$ almashtirish yordamida haqiqiy ildizlari

$$y_{1,2} = \pm\frac{\sqrt{2}}{2}$$

bo'lgan

$$24y^4 - 10y^2 - 1 = 0$$

bikvadrat tenglamani hosil qilish mumkin.

Demak, $\cos 2x = \pm\frac{\sqrt{2}}{2}$ bo'lib, berilgan tenglamaning yechimi $x=\frac{2k+1}{8}\pi$, ($k\epsilon Z$) bo'ladi.

4.4.10. $\sin x + 2\sin 2x = 3 + \sin 3x$ tenglamani yeching.

Yechish. Berilgan tenglamadan unga teng kuchli

$2\sin 2x + \sin x - \sin 3x = 3$, $2\sin 2x - 2\sin x \cdot \cos 2x = 3$

yoki

$$\sin 2x - \sin x \cdot \cos 2x = \frac{3}{2} \qquad (*)$$

tenglamani hosil qilish mumkin.

Koshi-Bunyakovskiy (9) tengsizligidan foydalanib $(*)$ tenglama chap tomoni uchun

$(\sin 2x - \sin x \cdot \cos 2x)^2 \le (1+\sin^2 x)(\sin^2 2x + \cos^2 2x) \le 2$

tengsizlikni hosil qilamiz.

Bundan $2\sin 2x - \sin x \cdot \cos 2x \le \sqrt{2}$, ya'ni

$\sin 2x - \sin x \cdot \cos 2x < \frac{3}{2}$.

Demak, $(*)$ tenglama va berilgan tenglama ildizlarga ega emas.

4.4.11. $\sqrt{2}(\sin x + \cos x) = tgx + ctg\,x$ tenglamani yeching.

Yechish. $\sin x + \cos x = \sqrt{2}\sin\left(x + \frac{\pi}{4}\right)$ bo'lib, tenglama chap

tomoni uchun $-2 \le \sqrt{2}(\sin x + \cos x) \le 2$ tengsizlik o'rinli bo'ladi.

$tgx + ctgx = tgx + \frac{1}{tgx}$ tenglikdan Koshi (3) va (4) tengsizligiga ko'ra: $|tgx + ctgx| \ge 2$ bo'lib, berilgan tenglama uning ikkala tomoni bir vaqtda -2 yoki 2 ga teng bo'lganda o'rinli bo'ladi. Natijada, $\begin{cases}\sqrt{2}(\sin x + \cos x) = -2,\\ tgx + ctgx = -2\end{cases}$ va $\begin{cases}\sqrt{2}(\sin x + \cos x) = 2,\\ tgx + ctgx = 2\end{cases}$ tenglamalar sistemasi hosil bo'ladi. Birinchi tenglamalar sistemasidan $\sin x + \cos x = -\sqrt{2}$ yoki $\sin x = \cos x = -\frac{\sqrt{2}}{2}$ va $tgx = -1$ bo'ladi. Sistema umumiy yechimga ega emas, ikkinchi tenglamalar sistemasidan $\sin x = \cos x = \frac{\sqrt{2}}{2}$ va $tgx = 1$ bo'lib, bu tenglamalarning ildizlari n-butun son uchun $x_1 = \frac{\pi}{4} + 2\pi n$ bo'ladi.

4.4.12. $\sin(\frac{\pi}{10} + \frac{3x}{2}) = 2\sin(\frac{3\pi}{10} - \frac{x}{2})$ tenglamani yeching.

Yechish. $\frac{3\pi}{10} - \frac{x}{2} = y$ bo'lsin, u holda:

$\frac{\pi}{10} + \frac{3x}{2} = \pi - 3\left(\frac{3\pi}{10} - \frac{x}{2}\right) = \pi - 3y$ bo'lib, berilgan tenglama quyidagi ko'rinishga keladi:

$$\sin 3y = 2\sin y,$$
$$\sin y\,(4sin^2 y - 1) = 0.$$

Oxirgi tenglamaning yechimlari:

$$y_1 = \pi k, y_2 = (-1)^k \frac{\pi}{6} + \pi k, y_3 = (-1)^{k+1} \frac{\pi}{6} + \pi k, k \epsilon Z$$

bo'ladi. Mos ravishda berilgan tenglamaning yechimlari:

$$x_1 = \frac{3\pi}{5} + \pi n, x_2 = \frac{3\pi}{5} + (-1)^n \frac{\pi}{3} + \pi n,$$
$$x_3 = \frac{3\pi}{5} + (-1)^{n+1} \frac{\pi}{3} + \pi n, n \epsilon Z$$

4.4.13. $\cos^2 x - \cos^4 x = \sin^2 x \cdot \sin 3x - 1$ tenglamani yeching.

Yechish. Tenglamaning chap va o'ng tomonlarini quyidagicha baholaymiz: $|\cos x| \le 1$ bo'lib, $\cos^4 x \le \cos^2 x$ va $\cos^2 x - \cos^4 x \ge 0$ shuningdek, $\sin^2 x \le 1$, $\sin 3x \le 1$ bo'lib, $\sin^2 x \cdot \sin 3x - 1 \le 0$. Demak, berilgan tenglama $\begin{cases}\cos^2 x - \cos^4 x = 0,\\ \sin^2 x \cdot \sin 3x = 1\end{cases}$ (*) tenglamalar sistemasiga teng kuchli bo'lib,

$\cos^2 x - \cos^4 x = \cos^4 x \cdot (1 - \cos^2 x) = \sin^2 x \cdot \cos^2 x$,

$0 \le \sin^2 x \le 1$ va $-1 \le \sin 3x \le 1$ bo'ladi. U holda (*) tenglamalar sistemasi $\begin{cases} \sin x \cdot \cos x = 0, \\ \sin^2 x = 1, \\ \sin 3x = 1 \end{cases}$ (**) tenglamalar sistemasiga teng kuchli bo'ladi.

Agar $\sin^2 x = 1$ bo'lsa, u holda $\cos x = 0$ va $\sin x \cdot \cos x = 0$ bo'lib, $\sin 3x = -4\sin^3 x + 3\sin x$ formulaga ko'ra (**) tenglamalar sistemasidan $\begin{cases} \sin x = \pm 1, \\ 4\sin^3 x - 3\sin x = -1 \end{cases}$ (***) tenglamalar sistemasi hosil bo'ladi va bu sistema $\sin x = -1$ tenglamaga teng kuchli bo'lib, uning, ya'ni berilgan tenglamaning ildizlari $x_1 = -\frac{\pi}{2} + 2\pi k$ bu yerda k – butun son.

4.4.14. $\left(\sin x + \sqrt{3}\cos x\right) \cdot \sin 4x = 2$ tenglamani yeching.

Yechish. Berilgan tenglamadan osongina

$\left(\frac{1}{2}\sin x + \frac{\sqrt{3}}{2}\cos x\right) \cdot \sin 4x = 1$ yoki $\sin\left(x + \frac{\pi}{3}\right) \cdot \sin 4x = 1$(*)

tenglamani hosil qilish mumkin.

Ma'lumki, $-1 \le \sin\left(x + \frac{\pi}{3}\right) \le 1$ va $-1 \le \sin 4x \le 1$ bo'lib, u holda (*) tenglamadan quyidagi ikkita

$$\begin{cases} \sin\left(x + \frac{\pi}{3}\right) = -1, \\ \sin 4x = -1 \end{cases} \text{ va } \begin{cases} \sin\left(x + \frac{\pi}{3}\right) = 1, \\ \sin 4x = 1 \end{cases} \quad (*)$$

tenglamalar sistemasi hosil bo'lib, birinchi sistemaning yechimlari m, n-butun sonlar uchun:

$$\begin{cases} x + \frac{\pi}{3} = -\frac{\pi}{2} + 2\pi n, \\ 4x = -\frac{\pi}{2} + 2\pi m \end{cases} \text{ yoki } \begin{cases} x = -\frac{5\pi}{6} + 2\pi k = \frac{\pi}{6}(12n - 5), \\ x = -\frac{\pi}{8} + \frac{\pi}{2}m = \frac{\pi}{8}(4m - 1) \end{cases}$$

ularning kesishmasi (umumiysi)ni topish uchun

$\frac{\pi}{6}(12n - 5) = \frac{\pi}{8}(4m - 1)$ yoki $4(12n - 5) = 3(4m - 1)$

bo'lib, oxirgi tenglikning chap tomoni doim juft, o'ng tomoni esa toq bo'ladi va (*) sistemaning birinchisi yechimga ega emas.

Ikkinchi tenglamalar sistemasining yechimlari butun k va l lar uchun: $\begin{cases} x + \frac{\pi}{3} = \frac{\pi}{2} + 2\pi k, \\ 4x = \frac{\pi}{2} + 2\pi l \end{cases}$ va $\begin{cases} x = \frac{\pi}{6} + 2\pi k = \frac{\pi}{6}(12k + 1), \\ x = \frac{\pi}{8} + \frac{\pi}{2}l = \frac{\pi}{8}(4l + 1) \end{cases}$

bo‘lib,bundan $\frac{\pi}{6}(12k+1)=\frac{\pi}{8}(4l+1)$ va $4(12k+1)=3(4l+1)$.

Yuqoridagi holga mos xulosa chiqarib berilgan tenglama haqiqiy ildizlarga ega emasligini ko‘rsatish mumkin.

4.4.15. $\sqrt{\frac{2\cos x}{3}}+\sqrt{\frac{2-2\cos x}{3}}=\frac{2}{3}\sin x+\frac{1}{2\sin x}$ tenglamani yeching.

Yechish. Shartga ko‘ra $\sin x>0$ va $\cos x\geq 0$ bo‘lib, butun n-soni uchun $2\pi n<x\leq\frac{\pi}{2}+2\pi n$ (x- birinchi chorakka tegishdi). Koshi-Bunyakovskiy (9) tengsizligiga ko‘ra berilgan tenglamaning chap tomoni:

$$\left(\sqrt{\frac{2\cos x}{3}}+\sqrt{\frac{2-2\cos x}{3}}\right)^2\leq(1^2+1^2)\left(\frac{2\cos x}{3}+\frac{2-2\cos x}{3}\right)=\frac{4}{3},$$

ya’ni $\sqrt{\frac{2\cos x}{3}}+\sqrt{\frac{2-2\cos x}{3}}\leq\frac{2}{\sqrt{3}}$ bo‘lib, $\sin x>0$ Koshi (1) tengsizligiga ko‘ra tenglamaning o‘ng tomoni:

$$\frac{2}{3}\sin x+\frac{1}{2\sin x}\geq 2\sqrt{\frac{2}{3}\sin x\cdot\frac{1}{2\sin x}}=\frac{2}{\sqrt{3}}\text{ bo‘ladi.}$$

Demak, $\frac{2}{3}\sin x+\frac{1}{2\sin x}=\frac{2}{\sqrt{3}}$ bo‘lib, $\sin x=\pm\frac{\sqrt{3}}{2}$ yoki $\cos x=\pm\frac{1}{2}$ ammo, shartga ko‘ra $\sin x>0$ va $\cos x\geq 0$ bo‘lib, $\sin x=\frac{\sqrt{3}}{2}$ va $\cos x=\frac{1}{2}$ topilgan $\sin x$ va $\cos x$ qiymatlarini berilgan tenglamaga qo‘yib uning ildizlari butun n-soni uchun $x_1=\frac{\pi}{3}+2\pi k$ ekanligiga ishonch hosil qilish mumkin.

4.4.16. $sin^3x+sin^32x+sin^33x=(\sin x+\sin 2x+\sin 3x)^3$ tenglamani yeching.

Yechish. Avvaldan ma’lum:

$$(x+y+x)^3-x^3-y^3-z^3=3(x+y)(z+x)(z+y)$$

ayniyatdan foydalanib berilgan tenglamada quyidagicha almashtirishlar bajarish mumkin:

$$(\sin x+\sin 2x)(\sin 2x+\sin 3x)(\sin x+\sin 3x)=0,$$

$$\sin\frac{3x}{2}\sin 2x\sin\frac{5x}{2}\cos x\cos^2\frac{x}{2}=0.$$

Oxirgi tenglikdagi har bir ko‘paytuvchini 0 ga tenglab quyidagi

beshta yechimni yozish mumkin:

1) $x=\frac{2n_1}{3}\pi$; 2) $x=\frac{n_2}{2}\pi$; 3) $x=\frac{2n_3}{5}\pi$; 4) $x=\frac{2n_4+1}{2}\pi$;

5) $x=(2n_5+1)\pi$. Bu yerda n_1, n_2, n_3, n_4, n_5-butun sonlar.

4 va 5 yechimlar 2-yechimda qatnashadi. Demak, berilgan tenglamaning yechimlari:

$x=\frac{2n_1}{3}\pi; x=\frac{n_2}{2}\pi; x=\frac{2n_3}{5}\pi$. Bu yerda n_1, n_2, n_3-butun sonlar.

4.4.17. $\left(\sin^2 x+\frac{1}{\sin^2 x}\right)^2+\left(\cos^2 x+\frac{1}{\cos^2 x}\right)^2=12+\frac{\sin y}{2}$

tenglamani yeching.

Yechish. $n=2$ bo'lganda (10) tengsizlikka ko'ra:

$a^2+b^2\geq\frac{(a+b)^2}{2}$ $(*)$. Bu tengsizlikdan foydalanib tenglamaning chap tomonini quyidagicha yozish mumkin:

$$\left(\sin^2 x+\frac{1}{\sin^2 x}\right)^2+\left(\cos^2 x+\frac{1}{\cos^2 x}\right)^2\geq$$

$$\geq\frac{1}{2}\left(\sin^2 x+\frac{1}{\sin^2 x}+\cos^2 x+\frac{1}{\cos^2 x}\right)^2=$$

$$=\frac{1}{2}\left(1+\frac{1}{\sin^2 x\cdot\cos^2 x}\right)^2=\frac{1}{2}\left(1+\frac{4}{\sin^2 2x}\right)^2\geq$$

$$\geq\frac{1}{2}(1+4)^2=12\frac{1}{2}.$$

Ikkinchi tomondan: $12+\frac{y}{2}\leq 12\frac{1}{2}$. Natijada,

$$\begin{cases}\sin^2 x+\frac{1}{\sin^2 x}=\cos^2 x+\frac{1}{\cos^2 x},\\ \sin y=1\end{cases}\qquad(**)$$

tenglamalar sistemasini hosil qilish mumkin. $0<x<1$ bo'lganda $f(z)=z^2+\frac{1}{z^2}$funksiyani qaraymiz u holda $(**)$ tenglamalar sistemasining birinchi tenglamasi $f(\sin x)=f(\cos x)$ ko'rinishidagi funksional tenglama bo'lib, $0<z<1$ uchun $f'(z)=\frac{2(z^4-1)}{z^3}<0$ bo'lib, $f(x)$ funksiya shu oraliqda kamayuvchi bo'ladi. Shuningdek, $f(z)$ – juft funksiya ekanligidan $f(\sin x)=f(\cos x)$ tenglama $\sin x=\pm\cos x$ tenglamaga teng kuchli bo'lib, bu tenglamaning ildizi n-butun son uchun $x_1=\frac{\pi}{4}(2n+1)$ bo'ladi. $(**)$ sistema ikkinchi tenglamasining ildizi k-butun son uchun $y_1=\frac{\pi}{2}(4k+1)$ bo'ladi.

4.4.18. $\cos x + cosy - \cos(x+y) = \frac{3}{2}$ tenglamani yeching.

Yechish. Tenglamani quyidagicha shakl almashtirish mumkin:

$$2\cos\frac{x+y}{2}\cdot\cos\frac{x-y}{2} - 2\cos^2\frac{x+y}{2} + 1 = \frac{3}{2},$$

$$2\cos^2\frac{x+y}{2} - 2\cos\frac{x+y}{2}\cdot\cos\frac{x-y}{2} + \frac{1}{2} = 0,$$

$$4\cos^4\frac{x+y}{2} - 4\cos\frac{x+y}{2}\cdot\cos\frac{x-y}{2} + $$

$$+\cos^2\frac{x-y}{2} - \cos^2\frac{x-y}{2} + 1 = 0.$$

$$\left(2\cos\frac{x+y}{2} - 2\cos\frac{x-y}{2}\right)^2 + \sin^2\frac{x-y}{2} = 0 \ (*)$$

$(*)$ tenglamadan $\begin{cases} 2\cos\frac{x+y}{2} = \cos\frac{x-y}{2}, \\ \sin\frac{x-y}{2} = 0 \end{cases}$ $(**)$ tenglamalar sistemasini hosil qilish mumkin.

$(**)$ sistemaning ikkinchi tenglamasidan $\cos\frac{x-y}{2} = \pm 1$ bo'lib, u holda $(**)$ tenglamalar sistemasi

$$\begin{cases} \cos\frac{x+y}{2} = \pm\frac{1}{2}, \\ \sin\frac{x-y}{2} = 0 \end{cases} \qquad (***)$$

tenglamalar sistemasiga teng kuchli bo'ladi. $(***)$ tenglamalar sistemasining yechimlari n va k butun sonlar uchun: $\frac{x+y}{2} = \frac{\pi}{3}(3n \pm 1)$ va $\frac{x-y}{2} = \pi k$ bo'ladi.

Natijada, berilgan tenglama ildizlari butun n va k butun sonlar uchun $x_1 = \frac{\pi}{3}(3n + 3k + 1)$, $y_1 = \frac{\pi}{3}(3n - 3k + 1)$,

$x_2 = \frac{\pi}{3}(3n + 3k - 1)$, $y_2 = \frac{\pi}{3}(3n - 3k - 1)$ bo'ladi.

4.4.19. $\frac{\sin^{10}x + \cos^{10}x}{4} = \frac{\sin^6 x + \cos^6 x}{4\cos^2 2x + \sin^2 2x}$ tenglamani yeching.

Yechish. Tenglamaning chap tomonini quyidagicha shakl almashtirish mumkin: $\frac{\sin^6 x + \cos^6 x}{4\cos^2 2x + \sin^2 2x} = \frac{\sin^6 x + \cos^6 x}{4(\cos^2 x + \sin^2 x) + 4\sin^2 x\cdot\cos^2 x} =$

$= \frac{(\sin^2 x + \cos^2 x)(\sin^4 x - \sin^2 x\cdot\cos^2 x + \cos^2 x)}{4(\sin^4 x - \sin^2 x\cdot\cos^2 x + \cos^4 x)} = \frac{1}{4}$. U holda berilgan tenglama: $\sin^{10}x + \cos^{10}x = 1$ $(*)$ ko'rinishga keladi.

$sin^2x + cos^2x = 1$ ayniyatdan foydalanib $(*)$ tenglamani quyidagi

ko‘rinishda yozish mumkin: $\sin^{10}x+\cos^{10}x=\sin^2x+\cos^2x$, $\sin^2x-\sin^{10}x+\cos^2x-\cos^{10}x=0$ yoki $\sin^2x\cdot(1-\sin^8x)+\cos^2x\cdot(1-\cos^8x)=0$ (**) Ma’lumki, $\sin^2x\geq 0$, $\cos^2x\geq 0$, $\sin^8x\leq 1$, $\cos^8x\leq 1$ va $1-\sin^8x\geq 0$, $1-\cos^8x\geq 0$ bo‘lib, ularga ko‘ra (**) tenglamadan $\begin{cases}\sin x=\pm 1,\\ \cos x=0\end{cases}$ yoki $\begin{cases}\sin x=0,\\ \cos x=\pm 1\end{cases}$ bo‘lib, ildizlari n-butun son uchun $x_1=\frac{\pi}{2}n$ bo‘ladi.

4.4.20. $(\cos 4x-\cos 2x)^2=\sin 3x+5$ tenglamani yeching.

Yechish. Ma’lumki,

$$|\cos 4x-\cos 2x|\leq 2\ va\ \sin 3x+5\geq 4$$

munosabatlar o‘rinli bo‘lib,berilgan tenglama yechimga ega bo‘lishi uchun

$$|\cos 4x-\cos 2x|=2\ va\ \sin 3x=-1$$

tengliklar bajarilishi lozim.Bunda ikki holni qarashga to‘g‘ri keladi:

$$a)\ \cos 2x=1, x=\pi k;$$

$$\cos 4x=-1, x=\left(\frac{n}{2}+\frac{1}{4}\right)\pi;$$

$$\sin 3x=-1, x=-\frac{\pi}{6}+\frac{2\pi}{3}l.$$

n,k,l-butun sonlar. Ushbu holda umumiy yechim mavjud emas.

$$b)\ \cos 4x=1, x=\frac{n}{2}\pi;$$

$$\cos 2x=-1, x=\left(k+\frac{1}{2}\right)\pi;$$

$$\sin 3x=-1, x=-\frac{\pi}{6}+\frac{2}{3}\pi l=\frac{4l-1}{6}\pi.$$

n,k,l-butun sonlar. Bu holda umumiy yechim:

$$x=\left(2m+\frac{1}{2}\right)\pi, m\epsilon Z.$$

4.4.21. $1+\cos^6x=2\cdot\sqrt[3]{\cos 2x}$ tenglamani yeching.

Yechish. Ma’lumki, $\cos 2x=2\cos^2x-1$ bo‘lib, u holda berilgan tenglamani $1+\cos^6x=2\cdot\sqrt[3]{2\cos^2x-1}$ ko‘rinishda yozib $\cos^2x=y$ almashtirish yordamida $0\leq y\leq 1$ bo‘lganda:

$1+y^3=2\cdot\sqrt[3]{2y-1}$ (*) tenglamani yozish mumkin.U holda, $1+y^3\geq 1$ bo‘lib, (*) tenglamadan $2\cdot\sqrt[3]{2y-1}\geq 1$ yoki $y\geq\frac{9}{16}$.

Shunday qilib, $\frac{9}{16} \le y \le 1$ va $(*)$ tenglama $y = \sqrt[3]{2 \cdot \sqrt[3]{2y-1} - 1}$ $(**)$ ko'rinishni oladi.

Agar $f(z) = \sqrt[3]{2z-1}$ bo'lsa $(**)$ tenglama $u = f(f(y))$ ko'rinishdagi funksional tenglama bo'lib, $f(z) = \sqrt[3]{2z-1}$ funksiya oz o'qida o'suvchi, u holda $y = f(f(y))$ tenglama o'zgaruvchi qabul qilishi mumkin bo'lgan qiymatlarida $f(y) = y$ tenglamaga teng kuchli, ya'ni $\frac{9}{16} \le y \le 1$ bo'lganda $y=\sqrt[3]{2y-1}$ tenglikning ikkala tomonini kubga ko'tarib, $y^3 - 2y + 1 = 0$ yoki $(y-1)(y^2+y-1) = 0$ bundan $\frac{9}{16} \le y \le 1$ shartga ko'ra $y_1 = 1$ va $y_2 = \frac{\sqrt{5}-1}{2}$.

Shuningdek, $\cos^2 x = y$ bo'lib u holda $\cos x = \pm 1$ va $\cos x = \pm\sqrt{\frac{\sqrt{5}-1}{2}}$ tengliklardan berilgan tenglamaning ildizlari butun n va k sonlar uchun $x_1 = \pi n$ va $x_2 = \pm arccos\sqrt{\frac{\sqrt{5}-1}{2}} + \pi k$ bo'ladi.

4.4.22. $\sqrt[4]{2\sin x - 1} + \sqrt[4]{3\sin x - 2} = 2$ tenglamani yeching.

Yechish. $\sin x \le 1$ bo'lib $2\sin x - 1 \le 1$ va $3\sin x - 2 \le 1$ bo'lganligidan $\sqrt[4]{2\sin x - 1} \le 1$, $\sqrt[4]{3\sin x - 2} \le 1$ va

$\sqrt[4]{2\sin x - 1} + \sqrt[4]{3\sin x - 2} \le 2$.

Demak, berilgan tenglama faqat va faqat $\sqrt[4]{2\sin x - 1} = 1$ va $\sqrt[4]{3\sin x - 2} = 1$ tengliklar bajarilgandagina o'rinli bo'ladi.

Demak, $\sin x = 1$ yoki $x_1 = \frac{\pi}{2} + 2\pi n$ bu yerda n- butun son .

4.4.23. $32cos^6 x - \cos 6x = 1$ tenglamani yeching.

Yechish.

$$cos^6 x = \left(\frac{1+\cos 2x}{2}\right)^3 = \frac{1}{8}(1 + 3\cos 2x + 3cos^2 2x + cos^3 2x)$$

va

$$\cos 6x = 4cos^3 2x - 3\cos 2x$$

ayniyatlardan foydalanib berilgan tenglamani quyidagi ko'rinishda yozish mumkin:

$$4cos^2 2x + 5\cos 2x + 1 = 0$$

yoki

$$(\cos 2x)_1 = -1, (\cos 2x)_2 = -\frac{1}{4}$$

bo‘lib,

$x_1 = \left(k + \frac{1}{2}\right)\pi, x_2 = \pm\frac{1}{2}\arccos\left(-\frac{1}{4}\right) + \pi k, k \epsilon Z$ yoki

$x = \frac{\pi}{6} + \frac{\pi n}{3}, n \in Z$

4.4.24. $2\sin^2 x + 2\sin^2 y = \sin x + \sin y + \sin x \cdot \sin y - 1$ tenglamani yeching.

Yechish. Tenglama ikkala tomonini 2 ga ko‘paytirib, almashtirishlar yordamida:

$$4sin^2x + 4sin^2y - 2\sin x - 2\sin y - 2\sin x \cos y + 2 = 0,$$
$$(\sin^2 x - 2\sin x + 1) + (\sin^2 y - 2\sin y + 1)$$
$$+ (\sin^2 x - 2\sin x \cdot \sin y + \sin^2 y) = 0,$$
$$(\sin x - 1)^2 + (\sin y - 1)^2 + (\sin x - \sin y)^2 = 0$$

tenglamani hosil qilish mumkin. Tenglama chap tomonidagi ko‘paytuvchilar nomanfiy bo‘lib tenglik $\sin x = \sin y = 1$ bo‘lganda bajariladi.

Demak, $x_1 = \frac{\pi}{2} + 2\pi n$ va $y_1 = \frac{\pi}{2} + 2\pi k$ bu yerda n, k- butun sonlar.

4.4.25. $2 + 2(\sin y + \cos y) \cdot \sin x = \cos 2x$ tenglamani yeching.

Yechish. $\cos 2x = 1 - 2\sin^2 x$ formuladan foydalanib berilgan tenglamani $\sin x$ ga nisbatan kvadrat tenglama ko‘rinishida yozish mumkin:

$$2sin^2x + 2(\sin y + \cos y)\sin x + 1 = 0 \qquad (*)$$

Ushbu tenglama ildizga ega bo‘lishi uchun uning diskriminanti nomanfiy bo‘lishi kerak, ya’ni, D=$((\sin y + \cos y)^2 - 2 \geq 0$.

Ammo, $-\sqrt{2} \leq \sin y + \cos y \leq \sqrt{2}$ ekanligidan $|\sin y + \cos y| = \sqrt{2}$ tenglik o‘rinli bo‘ladi.

Agar $\cos y + \sin y = \sqrt{2}$ bo‘lsa, u holda $y_1 = \frac{\pi}{4} + 2\pi n$ bo‘lib $(*)$ tenglamadan n va k butun sonlar uchun $2\sin^2 x + 2\sqrt{2}\sin x + 1 = 0$, $\left(\sqrt{2}\sin x + 1\right)^2 = 0, \sin x = -\frac{\sqrt{2}}{2}$ va $x_1 = (-1)^{k+1}\frac{\pi}{4} + \pi k$ bo‘ladi.

Agar $\cos y + \sin y = -\sqrt{2}$ bo‘lsa, u holda $y_2 = \frac{5\pi}{4} + 2\pi l, (l \in Z)$

bo‘lib, tenglama

$$2\sin^2 x - 2\sqrt{2}\sin x + 1 = 0$$

ko‘rinishni oladi. Bundan $\left(\sqrt{2}\sin x - 1\right)^2 = 0$, $\sin x = \frac{\sqrt{2}}{2}$ va $x_2 = (-1)^m \frac{\pi}{4} + \pi m$ bu yerda m-butun son.

4.4.26. $x^2 + 2\sin \pi x = 3x + \cos^2 \pi x - \frac{17}{4}$ tenglamani yeching.

Yechish. Berilgan tenglamani quyidagicha shakl almashtirish mumkin:

$$x^2 + 2\sin \pi x = 3x + 1 - \sin^2 \pi x - \frac{17}{4},$$

$$x^2 + 2\sin \pi x - 3x + \sin^2 \pi x + \frac{13}{4} = 0,$$

$$(\sin \pi x + 1)^2 + \left(x - \frac{3}{2}\right)^2 = 0.$$

Oxirgi tenglama $\begin{cases} \sin \pi x + 1 = 0, \\ x - \frac{3}{2} = 0 \end{cases}$ tenglamalar sistemasiga teng kuchli bo‘lib, u yagona $x_1 = \frac{3}{2}$ ildizga ega.

4.4.27. $1 - 2\sin^2 x + \sqrt{3} \cdot \sin 2x = 12x^2 - 4\pi x + \frac{\pi^2}{3} + 2$ tenglamani yeching.

Yechish. Tenglamaning o‘ng tomonida quyidagicha shakl almashtirishlarni bajarish mumkin:

$$1 - 2\sin^2 x + \sqrt{3}\sin 2x =$$

$$= \cos 2x + \sqrt{3} \cdot \sin 2x = 2 \cdot \left(\frac{1}{2}\cos 2x + \frac{\sqrt{3}}{2}\sin 2x\right)$$

$$= 2 \cdot \left(\sin \frac{\pi}{6} \cdot \cos 2x + \cos \frac{\pi}{6} \cdot \sin 2x\right) = 2\sin\left(2x + \frac{\pi}{6}\right).$$

Natijada, $12x^2 - 4\pi x + \frac{\pi^2}{3} + 2 = 2\sin\left(2x + \frac{\pi}{6}\right)$ tenglama hosil bo‘ladi va bundan:

$12x^2 - 4\pi x + \frac{\pi^2}{3} + 2 \le 2$, $\quad 36x^2 - 12\pi x + \pi^2 \le 0$ yoki $(6x - \pi)^2 \le 0$ va $(6x - \pi)^2 = 0$ yoki $x_1 = \frac{\pi}{6}$. x- ning bu qiymatini berilgan tenglamaga qo‘yib ko‘rib, $x_1 = \frac{\pi}{6}$ berilgan tenglama ildizi ekanligini ko‘rish mumkin.

4.4.28. $(\sin^4 x + \cos^2 2x) \cdot \left(\frac{1}{\sin^4 x} + \frac{1}{\cos^2 2x}\right) = 4\cos^2 \sqrt{\frac{\pi^2}{4} - x^2}$ tenglamani yeching.

Yechish. Tenglamaning aniqlanish sohasi $-\frac{\pi}{2} \le x < 0$ va $0 < 0 \le \frac{\pi}{2}$ oraliqlardan iborat.

$a > 0, b > 0$ bo'lganda $(a+b) \cdot \left(\frac{1}{a} + \frac{1}{b}\right) \ge 4$ $(*)$ tengsizlikdan foydalanib tenglamaning chap tomoni

$(\sin^4 x + \cos^2 2x)\left(\frac{1}{\sin^4 x} + \frac{1}{\cos^2 2x}\right) \ge 4$ bo'lib, o'ng tomoni esa $4\cos^2 \sqrt{\frac{\pi^2}{4} - x^2} \le 4$ bo'ladi.

Demak, $\begin{cases} (\sin^4 x + \cos^2 2x) \cdot \left(\frac{1}{\sin^4 x} + \frac{1}{\cos^2 2x}\right) = 4, \\ 4\sin^2 \sqrt{\frac{\pi^2}{3} - x^2} = 4 \end{cases}$ bo'lib, ushbu sistemadan tenglamaning aniqlanish sohasini hisobga olib $x_1 = \frac{\pi}{2}$ va $x_2 = -\frac{\pi}{2}$ ildizlarni topish mumkin. Ularni berilgan tenglamaga qo'yib ko'rib haqiqatan, ular berilgan tenglama ildizlari ekanligiga ishonch hosil qilish mumkin.

4.4.29. $\propto$-parametrning qanday qiymatlarida quyidagi tenglamalar sistemasi yechimga ega bo'ladi

$$\begin{cases} \sin x \sin y = \frac{5}{6}\sin \propto, \\ \cos x \cos y = \frac{5}{6}\cos \propto. \end{cases}$$

Yechish. Berilgan sistema tenglamalari ayirmasi va yig'indisidan:

$$\begin{cases} \cos(x+y) = \frac{5}{6}(\cos \propto - \sin \propto), \\ \cos(x-y) = \frac{5}{6}(\cos \propto + \sin \propto). \end{cases}$$

tenglamalar sistemasini hosil qilish mumkin.Ushbu sistema yechimga ega bo'lishining zaruriy va yetarli sharti:

$$\begin{cases} \left|\frac{5}{6}(\cos\alpha - \sin\alpha)\right| \le 1, \\ \left|\frac{5}{6}(\cos\alpha + \sin\alpha)\right| \le 1. \end{cases}$$

yoki

$$\begin{cases} \left|\cos\left(\alpha + \frac{\pi}{4}\right)\right| \le \frac{6}{5\sqrt{2}}, \\ \left|\cos\left(\alpha - \frac{\pi}{4}\right)\right| \le \frac{6}{5\sqrt{2}}. \end{cases}$$

kabi aniqlanadi. Bundan,

$$\begin{cases} arccos\frac{6}{5\sqrt{2}} + \pi k \le \alpha + \frac{\pi}{4} \le \pi - arccos\frac{6}{5\sqrt{2}} + \pi k, \\ arccos\frac{6}{5\sqrt{2}} + \pi m \le \alpha - \frac{\pi}{4} \le \pi - arccos\frac{6}{5\sqrt{2}} + \pi m, \end{cases}$$

yoki

$$\begin{cases} arccos\frac{6}{5\sqrt{2}} - \frac{\pi}{4} + \pi k \le \alpha \le \pi - arccos\frac{6}{5\sqrt{2}} - \frac{\pi}{4} + \pi k, \\ arccos\frac{6}{5\sqrt{2}} + \frac{\pi}{4} + \pi m \le \alpha \le \pi - arccos\frac{6}{5\sqrt{2}} + \frac{\pi}{4} + \pi m. \end{cases}$$

bo‘ladi.

Demak, m, k-butun sonlar uchun α parametr

$$arccos\frac{6}{5\sqrt{2}} + \frac{\pi}{4} + \pi n \le \alpha \le \frac{3\pi}{4} - arccos\frac{6}{5\sqrt{2}} + \pi n, n \epsilon Z$$

tengsizlikni qanoatlantirishi lozim bo‘ladi.

4.4.30. Quyidagi sistemani yeching:

$$\begin{cases} \sin x \cdot \cos y = -\frac{1}{2}, \\ \cos x \cdot \sin y = \frac{1}{2}. \end{cases}$$

Yechish. Tenglamalar sistemasining tenglamalarini qo‘shib va ikkinchi tenglamadan birinchisini ayirib: $\begin{cases} \sin(x+y) = 0, \\ \sin(y-x) = 1 \end{cases}$ tenglamalar sistemasini hosil qilish mumkin.

Bundan butun n, k lar uchun $x + y = \pi n$ va $y - x = \frac{\pi}{2} + 2\pi k$ bo‘lib natijada, berilgan tenglamaning ildizlari $x_1 = \frac{\pi}{4}(2n - 4k - 1)$,

$y_1 = \frac{\pi}{4}(2n + 4k + 1)$ bu yerda n, k butun sonlar bo'ladi.

4.4.31. Quyidagi sistemani yeching:

$$\begin{cases} ctgx + \sin 2y = \sin 2x, \\ 2\sin y \sin(x + y) = \cos x \end{cases}$$

Yechish. Qulaylik uchun $x=\frac{\pi}{2} - z$ almashtirish yordamida berilgan sistemani quyidagicha yozish mumkin:

$$\begin{cases} tgz + \sin 2y = \sin 2z, \\ 2\sin y \cos(z - y) = \sin z. \end{cases}$$

U holda sistemaning ikkinchi tenglamasi

$$\sin z + \sin(2y - z) = \sin z$$

yoki

$$\sin(2y - z) = 0$$

ko'rinishda bo'lib, $2y - z = \pi k, k \epsilon Z$ yoki $z=2y-\pi k$, $x=\frac{\pi}{2} - 2y + \pi k$.

U holda sistemaning birinchi tenglamasi:

$$tg2y + \sin 2y = \sin 4y,$$

$$\sin 2y\,(\cos 2y - 1)\left(\cos 2y + \frac{1}{2}\right) = 0,$$

$$\sin 2y = 0, \cos 2y = 1, \cos 2y = -\frac{1}{2}$$

ko'rinishga keladi va oxirgi tenglamaning birinchi ko'paytuvchisidan: $y=\frac{\pi n}{2}$ yoki $x=\frac{\pi}{2} + \pi l$. Ikkinchi ko'paytuvchisidan: $2y=2\pi n, y=\pi n$ yoki $x=\frac{\pi}{2} + \pi m$ va nihoyat, uchinchi ko'paytuvchisdan $2y=\pm\frac{2\pi}{3} + 2\pi m$ yoki $x=\frac{\pi}{2} \mp \frac{2\pi}{3} + \pi n$ yechimlarni olish mumkin.

Javob.

$$\begin{cases} x_1 = \frac{\pi}{2} + \pi l, \\ y_1 = \frac{\pi n}{2}. \end{cases} \begin{cases} x_2 = \frac{\pi}{2} \mp \frac{2\pi}{3} + \pi n, \\ y_2 = \pm\frac{\pi}{3} + \pi m. \end{cases}$$ m, n, l-butun sonlar

4.4.32. Quyidagi sistemani yeching:

$$\begin{cases} 4\sin x - 2\sin y = 3 \\ 2\cos x = \cos y = 0 \end{cases}$$

Yechish. Berilgan tenglamalar sistemasini quyidagicha yozish mumkin:

$$\begin{cases} \sin x = \frac{3}{4} + \frac{1}{2}\sin y, \\ \cos x = \frac{1}{2}\cos y \end{cases} \quad (*)$$

Bu tenglamalarni kvadratga ko'tarib qo'shilsa:

$1 = \frac{9}{16} + \frac{3}{4}\sin y + \frac{1}{4}\sin^2 y + \frac{1}{4}\cos^2 y$ yoki $\sin y = \frac{1}{4}$ va n- butun son uchun $y = (-1)^n \arcsin\frac{1}{4} + \pi n$ $(**)$ bo'lib, $(*)$ sistemaning birinchi tenglamasidan $\sin x = \frac{7}{8}$ va butun m- sonlari uchun

$x = (-1)^m \arcsin\frac{7}{8} + \pi m$ $(***)$.

$(*)$ tenglamalar sistemasini yozishda kvadratga ko'tarish amali bajarilganligi uchun chet ildizlar paydo bo'lib qolgan bo'lishi mumkin. Shuning uchun o'zgaruvchilarning $(**)$ va $(***)$ qiymatlarini $(*)$ sistemaning ikkinchi tenglamasiga qo'yganda $\cos x$ va $\cos y$ son qiymatlarining ishoralariga mos tushishi kerak. Natijada, juft m va n larda $(**)$ va $(***)$ formulaga mos $\cos x$ va $\cos y$ qiymatlarining ishoralari musbat m va n ning toq qiymatlarida esa manfiy bo'ladi.

Demak, berilgan tenglamalar sistemasining yechimlari butun m,n,k va l sonlari uchun

$$\begin{cases} x_1 = \arcsin\frac{7}{8} + 2\pi k, \\ y_1 = \arcsin\frac{1}{4} + 2\pi l \end{cases} \text{ va } \begin{cases} x_2 = -\arcsin\frac{7}{8} + \pi(2k+1), \\ y_2 = -\arcsin\frac{1}{4} + \pi(2l+1). \end{cases}$$

Mustaqil yechish uchun misollar

1. $sin^3 x \cos x - \sin x\, cos^3 x = \frac{1}{4}$, $x = -\frac{\pi}{8} + \frac{\pi k}{2}, k \epsilon Z$;
2. $\frac{1-tgx}{1+tgx} = 1 + \sin 2x$ $x = \pi k, k \epsilon Z$;
3. $1 + \sin x + \cos x + \sin 2x + \cos 2x = 0, x_1 = -\frac{\pi}{4} + \pi k, x_2 = \pm\frac{2\pi}{3} + 2\pi k$;
4. $1 + \sin x + \cos 3x = \cos x + \sin 2x + \cos 2x\, x_1 = \pi k, x_2 = (-1)^{k+1}\frac{\pi}{6} + \pi k, x_3 = \pm\frac{\pi}{3} + 2\pi k$;
5. $(\sin 2x + \sqrt{3}\cos 2x)^2 - 5 = \cos\left(\frac{\pi}{6} - 2x\right)$, $x = \frac{7\pi}{12} + \pi k$;

6. $2\sin 17x + \sqrt{3}\cos 5x + \sin 5x = 0, x_1 = -\frac{\pi}{66} + \frac{\pi k}{11}, x_2 = \frac{\pi}{36} + \frac{(2k+1)\pi}{12}$;

7. $sin^2x(tgx + 1) = 3\sin x(\cos x - \sin x) + 3,\ x_1 = -\frac{\pi}{4} + \pi k, x_2 = \frac{\pi}{3} + \pi k, x_3 = -\frac{\pi}{3} + \pi k$;

8. $sin^3x + cos^3x = 1 - \frac{1}{2}\sin 2x, x_1 = 2\pi k, x_2 = \frac{\pi}{2} + 2\pi k$;

9. $\frac{1}{sin^2x} - \frac{1}{cos^2x} - \frac{1}{tg^2x} - \frac{1}{ctg^2x} - \frac{1}{sec^2x} - \frac{1}{cosec^2x} = -3$, $x = \frac{1}{4}\pi + \frac{\pi k}{2}$;

10. $sin^4\frac{x}{3} + cos^4\frac{x}{3} = \frac{5}{8}$, $x = \frac{3n \pm 1}{2}\pi$;

11. $\frac{1}{2}(sin^4x + cos^4x) = sin^2xcos^2x + \sin x\cos x$,
$x = (-1)^k\frac{1}{2}arcsin\frac{\sqrt{5}-1}{2} + \frac{\pi k}{2}$;

12. $(1 + k)\frac{\cos x\cos(2x-\alpha)}{\cos(x-\alpha)} = 1 + k\cos 2x$,
$x = \frac{\alpha}{2} + (-1)^n\frac{1}{2}\arcsin(k\sin\alpha) + \frac{\pi n}{2}, |\ k\sin\alpha\ | \leq 1$;

13. $2 + \cos x = 2tg\frac{x}{2}$, $x = \frac{\pi}{2} + 2\pi k$;

14. $\operatorname{ctg}x - 2\sin 2x = 1 x_1 = \frac{\pi}{8} + \frac{\pi k}{2}, x_2 = \frac{3\pi}{4} + \pi k$;

15. $2\cos x\cos(\beta - x) = \cos\beta$, $x = \pm\frac{\pi}{4} + \frac{\beta}{2} + \pi k$,

16. $\sin x + \sin(\varphi - x) + \sin(2\varphi + x) = \sin(\varphi + x) + \sin(2\varphi - x)$ $va\varphi$ burchak koordinata tekisligining uchinchi choragida yotishi ma'lum bo'lsa $\cos\varphi$ ni hisoblang

$$\cos\varphi = \frac{1-\sqrt{5}}{4}.$$

17. $\sin 2x - 12(\sin x - \cos x) + 12 = 0,\ x_1 = \pi + 2\pi, x_2 = \frac{\pi}{2} + 2\pi k$;

18. $1 + 2\operatorname{cosec}x = -\frac{sec^2\frac{x}{2}}{2}$, $x = -\frac{\pi}{2} + 2\pi k$;

19. $ctg^2x = \frac{1+\sin x}{1+\cos x}$, $x_1 = \frac{\pi}{4} + k\pi, x_2 = -\frac{\pi}{2} + 2k\pi$;

20. $2tg3x - 3tg2x = tg^2 2xtg3x$, $x = k\pi$;

21. $6\operatorname{tg}x + 5\operatorname{ctg}3x = \operatorname{tg}2x$ $x = \pm\frac{1}{2}arccos\frac{1}{3} + \pi k$, $x = \pm\frac{1}{2}(\pi - arccos\frac{1}{4}) + \pi k$

22. $sin^5x - cos^5x = \frac{1}{\cos x} - \frac{1}{\sin x}$ $x = \frac{\pi}{4} + \pi n$

23. $tg(x-\frac{\pi}{4})$tgxtg(x+$\frac{\pi}{4}$) $=\frac{4cos^2x}{tg\frac{x}{2}-ctg\frac{x}{2}}\emptyset$

24. $\cos\pi\frac{x}{31}\cos 2\pi\frac{x}{31}\cos 4\pi\frac{x}{31}\cos 8\pi\frac{x}{31}\cos 16\pi\frac{x}{31}=\frac{1}{32}x_1=2n, n\neq 31l, x_2=\frac{31}{33}(2n+1), n\neq 33l+16;l\epsilon Z$

25. $\cos 7x-\sin 5x=\sqrt{3}(\cos 5x-\sin 7x)$ x=$\frac{\pi}{12,}(12k+1)$,x=$\frac{\pi}{24}(4m+1)$ (k,mϵZ)

26. $2-(7+\sin 2x)\,sin^2x+(7+\sin 2x)sin^4x=0$ x=$\frac{\pi}{4}+\pi k$,kϵZ

27. $\begin{cases}\sin(x+y)=0,\\ \sin(x-y)=0,\\ 0\leq x\leq\pi, 0\leq x\leq\pi.\end{cases}$ (0;0),($\frac{\pi}{2};\frac{\pi}{2}$), (0;$\pi$), ($\pi$; 0), ($\pi;\pi$)

28. $\begin{cases}\sin x=cosecx+\sin y,\\ \cos x=secx+\cos y.\end{cases}x=\frac{2k+2n+1}{4}\pi, y=\frac{2k-6n}{4}\pi$ (k,nϵZ)

29. $\begin{cases}sin^3x=\frac{1}{2}\sin y,\\ cos^3x=\frac{1}{2}\cos y.\end{cases}x_1=\frac{\pi}{4}+2k\pi, y_1=\frac{\pi}{4}+2l\pi; x_2=\frac{\pi}{4}+(2k+1)\pi, y_2=\frac{\pi}{4}+(2l+1)\pi$

30. $x_3=\frac{3\pi}{4}+2k\pi, y_3=\frac{3\pi}{4}+2l\pi; x_4=\frac{3\pi}{4}+(2k+1)\pi, y_4=\frac{3\pi}{4}+(2l+1)\pi$ (k,lϵ)

31. $\begin{cases}tgx+tgy=1,\\ \cos x\cos y=\frac{1}{\sqrt{2}}.\end{cases}x_1=\frac{\pi}{4}+(k+l)\pi, y_1=(k-l)\pi; x_2=(k+l)\pi, y_2=\frac{\pi}{4}+(k-l)\pi(k,l\epsilon Z)$

32. $\begin{cases}\sin x\sin y=\frac{1}{4\sqrt{2}},\\ tgxtgy=\frac{1}{3}.\end{cases}x_{1,2}=(k+l)\pi+\frac{1}{2}arccos\frac{1}{2\sqrt{2}}\pm\frac{\pi}{8}, y_{1,2}=(l-k)\pi+\frac{1}{2}arccos\frac{1}{2\sqrt{2}}\mp\frac{\pi}{8};$

$$x_{3,4}=(k+l)\pi-\frac{1}{2}arccos\frac{1}{2\sqrt{2}}\pm\frac{\pi}{8}, y_{3,4}$$
$$=(l-k)\pi-\frac{1}{2}arccos\frac{1}{2\sqrt{2}}\mp\frac{\pi}{8}$$

33. $\begin{cases} tgx + \frac{1}{tgx} = 2\sin(y + \frac{\pi}{4}), \\ tgy + \frac{1}{tgy} = 2\sin(x - \frac{\pi}{4}). \end{cases}$ $X=\frac{3}{4}\pi + 2k\pi, y = -\frac{3}{4}\pi + 2n\pi$

34. $\begin{cases} tgx = tg^3 y, \\ \sin x = \cos 2y. \end{cases}$ $x_1 = \frac{\pi}{2} - 2arctg\frac{1}{\sqrt{2}} + 2m\pi, y_1 = arctg\frac{1}{\sqrt{2}} + n\pi;$ $x_2 = \frac{\pi}{2} + 2arctg\frac{1}{\sqrt{2}} + 2m\pi, y_2 = -arctg\frac{1}{\sqrt{2}} + n\pi (m, n \epsilon Z)$

35. $\begin{cases} \sin x + \sin y = \sin(x + y), \\ |x| + |y| = 1. \end{cases}$ (1;0),(-1;0), (0;1), (0;-1)

36. $\begin{cases} \sin(u - 3x) = 2sin^3 x, \\ \cos(u - 3x) = 2cos^3 x \end{cases}$ $x=\frac{\pi}{4}(2n + 1), y = \pi(2m +$ 1) $(m, n \epsilon Z)$

4.5. Trigonometrik tengsizliklar

4.5.1. Ixtiyoriy x($x \neq \frac{\pi}{2}k$, k- butun son)lar uchun

$$\left(1 + \frac{1}{\sin^2 x}\right)\left(1 + \frac{1}{\cos^2 x}\right) \geq 9.$$

tengsizlikni isbotlang.

Yechish. Tengsizlikning chap tomonida quyidagi almashtirishlarni bajarish mumkin:

$$\left(1 + \frac{1}{\sin^2 x}\right)\left(1 + \frac{1}{\cos^2 x}\right) =$$

$$= \left(1 + \frac{\sin^2 x + \cos^2 x}{\sin^2 x}\right)\left(1 + \frac{\sin^2 x + \cos^2 x}{\cos^2 x}\right) =$$

$$= (2 + ctg^2 x)(2 + tg^2 x) =$$

$$= 5 + 2(tg^2 x + ctg^2 x) = 9 + 2(tg^2 x - 2tgx \cdot ctgx + ctg^2 x) = 9 + 2(tgx - ctgx)^2 \geq 9.$$

4.5.2. Ixtiyoriy haqiqiy x lar uchun quyidagi tengsizlikni isbotlang

$$(\sin^2 x)^{\cos^2 x} + (\cos^2 x)^{\sin^2 x} \leq 1 + \frac{1}{2}\sin^2 2x.$$

Yechish. Berilgan tengsizlik n-butun son uchun $x = \frac{\pi}{2}n$ bo‘lganda to‘g‘ri.

Agar $x \neq \frac{\pi}{2}n$ bo‘lsa, u holda $0 < \sin^2 x < 1$ va $0 < \cos^2 x < 1$.

$\sin^2 x + \cos^2 x = 1$ ayniyatning chap tomoniga Bernulli (8) tengsizligini qo'llab, ko'zlangan natijaga erishish mumkin:

$$(\sin^2 x)^{\cos^2 x} + (\cos^2 x)^{\sin^2 x} = (1 - \cos^2 x)^{\cos^2 x} + (1 - \sin^2 x)^{\sin^2 x}$$
$$\leq 1 - cos^4x + 1 - sin^4x =$$
$$= 2 - (cos^4x + sin^4x) = 2 - ((cos^2x + sin^2x)^2 -$$
$$2cos^2xsin^2x) = 2 - \left(1 - \frac{1}{2}sin^2 2x\right) = 1 + \frac{1}{2}sin^2 2x.$$

4.5.3. $(0;\pi)$ oraliqda quyidagi tengsizlikni isbotlang

$$x - \frac{x^3}{4} < \sin x.$$

Yechish.

$$\sin x = 2tg\frac{x}{2}cos^2\frac{x}{2} = 2tg\frac{x}{2}(1 - sin^2\frac{x}{2})$$

bo'lib, avvaldan ma'lum:

$$\sin x \leq x \leq tgx$$

ayniyatdan

$$tg\frac{x}{2} \geq \frac{x}{2} \; va \; 1 - sin^2\frac{x}{2} \geq 1 - \frac{x^2}{4}$$

ayniyatlarni hosil qilish mumkin. Ushbu tengsizliklardan berilgan tengsizlikning to'g'riligi kelib chiqadi.

4.5.6. Tengsizlikni yeching

$$(\sin^2(x + y) + 2\sin(x + y) + 2) \cdot \log_2(3^x + 3^{-x}) \leq 1.$$

Yechish. Berilgan shartga ko'ra:

$((sin(x + y) + 1)^2 + 1) \cdot \log_2(3^x + 3^{-x}) \leq 1$ (*) bo'lib, $(\sin(x + u) + 1)^2 + 1 \geq 1$ Koshi (3) tengsizligiga ko'ra:

$3^x + 3^{-x} = 3^x + \frac{1}{3^x} \geq 2$, u holda:

$$\log_2(3^x + 3^{-x}) \geq 1.$$

Natijada,

$$((sin(x + y) + 1)^2 + 1) \cdot \log_2(3^x + 3^{-x}) \geq 1$$

tengsizlik hosil bo'lib, (*) tegnsizlikka asosan:

$$((sin(x + y) + 1)^2 + 1) \cdot \log_2(3^x + 3^{-x}) = 1$$

tenglikni yozish mumkin.

Ushbu tenglik, $sin(x + y) = -1$ va $3^x = 1$ bo'lganda bajariladi.

Demak $x_1 = 0$ va $y_1 = -\frac{\pi}{2} + 2\pi n$ bu yerda n-butun son .

4.5.7. Agar $\sin x + \sin y + \sin z \geq \sqrt{5}$ bo'lsa, u holda quyidagi

ttengsizlikni isbotlang

$$\cos x + \cos y + \cos z \leq 2.$$

Yechish. Teskarisidan faraz qilib, ya'ni:

$\sin x + \sin y + \sin z \geq \sqrt{5}$ bo'lganda tengsizlik bajarilmasin. U holda:

$\begin{cases} \sin x + \sin y + \sin z \geq \sqrt{5}, \\ \cos x + \cos\cos y + \cos z > 2 \end{cases}$ sistema o'rinli bo'lib, bundan

$$(\sin x + \sin y + \sin z)^2 + (\cos x + \cos y + \cos z)^2 > 9,$$

$$3 + 2(\sin x \cdot \sin y + \cos x \cdot \cos y) + 2(\sin y \cdot \sin z +$$

$$+ \cos y \cdot \cos z) + 2(\sin x \cdot \sin z + \cos x \cdot \cos z) > 9$$

yoki

$$\cos(x - y) + \cos(y - z) + \cos(x - z) > 3 \qquad (*)$$

Ammo, (*) tengsizlik noto'g'ri, chunki

$$\cos(x - y) \leq 1, \cos(y - z) \leq 1, \cos(x - z) \leq 1$$

bo'lib,

$$\cos(x - y) + \cos(y - z) + \cos(x - z) \leq 3.$$

Ushbu qarama-qarshilik olingan teskari faraz noto'g'riligini va $\cos x + \cos y + \cos z \leq 2$ tengsizlik x, y, z lar uchun $\sin x + \sin y + \sin z \geq \sqrt{5}$ shartda bajarilishini ko'rsatadi .

4.5.8. $\text{tg}\alpha = n\text{tg}\beta$ $(n > 0)$ tenglik to'g'ri bo'lsa,

$$tg^2(\alpha - \beta) \leq \frac{(n-1)^2}{4n}$$

tengsizlikni isbotlang.

Yechish.

$$tg(\alpha - \beta) = \frac{tg\alpha - tg\beta}{1 + tg\alpha tg\beta} \; va \; tg\alpha = ntg\beta$$

tengliklardan:

$$tg^2(\alpha - \beta) = \frac{(n-1)^2 tg^2\beta}{(1 + ntg^2\beta)^2} = \frac{(n-1)^2}{(ctg\beta + ntg\beta)^2}$$

bo'lib, Koshi tengsizligiga ko'ra:

$$(ctg\beta + ntg\beta)^2 = \frac{(1 + ntg^2\beta)^2}{tg^2\beta} \geq \frac{(2\sqrt{1 \cdot ntg^2\beta})^2}{tg^2\beta} = 4n$$

Demak, berilgan shartda berilgan tengsizlik isbotlandi.

4.5.9. Trigonometrik tengsizlikni yeching

$$\frac{sin^2x-\frac{1}{4}}{\sqrt{3}-(\sin x+\cos x)} > 0.$$

Yechish.

$$|\sin x + \cos x| = \sqrt{2}\left|\sin\left(x+\frac{\pi}{4}\right)\right| \le \sqrt{2}$$

bo'lib, kasrning maxraji har doim musbat. Berilgan tengsizlik o'rinli bo'lishi uchun kasrning surati ham musbat bo'lishi kerak,ya'ni

$$sin^2x > \frac{1}{4}, \qquad |\sin x| > \frac{1}{2},$$

$$\frac{\pi}{6}+\pi k < x < \frac{5\pi}{6}+\pi k, k\epsilon Z.$$

4.5.10. Trigonometrik tengsizlikni yeching

$$5 + 2\cos 2x \le 3|2\sin x - 1|.$$

Yechish. Berilgan tengsizlik quyidagi ikkita tengsizliklar sistemasuga teng kuchli bo'ladi:

$$\begin{cases} 5 + 2\cos 2x \le 3(2\sin x - 1), \\ 2\sin x - 1 \ge 0 \end{cases} va \begin{cases} 5 + 2\cos 2x \le 3(-2\sin x + 1), \\ 2\sin x - 1 < 0 \end{cases}$$

Birinchi sistemadan $\sin x \ge 1$ yoki $\sin x = 1$ bo'lib,

$x=\frac{\pi}{2} + 2\pi k, k\epsilon Z$. Ikkinchi sistemadan esa, $\sin x \le -\frac{1}{2}$ bo'lib,

$$-\frac{5\pi}{6} + 2\pi n \le x \le -\frac{\pi}{6} + 2\pi n, k\epsilon Z.$$

4.5.11. Trigonometrik tengsizlikni yeching

$$tg\frac{x}{2} > \frac{tgx-2}{tgx+2}$$

Yechish. $\text{tg}\frac{x}{2} = t$ almashtirish yordamida berilgan tengsizlikni quyidagicha yozish mumkin:

$$t > \frac{2t-2+2t^2}{2t+2-2t^2},$$

$$\frac{(t-1)(t^2+t+1)}{t^2-t-1} > 0,$$

$$\frac{t-1}{t^2-t-1} > 0.$$

Chunki $t^2 + t + 1 > 0, t\epsilon R$ bo'lib, $t^2 - t - 1$ uchhadning ildizlari $t_1 = \frac{1-\sqrt{5}}{2} t_2 = \frac{1+\sqrt{5}}{2}$.

Demak, berilgan tengsizlik bajarilishi uchun $\text{tg}\frac{x}{2} > \frac{1+\sqrt{5}}{2}$ va

$\frac{1-\sqrt{5}}{2} < tg\frac{x}{2} < 1$ tengsizliklar o'rinli bo'lishi lozim:

$$2k\pi + 2arctg\frac{1+\sqrt{5}}{2} < x < \pi + 2\pi r,$$

$$2\pi k - arctg\frac{\sqrt{5}-1}{2} < x < \frac{\pi}{2} + 2\pi k, k\epsilon Z.$$

4.5.12. Trigonometrik tengsizlikni yeching

$$3\sin 2x - 1 > \sin x + \cos x.$$

Yechish. $\sin 2x = (\sin x + \cos x)^2 - 1$ ayniyatdan foydalanib berilgan tengsizlikni quyidagicha yozish mumkin:

$$3(\sin x + \cos x)^2 - (\sin x + \cos x) - 4 > 0,$$

$$(3(\sin x + \cos x) - 4)(\sin x + \cos x + 1) > 0,$$

Oxirgi tengsizlik quyidagi ikkita tengsizlikka ajraladi:

$$\sin x + \cos x < -1, \sin x + \cos x > {}^4/_3;$$

$$\sqrt{2}\sin(x + \frac{\pi}{4}) < -1, \sqrt{2}\sin\left(x + \frac{\pi}{4}\right) > {}^4/_3;$$

$$\sin(x + \frac{\pi}{4}) < -\frac{1}{\sqrt{2}}, \sin(x + \frac{\pi}{4}) > \frac{4}{3\sqrt{2}}.$$

Birinchi tengsizlikning yechimi:

$$\frac{5\pi}{4} + 2\pi k < x + \frac{\pi}{4} < \frac{7\pi}{4} + 2\pi$$

yoki

$$\pi + 2\pi k < x < \frac{3\pi}{2} + 2\pi k; k\epsilon Z$$

bo'lib, ikkinchisiniki esa

$$arcsin\frac{2\sqrt{2}}{3} + 2\pi k < x + \frac{\pi}{4} < \pi - arcsin\frac{2\sqrt{2}}{3} + 2\pi k \; yoki$$

$$arcsin\frac{2\sqrt{2}}{3} - \frac{\pi}{4} + 2\pi k < x < \frac{3\pi}{4} - arcsin\frac{2\sqrt{2}}{3} + 2\pi k; k\epsilon Z$$

bo'ladi.

Mustaqil yechish uchun misollar

1. α vaβ o'tkir burchaklar uchun quyidai tengsizliklarni isbotlang

$$a)\ \sin(\alpha + \beta) < \sin\alpha + \sin\beta,$$

$$b)\ \sin\frac{\alpha+\beta}{2} \geq \frac{\sin\alpha+\sin\beta}{2},$$

$$s)\ tg\frac{\alpha+\beta}{2} \leq \frac{tg\alpha+tg\beta}{2}.$$

2. $1+tgx+tg^2x+tg^2x=\frac{\sqrt{2}\cos(\frac{\pi}{4}-x)}{cos^3x}$

3. $\frac{1-2sin^2\alpha}{1+\sin 2\alpha}=\frac{1-tg\alpha}{1+tg\alpha}$

4. $\frac{3-4\cos 2\alpha+\cos 4\alpha}{3+4\cos 2\alpha+\cos 4\alpha}=tg^4\alpha$

5. $\frac{\cos 6\alpha-\cos 7\alpha-\cos 8\alpha+\cos 9\alpha}{\sin 6\alpha-\sin 7\alpha-\sin 8+\sin 9\alpha}=ctg\frac{15}{2}\alpha$

6. $ctg\frac{\alpha}{2}>1+ctg\alpha, 0<\alpha<\frac{\pi}{2}$

7. $(1+tg^2x)(1-3tg^2x)(1+tg2xtg3x)>0$

8. $\frac{\sin x-1}{\sin x-2}+\frac{1}{2}\geq\frac{2-\sin x}{3-\sin x}$

9. $\cos\sin\propto>\sin\cos\propto, 0\leq\propto\leq\frac{\pi}{2}$

10. $sin^3\propto-sin^6\propto\leq\frac{1}{4}$

11. $\frac{\sin(\alpha+\beta)}{2\sin\alpha\sin\beta}\geq ctg\frac{\alpha+\beta}{2}\ (\alpha>0,\beta>0,\alpha+\beta<\pi)$

12. $\cos x<1-\frac{x^2}{2}+\frac{x^4}{16}\ (0<x<\pi)$

13. $x-\frac{x^2}{2}<tgx\ 0<x<\frac{\pi}{2}$

14. $1-\cos x\leq\frac{x^2}{2}$

Tengsizliklarni yeching(15-22)

15. $5sin^2x+sin^22x>4\cos 2x\ \frac{\pi}{6}+\pi k<x<\frac{5\pi}{6}+\pi k, k\epsilon Z$

16. $|\ tgx+ctgx\ |<\frac{4}{\sqrt{3}}\ \frac{\pi}{6}+\frac{\pi k}{2}<x<\frac{\pi}{3}+\frac{\pi k}{2}, k\epsilon Z$

17. $\sqrt{\frac{3(\sin x+\cos x)-\sqrt{2}}{2\sqrt{2}-(\sin x+\cos x)}}>1-\frac{\pi}{4}+arcsin\frac{3}{4}+2\pi k<x<-\frac{\pi}{4}-arcsin\frac{3}{4}+(2k+1)\pi, k\epsilon Z$

18. $\frac{\sin 3x\cos\left(2x-\frac{\pi}{6}\right)}{\sin 2x}\leq 0$ $x=\frac{\pi}{3}+2\pi k,\frac{\pi}{2}+2\pi k<x\leq\frac{2\pi}{3}+2\pi k,\frac{5}{6}\pi+2\pi k\leq x<\pi+2\pi k,\pi+2\pi k<x\frac{5}{6}\pi+2\pi k,$

$$\frac{3}{2}\pi + 2\pi k \le x \le \frac{11}{6}\pi + 2\pi k, k\epsilon Z$$

19. $(3\sqrt{2}\cos x - \sqrt{2}\sin x - 2)(2\sin 2x - 1) \ge 0$, $x = \frac{\pi}{12} + 2\pi k$, $\frac{5\pi}{12} + 2\pi k \le x \le \frac{13\pi}{12} + 2\pi k$, $\frac{17\pi}{12} + 2\pi k \le x \le \frac{23\pi}{12} - 2arctg\frac{1}{3} + 2\pi k, k\epsilon Z$.

20. $ctg^3x + ctg^2x - ctgx - 1 < 0 \frac{\pi}{4} + \pi k < x < \frac{3\pi}{4} + \pi k, k\epsilon Z$

21. $\sqrt{3 - 4cos^2x} > 2\sin x + 1 \frac{7\pi}{6} + 2\pi k < x < \frac{11\pi}{6} + 2\pi k, k\epsilon Z$

22. $cos^3x \sin 3x + sin^3x \cos 3x < \frac{3}{8}\frac{5\pi}{24} + \frac{\pi k}{2} < x < \frac{\pi}{24} + \frac{\pi(k+1)}{2}$

4.6. Aralash tenglama va tengsizliklar

4.6.1. $x + y + z + \frac{1}{xy} = \cos(\sqrt{x} - \sqrt{y}) + cos\sqrt{z} + \cos(xyz)$ tenglamani yeching.

Yechish. Tenglamaning aniqlanish sohasi $x > 0, y > 0$ va $z \ge 0$.

$z \ge 0$ shartdan $x + y + z + \frac{1}{xu} \ge x + y + \frac{1}{xu}$. $(*)$ n=3 bo'lganda, Koshi (1) tengsizligiga asosan:

$$x + y + \frac{1}{xy} \ge 3 \cdot \sqrt[3]{xy\frac{1}{xy}} = 3.$$

Demak, $(*)$ va oxirgi tengsizlikdan berilgan tenglama ikkala tomoni ham 3 ga teng bo'lsagina o'rinli bo'ladi.

Natijada, $x + y + z + \frac{1}{xy} \ge 3$ $(*)$ tengsizlik o'rinli bo'lib, $\cos(\sqrt{x} - \sqrt{y}) \le 1$, $\cos\sqrt{z} \le 1$ va $\cos(xyz) \le 1$ munosabatdan $\cos(\sqrt{x} - \sqrt{y}) + \cos\sqrt{z} + \cos(xyz) \le 3$ bo'ladi.

Tenglamaning chap tomoni $x_1 = y_1 = 1$ va $z_1 = 0$ bo'lganda eng kichik bo'lib, 3 ga teng, o'ng tomoni esa o'zgaruvchilarning ayni shu qiymatlarida eng katta bo'lib, u ham 3 ga teng bo'ladi, ya'ni (1, 1, 0) berilgan tenglamaning yechimi bo'ladi.

4.6.2. $\cos^2(xsin\,x) = 1 + lg^2\sqrt{x^2 + x + 1}$ tenglamani yeching.

Yechish. Berilgan tenglama butun sonlar o'qida aniqlangan bo'lib, $cos^2(xsinx) \le 1$ va $1 + lg^2\sqrt{x^2 + x + 1} \ge 1$ tengsizliklar

o‘rinli. U holda berilgan tenglama $\begin{cases} \cos^2(x\sin x)=1, \\ lg^2\sqrt{x^2+x+1}=1 \end{cases}$ tenglamalar sistemasiga teng kuchli bo‘ladi. Sistemaning ikkinchi tenglamasi $x_1=0,\ x_2=-1$ ildizlarga ega. $x_1=0$ ildiz birinchi tenglamani qanoatlantiradi. Demak, $x_1=0$ berilgan tenglama ildizi bo‘ladi.

4.6.3. $\log_5\sin x>\log_{125}(3\sin x-2)$ tenglamani yeching.

Yechish. Logarifmik va trigonometric funksiyalarning xossalaridan foydalanib quyidagilarni yozish mumkin:

$$\begin{cases} sin^3x>3\sin x-2, \\ 3\sin x-2>0; \end{cases}\Rightarrow\begin{cases} sin^3x-3\sin x+2>0, \\ \sin x>\frac{2}{3}; \end{cases}\Rightarrow$$

$$\Rightarrow\begin{cases} (\sin x-1)^2(\sin x+2)>0, \\ \sin x>\frac{2}{3}. \end{cases}\Rightarrow\frac{2}{3}<\sin x<1.$$

Demak, $arcsin\frac{2}{3}+\pi k<x<\frac{\pi}{2}+\pi k$ va

$\frac{\pi}{2}+\pi k<x<\pi-arcsin\frac{2}{3}+\pi k, k\in Z$

4.6.4. Tenglamani yeching

$$\lg(x^2-4x+14)+\sin(xy)=0$$

Yechish. Berilgan tenglamani $\lg(x^2-4x+14)=-\sin(xy)$ $(*)$ ko‘rinishda yozish mumkin. U holda:

$x^2-4x+14=(x-2)^2+10\geq 10$ va $\lg(x^2-4x+14)\geq 1$ shuningdek, $-\sin(xy)\leq 1$ bo‘lib, $(*)$ tenglama uning ikkala tomoni ham 1 ga teng bo‘lganda o‘rinli bo‘ladi.

Ya’ni, quyidagi sistema hosil bo‘ladi: $\begin{cases} \lg(x^2-4x+14)=1, \\ \sin(xy)=-1 \end{cases}$ $(**)$

sistemaning birinchi tenglamasidan:

$x^2-4x+14=10$ yoki $x_1=2$ bo‘lib, ikkinchi tenglamadan y ning unga mos qiymati $\sin 2y=-1$ va $y=-\frac{\pi}{4}+\pi n$ bu yerda n-butun son.

4.6.5. $\log_2\left(17-\left|\sin\frac{\pi x}{2}\right|\right)=\sqrt{2x+15-x^2}$ tenglamani yeching.

Yechish. Tenglamaning chap tomonida $\left|\sin\frac{\pi x}{2}\right|\leq 1$ bo‘lib, u holda:

$$17 - \left|\sin\frac{\pi x}{2}\right| \geq 16 \text{ va } \log_2\left(17 - \left|\sin\frac{\pi x}{2}\right|\right) \geq 4.$$

Shuningdek, tenglamaning o'ng tomoni uchun:

$\sqrt{2x + 15 - x^2} = \sqrt{16 - (x-1)^2} \leq 4$ bo'ladi.

Demak, berilgan tenglama o'rinli bo'lishi uchun uning chap va o'ng qismi bir vaqtda 4 ga teng bo'lishi lozim, ya'ni $\begin{cases}\log_2\left(17 - \left|\sin\frac{\pi x}{2}\right|\right) = 4,\\ \sqrt{2x + 15 - x^2} = 4\end{cases}$ yoki $\begin{cases}\left|\sin\frac{\pi x}{2}\right| = 1,\\ x - 1 = 0\end{cases}$ oxirgi tenglamalar sistemasi va demak, berilgan tenglama yagona $x_1 = 1$ ildizga ega.

4.6.6. $(\log_{\cos x}\sin^2 x)\cdot(\log_{\cos^2 x}\sin x) = 4$ tenglamani yeching.

Yechish. Berilgan tenglamada o'zgaruvchi x ning qabul qilishi mumkin bo'lgan qiymatlari $\begin{cases}\sin x > 0,\\ \cos x > 0\end{cases}$ tensizlik bilan aniqlanib, uning yechimi l- butun son uchun $2\pi l < x < \frac{\pi}{2} + 2\pi l$ bo'ladi.

Berilgan tenglamadan $(2\log_{\cos x}\sin x)\cdot\left(\frac{1}{2}\cdot\log_{\cos x}\sin x\right) = 4$, $log^2_{\cos x}\sin x = 4$, yoki $\log_{\cos x}\sin x = \pm 2$ bo'ladi.

Agar $\log_{\cos x}\sin x = 2$ bo'lsa, u holda $\sin x = \cos^2 x$ bo'lib, $\sin^2 x + \sin x - 1 = 0$ va $(\sin x)_{1,2} = \frac{-1\pm\sqrt{5}}{2}$ bo'ladi hamda $0 < \sin x < 1$ va $2\pi l < x < \frac{\pi}{2} + 2\pi l$ shartlarga ko'ra n-butun son uchun

$$x_1 = arcsin\frac{\sqrt{5}-1}{2} + 2\pi n.$$

Agar $\log_{\cos x}\sin x = -2$ bo'lsa, u holda $\sin x = \frac{1}{\cos^2 x}$, $\sin x = \frac{1}{1-\sin^2 x}$ yoki $\sin^3 x - \sin x + 1 = 0$ bo'lib, bu tenglama haqiqiy ildizga ega emas.

Shunday qilib, berilgan tenglama n-butun son uchun $x_1 = arcsin\frac{\sqrt{5}-1}{2} + 2\pi n$ ildizlarga ega.

4.6.7. $\log_{(2\cos x)/\sqrt{3}}\sqrt{1 + 2\cos 2x} < 1$ tengsizlikni yeching.

Yechish. Ikkilangan burchak kosinusi formulasidan foydalanib va

y=cos x almashtirish yordamida berilgan tengsizlikni quyidagicha yozish mumkin:

$$\log_{2y/\sqrt{3}} \sqrt{4y^2-1} < 1.$$

Ushbu tengsizlikning aniqlanish sohasi: $\frac{1}{2} < y \leq 1$ va $y \neq \frac{\sqrt{3}}{2}$ tengsizliklar bilan aniqlanadi.

Bunda ikki hol bo'ladi:

1) Agar $\frac{1}{2} < y < \frac{\sqrt{3}}{2}$ bo'lsa, $4\sqrt{3}y^2 - 2y - \sqrt{3} > 0$ bo'lib,

$\frac{1}{2} < y < \frac{1+\sqrt{13}}{4\sqrt{3}}$ yoki $-\frac{\pi}{3} + 2\pi k < x < 2\pi - arccos\frac{1+\sqrt{13}}{4\sqrt{3}} + 2\pi k$

va $\arccos\frac{1+\sqrt{13}}{4\sqrt{3}} + 2\pi k < x < \frac{\pi}{3} + 2\pi k, k \epsilon Z.$

2) Agar $\frac{\sqrt{3}}{2} < y < 1$ bo'lsa, $4\sqrt{3}y^2 - 2y - \sqrt{3} < 0$ bo'lib ushbu tengsizlik qaralayotgan oraliqda yechimga ega emas,

4.6.8. Tengsizlikni yeching

$$(3 - \cos^2 x - 2\sin x)(lg^2 y + 2lgy + 4) \leq 3.$$

Yechish. Berilgan tenglamani quyidagicha yozish mumkin:

$$((\sin x - 1)^2 + 1) \cdot ((lgy + 1)^2 + 3) \leq 3 \qquad (*)$$

$(\sin x - 1)^2 \geq 0$ va $(lgy + 1)^2 \geq 0$ bo'lib u holda,

$((\sin x - 1)^2 + 1) \cdot ((lgy + 1)^2 + 3) \geq 3$ oxirgi va $(*)$ tengsizlikdan $((\sin x - 1)^2 + 1) \cdot ((lgy + 1)^2 + 3) = 3$ bo'lib, tenglik faqat $\sin x = 1$ va $lgy = -1$ bo'lganda bajariladi.

Demak, $x_1 = \frac{\pi}{2} + 2\pi n$, bu yerda n-butun son va $y_1 = \frac{1}{10}$.

4.6.9. ABC uchburchak burchaklari uchun

$\cos A \cdot \cos B \cdot \cos C \leq \frac{1}{8}$ tengsizlikni isbotlang

Yechish. $n = 3$ bo'lganda Koshi (1) tengsizligiga asosan: $\cos A \cdot \cos B \cdot cosC \leq \left(\frac{\cos A + \cos B + cosC}{3}\right)^3$ $(*)$ bo'lib quyidagi yordamchi tengsizlikni isbotlash kerak bo'ladi.

$$\cos A + \cos B + cosC \leq \frac{3}{2} \quad (**)$$

$(**)$ tengsizlik chap tomonida quyidagicha shakl almashtirishlarni bajarish mumkin:

$$\cos A + \cos B + cosC = cosA + 2cos\frac{B+C}{2} \cdot \cos\frac{B+C}{2} \quad (***)$$

ABC uchburchak burchaklari $A + B + C = \pi$, $0 < \frac{B+C}{2} < \frac{\pi}{2}$ va $-\frac{\pi}{2} < \frac{B-C}{2} < \frac{\pi}{2}$ bo'lib, bundan $0 < cos\frac{B+C}{2} < 1$, $0 < cos\frac{B-C}{2} < 1$ va $\cos\frac{B+C}{2} \cdot \cos\frac{B-C}{2} \leq \cos\frac{B+C}{2}$, $B+C = \pi - A$ va (***) tengsizlik quyidagi ko'rinishga keladi:

$$\cos A + \cos B + cosC \leq \cos A + 2\cos\frac{C+B}{2} = \cos A + 2\cos(\frac{\pi}{2} - \frac{A}{2})$$

$$= 1 - 2sin^2\frac{A}{2} + 2\sin\frac{A}{2} = \frac{3}{2} - 2(\sin\frac{A}{2} - \frac{1}{2})^2 \leq \frac{3}{2}$$

(**) tengsizlik va berilgan tengsizlik isbotlandi.

Mustaqil yechish uchun misollar

Tengsizliklarni yeching (1-3)

1. $|\sin x| \leq \log_3 x \leq \cos x$, $\emptyset$;
2. $4(x^3 - 2x + 1)(\sin x + 2\cos x) \geq 9 \mid x^3 - 2x + 1 \mid$ x=1, x=$\frac{-1\pm\sqrt{5}}{2}$;
3. $4^{sin^2\pi x} + 3 \cdot 4^{cos^2\pi x} \leq 8\frac{1}{4} + k \leq x \leq \frac{3}{4} + k$, $k \epsilon Z$;

Tenglamani yeching

4. $10sin^2x^2 + 4\cos 2x + 3\sin 2x = 5$

$x_{1,2} = \frac{-1\pm\sqrt{1+4(arccos0,8+\pi k)}}{2}$, $x_{3,4} = \frac{1\pm\sqrt{1+4(\pi l - arccos0,8)}}{2}$, $k = 0, k \epsilon N; l \epsilon N$;

5. $(x+4)(x^2 + 4) = 5 - 2^{x+4} - 16(\sqrt{2})^x$ x=4;
6. $\log_{1/3}(1 + (x^2 - 3x + 2)^2) = \sqrt{x^2 - 6x + 8}$ x=2;
7. $\sqrt{lgx} + \sqrt{1 - x^2} + \sqrt{1 - y^2} = 1 + |\sin y|$ (1;0);
8. Quyidagi tenglamani qanoatlantiruvchi barcha haqiqiy sonlar jo'ftliklarini toping

$$x^2 + 6x\sin(xy) + 9 = 0$$

$$x_1 = -3, y_1 = -\frac{\pi}{6} + \frac{2\pi k}{3}, k \epsilon Z$$

$$x_2 = 3, y_2 = -\frac{\pi}{6} + \frac{2\pi l}{3}, l \epsilon Z$$

9. Agar a,b,c musbat sonlar biror geometrik progressiyaning ketma-ket hadlari bo'lsa, u holda $a^n + c^n > 2b^n$ tengsizlik ixtiyoriy natural n larda bajarilishini isbotlang

10. Tenglamani yeching

a)$cos^2(xy) + 2x(1 + \cos(xy)) + 2x^2 + 1 = 0$ x=−1, y=$2\pi k, k\epsilon Z$

b) $x^2 + 2x\sin(xy) + 1 = 0, x_1 = -1, y_1 = \frac{3\pi}{2} + 2\pi n, n\epsilon Z;\ x_2 = 1, y_2 = \frac{3\pi}{2} + 2\pi m, m\epsilon Z$

11. Biror arifmetik progressiyaning $a_1, a_2, a_3, \cdots, a_n$ hadlari va d-ayirmasi uchun

$$S = \frac{1}{\sqrt{a_1} + \sqrt{a_2}} + \frac{1}{\sqrt{a_2} + \sqrt{a_3}} + \cdots + \frac{1}{\sqrt{a_{n-1}} + \sqrt{a_n}} = \frac{\sqrt{a_n} - \sqrt{a_1}}{d}$$

tenglikni isbotlang.

5-BOB. HOSILA, INTEGRAL VA ODDIY DIFFERENSIAL TENGLAMALAR

5.1. Funksiyaning ekstremal qiymatlari

5.1.1.

$$F(x,y,z) = x + y + z - xy - xz - yz + xyz$$

funksiyaning maksimal qiymatlarini toping bu yerda

$$x^2 + y^2 + z^2 = 1.$$

Yechish. Berilgan funksiyani

$$F(x,y,z) = (x-1)(y-1)(z-1) + 1 \qquad (**)$$

ko'rinishida yozish mumkin.

Shartga ko'ra, $x^2 + y^2 + z^2 = 1$ bo'lib, u holda $x \le 1$, $y \le 1$, $z \le 1$ va $x - 1 \le 0, y - 1 \le 0, z - 1 \le 0$ demak,

$$(x-1)(y-1)(z-1) \le 0 \text{ va } F(x,y,z) \le 1$$

bo'lib, tenglik o'rinli bo'lishi uchun $(x-1)(y-1)(z-1) = 0$ bo'lib, agar $x_0 = 1$ bo'lsa, $x^2 + y^2 + z^2 = 1$ shartdan $y_0 = z_0 = 0$. Demak, $maxF(x,y,z) = F(1,0,0)$=F(1,0,0)=F(0,1,0)=F(0,0,1)=1.

5.1.2. Hadlari musbat va o'zaro teng bo'lmagan

$$a : b = c : d$$

proporsiyaning eng katta va eng kichik hadlari yig'indisi qolgan ikkita hadi yig'indisidan katta ekanligini isbotlang

Yechish. Ma'lumki, proporsiya hadlari o'rinlarini almashtirib, eng katta hadini har doim birinchi o'ringa qo'yish mumkin.Agar a-soni proporsiyaning eng katta hadi bo'lsa, u holda d-soni eng kichik hadi bo'ladi.

Haqiqatdan,

$$d = b \cdot \frac{c}{a} = c \cdot \frac{b}{a}$$

bo'lib, $\frac{c}{a} < 1$ va $\frac{b}{a} < 1$ yoki d< b va d< c bo'ladi.

Quyidagi hosila proporsiyadan foydalanamiz:

$$\frac{a-b}{c-d} = \frac{a}{c} > 1,$$

$$\frac{a-b}{c-d} > 1,$$

$$a-b>c-d>0,$$
$$a+d>b+c.$$

Shuni isbotlash talab etilgan edi.

5.1.3. Uch xonali sonning uning raqamlari yig'indisiga nisbatining eng katta qiymatini toping

Yechish. Berilgan uch xonali son-$\overline{xyz}=100x+10y+1$ bo'lib,

$$100x+10y+z<100(x+1)\ va\ x+y+z\geq x+1,$$
$$\frac{100x+10y+z}{x+y+z}<\frac{100(x+1)}{x+1}=100.$$

Shunday qilib, y yoki z raqamlardan birortasi 0-dan farqli bo'lsa, bu nisbat 100-dan kichik va agar y=z=0 bo'lsa,

$$\frac{100x}{x}=100.$$

Demak, bu nisbatning eng katta qiymati 100-ga teng bo'lib, u 100; 200, $\cdots$, 900 sonlari uchun o'rinli bo'ladi.

5.1.4. Quyidagi funksiyaning eng katta qiymatini toping

$$f(x)=\sqrt{1-\frac{x}{3}}+\sqrt[6]{x+1}$$

Yechish. Funksiyaning aniqlanish sohasi: $-1\leq x\leq 3$ bo'lib, $-1<x<3$ bo'lganda $f(x)$ funksiya uchun Bernulli (8) tengsizligini qo'llab:

$$\sqrt{1-\frac{x}{3}}+\sqrt[6]{x+1}=\left(1-\frac{x}{3}\right)^{\frac{1}{2}}+(1+x)^{\frac{1}{6}}\leq 1-\frac{x}{6}+1+\frac{x}{6}=2$$

va bundan:

$f(0)=2, f(-1)=\frac{2}{\sqrt{3}}$ va $f(3)=\sqrt[3]{2}$ .U holda $f_{max}=f(0)=2.$

5.1.5. Funksiyaning eng kichik qiymatini toping

$$f(x)=\frac{x^2}{8}+x\cdot\cos x+\cos 2x.$$

Yechish. Berilgan funksiyani quyidagicha shakl almashtirish mumkin:

$f(x)=\frac{x^2}{8}+x\cos x+2\cos^2 x-1=\left(\sqrt{2}\cos x+\frac{x}{2\sqrt{2}}\right)^2-1\geq -1$

bo'lib, $\sqrt{2}\cos x+\frac{x}{2\sqrt{2}}=0$ bo'lganda berilgan funksiya o'zining eng kichik qiymatiga ega $\sqrt{2}\cos x+\frac{x}{2\sqrt{2}}=0$ yoki $\cos x+\frac{x}{4}=0$ tenglama

ildizga ega bo'lib, $f_{min} = -1$ ($\cos x = -\frac{x}{4}$ tenglama ildizga ega ekanligini grafik usulda ko'rsatish mumkin).

5.1.6. Funksiyaning eng kichik qiymatini toping

$$f(x) = x^4 + 2x^2 + \frac{1}{(x^2+1)^2} + 5.$$

Yechish. $u = f(x)$ funksiyani quyidagicha yozish mumkin:

$f(x) = (x^2+1)^2 + \frac{1}{(x^2+1)^2} + 4$(*) Koshi (3) tengsizligiga asosan: $(x^2+1)^2 + \frac{1}{(x^2+1)^2} \geq 2$ bo'lib, (*) tenglikdan $f(x) \geq 6$ va tenglik $x = 0$ bo'lganda o'rinli bo'ladi, ya'ni $f_{min} = f(0) = 6$.

5.1.7. Funksiyaning eng katta va eng kichik qiymatlarini toping

$$f(x, y) = 6\sin x \cdot \cos y + 2\sin x \cdot \sin y + 3\cos x.$$

Yechish. Koshi-Bunyakovskiy (9) tengsizligiga asosan:

$$f(x, y) = 6\sin x \cdot \cos y + 2\sin x \cdot \sin y + 3\cos x$$

ifoda uchun

$$f^2(x, y) \leq (6^2 + 2^2 + 3^2) \cdot (\sin^2 x \cdot \cos^2 y + \sin^2 x \cdot \sin^2 y + \cos^2 x)$$
$$= 49 \cdot (\sin^2 x \cdot (\cos^2 y + \sin^2 y) + \cos^2 x) =$$
$$= 49 \cdot (\sin^2 x + \cos^2 x) = 49$$

bo'lib, bundan $-7 \leq f(x, y) \leq 7$ endi $f(x, y)$ o'zining eng katta va eng kichik qiymatlariga teng bo'lishi mumkinligini ko'rsatish kerak bo'ladi, ya'ni:

$$f_{min} = -7, f_{max} = 7.$$

Buning uchun Koshi-Bunyakovskiy (9) tengsizligi tenglikka aylanadigan holatni qarash lozim bo'ladi. a-o'zgarmas soni uchun

$$\begin{cases} 6 = a\sin x \cos y \\ 2 = a\sin x \cdot \sin y, \\ 3 = a\cos x \end{cases} \quad (*)$$

tenglamalar sistemasini qaraymiz

(*) tenglamalarni kvadratga ko'tarib, qo'shib: $a^2 = 49$ tenglikni hosil qilish mumkin.

$a = 7$ bo'lsin, u holda: $\cos x = \frac{3}{7}$, $\sin x = \pm\sqrt{1 - \frac{9}{49}} = \pm\frac{2\sqrt{10}}{7}$,

$\sin y = \frac{2}{7 \cdot \left(\pm\frac{2\sqrt{10}}{7}\right)} = \pm\frac{1}{\sqrt{10}}$, $\cos y = \frac{6}{7 \cdot \left(\pm\frac{2\sqrt{10}}{7}\right)} = \pm\frac{3}{\sqrt{10}}$

Agar $\cos x = \frac{3}{7}$ bo'lsa, $\sin x = \frac{2\sqrt{10}}{7}$, $\sin y = \frac{1}{\sqrt{10}}$ va $\cos y = \frac{3}{\sqrt{10}}$ bo'lib, u holda:

$$f_{max} = 6 \cdot \frac{2\sqrt{10}}{7} \cdot \frac{3}{\sqrt{10}} + 2 \cdot \frac{2\sqrt{10}}{7} \cdot \frac{1}{\sqrt{10}} + 3 \cdot \frac{3}{7} = 7$$

Agar $\cos x = -\frac{3}{7}$ bo'lsa, $\sin x = -\frac{2\sqrt{10}}{7}$, $\sin y = \frac{1}{\sqrt{10}}$ va $\cos y = \frac{3}{\sqrt{10}}$ bo'lib, u holda:

$$f_{min} = 6 \cdot \left(-\frac{2\sqrt{10}}{7}\right) \cdot \frac{3}{\sqrt{10}} + 2 \cdot \left(-\frac{2\sqrt{10}}{7}\right) \cdot \frac{1}{\sqrt{10}} + 3 \cdot \left(-\frac{3}{7}\right) = -7$$

5.1.8. $x^2 + 3y^2 + z^2 = 2$ bo'lsa $f(x, y, z) = 2x + y - z$ funksiyaning eng katta va eng kichik qiymatlarini toping

Yechish. Koshi-Bunyakovskiy (9) tengsizligiga ko'ra:

$$f^2(x, y, z) = (2x + y - z)^2 = \left(2x + \frac{1}{\sqrt{3}} \cdot \sqrt{3}y + (-1) \cdot z\right)^2 \leq$$

$$\leq \left(4 + \tfrac{1}{3} + 1\right) \cdot (x^2 + 3y^2 + z^2) = \frac{16}{3} \cdot 2 = \frac{32}{3}.$$

Bundan:

$|f(x, y, z)| \leq \frac{4}{3}\sqrt{6}$ bo'lib, u holda: $f_{min}(x, y, z) = -\frac{4\sqrt{6}}{3}$ ekanligini ko'rsatish talab etiladi. (9) tengsizlikda tenglik bajarilishi uchun faqat va faqat $\frac{x}{2} = \frac{\sqrt{3}u}{\frac{1}{\sqrt{3}}} = \frac{z}{-1} = a$ shart bajarilishi kerak. U holda: $x = 2a$, $y = \frac{a}{3}$ va $z = -a$ bo'lib, $x^2 + 3y^2 + z^2 = 2$ shartdan:

$4a^2 + \frac{a^2}{3} + a^2 = 2$ va $a = \pm\frac{\sqrt{6}}{4}$.

Demak,

$$f_{min}(x, y, z) = f\left(-\frac{\sqrt{6}}{2}, -\frac{\sqrt{6}}{12}, \frac{\sqrt{6}}{4}\right) == 2 \cdot \left(-\frac{\sqrt{6}}{2}\right) + \left(-\frac{\sqrt{6}}{12}\right) - \frac{\sqrt{6}}{4}$$

$$= -\frac{4\sqrt{6}}{3}$$

Eslatma. Yuqoridagiga o'xshash mulohazalar yordamida $f_{max}(x, y, z) = f\left(\frac{\sqrt{6}}{2}, \frac{\sqrt{6}}{12}, -\frac{\sqrt{6}}{4}\right) = \frac{4\sqrt{6}}{3}$ tenglikni ko'rsatish mumkin.

5.1.9. Agar $2x^2 + y^2 + 2xy - 6x - y = 0$ bo'lsa, $f(x,y) = 2x + y$ funksiyaning eng katta va eng kichik qiymatlarini toping

Yechish. $2x + y = p$ bo'lsin, u holda quyidagi tenglamalar sistemasi yechimga ega bo'ladigan p ning eng katta va eng kichik qiymatlarini topish kerak bo'ladi $\begin{cases} 2x^2 + y^2 + 2xy - 6x - y = 0, \\ 2x + y = p. \end{cases}$

Bu sistemaning ikkinchi tenglamasidan $y = p - 2x$ ni birinchi tenglamasiga qo'yib x o'zgaruvchiga nisbatan p parametrli kvadrat tenglama hosil qilish mumkin: $2x^2 - 2x(p+2) + p^2 - p = 0$

Ushbu tenglama haqiqiy ildizga ega bo'lishi uchun uning diskriminanti nomanfiy bo'lishi kerak, ya'ni:

$(p+2)^2 - 2(p^2 - p) \geq 0$ yoki $p^2 - 6p - 4 \leq 0$,

tengsizlikning yechimi $3 - \sqrt{13} \leq p \leq 3 + \sqrt{13}$ bo'lib $f_{min} = 3 - \sqrt{13}$ va $f_{max} = 3 + \sqrt{13}$

Eslatma. Yuqoridagi masalani quyidagicha ham yechish mumkin. sistemaning ikkinchi tenglamasidan $x = \frac{p-y}{2}$ ni birinchi tenglamasiga qo'yib:

$$2\left(\frac{p-y}{2}\right)^2 + y^2 + 2y\left(\frac{p-y}{2}\right) - 6\left(\frac{p-y}{2}\right) - y = 0,$$

$$\frac{p^2-2py+y^2}{2} + y^2 + py - y^2 - 3p + 3y - y = 0,$$

$$y^2 + 4y + p^2 - 6p = 0$$

tenglamadan $D = 4 - p^2 - 6p \geq 0$ yoki $3 - \sqrt{13} \leq p \leq 3 + \sqrt{13}$.

Mustaqil yechish uchun misollar

1. $y=3x-4\sqrt{1-x^2}$ funksiya qaysi nuqtada o'zining eng katta qiymatiga erishadi (0,6;0,2)

2. Quyidagi funksiyalarning eng katta va eng kichik qiymatlarini toping:

a) $y=5\cos 2x - 4\sin 2x$ b) $y=3^{x-1} + 3^{-x-1}$

c) $y=-\frac{x^2}{x^4+1}$ d) $y=\frac{2x}{x^2+1}$

3. Ifodaning eng katta qiymatini toping: $\log_3^2 x + 2\log_3 x \log_3 \frac{3}{x}$

4. x va y-o‘zgaruvchilarning qanday qiymatlarida

$$x^2 + 2xy + 2y^2 + 2x + 4y + 3$$

Ifoda eng kichik qiymat qabul qiladi

5. k-ning qanday qiymatlarida ixtiyoriy x-uchun

$u = x^2 + (3k + 1)x + 3k$ funksiyaning qiymatlari

v=$(k + 1)x^2 + 4kx - 4k$ funksiya qiymatlaridan katta bo‘ladi.

6. $y_1 = (\log_6 7)^{\sin x}$ va $y_2 = (\log_7 6)^{\cos x}$ funksiyalarning eng katta qiymatlari o‘zaro teng ekanligini isbotlang

7. y=x+$\frac{1}{x-2}$ aunksiyaning (2;+∞) oraliqdagi eng kichik qiymatini toping.

8. Agar x va y musbat sonlar uchun x+y=2a tenglik o‘rinli bo‘lsa,

$$z = \frac{1}{x} + \frac{1}{y}$$

funksiyaning eng kichik qiymatini toping.

9. a-o‘zgarmas son uchun x+y+z=3a bo‘lsa,

$$u = x^2 + y^2 + z^2$$

Ifodaning eng kichik qiymatini toping.

10. Agar $x^2 + y^2 = 1$ bo‘lsa, u holda:

$$x^6 + y^6$$

Ifodaning eng katta va eng kichik qiymatlarini toping.

11. x va y o‘zgaruvchining qanday qiymatlarida

$$z = 5x^2 + 2y^2 - 2xy - 6x - 6y + 14$$

funksiyaning qiymati eng kichik bo‘ladi.

5.2. Hosilaning qo‘llanilishi

5.2.1. Quyidagi funksiyaning o‘sish va kamayish oraliqlarini toping

$$f(x) = x \cdot e^{-3x}$$

Yechish. Berilgan funksiyaning hosilasi:

$$f'(x) = e^{-3x}(1 - 3x)$$

bo‘lib, $f'(x) \geq 0$ yoki $1 - 3x \geq 0$, x≤$\frac{1}{3}$, ya’ni $(-\infty; \frac{1}{3}]$ oraliqda funksiya o‘suvchi va $[\frac{1}{3}; +\infty)$ oraliqda esa kamayuvchi bo‘ladi.

5.2.2. Quyidagi funksiyaning [1;6] kesmadagi eng katta va eng kichik qiymatlarini toping

$$f(x) = \frac{x}{8} + \frac{2}{x}$$

Yechish. $f'(x) = \frac{1}{8} - \frac{2}{x^2} = 0$ tenglamaning berilgan oraliqqa tegishli ildizi x=4. Berilgan funksiyaning oraliqning chetlaridagi va x=4 nuqtadagi qiymatlari:

$$f(1) = 2\frac{1}{8}, f(4) = 1, f(6) = 1\frac{1}{12}$$

bo'lib,

$$\max_{[1;6]} f(x) = f(1) = 2\frac{1}{8} \text{ va } \min_{[1;6]} f(x) = f(4) = 1.$$

5.2.3. Uchta musbat sonning yig'indisi 26 ga teng. Agar ikkinchi son birinchisidan uch marta katta bo'lib, kvadratlari yig'indisi eng kichik qiymat qabul qiladigan sonlarni toping.

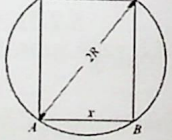
Yechish. Masalaning shartiga ko'ra: $\begin{cases} x + y + z = 26, \\ y = 3x \end{cases}$ bo'lib, z=26−4x. U holda berilgan masala quyidagi ifodaning eng katta qiymatini topish masalasiga keladi:

$S = x^2 + y^2 + z^2 = x^2 + (3x)^2 + (26 - 4x)^2, 0 < x < 3{,}25$

$S' = 52x - 208 = 0$, $x = 4 \epsilon (0; 3{,}25)$ $x < 4$ bo'lganda $S' > 0$ va $x > 4$ bo'lganda $S' < 0$ bo'ladi, ya'ni S funksiya x=4 nuqtada maksimumga ega bo'ladi.Demak, x=4, y=12, z=10.

5.2.4. R-radiusli aylanaga ichki chizilgan eng katta yuzaga ega bo'lgan to'g'ri to'rtburchak tomonlarini toping

Yechish. Pifagor teoremasiga ko'ra:

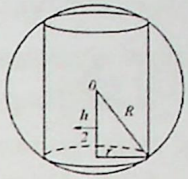


$BC = \sqrt{4R^2 - x^2}$. To'g'ri to'rtburchak yuzasi $S = x \cdot \sqrt{4R^2 - x^2}$ ekanligidan $y = S^2 = x^2(4R^2 - x^2)$ funksiyaning maksimumini topamiz. $y' = 8R^2x - 4x^3 = 0$ bundan $x = \sqrt{2}R$.

Javob: Tomoni $\sqrt{2}R$ ga teng bo'lgan kvadrat.

5.2.5. *R*-radiusli sharga eng katta hajmga ega bo'lgan silindr ichki chizilgan bo'lsa, silindrning hajmini toping.

Yechish. Silindr hajmi $V = \pi r^2 h$. Masalaning shartiga ko‘ra silindr balandligini berilgan shar radiusi va silindr radiusi orqali ifodalab silindr hajmini quyidagicha yozish mumkin:



$$V = 2\pi r^2\sqrt{R^2 - r^2}.$$

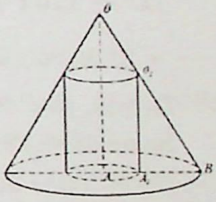
$y = V^2 = 4\pi^2 r^4(R^2 - r^2)$ funksiyani qaraymiz.

$y' = 4\pi^2(4r^3R^2 - 6r^5) = 0$ tenglamadan $r = \sqrt{\frac{2}{3}}R$ bo‘lganda y –funksiya maksimumga erishadi.

Javob. Silindrning hajmi: $V = \frac{4\pi R^3}{3\sqrt{3}}$.

5.2.6. Asosining radiusi R, balandligi H ga teng konusga yon sirti eng katta bo‘lgan silindr ichki chizilgan bo‘lsa, uning o‘lchamlarini toping.

Yechish. $S = 2\pi rh$.Endi OAB va O_1A_1B uchburchaklar o‘xshashligidan:



$\frac{h}{H} = \frac{R-r}{R}$. (h va r mos ravishda silindrning balandligi va asosining radiusi)

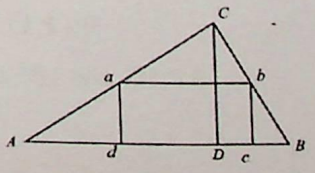
$h = \frac{H}{R}(R - r)$ topilgan balandlikni yuza formulasiga qo‘yib: $S = \frac{2\pi H}{R}(Rr - r^2)$.

$S' = 0$ tenglamadan $r = \frac{R}{2}$. Demak, $\frac{h}{H} = \frac{1}{2}$.

Javob.$r = \frac{R}{2}$, $h = \frac{H}{2}$.

5.2.7. ABC uchburchkning AB asosiga parallel to‘g’ri chiziq AC va CB yon tomonlarni mos ravishda a va b nuqtalarda kesib o‘tadi. AB asosda olingan d va c nuqtalar uchun $abcd$ to‘g’rito‘rtburchakning yuzi eng katta bo‘lsa, uning tomonlarini toping.

Yechish. Masala shartiga ko‘ra CD balandlik uzunligini h deb olib, $\Delta aCb \sim \Delta ACB$ bo‘lganligidan, $\frac{h-x}{h} = \frac{y}{AB}$ bo‘lib, to‘g’rito‘rtburchakning tomonlari



$ad = x$ va $ab = y$ ga teng bo'lsin, u holda

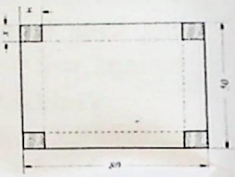
$S = \frac{AB}{h}(xh - x^2)$, $S' = 0$ tenglamadan $x = \frac{h}{2}$ bo'lganda yuza maksimal qiymatga ega bo'ladi.

Javob. To'g'rito'rtburchak tomonlari $\frac{h}{2}$ va $\frac{AB}{2}$.

5.2.8. O'lchamlari $80sm \times 50sm$ bo'lgan to'g'ri to'rtburchak shaklidagi temir listning burchaklaridan kvadratlar qirqib olinib qolgan chetki qismlari bukilib eng katta sig'imga ega bo'lgan usti ochiq quti yasalgan. Shu qutining balandligini toping.

Yechish. Masala shartiga ko'ra qirqilgan kvadratlar tomonining uzunligi x bo'lsin. Hosil bo'lgan to'g'rito'rtburchakli usti ochiq prizmaning hajmi

$$V = x(50 - 2x)(80 - 2x), \text{ yoki}$$
$$V = 4(x^3 - 65x^2 + 1000x).$$



U holda $V' = 0$ yoki

$3x^2 - 130x + 1000 = 0$ bo'lib,

$$x_1 = 10sm, x_2 = \frac{100}{3}sm.$$

Demak, $x = 10sm$ bo'lsa, hajm maksimal bo'ladi.

Javob. $x = 10sm$.

5.2.9. ln 1,1 ni 0,0001 aniqlik bilan hisoblang.

Yechish. y=ln(1 + x) funksiya uchun Makloren qatori quyidagicha bo'ladi:

$$\ln(1 + x) = x - \frac{x^2}{2} + \frac{x^3}{3} + \cdots + (-1)^{n-1}\frac{x^n}{n} \cdots$$

yoki

$$\ln 1{,}1 = \ln(1 + 0{,}1) = 0{,}1 - \frac{(0{,}1)^2}{2} + \frac{(0{,}1)^3}{3} - \frac{(0{,}1)^4}{4} + \cdots$$

ln 1,1 ni 0,0001 aniqlikda hisoblash uchun qatorning uchta hadini olish kifoya qiladi.

Demak,

$$\ln(1{,}1) \approx 0{,}1 - \frac{0{,}01}{2} + \frac{0{,}0001}{3} \approx 0{,}0953$$

5.2.10. cos 18^0 ni 0,0001 aniqlik bilan hisoblang.

Yechish. f(x)=cos x funksiyaning Makloren qatoriga yoyilmasi:

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots$$

bo'lib, $\cos 18^0 = \cos\frac{\pi}{18} \approx 1 - \frac{\left(\frac{\pi}{18}\right)^2}{2!} + \frac{\left(\frac{\pi}{18}\right)^4}{4!} - \frac{\left(\frac{\pi}{18}\right)^6}{6!} + \frac{\left(\frac{\pi}{18}\right)^8}{8!} + \cdots$ tenglikni 0,0001 aniqlikda hisoblash uchun yoyilmaning ikkita hadini olish kifoyadir.

Shunday qilib,

$$\cos 18^0 \approx 1 - \frac{0{,}745^2}{2!} \approx 0{,}9848.$$

5.2.11. $\sqrt[4]{630}$ ni 0,001 aniqlik bilan hisoblang

Yechish. $\sqrt[4]{630} = \sqrt[4]{625(1+\frac{5}{625})} = 5\cdot(1+0{,}008)^{1/4}$ bo'lib,

$$(1+x)^m = 1 + \frac{m}{1!}x + \frac{m(m-1)}{2!}x^2 + \cdots + \frac{m(m-1)(m-2)\cdots(m-n+1)}{n!}x^n + \cdots$$

formulaga asosan:

$$(1+0{,}008)^{1/4} = 1 + 0{,}002 - 0{,}00006 + 0{,}0000028 - \cdots$$

Ishoralari almashinuvchi sonli qator hosil bo'ladi. $(1+0{,}008)^{1/4}$ ni hisoblashda qatorning dastlabki uchta hadini olish yetarli.

$$630^{1/4} = 5\cdot(1+0{,}008)^{1/4} \approx 5\cdot(1+0{,}002-0{,}00006) = 5\cdot 0{,}0097 \approx 0{,}01$$

Mustaqil yechish uchun misollar

1. Quyidagi funksiyalarni avval soddalashtirib, so'ngra hosilasini toping

a) $f(x)=\frac{\sqrt[3]{x+\sqrt{2-x^2}}\cdot\sqrt[6]{1-x\sqrt{2-x^2}}}{\sqrt[3]{1-x^2}}$ b) $f(x)=\frac{\left[(1-x^2)^{-1/2}+1+\frac{1}{(1-x^2)^{-1/2}-1}\right]^{-2}}{2-x^2-2\sqrt{1-x^2}}$

c) $f(x)=\frac{\sqrt[3]{2x+2\sqrt{x^2-1}}}{\left(\frac{\sqrt{x-1}}{\sqrt{x+1}}+\frac{\sqrt{x+1}}{\sqrt{x-1}}+2\right)^{1/3}}$ d) $f(x)=\frac{2x\sqrt{\frac{1}{4}\left(\frac{1}{\sqrt{x}}+\sqrt{x}\right)^2-1}}{2\sqrt{\frac{1}{4}\left(\frac{1}{\sqrt{x}}+\sqrt{x}\right)^2-1}-\frac{1}{2}\left(\frac{1}{\sqrt{x}}-\sqrt{x}\right)}$

e) $f(x)=\left(\frac{\sqrt[4]{ax^3}-\sqrt[4]{a^3x}}{\sqrt{a}+\sqrt{x}}+\frac{1+\sqrt{ax}}{\sqrt[4]{ax}}\right)^{-2}\cdot\sqrt{1+2\sqrt{\frac{a}{x}}+\frac{a}{x}}$

f) $f(x)=\sqrt{x+2\sqrt{2x-4}}+\sqrt{x-2\sqrt{2x-4}}$

2. Quyidagi funksiyalarning o'sish va kamayish oraliqlarini toping

a) $f(x)=\frac{x^2-2}{2x+3}$ b) $f(x)=\arccos\left(\frac{1-x^2}{1+x^2}\right)$

c) $f(x)=\frac{x}{\ln x}$ d) $f(x)=x\cdot\left(1+\frac{1}{x}\right)^x$

e) $f(x)=\frac{1}{2}ctg^2x+\ln(\sin x)$ f) $f(x)=\frac{x^2}{2^x}$

3. a-parametrning qanday qiymatlarida quyidagi funksiya x-ning ixtiyoriy haqiqiy qiymatlarida o'suvchi bo'ladi

a) $f(x)=\sin 2x-8(a-1)\sin x+(4a^2+8a-14)x$

b) $f(x)=8ax-\sin 6x-7x-\sin 5x$

4. Quyidagi funksiyalarning o'sish va kamayish oraliqlarini toping

a) $f(x)=\frac{2x-1}{(x-1)^2}$ b) $f(x)=\ln tg\left(\frac{x}{2}+\frac{\pi}{4}\right)$

c) $f(x)=\frac{3}{2}x-sin^2x$ d) $f(x)=2\ln(x-2)-x^2+4x+1$

5. Cheksiz kamayuvchi geometrik progressiyaning yig'indisi $f(x)=x^3+3x-9$ funksiyaning [-2;3] oraliqdagi eng katta qiymatiga teng.Agar progressiyaning birinchi va ikkinchi hadlari ayirmasi $f'(0)$ ga teng bo'lsa uning maxrajini toping.

6. Cheksiz kamayuvchi geometrik progressiyaning yig'indisi $f(x)=3x^2-x+\frac{25}{12}$ funksiyaning eng kichik qiymatiga teng.Agar progressiyaning birinchi hadi maxrajining kvadratiga teng bo'lsa,uning maxrajini toping.

7. a-parametrning qanday qiymatilarida

$f(x)=x^3-6x^2+9x+a$ funksiyaning x=2 nuqtadagi va ekstremum qiymatlari biror tartibda geometrik progressiya tashkil qiladi?

8. $\frac{4}{\sin x}+\frac{1}{1-\sin x}=a$ tenglama $(0;\frac{\pi}{2})$ oraliqda kamida bitta ildizga ega bo'ladigan a-parametrning eng kichik qiymatini toping.

9. Kemaning tezligi-v_k,daryo oqimining tezligi-v_d.Daryoning narigi qirg'og'iga "oqmasdan" o'tish uchun kemani qirg'oqqa nisbatan qanday burchak ostida yo'naltirish kerak?

10. Gorizontga $\propto$-burchak ostida v_0-boshlang'ich tezlik bilan otilgan jism $\propto$-ning qanday qiymatida eng uzoq masofani o'tadi.

11. Quyidagi funksiyalarning ko'rsatilgan oraliqdagi eng katta va eng kichik qiymatlarini toping

a) f(x)=$x^5 - x^3 + x + 2, [-1; 1]$ b) f(x)=$3x^4 + 4x^3 + 1, [-2; 1]$

c) f(x)=$\frac{x}{2} - \frac{1}{4}\sin 2x + \frac{1}{3}cos^3x - \cos x, [-\frac{\pi}{2}; \frac{\pi}{2}]$

d) f(x)=$| x^2 + x | + | x^2 + 5x + 6 |, [-\frac{5}{2}; \frac{1}{2}]$

12. cos 0,4 ni 0,001 aniqlik bilan hisoblang

13. arctg0,4 ni 0,001 aniqlik bilan hisoblang

14. Quyidagilarni 0,001 aniqlik bilan hisoblang

a) $\sqrt[3]{500}$; b) $\sqrt[4]{17}$; c) $\cos 12^0$

5.3. Aniqmas integral

5.3.1. $f(x) = \frac{(x^m - x^n)^2}{\sqrt{x}}, (m, n \in Z)$ funksiyaning boshlang'ich funksiyalarini toping.

Yechish. Berilgan funksiyani quyidagicha yozib olamiz:

$$f(x) = x^{2m-\frac{1}{2}} - 2x^{m+n-\frac{1}{2}} + x^{2n-\frac{1}{2}}.$$

Yuqorida keltirilgan integrallar jadvalidan foydalanib,

$$\int \left(x^{2m-\frac{1}{2}} - 2x^{m+n-\frac{1}{2}} + x^{2n-\frac{1}{2}}\right) dx =$$

$$= \frac{1}{2m+\frac{1}{2}} x^{2m+\frac{1}{2}} - \frac{2}{m+n+\frac{1}{2}} x^{m+n+\frac{1}{2}} + \frac{1}{2n+\frac{1}{2}} x^{2n+\frac{1}{2}} + C =$$

$$= \frac{2x^{2m}\sqrt{x}}{4m+1} - \frac{4x^{m+n}\sqrt{x}}{2m+2n+1} + \frac{2x^{2n}\sqrt{x}}{4n+1} + C.$$

Javob. $\int \frac{(x^m - x^n)^2}{\sqrt{x}} dx = \frac{2x^{2m}\sqrt{x}}{4m+1} - \frac{4x^{m+n}\sqrt{x}}{2m+2n+1} + \frac{2x^{2n}\sqrt{x}}{4n+1} + C.$

5.3.2. $\int \frac{xdx}{x^2+a^2}$ aniqmas integralni hisoblang.

Yechish. Berilgan integral osti ifodada $x^2 + a^2 = t$ almashtirish olamiz. Bundan $2xdx = dt$. $\int \frac{xdx}{x^2+a^2} = \frac{1}{2}\int \frac{dt}{t} = \frac{1}{2} ln|t| + C$.

Javob. $\frac{1}{2} ln(x^2 + a^2) + C$.

5.3.3. $\int e^{cosx} \cdot sinxdx$ aniqmas integralni hisoblang.

Yechish. Berilgan integralni $cosx = t$ almashtirish yordamida quyidagicha hisoblash mumkin:

$$\int e^{cosx} \cdot sinxdx = -\int e^{cosx} d(cosx) = -\int e^t dt = -e^t + C.$$

Javob. $-e^{cosx} + C$.

5.3.4. $\int x \cdot e^x dx$ aniqmas integralni hisoblang.

Yechish. Agar $u = x, dv = e^x dx$ bo'lsa, u holda $du = dx$, $v = \int e^x dx = e^x$ bo'ladi va $\int x \cdot e^x dx = x \cdot e^x - \int e^x dx$. Demak,

$$\int x \cdot e^x dx = x \cdot e^x - e^x + C = e^x(x - 1) + C.$$

Javob. $e^x(x - 1) + C$.

5.3.5. $\int lnxdx$ aniqmas integralni hisoblang.

Yechish. Agar $u = lnx, dv = dx$ bo'lsa, u holda $du = \frac{1}{x}dx$, $v = x$ bo'lib,

$$\int lnxdx = x \cdot lnx - \int x \cdot \frac{1}{x}dx = x \cdot lnx - x + C = x \cdot ln\frac{x}{e} + C.$$

Javob. $x \cdot ln\frac{x}{e} + C$.

5.3.6. $\int \frac{2x-1}{x^2-5x+6}dx$ aniqmas integralni hisoblang.

Yechish. $x^2 - 5x + 6 = (x - 3)(x - 2)$ ko'paytma ko'rinishida yozib, integral ostidagi ifodani yanada soddaroq kasrlar yig'indisi ko'rinishida yozamiz:

$$\frac{2x - 1}{x^2 - 5x + 6} = \frac{A}{x - 3} + \frac{B}{x - 2}$$

$$= \frac{x(A + B) - (2A + 3B)}{x^2 - 5x + 6}, \begin{cases} A + B = 2, \\ 2A + 3B = 1. \end{cases}$$

Bundan $A = 5, B = -3$.

Demak,

$$\int \frac{2x-1}{x^2-5x+6}dx = 5\int \frac{dx}{x-3} - 3\int \frac{dx}{x-2} =$$
$$= 5ln|x-3| - 3ln|x-2| + C.$$

Javob. $5ln|x-3| - 3ln|x-2| + C.$

5.3.7. $\int \frac{dx}{x^4-1}$ aniqmas integralni hisoblang.

Yechish. $\frac{1}{x^4-1} = \frac{A}{x-1} + \frac{B}{x+1} + \frac{Cx+D}{x^2+1} =$

$$= \frac{A(x^3+x^2+x+1) + B(x^3-x^2+x-1) + C(x^3-x) + D(x^2-1)}{(x-1)(x+1)(x^2+1)}.$$

$A+B+C=0,\ A-B+D=0,\ A+B-C=0,\ A-B-D=0$

tenlamalardan $A=\frac{1}{4},\ B=-\frac{1}{4},\ C=0,\ D=-\frac{1}{2}$ bo'lib,

$$\int \frac{dx}{x^4-1} = \frac{1}{4}\int \frac{dx}{x-1} - \frac{1}{4}\int \frac{dx}{x+1} - \frac{1}{2}\int \frac{dx}{x^2+1} =$$
$$= \frac{1}{4} ln\left|\frac{x-1}{x+1}\right| - \frac{1}{2} arctgx + C.$$

Javob. $\frac{1}{4} ln\left|\frac{x-1}{x+1}\right| - \frac{1}{2} arctgx + C.$

5.3.8. $\int \frac{x^3dx}{x^8-2}$ aniqmas integralni hisoblang.

Yechish. $\int \frac{x^3dx}{x^8-2} = \frac{1}{4}\int \frac{d(x^4)}{(x^4)^2-(\sqrt{2})^2} = \frac{1}{8\sqrt{2}} ln\left|\frac{x^4-2}{x^4+2}\right| + C.$

5.3.9. $\int \frac{x^2-1}{x^4+1}dx$ aniqmas integralni hisoblang.

Yechish.

$$\int \frac{x^2-1}{x^4+1}dx = \int \frac{1-\frac{1}{x^2}}{x^2+\frac{1}{x^2}}dx = \int \frac{d\left(x+\frac{1}{x}\right)}{\left(x+\frac{1}{x}\right)^2-\left(\sqrt{2}\right)^2} =$$
$$= \frac{1}{2\sqrt{2}} \ln\left|\frac{x+\frac{1}{x}-\sqrt{2}}{x+\frac{1}{x}+\sqrt{2}}\right| + C.$$

Javob. $\frac{1}{2\sqrt{2}} ln\left|\frac{x^2-\sqrt{2}x+1}{x^2+\sqrt{2}x+1}\right| + C.$

5.3.10. $\int \frac{x^2+1}{x^4+1}dx$ aniqmas integralni hisoblang.

Yechish. $\int \frac{x^2+1}{x^4+1}dx = \int \frac{1+\frac{1}{x^2}}{x^2+\frac{1}{x^2}}dx = \int \frac{d\left(x-\frac{1}{x}\right)}{\left(x-\frac{1}{x}\right)^2-(\sqrt{2})^2} =$

$$= \frac{1}{\sqrt{2}} arctg \frac{x-\frac{1}{x}}{\sqrt{2}} + \frac{\pi}{2\sqrt{2}} sgnx + C.$$

5.3.11. $\int x \cdot (1-x)^{10}dx$ aniqmas integralni hisoblang.

Yechish. Oddiy $x = 1-(1-x)$ tenglikdan foydalanamiz.

$$\int x \cdot (1-x)^{10}dx = \int (1-x)^{10}dx - \int (1-x)^{11}dx =$$

$$- \int (1-x)^{10}d(1-x) +$$

$$+ \int (1-x)^{11}d(1-x) = -\frac{1}{11}(1-x)^{11} + \frac{1}{12}(1-x)^{12} + C.$$

Javob. $-\frac{1}{11}(1-x)^{11} + \frac{1}{12}(1-x)^{12} + C.$

5.3.12. $\int \frac{dx}{(x^3+1)^2}$ aniqmas integralni hisoblang.

Yechish.

$\int \frac{dx}{(x^3+1)^2} = \frac{Ax^2+Bx+C}{x^3+1} + D\int \frac{dx}{(x+1)} + \int \frac{Ex+F}{x^2-x+1}dx$ ko'rinishda ifodalaymiz. Endi yuqoridagi tenglikning ikkala tomoniini differensiallab, umumiy maxrajga keltiramiz. Hosil bo'lgan tenglik ikkala tomonining maxrajlari birxil. Demak,

$$1 \equiv -Ax^4 - 2Bx^3 - 3Cx^2 + 2Ax + B$$
$$+ D(x^5 - x^4 + x^3 + x^2 - x + 1) +$$
$$+(Ex+F)\cdot(x^4 + x^3 + x + 1)$$

tenglik o'rinli bo'lib ikki ko'phadning aynan tengligidan quyidagi tenglamalar sistemasini hosil qilish mumkin:

$$\begin{cases} D+E=0, \\ -A-D+E+F=0, \\ -2B+D+F=0, \\ 2A-D+E+F=0, \\ B+D+F=1. \end{cases} \begin{cases} A=C=0, \\ B=\frac{1}{3}, \\ D=-E=\frac{2}{9}, \\ F=\frac{4}{9}. \end{cases}$$

$$\int \frac{dx}{(x^3+1)^2} = \frac{x}{x^3+1} + \frac{2}{9}\ln|x+1| - \frac{2}{9}\int \frac{x-2}{x^2-x+1}dx = \frac{x}{3(x^3+1)} +$$
$$+\frac{1}{9}\ln\frac{(x+1)^2}{x^2-x+1} + \frac{2}{3\sqrt{3}} arctg\frac{2x-1}{\sqrt{3}} + C, x \neq -1.$$

5.3.13. $\int \frac{x^5-x}{x^8+1}dx$ aniqmas integralni hisoblang.

Yechish. $\int \frac{x^5-x}{x^8+1}dx = \frac{1}{2}\int \frac{x^4-1}{x^8+1}d(x^2) = \frac{1}{2}\int \frac{1-\frac{1}{x^4}}{x^4+\frac{1}{x^4}}d(x^2) =$

$$= \frac{1}{2}\int \frac{d\left(x^2+\frac{1}{x^2}\right)}{\left(x^2+\frac{1}{x^2}\right)^2 - \left(\sqrt{2}\right)^2}.$$

$x^2 + \frac{1}{x^2} = t$ almashtirish olsak,

$$\frac{1}{2}\int \frac{d\left(x^2+\frac{1}{x^2}\right)}{\left(x^2+\frac{1}{x^2}\right)^2 - \left(\sqrt{2}\right)^2} = \frac{1}{2}\int \frac{dt}{t^2-\left(\sqrt{2}\right)^2} =$$

$$= \frac{1}{4\sqrt{2}} ln\left|\frac{t-\sqrt{2}}{t+\sqrt{2}}\right| + C.$$

Javob. $\frac{1}{4\sqrt{2}} ln\left|\frac{x^4-\sqrt{2}x^2+1}{x^4+\sqrt{2}x^2+1}\right| + C.$

5.3.14. $\int \frac{x\sqrt[3]{2+x}}{x+\sqrt[3]{2+x}}dx$ aniqmas integralni hisoblang.

Yechish. $x \neq -1$ bo'lganda $x + 2 = t^3$ bo'lsin, u holda $dx = 3t^2dt$ va berilgan integral quyidagicha hisoblanadi:

$$\int \frac{(t^3-2)t \cdot 3t^2dt}{(t^3-2)+t} = 3\int \left(t^3 - t + \frac{t^2-2t}{(t-1)(t^2+t+2)}\right)dt =$$

$$= \frac{3}{4}t^4 - \frac{3}{2}t^2 + \int \frac{3t^2-6t}{(t-1)(t^2+t+2)}dt.$$

Yuqorida qaralgan noma'lum koeffisientlar usuliga ko'ra:

$\frac{3t^2-6t}{(t-1)(t^2+t+2)} = \frac{A}{t-1} + \frac{Bt+C}{t^2+t+2}$ bo'lib, $A = -\frac{3}{4}, B = \frac{15}{4}, C = -\frac{3}{2}$ larni topamiz.

U holda: $\int \frac{3t^2-6t}{(t-1)(t^2+t+2)}dt = -\frac{3}{4}\int \frac{dt}{t-1} + \frac{15}{4}\int \frac{t-\frac{2}{5}}{t^2+t+2}dt =$

$$= -\frac{3}{4}ln|t-1| + \frac{15}{8}ln|t^2+t+2| - \frac{27}{4\sqrt{7}} arctg\frac{2t+1}{\sqrt{7}} + C.$$

$(t = \sqrt[3]{x+2})$

Javob. $\frac{3}{4}t^4 - \frac{3}{2}t^2 - \frac{3}{4}ln|t-1| + \frac{15}{8}ln|t^2+t+2| -$
$- \frac{27}{4\sqrt{7}} arctg \frac{2t+1}{\sqrt{7}} + C,$

5.3.15. $\int \frac{x^3}{\sqrt{1+2x-x^2}} dx$ aniqmas integralni hisoblang.

Yechish. Ostragradskiy usulidan foydalanib berilgan integralni quyidagicha yozib olish mumkin:

$$\int \frac{x^3}{\sqrt{1+2x-x^2}} dx =$$
$$= (Ax^2+Bx+C)\sqrt{1+2x-x^2} + \lambda \int \frac{dx}{1+2x-x^2}.$$

U holda tenglikning ikkala tomonini differensiallab, ma'lum shakl almashtirishlar bajarib quyidagi tenglikni hosil qilish mumkin:

$$x^3 = -3Ax^3 + (5A-2B)x^2 + (2A+3B-C)x + B + C + \lambda.$$

Natijada,

$$\begin{cases} -3A = 1, \\ 5A - 2B = 0, \\ 2A + 3B - C = 0, \\ B + C + \lambda = 0. \end{cases} \begin{cases} A = -\frac{1}{3}, \\ B = -\frac{5}{6}, \\ C = -\frac{19}{3}, \\ \lambda = 4. \end{cases}$$

bo'lib,

$$\int \frac{x^3}{\sqrt{1+2x-x^2}} dx = -\frac{2x^2+5x+19\sqrt{1+2x-x^2}}{5} +$$
$$+4arcsin\frac{x-1}{\sqrt{2}} + C$$

Javob. $-\frac{2x^2+5x+19\sqrt{1+2x-x^2}}{6} + +4arcsin\frac{x-1}{\sqrt{2}} + C.$

5.3.16. $\int \frac{x^2dx}{(4-2x+x^2)\cdot\sqrt{2+2x-x^2}}$ aniqmas integralni hisoblang.

Yechish.

$$\int \frac{x^2dx}{(4-2x+x^2)\cdot\sqrt{2+2x-x^2}} = \int \frac{dx}{\sqrt{2+2x-x^2}} + \int \frac{(2x-4)dx}{(4-2x+x^2)\cdot\sqrt{2+2x-x^2}}$$

tenglik o'rinli bolib,

$\int \frac{dx}{\sqrt{2+2x-x^2}} = \int \frac{dx}{\sqrt{(\sqrt{3})^2-(x-1)^2}} = arcsin\frac{x-1}{\sqrt{3}} + C_1$ va ikkinchi integralni z =$x-1$ almashtirish yordamida quyidagicha yozish mumkin:

$\int \frac{(2x-4)dx}{(4-2x+x^2)\cdot\sqrt{2+2x-x^2}} = \int \frac{(2z-2)dz}{(3+z^2)\sqrt{3-z^2}} = \int \frac{2zdz}{(3+z^2)\sqrt{3-z^2}} -$

$-2\int \frac{dz}{(3+z^2)\sqrt{3-z^2}}$. t=$\sqrt{3-z^2}$ bo'lsin, u holda $-2tdt=2zdz$ va berilgan integralning birinchisi quyidagicha hisoblanadi:

$$\int \frac{2zdz}{(3+z^2)\sqrt{3-z^2}} = 2\int \frac{dt}{6-t^2} = -\frac{1}{\sqrt{6}} ln\left|\frac{\sqrt{6}+t}{\sqrt{6}-t}\right| + C_2$$

yoki x – o'zgaruvchiga qaytsak,

$$-\frac{1}{\sqrt{6}} ln\left|\frac{\sqrt{6}+\sqrt{2+2x-x^2}}{\sqrt{6}-\sqrt{2+2x-x^2}}\right| + C_2.$$

Endi $-2\int \frac{dz}{(3+z^2)\sqrt{3-z^2}}$ integralni hisoblaymiz. Bunda $\frac{z}{\sqrt{3-z^2}} = l$ almashtirish olamiz va $-\frac{2}{3}\int \frac{dl}{2l^2+1} = -\frac{\sqrt{2}}{3} arctg\sqrt{2}l + C_3$.

x – o'zgaruvchiga qaytsak, $-\frac{2}{3} arctg\frac{\sqrt{2}(x-1)}{\sqrt{2+2x-x^2}} + C_3$. Barcha hisoblanganlarni qo'shib yozamiz.

Javob. $arcsin\frac{x-1}{\sqrt{3}} - \frac{1}{\sqrt{6}} ln\left|\frac{\sqrt{6}+\sqrt{2+2x-x^2}}{\sqrt{6}-\sqrt{2+2x-x^2}}\right| - \frac{2}{3} arctg\frac{\sqrt{2}(x-1)}{\sqrt{2+2x-x^2}} + C.$

Ba'zi trigonometrik funksiyalarning aniqmas integrallarini hisoblash.

$\int sin^m x \cdot cos^n x dx\ (m,n \in Z)$ ko'rinishdagi integrallar ko'pincha belgilash kiritish yoki darajani pasaytirish usullaridan foydalanib yechiladi.

5.3.16. $\int \frac{cos^4 x}{sin^3 x} dx$ aniqmas integralni hisoblang.

Yechish. $\int \frac{cos^4 x}{sin^3 x} dx = -\frac{1}{2}\int cos^3 x\, d\left(\frac{1}{sin^2 x}\right) = -\frac{1}{2}\left(\frac{cos^3 x}{sin^2 x} +\right.$

$\left.+3\int \frac{cos^2 x}{sinx} dx\right)$.

$$-\frac{3}{2}\int \frac{cos^2 x}{sinx} dx = -\frac{3}{2} cosx - \frac{3}{2} ln\left|tg\frac{x}{2}\right| + C.$$

Demak, $\int \frac{cos^4 x}{sin^3 x} dx = -\frac{cos^3 x}{2sin^2 x} - -\frac{3}{2} cosx - \frac{3}{2} ln\left|tg\frac{x}{2}\right| + C.$

5.3.17. $\int \frac{1}{\cos x \cdot \sqrt[3]{\sin^2 x}} dx$ aniqmas integralni hisoblang.

Yechish. $\int \frac{1}{\cos x \sqrt[3]{\sin^2 x}} dx = \int \frac{d(\sin x)}{(1-\sin^2 x)\sqrt[3]{\sin^2 x}}$, $t^3 = \sin x$, $x \neq \frac{\pi}{2}k$ bo'lsin, u holda berilgan integralni quyidagicha hisoblash mumkin:

$$\int \frac{d(\sin x)}{(1-\sin^2 x)\cdot \sqrt[3]{\sin^2 x}} = 3\int \frac{dt}{1-t^6} = \frac{3}{2}\int \frac{dt}{1-t^3} + \frac{3}{2}\int \frac{dt}{1+t^3} =$$

$$= \frac{3}{2}\left(\int \frac{dt}{1-t^3} + \int \frac{dt}{1+t^3}\right).$$

$$\frac{3}{2}\left(\int \frac{dt}{1-t^3} + \int \frac{dt}{1+t^3}\right) =$$

$$= -\frac{1}{4} ln \frac{(1-t)^2}{t^2+t+1} + \frac{\sqrt{3}}{2} arctg \frac{2t+1}{\sqrt{3}} + \frac{1}{4} ln \frac{(1+t)^2}{t^2-t+1}$$

$$+ \frac{\sqrt{3}}{2} arctg \frac{2t-1}{\sqrt{3}}.$$

Javob. $\frac{1}{4} ln \frac{(1+t)^2(t^2+t+1)}{(1-t)^2(t^2-t+1)} + \frac{\sqrt{3}}{2} arctg \frac{\sqrt{3}t}{1-t^2} + C$, $t = \sqrt[3]{sinx}$, $x \neq \frac{k\pi}{2}$.

5.3.18. $\int tg^5 x dx$ aniqmas integralni hisoblang.

Yechish.

$$\int tg^5 x dx = \int tg^3 x \cdot \left(\frac{1}{cos^2 x} - 1\right) dx$$

$$= \frac{tg^4 x}{4} - - \int tgx \left(\frac{1}{cos^2 x} - 1\right) dx.$$

Javob. $\frac{tg^4 x}{4} - \frac{tg^2 x}{2} - ln|cosx| + C$.

5.3.19. $\int ctg^6 x dx$ aniqmas integralni hisoblang.

Yechish. $\left(\frac{1}{sin^2 x} - 1\right) = ctg^2 x, x \neq k\pi$ ayniyatdan foydalanamiz.

$$\int ctg^6 x dx = \int ctg^4 x \cdot \left(\frac{1}{sin^2 x} - 1\right) dx = -\frac{ctg^5 x}{5} -$$

$$- \int ctg^2 x \left(\frac{1}{sin^2 x} - 1\right) dx = -\frac{ctg^5 x}{5} + \frac{ctg^3 x}{3} - ctgx - x + C.$$

Javob. $-\frac{ctg^5x}{5}+\frac{ctg^3x}{3}-ctgx-x+C,\ x\neq k\pi.$

5.3.20. $\int\frac{1}{3+sinx}dx$ aniqmas integralni hisoblang.

Yechish. $sinx=\frac{2tg\frac{x}{2}}{1+tg^2\frac{x}{2}}, t=tg\frac{x}{2}, dx=\frac{2}{1+t^2}dt$ almashtirish olamiz.

Natijada, berilgan integralni quyidagicha yozish mumkin:

$$\int\frac{dx}{3+sinx}=\int\frac{1}{3+\frac{2t}{1+t^2}}\cdot\frac{2dt}{1+tg^2\frac{x}{2}}=2\int\frac{dt}{3t^2+2t+3}=$$

$$=\frac{2}{3}\int\frac{d\left(t+\frac{1}{3}\right)}{\left(t+\frac{1}{3}\right)^2+\left(\frac{2\sqrt{2}}{3}\right)^2}.$$

$\frac{2}{3}\int\frac{d\left(t+\frac{1}{3}\right)}{\left(t+\frac{1}{3}\right)^2+\left(\frac{2\sqrt{2}}{3}\right)^2}=\frac{\sqrt{2}}{2}arctg\frac{3t+1}{2\sqrt{2}}+C.$ x –o‘zgaruvchiga qaytamiz.

Javob. $\frac{\sqrt{2}}{2}arctg\frac{3tg\frac{x}{2}+1}{2\sqrt{2}}+C.$

5.3.21. $\int\left(1-\frac{2}{x}\right)^2\cdot e^xdx$ aniqmas integralni hisoblang.

Yechish. $\int\left(1-\frac{2}{x}\right)^2\cdot e^xdx=\int\left(1-\frac{4}{x}+\frac{4}{x^2}\right)\cdot e^xdx=e^x-4\int\frac{e^xdx}{x}+4\int\frac{e^xdx}{x^2}.$

$-4\int\frac{e^xdx}{x}$ integralni bo‘laklab integrallaymiz:

$$-4\int\frac{e^xdx}{x}=-4\frac{e^x}{x}-4\int\frac{e^xdx}{x^2}.$$

Hisoblangan integralni o‘rniga qo‘ysak hisoblanmagan $\int\frac{e^xdx}{x^2}$ integral yo‘qoladi.

Javob. $e^x\left(1-\frac{4}{x}\right)+C,\ x\neq 0.$

Aniq integral.

5.3.22. Hisoblang

$$\int_0^{\frac{\pi}{2}} cos^4 x dx$$

Yechish. Integral ostidagi ifodani quyidagicha shakl almashtirish mumkin:

$$cos^4 x = (\frac{1+\cos 2x}{2})^2 = \frac{1}{4}(1 + 2\cos 2x + cos^2 2x) =$$
$$= \frac{3}{8} + \frac{1}{2}\cos 2x + \frac{1}{8}\cos 4x.$$

Demak,

$$\int_0^{\frac{\pi}{2}} cos^4 x dx = \int_0^{\frac{\pi}{2}} \left(\frac{3}{8} + \frac{1}{2}\cos 2x + \frac{1}{8}\cos 4x\right) dx =$$

$$= \left(\frac{3}{8}x + \frac{1}{4}\sin 2x + \frac{1}{32}\sin 4x\right)\Big|_0^{\frac{\pi}{2}} = \frac{3\pi}{16}.$$

5.3.23. Hisoblang

$$\int_1^2 \ln x \, dx.$$

Yechish. Bo'laklab integrallash usuliga ko'ra: u(x)=ln x va dv=dx bo'lsin, u holda $du = \frac{1}{x}dx$ va v(x)=x bo'lib,

$$\int_1^2 \ln x \, dx = x \ln x \Big|_1^2 - \int_1^2 x \cdot \frac{1}{x} dx = (x \ln x - x)\Big|_1^2 = 2\ln 2 - 1.$$

5.3.24. Quyidagi funksiyaning [2;3] oraliqdagi eng kichik va eng katta qiymatlarini toping

$$F(x) = \int_a^x (t+1)dt$$

Yechish. $F'(x) = x + 1$ bo'lib, F(x)-funksiya x> -1 bo'lganda, ya'ni, [2;3] oraliqda o'suvchi bo'lib:

$$\max_{[2;3]} F(x) = F(3) = \int_0^3 (x+1)dx = (\frac{x^2}{2} + x)\Big|_0^3 = 7{,}5.$$

$$\min_{[2;3]} F(x) = F(2) = \int_0^2 (x+1)dx = \left(\frac{x^2}{2} + x\right)\Big|_0^2 = 4.$$

5.3.25. To‘g’ri chiziq bo‘ylab harakatlanayotgan jismning tezligi

$$v(t) = \sqrt{t} + 2t \ (^{km}/_{s})$$

qonuniyat bo‘yicha o‘zgaradi. Agar jism 1 s. dan keyin harakat trayektoriyasidagi A punktdan 5 km uzoqlashgan bo‘lsa 3 s. dan keyin A punktdan qanday uzoqlikda bo‘ladi.

Yechish. To‘g’ri chiziq bo‘ylab harakatlanayotgan jismning tezligi v(t) bo‘lsin, u holda t_0 vaqtdan t($t > t_0$) vaqtgacha bosib o‘tgan yo‘l quyidagicha hisoblanadi:

$$S(t) = \int_{t_0}^{t} v(x)dx.$$

Demak,

$$S = \int_1^3 (\sqrt{t} + 2t)dt = \left(\frac{2t^{3/2}}{3} + t^3\right)\Big|_1^3 = 7\frac{1}{3} + 2\sqrt{3}\ (km).$$

5.3.26. Quyidagi funksiya grafigining absissalari $x_1 = 0$ va $x_2 = b, (b > 0)$ bo‘lgan nuqtalari orasidagi egri chiziq uzunligini hisoblang

$$f(x) = \frac{a}{2}(e^{x/a} + e^{-x/a}).$$

Yechish. Berilgan funksiyaning hosilasi:

$$f'(x) = \frac{a}{2}\left(\frac{1}{a}e^{x/a} - \frac{1}{a}e^{-x/a}\right)$$

bo‘lib,

$$\sqrt{1 + (f'(x))^2} = \frac{1}{2}\left(e^{\frac{x}{a}} + e^{-\frac{x}{a}}\right).$$

$$L = \int_0^b \frac{1}{2}\left(e^{\frac{x}{a}} + e^{-\frac{x}{a}}\right)dx = \frac{a}{2}\left(e^{\frac{b}{a}} - e^{-\frac{b}{a}}\right).$$

5.3.27. y=0, $y=2^x$, x=−1, x=2 chiziqlar bilan chegaralangan soha yuzini toping

Yechish. $S = \int_{-1}^{2} 2^x dx = \frac{1}{\ln 2} 2^x \Big|_{-1}^{2} = \frac{7}{2\ln 2}$

5.3.28. $y=x^2+1, y=0, x=1, x=2$ chiziqlar bilan chegaralangan egri chiziqli trapetsiyani a) OX; b) OY o'qi atrofida aylanishidan hosil bo'lgan jismning hajmini toping

Yechish. a) Ox o'qi atrofida aylanishidan hosil bo'lgan jismning hajmi:

$$V_{ox} = \pi \int_1^2 (x^2+1)^2 dx = \pi \int_1^2 (x^4+2x^2+1)dx = \frac{178\pi}{15}(h.b).$$

b) OY o'qi atrofida aylanishidan hosil bo'lgan jismning hajmi:

$V_{oy} = 2\pi \int_1^2 x \cdot (x^2+1)dx = \frac{21\pi}{2}$ $(h.b)$.

Mustaqil yechish uchun misollar.

Yuqoridagi integrallash usullaridan foydalanib, quyidagi aniqmas integrallarni hisoblang.

1. $\int \frac{xdx}{(x-1)^2(x+1)^3}$.
2. $\int \frac{x^2+3x-2}{(x-1)(x^2+x+1)^2}dx$.
3. $\int \frac{4x^5-1}{(x^5+x+1)^2}dx$.
4. $\int \frac{dx}{x^4+2x^3+3x^2+2x+1}$.
5. $\int \frac{1-x^7}{x(1+x^7)}dx$.
6. $\int \frac{x^2-1}{x^4+x^3+x^2+x+1}dx$.
7. $\int \frac{dx}{(1+\sqrt[4]{x})^3 \cdot \sqrt{x}}$.
8. $\int \frac{1}{x(1+2\sqrt{x}+\sqrt[3]{x})}dx$.
9. $\int \frac{\sqrt{x^2+2x+2}}{x}dx$.
10. $\int \frac{xdx}{(1+x)\cdot\sqrt{1-x-x^2}}$.
11. $\int \frac{dx}{(x^2+1)\cdot\sqrt{x^2-1}}$.
12. $\int \frac{1}{(1-x^4)\cdot\sqrt{1+x^2}}dx$.
13. $\int \frac{\sqrt{x(x+1)}}{\sqrt{x}+\sqrt{x+1}}dx$.
14. $\int \frac{x+\sqrt{1+x+x^2}}{1+x+\sqrt{1+x+x^2}}dx$.
15. $\int \frac{dx}{\sqrt[4]{1+x^4}}$.
16. $\int \frac{dx}{x\cdot\sqrt[6]{1+x^4}}$.
17. $\int sin^4x \cdot cos^5x\, dx$.
18. $\int sin^5x \cdot cos^5x\, dx$.
19. $\int \frac{dx}{\sqrt{tgx}}$.
20. $\int \frac{dx}{\sqrt[3]{tgx}}$.

21. $\int \frac{sinx \cdot cosx}{sinx + cosx} dx.$

22. $\int \frac{1}{(sin^2x + 2cos^2x)^2} dx.$

23. $\int \frac{sinx + 2cosx - 3}{sinx - 2cosx + 3} dx.$

24. $\int \frac{2sinx + cosx}{3sinx + 4cosx - 2} dx.$

25. $\int x^3 e^{3x} dx.$

26. $\int (x^2 - 2x + 2) \cdot e^{-x}\, dx.$

27. $\int x^7 e^{-x^2} dx.$

28. $\int x^2 e^{\sqrt{x}} dx.$

29. $\int \sqrt{\frac{e^x - 1}{e^x + 1}} dx.$

30. $\int \frac{1}{\sqrt{1 + e^x} + \sqrt{1 - e^x}} dx.$

31. $\int x^3 ln^3 x dx.$

32. $\int ln\left(\sqrt{1 - x} + \sqrt{1 - x}\right) dx.$

33. $\int \sqrt{x} arctg\sqrt{x} dx.$

34. $\int x arcsin(1 - x) dx.$

35. $\int arcsin\sqrt{x} dx.$

36. $\int (2x + 3) arccos(2x - 3) dx.$

Aniq integrallarni hisoblang

37. $\int_1^4 \frac{4x - 2\sqrt{x}}{x} dx.$

38. $\int_{0,5}^{1} \left(4x - \frac{1}{2x}\right) dx.$

39. $\int_0^{\frac{\pi}{2}} cos\, x\, sin\, x\, dx.$

40. $\int_0^{\pi} \cos x \sin 3x dx.$

41. $\int_{-8}^{8} \frac{dx}{\sqrt{5 + \frac{x}{2}}}.$

42. $\int_{-4}^{2} \frac{x\, dx}{\sqrt{2 - \frac{x}{2}}}.$

43. $\int_{\frac{1}{3}}^{\frac{5}{3}} (x - 2)\sqrt{3x - 1} dx.$

44. $\int_1^2 \frac{dx}{\sqrt{x - 1} + \sqrt{1 + x}}.$

45. $\int_2^9 \sqrt[3]{x-1}dx.$

46. $\int_0^{\frac{7}{3}} \frac{x+1}{\sqrt[3]{3x+1}}\,dx.$

47. $\int_0^{\frac{\pi}{4}} (\sin 2t - \cos 2t)^2\,dt.$

48. $\int_0^1 \frac{x\,dx}{(x+1)^2}.$

49. $\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} (tg\,x + c\,tg\,x)^{-1}\,dx.$

50. $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \left[1 - \frac{1}{1-\sin^{-1}\left(2x+\frac{3\pi}{2}\right)}\right] dx$

51. $\int_0^{\pi} \left(\cos^2\left(\frac{3\pi}{8} - \frac{x}{4}\right) - \cos^2\left(\frac{11\pi}{8} + \frac{x}{4}\right)\right) dx.$

52. $\int_{-1}^{1} \sqrt{x^2-2x+1}\,dx.$

53. $\int_0^2 \sqrt{x^2-2x+1}\,dx.$

54. $\int_{-\frac{\pi}{4}}^{\frac{\pi}{2}} \sqrt{1-\cos^2 x}\,dx.$

55. $\int_0^{\pi} \sqrt{1-\sin 2x}\,dx.$

56. $\int_3^5 \left(\sqrt{x+2\sqrt{2x-4}} + \sqrt{x-2\sqrt{2x-4}}\right) dx.$

Quyidagi funksiyalarning berilgan sonly oraliqdagi eng katta va eng kichik qiymatlarini toping.

57. $F(x) = \int_0^x \sin t\, dt, x \in \left[0; \frac{\pi}{2}\right]$.

58. $F(x) = \int_0^x (2t-5)dt, x \in [-1; 3]$.

59. $F(x) = \int_0^x (t^2 - 5t + 6)dt, x \in [0; 4]$.

4. $F(x) = \int_1^x |t|\, dt, x \in \left[-\frac{1}{2}; \frac{1}{2}\right]$.

60. Quyidagi funksiyaning Ox o'qi bilan kesishish nuqtalaridagi urinma tenglamalarini yozing.

$$F(x) = \int_2^x (2t-5)dt.$$

61. Quyidagi funksiyalar kesishish nuqtalarining absissalarini toping.

$$F_1(x) = \int_2^x (2t-5)dt, \quad F_2(x) = \int_3^x (2t-5)dt.$$

62. Quyidagi funksiyalar kesishish nuqtalari koordinatalarin toping.

$$F_1(x) = \int_2^x (2t-5)dt, \quad F_2(x) = \int_0^x 2tdt.$$

63. Quyidagi funksiyaning koordinatalar boshidan o'tuvchi boshlang'ich funksiyasini toping.

$$F(x) = \int_3^x (2t-5)dt.$$

64. Quyidagi funksiyaning grafigiga koordinatalar tekisligi birinchi choragi bessektrisasiga parallel qilib o'tkazilgan urinmalari tenglamalarini yozing.

$$F(x) = \int_0^x 2|t|dt.$$

65. Jism harakatlanish tezligi vaqtning kvadratiga proporsional. $t = 0$ dan dastlabki $3s$ da $18sm$ yo'l yurgani ma'lum bo'lsa, O'ilgan yo'l va vaqt orasidagi bog'lanishni toping.

66. Ikki jism bir nuqtadan bir xil yo'nalishda to'g'ri chiziq bo'ylab bir paytda harakatlanishni boshlashdi. Jismlar tezliklari $v_1 = 3t^2\ m/s$ va $v_2 = 2t\ m/s$ ga teng bo'lsa, harakat boshlanganidan necha sekund o'tgach jismlar orasidagi masofa $216\ m$ bo'ladi?

67. Agar $f(x) = Ax^2 + Bx + C$ funksiya berilgan bo'lib, $f'(1) = 8$,

$f(2) + f''(2) = 33$, $\int_0^1 f(x) = \frac{7}{3}$ shartlar bajarilsa, A, B, C larning qiymatlarini toping.

68. Quyidagi tenglik bajariladigan a –parametrning musbat qiymatlarini toping.

$$\int_0^a (3x^2 + 4x - 5)dx = a^3 - 2.$$

Quyidagi funksiyalar bilan berilgan egri chiziqlarning ko'rsatilgan oraliqdagi uzunliklarini toping.

$a)\ x = \frac{1}{4}y^2 - \frac{1}{2}lny, \qquad (1 \le y \le e).$

$b)\ y = aln\frac{a^2}{a^2 - x^2}, (0 \le x \le b < a).$

$c)\ y = lncosx \ \left(0 \le x \le a < \frac{\pi}{2}\right).$

$d)\ y = \sqrt{1 - x^2} + arcsinx, \left(0 \le x \le \frac{7}{9}\right).$

$e)\ y = \sqrt{1 - x^2} + arcsinx, \left(0 \le x \le \frac{7}{9}\right).$

$f)\ y = arccose^{-x}, (0 \le x \le 1).$

69. Quyidagi misollarda berilgan chiziqlar bilan chegaralangan sohalar yuzini toping.

1. $y = x^2 + x, \qquad y = x + 1.$ 2. $y = -2x^2 + 3x + 6, \qquad y = x + 2$

3. $y = 0, \quad y = 20 - 2x^2 - 6x.$

4. $y = x^2, y = \frac{1}{x}, y = 0, x = 2.$

5. $y = \left(\frac{1}{2}\right)^x, x - 2y + 2 = 0, x = 2.$

6. $y = 4x - x^2, y - x = 0.$

7. $y = \frac{5}{x}, y = 6 - x.$

8. $y = x^3, y = \frac{1}{x}, x = 2.$

9. $y = x^2 + 1, y = -x^2 + 3.$

10. $y = \frac{1}{\cos^2 x}, y = 0, x = 0, x = \frac{\pi}{4}.$

11. $y = 2^x, y = 2, x = -1.$

12. $xy = 7, y = 0, x = 4, x = 12.$

13. $y = (x-1)^2, y = x + 1.$

14. $y = -x^2 + \frac{7}{2}x + 1, y = 2^{-x}, x = 2 \ (x \le 2).$

15. $x = 1, x = 2, y = 0, log_2 x + log_2 y = 0.$

16. $y = 2x^2 + 1, y = x + 2, y = 1{,}5.$

17. $y = 2^x, y = 4^x, x = 1.$

18. $y = x^2, y = 2\sqrt{2x}.$

19. $2yx = 16 + x^2, y = 5.$

20. $y = -1 + 8x^2 - x^4, y = 15, x = 1 \ (x \ge 1).$

21. $y = \frac{1}{(1+x^2)}, y = \frac{x^2}{2}.$

22. $3y = -x^2 + 8x - 7, y + 1 = \frac{4}{(x-3)}.$

23. $y = \sqrt{x}, y = \sqrt{4 - 3x}, y = 0.$

70. $\begin{cases} x^2 + y^2 \le r^2, r > 0, \\ x - y \le 0, y \ge 0. \end{cases}$ tengsizliklar sistemasini qanoatlantiruvchi nuqtalar to‘plamidan iborat soha yuzini toping.

71. $max\{x; y\} = 1, x^2 + y^2 = 1$ chiziqlar qismlari va koordinatalar tekisligining birinchi choragida $max\{x; y\} = \begin{cases} x \ge y \text{ bo'lganda}, x, \\ x < y \text{ bo'lganda}, y \end{cases}$ shart bilan chegaralangan soha yuzini toping.

72. $y = x^2$ va $y = 2x - x^2$ funksiyalar grafiklari bilan chegaralangan soha yuzini toping.

73. $y = 2x^2$ funksiya grafigi, shu funksiya grafigiga $x = 2$ nuqtada o‘tkazilgan urinma va Ox o‘qi bilan chegaralangan soha yuzini toping.

74. $y = x^2 - 2x + 2$ funksiya grafigi, shu funksiya grafigining $M(3; 5)$ nuqtasiga o‘tkazilgan urinma va koordinata o‘qlari bilan chegaralangan soha yuzini toping.

33. $y = \frac{1}{x} + 1$funksiya grafigi, shu funksiya grafigining $M\left(2;\frac{3}{2}\right)$ nuqtasiga o'tkazilgan urinma va $x = 1$ to'g'ri chiziqlar bilan chegaralangan soha yuzini toping.

75. $y = x^2 - 4x + 5$ funksiya grafigi, shu funksiya grafigining $x_1 = 1$ va $x_2 = 4$ nuqtalariga o'tkazilgan urinmalar bilan chegaralangan yuzani toping.

77. $xy = 2, x = 1, x = 2$ chiziqlar bilan chegaralangan egri chiziqli trapetsiyani Ox o'qi atrofida aylanishidan hosil bo'lgan jismning hajmini toping.

78. $y^2 = x, y = x^2$ egri chiziqlar bilan chegaralangan figurani Ox o'qi atrofida aylanishidan hosil bo'lgan jismning hajmini toping.

79. $y = \frac{e^x + e^{-x}}{2}, x_1 = a, x_2 = b$ chiziqlar bilan chegaralangan figurani Ox o'qi atrofida aylanishidan hosil bo'lgan jismning hajmini toping.

80. $y = 2x - x^2$vaOx o'qi bilan chegaralangan sohani Oy o'qi atrofida aylanishidan hosil bo'lgan jismning hajmini toping.

81. $y = arcsinx, y = \frac{\pi}{2}, x = 0$ chiziqlar bilan chegaralangan egri chiziqli trapetsiyani Oy o'qi atrofida aylanishidan hosil bo'lgan jismning hajmini toping.

82. $y = ln2, y = lnx, y = 0, x = 0$ chiziqlar bilan chegaralangan sohani Oy o'qi atrofida aylanishidan hosil bo'lgan jismning hajmini toping.

5.4. Oddiy differensial tenglamalar.

Agar F –berilgan funksiya barcha argumentlari bo'yicha uzluksiz bo'lib, quyidagi ko'rinishdagi $F\left(x, y, y', y'', \dots, y^{(n)}\right) = 0$ tenglamaga n tartibli oddiy differensial tenglama deyiladi. Bunda y=y(x) noma'lum fuhksiya va $y^{(n)} = y^{(n)}(x)$ –noma'lum funksiyaning $n-$ tartibli hosilasi.

Yuqoridagi $n-$ tartibli oddiy differensial tenglamaning umumiy yechimi istalgan n- ta constantalar(o'zgarmas sonlar)ga bog'liq bo'ladi.

5.4.1. $y = cosx$ funksiya $y'' + y = 0$ tenglamaning yechimi bo'lishini ko'rsating.

Yechish: Berilgan funksiyaning ikkinchi tartibli hosilasini hisoblab berilgan tenglamaga qo'yib: $(y' = -sinx,\ y'' = -cosx)$, $-cosx + cosx = 0$ tenglamani hosil qilish mumkin. Demak, $y = cosx$ funksiya berilgan differensial tenglamaning yechimi bo'lar ekan.

Differensial tenglama qo'shimcha shart bilan berilsa, boshlang'ich shartlar bilan berilgan Koshi masalasi deyiladi. Qo'shimcha shartlar bir nechta nuqtalarda berilgan bo'lsa, chegaraviy masala deyiladi.

5.4.2. $y(0) = y'(0) = 0$ boshlang'ich shart bilan berilgan $y'' = x$ tenglamani yeching.

Yechish. Berilgan tenglamani ketma-ket ikki marta integrallab, mos ravishda $y' = \frac{x^2}{2} + C_1\ va\ \ y = \frac{x^3}{6} + C_1x + C_2$ tengliklarni hosil qilish mumkin.

$y(0) = y'(0) = 0$ boshlang'ich shartdan $C_1 = C_2 = 0$ ekanligi kelib chiqadi.

Javob: $y = \frac{x^3}{6}$.

5.4.3. $y(1) = y(2) = 0$ chegaraviy shartlar bilan berilgan $y'' = 12x^2 + 18x$ tenglamani yeching.

Yechish. Berilgan tenglamani ketma-ket ikki marta integrallab, mos ravishda

$y' = 4x^3 + 9x^2 + C_1$ va $y = x^4 + 3x^3 + C_1x + C_2$ tengliklarni hosil qilish mumkin. $y(1) = y(2) = 0$ chegaraviy shartlardan quyidagi sistema hosil bo'ladi:

$$\begin{cases} 1 + 3 + C_1 + C_2 = 0, \\ 16 + 24 + 2C_1 + C_2 = 0. \end{cases}$$

Bundan: $C_1 = -36, C_2 = 32$.

Javob: $y = x^4 + 3x^3 - 36x + 32$.

Ta'rif.

$$y' = f_1(x)f_2(y) \text{ yoki } f_1(x)dx + f_2(y)dy = 0 \quad (1)$$

ko'rinishga keladigan differensial tenglamalarga o'zgaruvchilari ajraladigan differensial tenglamalar deyiladi.

Bu tenglamaning yechimi y=$\varphi(x)$ bo'lsin, u holda dy= $\varphi'(x)dx$ bo'lib, (1) tenglama:

$$f_1(x)dx + f_2[\varphi(x)]\varphi'(x)dx = 0$$

ko'rinishga keladi va berilgan tenglamaning yechimi quyidagicha yoziladi:

$$\int f_1(x)dx + \int f_2[\varphi(x)]\varphi'(x)dx = 0 = 0.$$

5.4.4. $cos^2y \cdot ctgx\, dx + sin^2x \cdot tgy\, dy = 0$ tenglamani yeching.

Yechish. Berilgan tenglamaning ikkala tomonini $cos^2y \cdot sin^2x$ ga bo'lib,

$\frac{ctgx}{sin^2x}dx + \frac{tgy}{cos^2y}dy = 0$ tenglikni va uni integrallab,

$\int \frac{ctgx}{sin^2x}dx + \int \frac{tgy}{cos^2y}dy = C$ yoki $-\frac{ctg^2x}{2} + \frac{tg^2y}{2} = c$ (c –ixtiyoriy o'zgarmas son) tenglikni hosil qilish mumkin.

C=2c deb olish mumkin.

Javob: $tg^2y - ctg^2x = C.$

5.4.5. $xy' - \frac{y}{lnx} = 0$ tenglamaning umumiy yechimini toping.

Yechish. Berilgan tenglamani $\frac{1}{xy}$ ga ko'paytirib, $\frac{dy}{y} = \frac{dx}{x \cdot lnx}$ tenglamani va undan

$\int \frac{dy}{y} = \int \frac{dx}{x \cdot lnx}$, $ln|y| = \int \frac{d(lnx)}{lnx}$, $ln|y| = ln|lnx| + c$ $(C > 0)$,

$c = lnC$ bo'ladi.

Demak, tenglamaning umumiy yechimi $|y| = C \cdot |lnx|$.

Javob: $y = C \cdot lnx$.

5.4.6. $(1 + e^x)y \cdot y' = e^x$ differensial tenglamaning $y(0) = 1$ boshlang'ich shart bajariladigan xususiy yechimini toping.

Yechish: Berilgan tenglamani $ydy = \frac{e^x}{1+e^x}dx$ ko'rinishda yozib, undan|

$$\int ydy = \int \frac{e^x}{1+e^x}dx \quad yoki \quad \frac{y^2}{2} = \ln(1+e^x) + C$$

tenglikni olish mumkin. Berilgan boshlang'ich shartdan:

$\frac{1}{2} = \ln(1+e^0) + C$ yoki C= $\frac{1}{2} - \ln 2$. Demak,

$$y^2 = 1 + 2ln\frac{1+e^x}{2} = 1 + ln\left(\frac{1+e^x}{2}\right)^2 \quad yoki \; y = \pm\sqrt{1 + ln\left(\frac{1+e^x}{2}\right)^2}.$$

Bunda boshlang'ich shartga ko'ra: $y > 0$ $(y(0) = 1)$

Javob: $y = \sqrt{1 + ln\left(\frac{1+e^x}{2}\right)^2}$.

Ta'rif.

$$y' = f\left(\frac{y}{x}\right)$$

ko'rinishdagi differensial tenglamalarga bir jinsli differensial tenglama deyiladi.

Oddiy bir jinsli differensial tenglamalar $y(x) = x \cdot u(x)$ almashtirish yordamida o'zgaruvchilari ajraladigan differensial tenglamaga keltiriladi.

$y'(x) = u(x) + xu'(x)$ bo'lib, berilgan tenglama $u + xu' = f(u)$ ko'rinishga, yoki $\frac{du}{f(u)-u} = \frac{dx}{x}$ ko'rinishga keladi va uning yechimi $\int \frac{du}{f(u)-u} = \int \frac{dx}{x}$ yoki $\int \frac{du}{f(u)-u} = \ln|x|$ kabi yoziladi. Oxirgi tenglikdan u(x) funksiyani topib berilgan tenglamaning yechimi $y(x) = x \cdot u(x)$ ekanligiga ishonch hosil qilish mumkin.

5.4.7. $(x^2 + y^2)dx - xydy = 0$ tenglamaning umumiy yechimini toping.

Yechish. Berilgan tenglamadan $y' = \frac{x^2+y^2}{xy}$ bo'lib, y=ux almashtirish yordamida $x\frac{du}{dx} + u = \frac{1+u^2}{u}$ yoki $\frac{dx}{x} = udu$ tenglamani hosil qilish mumkin. Bundan,

$$ln|x| = \frac{u^2}{2} + lnC, \; ln\frac{x}{C} = \frac{u^2}{2} \text{ ya'ni } x = Ce^{u^2/2}.$$

Javob: $x = Ce^{\frac{y^2}{2x^2}}$.

5.4.8. Differensial tenglamani yeching:

$$\frac{dy}{dx} = \frac{x^2 + y^2}{2xy}.$$

Yechish. $y = xu$ almashtirish yordamida berilgan tenglamani quyidagicha yozish mumkin:

$x\frac{du}{dx} + u = \frac{1+u^2}{2u}$. Bundan $x\frac{du}{dx} = \frac{1+u^2}{2u} - u = \frac{1-u^2}{2u}$ $yoki$ $\frac{dx}{x} = \frac{2udu}{1-u^2}$;

$$ln|x| = -ln|1-u^2| + ln|C|; x = \frac{C}{1-u^2}; 1 = \frac{Cx}{x^2 - y^2};$$

$$x^2 - y^2 = Cx;$$

Jabob: $y = \pm\sqrt{x^2 - Cx}$.

5.4.9. Quyidagi tenglamani yeching:

$$x^2y' = y^2 + yx.$$

Yechish. Berilgan tenglamaning ikkala tomonini x^2 ga bo'lib, $y(x) = x \cdot u(x)$ almashtirish yordamida hosil bo'lgan: $\frac{du}{u^2} = \frac{dx}{x}$ tenglikni integrallab,

$$u = -\frac{1}{ln|x| + C} \; va \; y = -\frac{x}{ln|x| + C}.$$

Javob: $y = -\frac{x}{ln|x|+C}$.

$$A(x) \cdot y' + B(x)y = f(x)$$

ko'rinishdagi tenglamalarga oddiy chiziqli differensial tenglamalar deyiladi. Bu yerda $A(x), B(x)$ va $f(x)$ berilgan uzluksiz funksiyalar.

$A(x) \neq 0$ deb, berilgan tenglamani ikkala tomonini $A(x)$ ga bo'lib, hosil bo'lgan:

$$\frac{dy}{dx} + P(x) \cdot y = Q(x). \left(bu \text{ yerda } P(x) = \frac{B(x)}{A(x)}, Q(x) = \frac{f(x)}{A(x)}\right).$$

tenglamani $y = uv$ $(\frac{dy}{dx} = v\frac{du}{dx} + u\frac{dv}{dx})$ almashtirish yordamida o'zgaruvchilari ajraladigan tenglamaga keltirish mumkin:

$$v\frac{du}{dx} + u\left[\frac{dv}{dx} + P(x)v\right] = Q(x).$$

Endi berilgan tenglamaning $v = v(x)$ xususiy yechimini shunday tanlash kerakki,

$\frac{dv}{dx} + P(x)v = 0$ bo'lsin, u holda berilgan tenglama quyidagi ko'rinishga keladi:

$$v\frac{du}{dx} = Q(x).$$

Ushbu tenglamaning umumiy yechimi $u = u(x, C)$.Demak, berilgan chiziqli differensial tenglamaning umumiy yechimi

$y = u(x, C)v(x)$ ko'rinishda bo'ladi.

5.4.10. Chiziqli differensial tenglamani yeching:

$$y' - y\,tgx = sinx.$$

Yechish. $y = uv$ va $y' = u'v + uv'$ tengliklarlarga ko'ra, berilgan tenglamani quyidagicha yozish mumkin:

$$u'v + uv' - uv\,tgx = sinx,$$

$$u'v + u(v' - v\,tgx) = sinx. (1)$$

$v' - v\,tgx = 0$ bo'lib, $\frac{dv}{dx} = v\,tgx$; $\frac{dv}{v} = tgx\,dx$; $ln|v| = -ln|cosx|$,

$v = \frac{1}{cosx}$ xususiy yechimni (1) tenglamaga qo'yib hosil bo'lgan tenglamani quyidagicha yechish mumkin:

$$u' \cdot \frac{1}{\cos x} = \sin x,$$

$$du = \sin x \cos x\,dx,$$

$$u = \frac{sin^2x}{2} + C.$$

Demak, $y = uv$ tenglikka ko'ra berilgan tenglamaning umumiy yechimi: $y=(\frac{sin^2x}{2} + C) \cdot \frac{1}{\cos x}$.

Javob: $y = \left(\frac{sin^2x}{2} + C\right) \cdot \frac{1}{cosx}$.

5.4.11. Differensial tenglamani yeching:

$$xy' = 3y + x^2.$$

Yechish. Dastlab, berilgan tenglamaning bir jinsli bo'lgan

$$xy' = 3y$$

qismining umumiy yechimini topamiz:

$$\int \frac{dy}{y} = 3\int \frac{dx}{x}, \qquad ln|y| = ln|c_1x^3|, \qquad y_1 = Cx^3.$$

Endi berilgan tenglamaning xususiy yechimini $y = z(x)x^3$ ko'rinishida izlaymiz, u holda berilgan tenglama

$$x^2z' = 1$$

ko'rinishga kelib, uning yechimlaridan biri: $z = -\frac{1}{x}$ yoki $y_0 = -x^2$. Demak, $y = y_1 + y_0$ bo'lib, berilgan tenglamaning umumiy yechimi y=$Cx^3 - x^2$ bo'ladi.

Javob: $y = Cx^3 - x^2$.

Ba'zi hollarda chiziqli bo'lmagan tenglamalarda y ni erkli o'zgaruvchi deb qarab, x ni izlanayotgan funksiya deb olish bilan chiziqli tenglama ko'rinishiga keltirish mumkin. Masalan, chiziqli bo'lmagan $y = (2x + y^3) \cdot y'$ tenglamani

$$ydx - (2x + y^3)dy = 0$$

yoki

$$\frac{dx}{dy} - \frac{2}{y}x = y^2$$

ko'rinishdagi chiziqli tenglamaga keltirish mumkin.

Bernulli tenglamasi

$$y' + a(x)y = b(x)y^m. \ (m \neq 0, m \neq 1)$$

Agar $m = 0$ bolsa, tenglama chiziqli va $m = 1$ bo'lsa, tenglama o'zgaruvchilari ajraladigan differensial tenglama bo'ladi. Ushbu tenglamani $y = uv$ yoki $z = y^{1-m}$ almashtirish yordamida chiziqli tenglamaga keltirish mumkin.

5.4.12. Differensial tenglamani yeching:

$$y' = \frac{y}{x} + \frac{x^2}{y}.$$

Yechish. $y = uv$ bo'lsin, u holda $y' = u'v + uv'$ bo'lib, berilgan tenglama quyidagi ko'rinishga keladi:

$$v\left(\frac{du}{dx} - \frac{u}{x}\right) + \left(\frac{dv}{dx}u - \frac{x^2}{uv}\right) = 0.$$

$$\frac{du}{dx} - \frac{u}{x} = 0$$

tenglamaning umumiy yechimi u= Cx bo'lib, xususiy yechimlaridan biri $u_1 = x$ uchun: $\frac{dv}{dx}x - \frac{x^2}{xv} = 0$ yoki $\frac{dv}{dx} = \frac{1}{v}$ va bu tenglamaning umumiy yechimi

$v = \sqrt{2x + C}$. Demak, berilgan tenglamani umumiy yechimi: y=x$\sqrt{2x + C}$.

Javob: $y = x\sqrt{2x + C}$.

5.4.13. Quyidagi Bernulli tenglamasini x=x(y) ga nisbatan yeching

$$\frac{dx}{dy} = \frac{x}{2y} - \frac{1}{2x}$$

Yechish. $x = uv$ almashtirish yordamida berilgan tenglamani quyidagicha yozish mumkin:

$$v\left(\frac{du}{dy} - \frac{u}{2y}\right) + \left(\frac{dv}{dy}u + \frac{1}{2uv}\right) = 0.$$

$\frac{du}{dy} - \frac{u}{2y} = 0$ tenglamaning $u_1 = \sqrt{y}$ xususiy yechimini $\left(\frac{dv}{dy}u + \frac{1}{2uv}\right) = 0$ tenglamaga qo'yib, tenglamaning umumiy $v^2 = ln\frac{C}{y}$ yechimini topish mumkin. $u_1^2 = y$ va $v^2 = ln\frac{C}{y}$ tengliklarni o'zaro ko'paytiramiz.

Demak, $x^2 = y \cdot ln\frac{C}{y}$.

Javob: $x^2 = y \cdot ln\frac{C}{y}$.

$y'' = f(x)$ ko'rinishdagi ikkinchi tartibli differensial tenglamalar.

$$y'' = f(x)$$

ko'rinishdagi ikkinchi tartibli differensial tenglamalar ikki marta ketma – ket integrallash yordamida yechiladi.

5.4.14. Tenglamaning umumiy yechimini toping

$$y'' = 1 - x^2.$$

Yechish. $\frac{dy}{dx} = z$ bo'lsin, u holda: $\frac{dz}{dx} = 1 - x^2$, $dz = (1 - x^2)dx$ bo'lib, $z = \int(1 - x^2)\,dx = x - \frac{x^3}{3} + C_1$. U holda: $\frac{dy}{dx} = x - \frac{x^3}{3} + C_1$ yoki $y = \int\left(x - \frac{x^3}{3} + C_1\right)dx = \frac{x^2}{2} - \frac{x^4}{12} + C_1x + C_2$.

Javob. $y = \frac{x^2}{2} - \frac{x^4}{12} + C_1x + C_2$.

O‘zgarmas koeffitsientli ikkinchi tartibli bir jinsli chiziqli differensial tenglamalarni yechish.

$$y'' + py' + qy = 0.$$

Ko‘rinishdagi o‘zgarmas koeffitsientli ikkinchi tartibli bir jinsli chiziqli differensial tenglamani yechishda uning xarakteristik $r^2 + pr + q = 0$ tenglamasi tuziladi. (izlanayotgan funksiya hosilasi tartibi r ning mos darajasi bilan va y funksiya 1 bilan almashtiriladi) Berilgan tenglamaning umumiy yechimi, xarakteristik tenglamaning r_1, r_2-ildizlariga bog’liq bo‘ladi.

1-hol. Agar $r^2 + pr + q = 0$ tenglamaning ildizlari r_1, r_2 –lar turli haqiqiy sonlar bo‘lsa, u holda berilgan differensial tenglamaning umumiy yechimi quyidagi ko‘rinishda bo‘ladi:

$$y = C_1 e^{r_1 x} + C_2 e^{r_2 x}.$$

2-hol. Agar $r^2 + pr + q = 0$ tenglamaning ildizlari r_1, r_2 -lar haqiqiy va o‘zaro teng($r_1 = r_2 = r$) bo‘lsa, u holda berilgan differensial tenglamaning umumiy yechimi quyidagi ko‘rinishda bo‘ladi:

$$y = (C_1 + C_2 x) \cdot e^{rx}.$$

3-hol. Agar $r^2 + pr + q = 0$ tenglamaning ildizlari r_1, r_2 -lar o‘zaro qo‘shma kompleks sonlar($r_1 = \alpha + \beta i$, $r_2 = \alpha - \beta i$) bo‘lsa, u holda differensial tenglamaning umumiy yechimi quyidagi ko‘rinishda bo‘ladi:

$$y = e^{\alpha x}(C_1 cos\beta x + C_2 sin\beta x).$$

5.4.15. Tenglamaning umumiy yechimini toping

$$y'' - 5y' - 6y = 0.$$

Yechish. y'', y', y larni yuqorida aytilgandek, mos ravishda $r^2, r, 1$ larga almashtirib, hosil bo‘lgan: $r^2 - 5r - 6 = 0$ xarakteristik tenglamaning ildizlari $r_1 = -1, r_2 = 6$ bo‘ladi.

Demak, $y = C_1 e^{r_1 x} + C_2 e^{r_2 x}$ (1-hol) formulaga ko‘ra, berilgan tenglamaning umumiy yechimini yozish mumkin:

$$y = C_1 e^{-x} + C_2 e^{6x}.$$

Javob: $y = C_1 e^{-x} + C_2 e^{6x}$.

5.4.16. Tenglamaning umumiy yechimini toping

$$y'' - 4y' + 4y = 0.$$

Yechish. Berilgan tenglama xarakteristik tenglamasi: $r^2 - 4r + 4 = 0$ –ning ildizlari $r_1 = r_2 = 2$ bo'lib, u holda (2-hol) tenglamaning umumiy yechimi:

$$y = (C_1 + C_2x) \cdot e^{2x}.$$

Javob: $y = e^{2x} \cdot (C_1 + C_2x)$.

5.4.17. Tenglamaning umumiy yechimini toping

$$y'' + 6y' + 25y = 0.$$

Yechish. $r^2 + 6r + 25 = 0$ xarakteristik tenglamaning ildizlari $r_1 = r_2 = -3 \pm 4i$ bo'ladi. Demak, (3-hol) berilgan tenglamaning umumiy yechimi:

$$y = e^{-3x}(C_1 cos4x + C_2 sin4x)$$

ko'rinishga bo'ladi.

Javob. $y = e^{-3x}(C_1 cos4x + C_2 sin4x)$.

5.4.18. Tenglamaning $y(0) = 1$, $y'(0) = -1$ boshlang'ich shartlarni qanoatlantiruvchi xususiy yechimini toping

$$y'' - 3y' + 25y = 0.$$

Yechish. $r^2 - 3r + 2 = 0$ xarakteristik tenglamaning ildizlari $r_1 = 1, r_2 = 2$ bo'ladi. Demak, berilgan tenglamaning umumiy yechimi

$$y = C_1e^x + C_2e^{2x}.$$

Berilgan boshlang'ich shartlarga ko'ra, yechimlari: $C_1 = -2$ va $C_2 = 3$ bo'lgan quyidagi tenglamalar sistemasini hosil qilish mumkin:

$$\begin{cases} C_1 + C_2 = 1, \\ C_1 + 2C_2 = -1. \end{cases}$$

Demak, berilgan tenglamaning boshlang'ich shartlarni (y(0)=1, $y' = -1$) qanoatlantiruvchi xususiy yechimi: $y=3e^x - 2e^{2x}$.

Javob: $y = 3e^x - 2e^{2x}$.

O'zgarmas koeffitsientli ikkinchi tartibli bir jinsli bo'lmagan chiziqli differensial tenglamalarni yechish.

$$y'' + py' + qy = f(x). (1)$$

ko'rinishdagi tenglamalarga o'zgarmas koeffitsientli ikkinchi tartibli bir jinsli bo'lmagan chiziqli differensial tenglamalar deyiladi.

Berilgan tenglamaga mos bir jinsli: $y'' + py' + qy = 0$ (2) tenglamaning umumiy yechimi $\tilde{y}$ – bo'lsin va $y'' + py' + qy = f(x)$

tenglamaning biror xususiy yechimi $\dot{y}$-bo'lsin, u holda (1) tenglamaning umumiy yechimi ularning yig'indisi, ya'ni $y = \tilde{y} + \dot{y}$ ko'rinishida bo'ladi.

$y'' + py' + qy = f(x)$ tenglamaning $\dot{y}$ xususiy yechimini ikki usulda topish mumkin:

I) Agar berilgan qlidifferensial tenglamaning o'ng tomoni $f(x) = e^{kx}P_n(x)$ ko'rinishda bo'lsa, k-(2) tenglama xarakteristik tenglamasi $(r^2 + pr + q = 0)$ ning ildizi emas, bu holda $\dot{y}$ ni $\dot{y} = e^{kx}Q_n(x)$ ko'rinishda izlash mumkin. Agar k-xarakteristik tenglamaning ildizi bo'lsa, u holda xususiy yechim $\dot{y} = x^m e^{kx}Q_n(x)$ ko'rinishda izlanadi.

II) $f(x) = a\cos\lambda x + b\sin\lambda x$ ko'rinishda bo'lib, k-xarakteristik tenglamaning ildizi bo'lmasa, u holda (1) tenglamaning xususiy yechimi $\dot{y} = A\cos\lambda x + B\sin\lambda x$ ko'rinishda, agar k-xarakteristik tenglamaning ildizi bo'lsa, u holda berilgan tenglamaning xususiy yechimi: $\dot{y} = x(A\cos\lambda x + B\sin\lambda x)$ ko'rinishda izlanadi.

5.4.19. Quyidagi tenglamaning umumiy yechimini toping:

$$y'' + 4y' + 3y = (8x^2 + 84x)e^x.$$

Yechish. $y'' + 4y' + 3y = 0$ tenglamaning $\tilde{y}$-umumiy yechimi $\tilde{y} = C_1e^{-3x} + C_2e^{-x}$ va $k = 1$ xarakteristik tenglamaning ildizi emas. Demak, xususiy yechimni quyidagi ko'rinishda izlash kerak bo'ladi.

$\dot{y} = (Ax^2 + Bx + C)e^x$. A, B, C noma'lum koeffitsentlar quyidagicha topiladi:

$$\dot{y}' = (Ax^2 + Bx + C)e^x + (2Ax + B)e^x =$$
$$= (Ax^2 + 2Ax + Bx + B + C)e^x,$$
$$\dot{y}'' = \left((Ax^2 + 2Ax + Bx + B + C)e^x\right)' =$$
$$= (Ax^2 + 4Ax + Bx + 2A + 2B + C)e^x;$$

$\dot{y}, \dot{y}', \dot{y}''$ larni $y'' + 4y' + 3y = (8x^2 + 84x)e^x$ tenglamaga qo'yib, hosil bo'lgan:

$$Ax^2 + (12A + 8B)x + (2A + 6B + 8C) = 8x^2 + 84x$$

tenglamadan quyidagi tenglamalar sistemasi hosil bo'ladi.

$$\begin{cases} 8A = 8, \\ 12A + 8B = 84, \\ 2A + 6B + 8c = 0. \end{cases} \quad \begin{cases} A = 1, \\ B = 9, \\ C = -7. \end{cases}$$

Demak, $\dot{y} = (x^2 + 9x - 7)e^x$ bo'lib,

$y=C_1e^{-3x} + C_2e^{-x} + (x^2 + 9x - 7)e^x$

Javob: $y = C_1e^{-3x} + C_2e^{-x} + (x^2 + 9x - 7)e^x$.

5.4.10. Quyidagi tenglamaning umumiy yechimini toping:

$$y'' + 6y' + 9y = 14e^{-3x}.$$

Yechish. $y'' + 6y' + 9y = 0$ tenglamaning umumiy yechimi $(r^2 + 6r + 9 = 0, (r + 3)^2 = 0, r_1 = r_2 = -3)$:

$$\tilde{y} = (C_1 + C_2x)e^{-3x}.$$

Berilgan tenglamaning o'ng tomonidagi ifodada $p_0 = 14$, $k = -3$. Demak,

$$\dot{y} = Ax^2e^{-3x}.$$

U holda:

$$\dot{y}' = (-3Ax^2 + 2Ax)e^{-3x}, \dot{y}'' = (9Ax^2 - 12Ax)e^{-3x}.$$

Ushbu tengliklarni berilgan tenhlamaga qo'yib, A=7 ni topish mumkin.

$$\dot{y} = 7x^2e^{-3x}.$$

Demak, $y=(C_1 + C_2x)e^{-3x} + 7x^2e^{-3x}$.

Javob. $y = (C_1 + C_2x)e^{-3x} + 7x^2e^{-3x}$.

5.4.21. Quyidagi tenglamaning umumiy yechimini toping:

$$y'' - 4y' + 5y = 2cosx + 6sinx.$$

Yechish: Berilgan tenglamaga mos $r^2 - 4r + 5 = 0$ xarakteristik tenglamaning ildizlari $r_{1,2} = 2 \pm i$ bo'lib, $y'' - 4y' + 5y = 0$ tenglamaning umumiy yechimi quyidagicha bo'ladi:

$$\tilde{y} = (C_1cosx + C_2sinx)e^{2x}.$$

$a = 2, b = 6, \lambda = \pm i$ ekanligidan, berilgan tenglamaning xususiy yechimini $\dot{y} = Acosx + Bsinx$ ko'rinishida izlaymiz. $\dot{y}', \dot{y}''$ larni hisoblab, berilgan tenglamaga qo'yib, A, B koeffitsientni topamiz, ya'ni:

$$\dot{y}' = -Asinx + Bcosx,$$

$$\dot{y}'' = -Acosx - Bsinx.$$

$(4A - 4B)cosx + (4A + 4B)sinx = 2cosx + 6sinx$ tenglikdan noma'lum koeffitsientlarga nisbatan quyidagi tenglamalar sistemasi hosil bo'ladi:

$$\begin{cases} 4A - 4B = 2, \\ 4A + 4B = 6. \end{cases} \begin{cases} A = 1, \\ B = \frac{1}{2}. \end{cases}$$

Demak, $\dot{y} = \cos x + \frac{1}{2}\sin x$ bo'lib,

$y = (C_1 \cos x + C_2 \sin x)e^{2x} + \cos x + \frac{1}{2}\sin x.$

Javob: $y = (C_1 cosx + C_2 sinx)e^{2x} + cosx + \frac{1}{2}sinx.$

5.4.22. Quyidagi tenglamaning umumiy yechimini toping:

$$y'' + 4y = 12cos2x.$$

Yechish. $r^2 + 4 = 0$ xarakteristik tenglamaning $r_{1,2} = \pm 2i$ ildizlariga ko'ra, $y'' + 4y = 0$ tenglamaning umumiy yechimi quyidagicha bo'ladi:

$$\tilde{y} = C_1 cos2x + C_2 sin2x.$$

$a = 12, b = 0, \lambda = \pm 2i$ ekanligidan, xususiy yechimni:

$\dot{y} = x(Acosx + Bsinx)$ ko'rinishida izlaymiz. U holda:

$\dot{y}' = Acos2x + Bsin2x + x(-2Asin2x + 2Bcos2x),$

$\dot{y}'' = -4Asin2x + 4Bcos2x + x(-4Acos2x - 4Bsin2x)$

bo'lib,

$4Bcos2x - 4Asin2x = 12cos2x$ tenglamadan $A = 0, B = 3.$

Demak,

$$\dot{y} = 3x \cdot sin2x.$$

Javob: $y = C_1 cos2x + C_2 sin2x + 3x \cdot sin2x.$

5.4.23. Quyidagi tenglamaning umumiy yechimini toping:

$$y''' - 6y'' + 9y' = xe^{3x} + e^{3x}cos2x.$$

Yechish. $r^3 - 6r^2 + 9r = 0$ xarakteristik tenglamaning $r_1 = 0; r_{2,3} = 3$ ildizlariga ko'ra: $y''' - 6y'' + 9y' = 0$ tenglamaning umumiy yechimi quyidagicha bo'ladi:

$$\tilde{y} = (C_1 + C_2 x) \cdot e^{3x} + C_3.$$

Endi tenglikning o'ng tomonidagi har bir yig'indi uchun xususiy yechimlarni quyi-dagicha toppish mumkin:

(I) $y''' - 6y'' + 9y' = xe^{3x},$

(II) $y''' - 6y'' + 9y' = e^{3x}cos2x.$

(I) tenglamaning xususiy yechimi $\dot{y_1} = x^2(ax+b)e^{3x}$ ko'rinishda izlanadi.

$\dot{y_1}', \dot{y_1}'', \dot{y_1}'''$ larni hisoblab, berilgan tenglamaga qo'yib, $a = \frac{1}{18}, b = -\frac{1}{18}$ ekanligini topamiz. Demak,

$$\dot{y_1} = x^2\left(\frac{1}{18}x - \frac{1}{18}\right)e^{3x}.$$

(II) tenglamaning xususiy yechimi $\dot{y_2} = (Lcos2x + Dsin2x)e^{3x}$ ko'rinishda izlanadi. $\dot{y_2}', \dot{y_2}'', \dot{y_2}'''$larni hisoblab, berilgan tenglamaga qo'yib, $L = -\frac{3}{52}, D = -\frac{1}{26}$ ekanligini topamiz. Demak,

$$\dot{y_2} = \left(-\frac{3}{52}cos2x - \frac{1}{26}sin2x\right)e^{3x}.$$

U holda $y''' - 6y'' + 9y' = xe^{3x} + e^{3x}cos2x$ tenglamaning umumiy yechimi quyidagicha bo'ladi:

$$y = \tilde{y} + \dot{y_1} + \dot{y_2}.$$

Javob: $y = e^{3x}\left(\frac{1}{18}x^3 - \frac{1}{18}x + C_2x - \frac{3}{52}cos2x - \frac{1}{26}sin2x + C_1\right) + C_3.$

Differensial tenglamalarning masalalar yechishda qo'llanilishi.

5.4.24. Egri chiziqning biror nuqtasidan koordinatalar o'qlariga parallel to'g'ri chiziqlar o'tkazilganda, shu va koordinata o'qlari bilan chegaralangan to'g'ri to'rtburchak yuzasini1:2 nisbatda kesib o'tuvchi egri chiziqlarni toping.

Yechish. To'g'rito'rtburchak yuzasi $S = xy$ bo'lsin. Egri chiziq bilan kesilganda hosil bo'lgan yuza $S_1 = \frac{1}{3}S = \frac{1}{3}xy$ yoki $S_2 = \frac{2}{3}S = \frac{2}{3}xy$.

$S_1 = \int_{x_0}^{x} ydx = \frac{1}{3}xy$ tenglikning ikkala tomonini differensiallab quyidagilarni hosil qilish mumkin:

$$y = \frac{1}{3}y + \frac{1}{3}xy',$$
$$xy' = 2y,$$

$$\frac{dy}{y} = \frac{2dx}{x},$$
$$ln|y| = 2ln|x| + lnC,$$
$$y = Cx^2.$$

Xuddi shunday, $S_2 = \int_{x_0}^{x} ydx = \frac{2}{3}xy$ tenglikning ikkala tomoniini differensiallab:

$$y = \frac{2}{3}y + \frac{2}{3}xy'$$
$$2xy' = y,$$
$$\frac{2dy}{y} = \frac{dx}{x},$$
$$ln|x| = 2ln|y| + lnC.$$
$$x = Cy^2.$$

Javob: $y = Cx^2$, $x = Cy^2$.

5.4.25. Egri chiziqning istalgan nuqtalariga o'tkazilgan urinmalar Ox o'qi bilan kesishish nuqtalari, shu urinish nuqtalaridan va koordinata boshidan bir xil uzoqlikda joylashgan bo'lsa, $(-1; 1)$ nutadan o'tuvchi barcha egri chiziqlar tenglamalarini yozing.

Yechish. Masala shartiga ko'ra $OB = AB$, ikki nuqta orasidagi masofalarni tenglab, quyidagini yozish mumkin:

$$\sqrt{\left(\frac{y}{y'}\right)^2 + y^2} = \left|x - \frac{y}{y'}\right|; \quad \left(\frac{y}{y'}\right)^2 + y^2 = \left(x - \frac{y}{y'}\right)^2.$$

Bundan quyidagi bir jinsli differensial tenglama hosil bo'ladi:

$$y' = \frac{2xy}{x^2 - y^2}$$

$y = xu$ $(y' = u + xu')$ almashtirish yordamida ko'zlangan natijaga erishish mumkin:

$$\frac{dx}{x} = \frac{1-u^2}{u^3+u}du,$$
$$ln|x| = ln\left|\frac{Cu}{u^2+1}\right|,$$
$$x\left(\frac{y^2}{x^2} + 1\right) = \frac{Cy}{x},$$
$$y^2 + x^2 = Cy.$$

Javob: $y^2 + x^2 = Cy$.

5.4.26. Massasi 400 gr bo'lgan koptok 20 m/s boahlang'ich tezlik bilan yuqoriga vertical(tik) otilgan. Agar 1 m/s tezlik bilan harakatlanayotgan koptokka havoning qarshilik kuchi 5 H bo'lsa va erkin tushish tezlanishi 10 m/s^2 deb olib, koptok ko'- tarilishi mumkin bo'lgan maksimal balandlikni toping.

Yechish. Koptokning t-vaqtdagi oniy tezligi $v(t)$ bo'lsin. Koptokning harakatiga $F_g = mg$ -og'irlik kuchi va $F_r = \mu mv^2$- havoning qarshilik kuchi tasir qiladi. Bunda

$v = 1m/s, m = 400\ gr$ bo'lganda, $F_r = 5$ H ekanligidan $\mu = \frac{F_r}{mv^2}$;

$\mu = \frac{5}{400} = 0{,}0125m^{-1}.ma = -F_g - F_r$ tenglikdan $mv' = -10m - 0{,}0125mv^2$.

Endi hosil qilingan tenglikni ikkala tomonini m −ga qisqartirib, o'zgaruvchilarini ajratib, integrallash mumkin:

$$\int \frac{dv}{0{,}0125v^2 + 10} = \int -dt,$$

$$2\sqrt{2}arctg\frac{v}{20\sqrt{2}} + C = -t,$$

$$v = 20\sqrt{2}tg\frac{C - t}{2\sqrt{2}}.$$

Demak,$v = 0$, ya'ni $t = C$ bo'lganda koptok eng yuqori nuqtaga ko'tariladi.

$$h = 20\sqrt{2}\int_0^C tg\frac{C - t}{2\sqrt{2}}dt = 80ln\left|cos\frac{C - t}{2\sqrt{2}}\right|\Big|_{t=0}^{t=C}$$

$$= -80ln\left|cos\frac{C}{2\sqrt{2}}\right|.$$

$v(0) = 20$ bo'lib, umumiy yechimga ko'ra, $C = 2\sqrt{2}arctg\frac{1}{\sqrt{2}}$ ekanligini topamiz. Demak,

$$h = -80ln\left|cosarctg\frac{1}{\sqrt{2}}\right| = 40ln\frac{3}{2} \approx 16{,}22.$$

Javob.≈ 16 m 22 sm.

5.4.27. Boshlang'ich massasi 250 t bo'lgan reaktiv dvigatelli raketa vertical yuqoriga uchirildi. Yonilg'I sarfi va dvigateldan chiqayotgan gazning tezligi mos ravishda 1 t/s va 3 km/s. Agar havoning qarshiligi hisobga olinmasa va yer sirtidagi (startda) tezligi nolga teng bo'lsa, raketa bir minutda qanday balandlikka ko'tariladi?

Yechish. t vaqt momentida $m(t)$ –raketa massasi, $v(t)$ –raketa tezligi bo'lsin. Raketa harakatiga yerning tortishkuchi $F = mg$ va dvigatelning tortish kuchlari ta'sir qiladi. Demak,Misherskiy tenglamasini quyidagicha yozish mumkin:

$mv' = -0{,}01m - 3m'$ (erkin tushish tezlanishi g=0,01 km/s^2) $v' = -0{,}01 - 3\frac{m'}{m}$ tenglikning ikkala tomonini integrallab, $v(t) = -0{,}01t - 3lnm + C$ tenglikni hosil qilish mumkin.

$v(0) = 0$ va $m(0) = 250$t shartlardan:

$$C = 3ln250, v = 3ln\frac{250}{m} - 0{,}01t, m = C - t.$$

Topilganlarni $v(t) = -0{,}01t - 3lnm + C$ formulaga qo'yib, $v = -3ln(1 - 0{,}004t) - 0{,}01t$ tenglikni hosil qilish mumkin.

Demak,

$$h = -3\int_0^{60} \ln(1 - 0{,}004t) - 0{,}01\int_0^{60} tdt \approx 5{,}571\ km.$$

Javob: 5,571 km.

5.4.28. Agar har yili inflyatsiya (pulning qadrsizlanishi) 7% ni tashkil qilsa, necha yildan so'ng narx qiymatlari ikki marta oshadi?

Yechish. Masalani uzluksiz protsent hisoblash modeli bilan yechsak ham bo'ladi.

$a\left(1 + \frac{7}{100}\right)^n = 2a.t$ vaqtda $a(t)$ narxlar qiymatiga teng bo'lsa, $a' = 0{,}07a$ va

$a = C^{0{,}07t}; a = 2C$ bo'ladi. $2C = Ce^{0{,}07t}$ yoki $e^{0{,}07t} = 2$, $t = \frac{ln2}{0{,}07} \approx 10$.

Javob.≈ 10 yil.

Mustaqil yechish uchun misollar

Quyidagi oddiy differensial tenglamalarni yeching.

1. $y' = y - x^2$.
2. $2(y + y') = x + 3$.
3. $y' = \frac{x^2 + y^2}{2} - 1$.
4. $(y^2 + 1) \cdot y' = y - x$.
5. $y \cdot y' + x = 0$.
6. $x \cdot y' = 2y$.
7. $x \cdot y' + y = 0$.
8. $y' + y = (x - y')^3$.
9. $y' = x - e^y$.
10. $y(y' + x) = 1$.
11. $xy\,dx + (x + 1)dy = 0$.
12. $\sqrt{y^2 + 1}\,dx = xy\,dy$.
13. $2x^2y \cdot y' + y^2 = 2$.
14. $y' - xy^2 = 2xy$.
15. $e^{-y}\left(1 + \frac{dy}{dx}\right) = 1$.
16. $y' = 10^{x+y}$.
17. $x \cdot \frac{dx}{dt} + t = 1$.
18. $y' = \cos(y - x)$.
19. $y' - y = 2x - 3$.
20. $y' = \sqrt{4x + 2y - 1}$.
21. $(x^2 - 1)y' + 2xy^2 = 0;\ y(0) = 1$.
22. $y' = 3 \cdot \sqrt[3]{y^2};\ y(2) = 0$.
23. $xy' + y = y^2;\ y(1) = 0{,}5$.
24. $(x + 2y) \cdot y' = 1;\ y(0) = -1$.

Quyidagi bir jinsli oddiy differensial tenglamalarni yeching.

1. (x+2y)dx-xdy=0
2. $(x - y)dx + (x + y)dy = 0$.
3. $(y^2 - 2xy)dx + x^2dy = 0$.
4. $2x^3y' = y(2x^2 - y^2)$.
5. $y^2 + x^2y' = xyy'$
6. $(x^2 + y^2)y' = 2xy$.
7. $xy' - y = x \cdot tg\frac{y}{x}$.
8. $xy' = y - x \cdot e^{\frac{y}{x}}$.
9. $xy' - y = (x + y)ln\frac{x + y}{x}$.
10. $xy' = ycos\left(ln\frac{y}{x}\right)$.
11. $\left(y + \sqrt{xy}\right)dx = xdy$.
12. $xy' = \sqrt{x^2 - y^2} + y$.
13. $(2x + y + 1)dx + (x + y - 3)dy = 0$.
14. $(2x + y + 1)dx - (4x + 2y - 3)dy = 0$.
15. $y' = 2 \cdot \left(\frac{y + 2}{x + y - 1}\right)^2$.
16. $(y' + 1)ln\frac{y + x}{x + 3} = \frac{y + x}{x + 3}$.
17. $2xdy + (x^2y^4 + 1)ydx = 0$.
18. $ydx + x \cdot (2xy + 1)dy = 0$.
19. $\frac{2}{3}xyy' = \sqrt{x^6 - y^4} + y^2$.
20. $2y + (x^2y + 1)xy' = 0$.

21. $2y' + x = 4\sqrt{y}$.

22. $y' = y^2 - \frac{2}{x^2}$.

Quyidagi chiziqli differensial tenglamalarni yeching.

1. $xy' - 2y = 2x^4$.
2. $(2x + 1)y' = 4x + 2y$.
3. $(xy + e^x)dx - xdy = 0$.
4. $x^2y' + xy + 1 = 0$.
5. $y = x(y' - xcosx)$.
6. $2x(x^2 + y)dx = dy$.
7. $(xy' - 1)lnx = 2y$.
8. $xy' + (x + 1)y = 3x^2e^{-x}$.
9. $(x + y^2)dy = ydx$.
10. $(2e^y - x)y' = 1$.
11. $(sin^2y + xctgy)y' = 1$.
12. $(2x + y)dy = ydx + 4lnydy$.
13. $y' = y^4cosx + ytgx$.
14. $xy^2y' = x^2 + y^3$.
15. $xydy = (y^2 + x)dx$.
16. $xy' - 2x^2\sqrt{y} = 4y$.
17. $xy' + 2y + x^5y^3e^x = 0$.
18. $2y' - \frac{x}{y} = \frac{xy}{x^2 - 1}$.
19. $y'^{x^3}siny = xy' - 2y$.
20. $(2x^2y \cdot lny - x)y' = y$.
21. $(x^2 - 1)y'siny + 2xcosy = 2x - 2x^3$.
22. $x \cdot (e^y - y') = 2$.

Quyidagi differensial tenglamalarning umumiy yechimlarini toping.

1. $x^2y'' = y'^2$.
2. $2xy'y'' = y'^2 - 1$.
3. $y^3y'' = 1$.
4. $y'^2 + 2yy'' = 0$.
5. $y'' = 2yy'$.
6. $yy'' + 1 = y'^2$.
7. $2y'(y'' + 2) = xy''^2$.
8. $y^4 - y^3y'' = 1$.
9. $y'^2 = (3y - 2y')y''$.
10. $y''(2y' + x) = 1$.
11. $yy'' + y = y'^2$.
12. $xy'' = y' + x(y'^2 + x^2)$.
13. $yy''' + 3y'y'' = 0$.
14. $y'y''' = 2y''^2$.
15. $yy'' + y'^2 = 1$.
16. $y'' = xy' + y + 1$.
17. $x^2(2yy'' - y'^2) = 1 - 2xyy'$.
18. $yy' + xyy'' - xy'^2 = x^3$.

Quyidagi boshlang'ich shrtlar bilan berilgan diggerensial tenglamalarni yeching.

19. $2y''' - 3y'^2 = 0;\ y(0) = -3, y'(0) = 1, y''(0) = -1$.

20. $x^2y'' - 3xy' = \frac{6y^2}{x^2} - 4y;\ y(1) = 1, y'(1) = 4$.

21. $y''' = 3yy';\ y(0) = -2, y'(0) = 0, y''(0) = 4{,}5$.

22. $y''cosy + y'^2siny = y';\ y(-1) = \frac{\pi}{6}, y'(-1) = 2$.

Quyidagi differensial tenglamalarning umumiy yechimlarini toping.

23. $y'' - 4y' + 5y = 0.$
24. $y'' + 2y' + 10y = 0.$
25. $y'' + y' - 2y = 0.$
26. $y'' + 4y' + 3y = 0.$
27. $y^{IV} - 5y'' + 4y = 0.$
28. $y^{V} + 8y''' + 16y' = 0.$
29. $y'' - 2y' - 3y = e^{4x}.$
30. $y'' + y = 4xe^{x}.$
31. $y'' + y = 4sinx.$
32. $y'' - 3y' + 2y = sinx.$
33. $y'' + 3y' - 4y = e^{-4x} + xe^{-x}.$
34. $y'' - 4y' + 8y = e^{2x} + sin2x.$

Quyidagi boshlang'ich shrtlar bilan berilgan diggerensial tenglamalarni yeching.

35. $y'' - 2y' + y = 0;\ y(2) = 1, y'(2) = -2.$
36. $y'' + y = 4e^{x};\ y(0) = 4, y'(0) = -3.$
37. $y'' + 2y' + 2y = xe^{-x};\ y(0) = y'(0) = 0.$
38. $y''' - y' = 0;\ y(0) = 3, y'(0) = -1, y''(0) = 1.$

6-bob. Geometriyada tengsizlik va geometrik masalalar

6.1. Geometriyada tengsizlik

6.1.1. ABC uchburchak burchaklari uchun

$$\sin^2 A + \sin^2 B + \sin^2 C \le \frac{9}{4}$$

tengsizlikni isbotlang.

Yechish. Dastlab quyidagi yordamchi tengsizlikni isbotlaymiz $\cos 2A + \cos 2B + \cos 2C \ge -\frac{3}{2}$ $(*)$ O nuqta ABC uchburchakka tashqi chizilgan aylana markazi bo'lsin ya'ni $R = |\overline{OA}| = |\overline{OB}| = |\overline{OC}|$ bo'lib, $\bar{c} = \overline{OA} + \overline{OB} + \overline{OC}$ yig'indi vektor faqatgina muntazam uchburchak uchun $\bar{0}$ vektor bo'lib ushbu yig'indi uchun $(\overline{OA} + \overline{OB} + \overline{OC})^2 \ge 0$ tengsizlikni yozish mumkin

Skalyar ko'paytma formulasidan

$$\overline{OA} \circ \overline{OB} = R^2 \cos 2C,\ \overline{OA} \circ \overline{OC} = R^2 \cos 2B,$$

$$\overline{OB} \circ \overline{OC} = R^2 \cos 2A \text{ va } (\overline{OB})^2 = (\overline{OC})^2 = (\overline{OC})^2 = R^2$$

tenglikdan $(\overline{OA} + \overline{OB} + \overline{OC})^2 \ge 0$ tengsizlik

$$3R^2 = 2R^2 \cos 2A + 2R^2 \cos 2B + 2R^2 \cos 2C \ge 0$$

ko'rinishini olib, $(*)$ tengsizlikning to'g'riligi kelib chiqadi. U holda:

$$sin^2A + sin^2B + sin^2C =$$

$$= \frac{1}{2}(1 - \cos 2A) + \frac{1}{2}(1 - \cos 2B) + \frac{1}{2}(1 - \cos 2C) =$$

$$= \frac{3}{2} - \frac{1}{2}(\cos 2A + \cos 2B + \cos 2C).$$

bo'lib, $(*)$ tengsizlikdan foydalanib:

$$\sin^2 A + \sin^2 B + \sin^2 C \le \frac{3}{2} - \frac{1}{2}\left(-\frac{3}{2}\right) = \frac{9}{4}$$

tengsizlikni hosil qilish mumkin.

6.1.2. Tomolari uzunliklari a_1, a_2, a_3, a_4 (fazodagi to'rt bo'g'inli ixtiyoriy yopiq siniq chiziq) uchun quyidagi tengsizlikni isbotlang

$$a_1^2 + a_2^2 + a_3^2 > \frac{1}{3}a_4^2.$$

Yechish. Berilgan to'rtburchak(siniq chiziq) tomonlari (bo'g'inlari) $\overrightarrow{AB} = \overrightarrow{a_1}, \overrightarrow{BC} = \overrightarrow{a_2}, \overrightarrow{CD} = \overrightarrow{a_3}, \overrightarrow{DA} = \overrightarrow{a_4}$ bo'lsin, u holda ko'pburchakning xossasidan

$$\vec{a}_1 + \vec{a}_2 + \vec{a}_3 + \vec{a}_4 = \vec{0}$$

bo'lib,

$$\vec{a}_4 = -(\vec{a}_1 + \vec{a}_2 + \vec{a}_3),$$

$$\vec{a}_4^2 = \vec{a}_1^2 + \vec{a}_2^2 + \vec{a}_3^2 + 2\vec{a}_1\vec{a}_2 + 2\vec{a}_2\vec{a}_3 + 2\vec{a}_1\vec{a}_3$$

$(\vec{x} - \vec{y})^2 = \vec{x}^2 + \vec{y}^2 - 2\vec{x}\vec{y}$ tenglikdan foydalanib quyidagi tengliklarni yozish mumkin:

$$2\overrightarrow{a_1}\overrightarrow{a_2} = \overrightarrow{a_1}^2 + \overrightarrow{a_2}^2 - (\overrightarrow{a_1} - \overrightarrow{a_2})^2,$$

$$2\overrightarrow{a_2}\overrightarrow{a_3} = \overrightarrow{a_2}^2 + \overrightarrow{a_3}^2 - (\overrightarrow{a_2} - \overrightarrow{a_3})^2,$$

$$2\overrightarrow{a_3}\overrightarrow{a_1} = \overrightarrow{a_3}^2 + \overrightarrow{a_1}^2 - (\overrightarrow{a_3} - \overrightarrow{a_1})^2.$$

Natijada:

$$\vec{a}_4^2 = 3(\vec{a}_1^2 + \vec{a}_2^2 + \vec{a}_3^2) - (\vec{a}_1 - \vec{a}_2)^2 - (\vec{a}_2 - \vec{a}_3)^2 - (\vec{a}_3 - \vec{a}_1)^2$$

bo'lib,

$$(\vec{a}_1 - \vec{a}_2)^2 > 0, (\vec{a}_2 - \vec{a}_3-)^2 > 0, (\vec{a}_3 - \vec{a}_1)^2 > 0$$

bo'lganligidan:

$$\vec{a}_4^2 < 3(\vec{a}_1^2 + \vec{a}_2^2 + \vec{a}_3^2) \; yoki \; \vec{a}_1^2 + \vec{a}_2^2 + \vec{a}_3^2 > \frac{1}{3}\vec{a}_4^2.$$

6.1.3. Ixtiyoriy uchburchak burchaklari uchun

$$\left(\sin\frac{A}{2}\right)^{-1} + \left(\sin\frac{B}{2}\right)^{-1} + \left(\sin\frac{C}{2}\right)^{-1} \geq 6.$$

tengsizlikni isbotlang

Yechish. $n = 3$ bo'lganda Koshi (1) tengsizligiga ko'ra:

$$\frac{1}{\sin\frac{A}{2}} + \frac{1}{\sin\frac{B}{2}} + \frac{1}{\sin\frac{C}{2}} \geq \frac{3}{\sqrt[3]{\sin\frac{A}{2}\cdot\sin\frac{B}{2}\cdot\sin\frac{C}{2}}}. \qquad (*)$$

Dastlab, $\sin\frac{A}{2}\cdot\sin\frac{B}{2}\cdot\sin\frac{C}{2} \leq \frac{1}{8}$ $(**)$ tengsizlikni isbotlaymiz. $(**)$ tengsizlikning chap tomonida ayniy almashtirishlar yordamida

$$\sin\frac{A}{2}\cdot\sin\frac{B}{2}\cdot\sin\frac{C}{2} = \frac{1}{2}\left(\cos\frac{A-B}{2} - \cos\frac{A+B}{2}\right)\cdot\sin\left(\frac{\pi}{2} - \frac{A+B}{2}\right) =$$

$$= \frac{1}{2}\left(\cos\frac{A-B}{2} - \cos\frac{A+B}{2}\right)\cdot\cos\frac{A+B}{2} =$$

$$= \frac{1}{2}\left(\cos\frac{A-B}{2}\cdot\cos\frac{A+B}{2}\right) \leq \frac{1}{2}\left(\cos\frac{A-B}{2}\cdot\cos^2\frac{A+B}{2}\right) =$$

$$-\frac{1}{2}\left(\cos\frac{A+B}{2} - \frac{1}{2}\right)^2 + \frac{1}{8} \leq \frac{1}{8}$$

ni hosil qilish mumkin.

$(**)$ tengsizlikni isbotlashda uchburchak burchaklari uchun

quyidagi munosabatlardan foydalanish mumkin $0 < \frac{A+B}{2} < \frac{\pi}{2}$, $-\frac{\pi}{2} < \frac{A-B}{2} < \frac{\pi}{2}$, $0 < cos\frac{A+B}{2} < 1$, $0 < cos\frac{A-B}{2} \leq 1$ va $cos\frac{A-B}{2} \cdot cos\frac{A+B}{2} \leq cos\frac{A+B}{2}$.

Demak, (**) tengsizlik va (*) tengsizlik natijada, berilgan tengsizlik isbotlandi.

Eslatma. (**) tengsizlikning boshqacha isboti ABC uchburchakning tomonlari $AB = c$, $BC = a$ va $AC = b$ bo'lsin, u holda kosinuslar teoremasiga ko'ra:

$$a^2 = b^2 + c^2 - 2bc \cdot cosA = (b-c)^2 + 2bc \cdot (1 - cosA) =$$

$(b-c)^2 + 4bc \cdot \sin^2\frac{A}{2}$ bo'lib, $4bc \cdot \sin^2\frac{A}{2} \leq a^2$ yoki $\sin\frac{A}{2} \leq \frac{a}{2\sqrt{bs}}$ xuddi shunday, $\sin\frac{B}{2} \leq \frac{b}{2\sqrt{ac}}$ va $\sin\frac{C}{2} \leq \frac{c}{2\sqrt{ab}}$ tengsizliklarni hosil qilish mumkin.

Natijada, $\sin\frac{A}{2} \cdot \sin\frac{B}{2} \cdot \sin\frac{C}{2} \leq \frac{a}{2\sqrt{bc}} \cdot \frac{b}{2\sqrt{ac}} \cdot \frac{c}{2\sqrt{ab}} = \frac{abc}{8abc} = \frac{1}{8}$ bo'lib, (**) tengsizlik isbot bo'ldi.

6.1.4. Berilgan ABC uchburchak va M nuqta uchun (a=BC, b=AC, c=AB, $a_1 = MA$, $b_1 = MB$, $c_1 = MC$) quyidagi tengsizlikni isbotlang

$$a^2 + b^2 + c^2 \leq 3(a_1^2 + b_1^2 + c_1^2)$$

Yechish.

$$(\overrightarrow{MA} + \overrightarrow{MB} + \overrightarrow{MC})^2 \geq 0$$

bo'lib,

$$a_1^2 + b_2^2 + c_3^2 + 2\overrightarrow{MA} \cdot \overrightarrow{MB} + 2\overrightarrow{MB} \cdot \overrightarrow{MC} + 2\overrightarrow{MC} \cdot \overrightarrow{MA} \geq 0,$$

ammo

$$2\overrightarrow{MA} \cdot \overrightarrow{MB} = \overrightarrow{MA}^2 + \overrightarrow{MB}^2 - (\overrightarrow{MA} - \overrightarrow{MB})^2 = a_1^2 + b_1^2 - c^2,$$
$$2\overrightarrow{MB} \cdot \overrightarrow{MC} = \overrightarrow{MB}^2 + \overrightarrow{MC}^2 - (\overrightarrow{MB} - \overrightarrow{MC})^2 = b_1^2 + c_1^2 - a^2,$$
$$2\overrightarrow{MC} \cdot \overrightarrow{MA} = \overrightarrow{MC}^2 + \overrightarrow{MA}^2 - (\overrightarrow{MC} - \overrightarrow{MA})^2 = c_1^2 + a_1^2 - b^2.$$

Natijada,

$$a_1^2 + b_1^2 + c_1^2 + a_1^2 + b_1^2 - c^2 + b_1^2 + c_1^2 - a^2 + c_1^2 + a_1^2 - b^2 \geq 0,$$
$$a^2 + b^2 + c^2 \leq 3(a_1^2 + b_1^2 + c_1^2).$$

Tenglik faqat va faqat $\overrightarrow{MA} + \overrightarrow{MB} + \overrightarrow{MC} = \vec{0}$ bo'lganda, ya'ni O nuqta ABC uchburchakka tashqi chizilgan aylananing markazi

bo‘lganda o‘rinli bo‘ladi.

6.1.5. Uchburchak burchaklari uchun quyidagi tengsizlikni isbotlang

$$tg^2A + tg^2B + tg^2C \geq 9\,.$$

Yechish. Berilgan tengsizlikdan $(tg^2A + 1) + (tg^2B + 1) + (tg^2C + 1) \geq 12$ bo‘lib, $tg^2\alpha + 1 = \frac{1}{\cos^2\alpha}$ ayniyatga ko‘ra, quyidagi tengsizlikni hosil qilish mumkin: $\frac{1}{\cos^2 A} + \frac{1}{\cos^2 B} + \frac{1}{\cos^2 C} \geq 12$. $(*)$

Tengsizlikni isbotlash uchun $n = 3$ bo‘lganda Koshi (1) tengsizligidan foydalanib:

$\frac{1}{\cos^2 A} + \frac{1}{\cos^2 B} + \frac{1}{\cos^2 C} \geq \frac{3}{\sqrt[3]{(\cos A \cdot \cos B \cdot cosC)^2}}$ $(**)$ tengsizlikni hosil qilib yuqorida isbotlangan.

$\cos A \cdot \cos B \cdot cosC \leq \frac{1}{8}$ tengsizlikdan foydalanib $(**)$ tengsizlikdan $\frac{1}{\cos^2 A} + \frac{1}{\cos^2 B} + \frac{1}{\cos^2 C} \geq 12$ tengsizlikni hosil qilish mumkin.

6.1.6.

$$tg^2\frac{A}{2} + tg^2\frac{B}{2} + tg^2\frac{C}{2} \geq 1.$$

Yechish. 5 masala yechimiga o‘xshab berilgan tengsizlik $\frac{1}{cos^2\frac{A}{2}} + \frac{1}{cos^2\frac{B}{2}} + \frac{1}{cos^2\frac{C}{2}} \geq 4$ $(*)$ tengsizlikka teng kuchli bo‘ladi.

$n = 3$ bo‘lganda Koshi (1) tengsizligiga ko‘ra:

$\frac{1}{cos^2\frac{A}{2}} + \frac{1}{cos^2\frac{B}{2}} + \frac{1}{cos^2\frac{C}{2}} \geq \frac{3}{\sqrt[3]{\left(\cos\frac{A}{2}\cdot\cos\frac{B}{2}\cdot cos\frac{C}{2}\right)^2}}$ $(**)$ bo‘ladi.

$\cos\frac{A}{2} \cdot \cos\frac{B}{2} \cdot \cos\frac{C}{2} = \frac{3\sqrt{3}}{8}$ $(***)$ yordamchi tengsizlikni isbotlaymiz. $0 < A < \pi$, $0 < B < \pi$ va $0 < C < \pi$ bo‘lib, u holda $\cos\frac{A}{2} = \sqrt{\frac{1+\cos A}{2}}$, $\cos\frac{B}{2} = \sqrt{\frac{1+\cos B}{2}}$, $\cos\frac{C}{2} = \sqrt{\frac{1+\cos C}{2}}$ ayniyatlardan foydalanib $n = 3$ uchun Koshi (1) tengsizligiga ko‘ra:

$$\cos\frac{A}{2} \cdot \cos\frac{B}{2} \cdot \cos\frac{C}{2} = \sqrt{\frac{(1+\cos A)\cdot(1+\cos B)\cdot(1+\cos C)}{8}} \leq$$

$$\leq \sqrt{\frac{1}{8}\cdot\left(\frac{3+cosA+\cos B+\cos C}{3}\right)^3}\,.$$

$cosA + \cos B + \cos C \le \frac{3}{2}$ tengsizlikdan $(***)$ va u bilan birga $(*)$ hamda $(**)$ tengsizliklar isbotlandi.

6.1.7. a, b, c ixtiyoriy uchburchak tomonlari bo'lsa, u holda quyidagi tengsizlikni isbotlang

$$\frac{a^2+2bc}{b^2+c^2} + \frac{b^2+2ac}{a^2+c^2} + \frac{c^2+2ab}{a^2+b^2} > 3 .$$

Yechish. Uchburchak tengsizligidan $a>|c-b|$ yoki $a^2 > (c-b)^2$ bo'lib, $a^2 + 2bc > b^2 + c^2$ yoki $\frac{a^2+2bc}{b^2+c^2} > 1$ bo'ladi.

Xuddi shunday, $\frac{c^2+2ab}{b^2+a^2} > 1$ va $\frac{b^2+2ac}{a^2+c^2} > 1$ tengsizliklarni hosil qilish mumkin.

Hosil bo'lgan tengsizliklarni qo'shib berilgan tengsizlikni hosil qilish mumkin.

6.1.8. a, b, c uchburchak tomonlari h_a, h_b, h_c balandliklari S – esa uchburchakning yuzi bo'lsa, quyidagi tengsizlikni isbotlang

$$(a^2 + b^2 + c^2)(h_a^2 + h_b^2 + h_c^2) \ge 36S^2.$$

Yechish. Uchburchakning yuzi $S = \frac{ah_a}{2} = \frac{bh_b}{2} = \frac{ch_c}{2}$ bo'lib, $6S = ah_a + bh_b + ch_c$ Koshi-Bunyakovskiy tengsizligiga asosan:

$(6S)^2 = (ah_a + bh_b + ch_c)^2 \le (a^2 + b^2 + c^2)(h_a^2 + h_b^2 + h_c^2)$ bo'ladi.

Demak, berilgan tengsizlik isbotlandi.

6.1.9. Ixtiyoriy uchburchak tomonlari a, b, c va yuzi S uchun, quyidagi tengsizlikni isbotlang

$$a^2 + b^2 + c^2 \ge 4\sqrt{3} \cdot S .$$

Yechish. $n = 3$ bo'lganda Koshi (1) tengsizligiga ko'ra:

$$(p-a)(p-b)(p-c) \le \left(\frac{(p-a)+(p-b)+(p-c)}{3}\right)^3 = \frac{p^3}{27}$$

$$p(p-a)(p-b)(p-c) \le \frac{p^4}{27}$$

bo'lib, u holda

$$S = \sqrt{p(p-a)(p-b)(p-c)} \le \sqrt{\frac{p^4}{27}} = \frac{p^2}{3\sqrt{3}} = \frac{(a+b+c)^2}{12\sqrt{3}},$$

ya'ni $S \le \frac{(a+b+c)^2}{12\sqrt{3}}$ $(*)$ Koshi-Bunyakovskiy (9) tengsizligidan:

$(a+b+c)^2 \le 3(a^2+b^2+c^2)$ bo'lib, bundan va $(*)$ tengsizlikdan

$S \le \frac{a^2+b^2+c^2}{4\sqrt{3}}$ bo'ladi. Natijada, $a^2 + b^2 + c^2 \ge 4\sqrt{3} \cdot S$ tengsizlik isbot bo'ldi.

Eslatma. Isbotlangan tengsizlik tenglikka aylanishi uchun uchburchak tomonlari teng, ya'ni muntazam uchburchakning tomoni a bo'lsa, $S = \frac{\sqrt{3} \cdot a^2}{4}$.

6.1.10. Qavariq ABCD to'rtburchak tomonlari a, b, c, d bo'lsa, $S_{ABCD} \le \frac{a^2+b^2+c^2+d^2}{4}$ tengsizlikni isbotlang.

Yechish. ABCD to'rtburchak tomonlari $AB = \alpha$, $BC = b$, $CD = c$, $AD = d$ bo'lsin, u holda AC diagonal ABCD to'rtburchakni ikkita ABC va ACD uchburchaklarga ajratadi va $S_{ABCD} = S_{ABC} + S_{ACD}$.

Ma'lumki, $S_{ABC} = \frac{ab \cdot \sin B}{2} \le \frac{ab}{2}$ bo'lib Koshi (2) tengsizligidan $ab \le \frac{a^2+b^2}{2}$.

Demak, $S_{ABC} \le \frac{a^2+b^2}{4}$ xuddi shunday, $S_{ACD} \le \frac{c^2+d^2}{4}$ bo'lib $S_{ABCD} = S_{ABC} + S_{ACD}$ tenglikdan berilgan tengsizlik isbotlandi.

Eslatma. $S_{ABCD} \le \frac{a^2+b^2+c^2+d^2}{4}$ tengsizlikda tenglik ABCD kvadrat bo'lganda bajariladi.

6.1.11. a, b, c perimetri 2 ga teng bo'lgan uchburchakning tomonlari bo'lsa, $a^2 + b^2 + c^2 < 2(1 - abc)$ tengsizlikni isbotlang.

Yechish. Perimetri 2 ga teng bo'lgan uchburchak tomonlari uchun $a < 1$, $b < 1$ va $c < 1$ bo'ladi.

Demak, $(1 - a)(1 - b)(1 - c) > 0$ bo'lib,

$$1 - (a + b + c) + ab + ac + bc - abc =$$
$$= 1 - 2 + ab + ac + bc - abc =$$
$$= -1 + \frac{a^2 + b^2 + c^2}{2} + ab + ac + bc - \frac{a^2 + b^2 + c^2}{2}$$
$$- abc = -1 + \frac{(a^2 + b^2 + c^2)^2}{2} - \frac{a^2 + b^2 + c^2}{2} - abc =$$
$$= -1 + \frac{4}{2} - \frac{a^2 + b^2 + c^2}{2} - abc = 1 - \frac{a^2 + b^2 + c^2}{2} - abc > 0$$

bo'ladi.

$a^2 + b^2 + c^2 < 2(1 - abc)$ tengsizlik isbotlandi.

6.1.12. Agar uchburchakning tomonlari a, b, c va r unga ichki chizilgan aylana radiusi bo'lsa, $a+b+c \geq 6\sqrt{6}r$ tengsizlikni isbotlang.

Yechish. $p=\frac{1}{2}(a+b+c)$ bo'lib $n=3$ uchun Koshi (1) tengsizligiga ko'ra:

$$3\cdot\sqrt{(p-a)(p-b)(p-c)} \leq p-a+p-b+p-c=p$$

bundan $27p(p-a)(p-b)(p-c) \leq r^4$, ya'ni $27S^2 \leq p^4$. S-uchburchakning yuzi bo'lib ushbu tengsizlikdan $27\left(\frac{S}{p}\right)^2 \leq p^2$ yoki $27r^2 \leq p^2$, ya'ni $p \geq 3\sqrt{3}r$ bundan talab qilingan $a+b+c \geq 6\sqrt{3}r$ tengsizlikning to'g'riligi kelib chiqadi.

6.1.13. h_a, h_b, h_c uchburchak tomonlariga tushirilgan balandliklar r-unga ichki chizilgan aylana radiusi bo'lsa, $h_a+h_b+h_c \geq 9r$ tengsizlikni isbotlang

Yechish. h_a, h_b, h_c va r uchun yordamchi $\frac{1}{h_a}+\frac{1}{h_b}+\frac{1}{h_c}=\frac{1}{r}$ tenglikni isbotlaymiz.

$S=\frac{1}{2}ah_a=\frac{1}{2}bh_b=\frac{1}{2}ch_c=Pr$ bo'lib, (bu yerda $P=\frac{a+b+c}{2}$, r-uchbuchakka ichki chizilgan aylana radiusi)

Ulardan $\frac{1}{h_a}+\frac{1}{h_b}+\frac{1}{h_c}=\frac{a}{2S}+\frac{b}{2S}+\frac{c}{2S}=\frac{p}{S}=\frac{1}{r}$ bo'lib, (5)

tengsizlikka asosan: $(h_a+h_b+h_c)\cdot\left(\frac{1}{h_a}+\frac{1}{h_b}+\frac{1}{h_c}\right) \geq 9$ bo'ladi

Yuqorida isbotlangan $\frac{1}{h_a}+\frac{1}{h_b}+\frac{1}{h_c}=\frac{1}{r}$ tengsizlikka ko'ra $\frac{h_a+h_b+h_c}{r} \geq 9$ bo'ladi.

Demak, $h_a+h_b+h_c \geq 9r$.

6.1.14. Ixtiyoriy uchburchakda unga ichki chizilgan aylana diametri tashqi chizilgan aylana radiusidan katta bo'lolmasligini, ya'ni $2r \leq R$ tengsizlikni isbotlang

Yechish. Ixtiyoriy uchburchak uchun $r=\frac{S}{p}$ va $R=\frac{abc}{4S}$ formulalar o'rinli bo'ladi. Demak,

$$\frac{2r}{R}=\frac{8S^2}{abcp}=\frac{8p(p-a)(p-b)(p-c)}{abcp} \text{ yoki } \frac{2r}{R}=\frac{8(p-a)(p-b)(p-c)}{abc} \quad (*).$$

Koshi (2) tengsizligiga asosan,

$(p-a)(p-b) \le \left(\frac{p-a+p-b}{2}\right)^2 = \frac{a^2}{4}$, $(p-a)(p-c) \le \frac{b^2}{4}$ va $(p-b)(p-c) \le \frac{a^2}{4}$, ya'ni $(p-a)(p-b)(p-c) \le \frac{abc}{8}$ bo'lib, bulardan va $(*)$ tengsizlikdan $\frac{2r}{R} \le \frac{8abc}{8abc} = 1$ yoki $2r \le R$ ekanligi kelib chiqadi.

6.1.15. Yarim perimetri P, yuzi S, tashqi va ichki chizilgan aylana radiuslari mos ravishda R va r bo'lgan ixtiyoriy uchburchak uchun

$$\frac{2\sqrt{3}}{9} \cdot S \le r \cdot R \le \frac{2r^2}{27}$$

qo'sh tengsizlik o'rinli ekanini isbotlang.

Yechish. $S = p \cdot r$ va $S = \frac{abc}{4R}$ bo'lib, u holda $r = \frac{2S}{a+b+c}$, $R = \frac{abc}{4S}$ va $r \cdot R = \frac{2S}{a+b+c} \cdot \frac{abc}{4S} = \frac{abc}{2(a+b+c)}$.

n=3 bo'lganda Koshi tengsizligiga ko'ra:

$abc \le \left(\frac{a+b+c}{3}\right)^3$ bo'lib, $r \cdot R \le \frac{1}{2(a+b+c)} \cdot \left(\frac{a+b+c}{3}\right)^3 = \frac{(a+b+c)^2}{54} = \frac{2p^2}{27}$ yoki $r \cdot R \le \frac{2r^2}{27}$. Ixtiyoriy uchburchak uchun (yuqoridagi misollarga o'xshash) $\sin A + \sin B + \sin C \le \frac{3\sqrt{3}}{2}$ $(*)$ tengsizlikdan foydalanib, ma'lum: $a = 2R \cdot \sin A$, $b = 2R \cdot \sin B$ va $c = 2R \cdot \sin C$ tengliklar yordamida $(*)$ tengsizlikni quyidagicha yozish mumkin:

$$\frac{a}{2R} + \frac{b}{2R} + \frac{c}{2R} \le \frac{3\sqrt{3}}{2} \text{ yoki } a + b + c \le 3\sqrt{3} \cdot R. \qquad (**$$

$a + b + c = (2S)/r$ tenglikdan $(**)$ tengsizlik $(2S)/r = 3\sqrt{3} \cdot R$ yoki $r \cdot R \ge (2\sqrt{3})/9 \cdot S$ bo'lib berilgan tengsizlik isbotlandi.

Eslatma. Qo'sh tengsizlikda tenglik berilgan uchburchak muntazam uchburchak bo'lganda o'rinli bo'lib, tomoni a- bo'lgan muntazam uchburchak uchun $r \cdot R = a^2/6$ tenglik to'g'ri bo'ladi.

6.1.16. Yuzalari teng bo'lgan uchburchaklardan perimetri eng kichigi muntazam uchburchak ekanligini isbotlang

Yechish. Geron formulasiga asosan:

$$S = \sqrt{\frac{a+b+c}{2} \cdot \frac{a+b-c}{2} \cdot \frac{a+c-b}{2} \cdot \frac{b+c-a}{2}},$$

$$(a+b+c)(a+b-c)(a+c-b)(b+c-a) = 16S^2. \quad (*)$$

(a+b+c)- yig'indini quyidagicha yozib olish mumkin:

$$a+b+c=\frac{3}{4}\left(\frac{a+b+c}{3}+\frac{a+b-c}{1}+\frac{a+c-b}{1}+\frac{b+c-a}{1}\right).\ (**)$$

(*) tenglikni quyidagicha yozib olamiz:

$$\frac{a+b+c}{3}\cdot\frac{a+b-c}{1}\cdot\frac{a+c-b}{1}\cdot\frac{b+c-a}{1}=\frac{16S^2}{3}.\quad (***)$$

(***) tenglikning ko'paytuvchilari, yig'indisi o'zgarmas bo'lgan (**) tenlikning qo'shiluvchilaridir. Yig'indi eng kichik bo'lishi uchun faqat va faqat, qo'shiluvchilar o'zaro teng bo'lishlari kerak, ya'ni:

$$\frac{a+b+c}{3}=a+b-c=a+c-b=b+c-a.$$

Demak, $a=b=c$.

Mustaqil yechish uchun misollar

Ixtiyoriy uchburchak uchun quyidagi tengsizliklarni isbotlang (1-6).

1. $\cos 2A+\cos 2B-\cos 2S\le\frac{3}{2}$
2. $\sqrt{3}(\cos 2A-\cos 2C)+\cos 2B\le\frac{5}{2}$
3. $\sqrt{3}\cos 2A+2\cos 2B+2\sqrt{3}\cos 2C\ge -4$
4. $((\sqrt{5}+1)(\cos 2A+\cos 2B))-(3+\sqrt{5})\cos 2C\le 4+\sqrt{5}$
5. $\sin A+\sin B+\sin C\le\frac{3\sqrt{5}}{2}$
6. $\frac{\sin A+\sin B+\sin C}{\sin A\sin B\sin C}\ge 4$

7. Uchburchak ichidagi M nuqtadan o'tkazilgan MA, MB, MC to'g'ri chiziqlar uning tomonlarini mos ravishda A_1, B_1, C_1 nuqtalarda kesib o'tadi. Quyidagi tengsizliknlari isbotlang

a) $\frac{AM}{A_1M}+\frac{BM}{B_1M}+\frac{CM}{C_1M}\ge 6$

b) $\frac{AM}{A_1M}\cdot\frac{BM}{B_1M}\cdot\frac{CM}{C_1M}\ge 8$

8. Qavariq to'rtburchakning yuzi diagonallari kvadratlari yig'indisining to'rtdan biridan katta bo'lolmasligini isbotlang.

6.2. Geometrik masalalar

6.2.1. Uchburchak tomonlari kvadratlari yig'indisi m^2 to'rtinchi darajalari yig'indisi n^4 bo'lsa uning yuzini toping.

Yechish. ABC uchburchak tomonlari $AB = c$, $AC = b$, $BC = a$ va AC tomoniga tushirilgan balandlik $BK = h$ va $AK = x$ bo'lsin, u holda, $h^2 = c^2 - x^2$, $h^2 = a^2 - (b - x)^2$ bundan:

$$c^2 - x^2 = a^2 - (b - x)^2 \text{ va } x = \frac{b^2+c^2-a^2}{2b}.$$

Natijada, $h^2 = c^2 - x^2 = c^2 - \left(\frac{b^2+c^2-a^2}{2b}\right)^2$ $(*)$ bo'lib, $S = \frac{ah}{2}$ formulani inobatga olgan holda $(*)$ tenglikni quyidagicha yozish mumkin:

$$S^2 = \frac{b^2}{4}\left(c^2 - \left(\frac{b^2 + c^2 - a^2}{2b}\right)^2\right) =$$

$$= \frac{b^2}{16b^2}(4b^2c^2 - b^4 - c^4 - a^4 - 2b^2c^2 + 2a^2b^2 + 2a^2c^2)$$

$$= \frac{1}{16}(2a^2b^2 + 2a^2c^2 + 4b^2c^2 - b^4 - c^4) =$$

$$= \frac{1}{16}\left((a^2 + b^2 + c^2)^2 - 2(a^4 + b^4 + c^4)\right) =$$

$$= \frac{1}{16}(m^4 - 2n^4).$$

Demak, $S = \frac{\sqrt{m^4-2n^4}}{4}$.

6.2.2. Agar biror uchburchak tomonlari uzunliklarining kvadratlari arifmetik progressiyaning ketma-ket hadlaridan iborat bo'lsa, tomonlari shu uchburchak medianalari uzunliklariga teng bo'lgan uchburchak berilgan uchburchakka o'xshash ekanligini isbotlang.

Yechish. Berilgan uchburchak tomonlari a,b,c (aniqlik uchun $a \geq b \geq c$ bo'lsin) mos medianalari m_a, m_b, m_c bo'lsin. Arifmetik progressiyaning xossasidan: $2b^2 = a^2 + c^2$ tenglikni yozish mumkin.

Uchburchak medianalari formulalari

$$m_a = \frac{1}{2}\sqrt{2b^2 + 2c^2 - a^2}, m_b = \frac{1}{2}\sqrt{2a^2 + 2c^2 - b^2},$$

$$m_c = \frac{1}{2}\sqrt{2a^2 + 2b^2 - c^2}$$

va masala shartidan:

$$m_a = \frac{1}{2}\sqrt{a^2 + c^2 + 2c^2 - a^2} = \frac{c\sqrt{3}}{2},$$

$$m_b = \frac{1}{2}\sqrt{4b^2 - b^2} = \frac{b\sqrt{3}}{2},$$

$$m_c = \frac{1}{2}\sqrt{2a^2 + a^2 + c^2 - c^2} = \frac{a\sqrt{3}}{2}.$$

tengliklarni hosil qilish mumkin.

Demak, $m_c : m_b : m_a = \frac{a\sqrt{3}}{2} : \frac{b\sqrt{3}}{2} : \frac{c\sqrt{3}}{2}$ bo'lib, $a : b : c = m_c : m_b : m_a$ va demak,tomonlari m_c, m_b, m_a bo'lgan uchburchak berilgan uchburchakka o'xshash ekan.

6.2.3. O'lchamlari a,b,c bo'lgan to'g'ri burchakli parallelepiped uchun $3a + 4b + 10c = 500$ munosabat o'rinli va diagonali $d = 20\sqrt{5}$ bo'lsa, uning hajmini toping.

Yechish. To'g'ri burchakli parallelepiped uchun $a^2 + b^2 + c^2 = d^2$ bo'lib, $d = 20\sqrt{5}$. U holda:

$$a^2 + b^2 + c^2 = 2000.$$

Koshi-Bunyakovskiy tengsizligiga asosan:

$$(3a + 4b + 10c)^2 \le (9 + 16 + 100)(a^2 + b^2 + c^2) =$$
$$= 125 \cdot 2000 = 250000.$$

Misolning shartiga ko'ra:

$$3a + 4b + 10c = 500.$$

Berilgan tengsizlik tenglamaga aylanishi uchun Koshi-Bunyakovskiy tengsizligiga ko'ra $\frac{a}{3} = \frac{b}{4} = \frac{c}{10} = k$ tenglik bajarilishi lozim.

Natijada, $a = 3k$, $b = 4k$ va $s = 10k$ bo'lib, u holda

$3a + 4b + 10c = 500$ va $9k + 16k + 100k = 500$ yoki $k = 4$

Demak, $a = 16$, $b = 16$, $c = 40$ va parallelepipedning hajmi $V = abc = 7680$.

Eslatma. Masalani boshqacharoq yechish ham mumkin, ya'ni masala shartiga ko'ra: $\begin{cases} 3a + 4b + 10c = 500 \\ a^2 + b^2 + c^2 = 2000 \end{cases}$ bo'lib, bu sistemaning birinchi tenglamasini 8 ga ko'paytirib ikkinchisidan ayirsak:

$(a-12)^2+(b-16)^2+(c-40)^2=0$ bo'ladi, bundan esa $a=16$, $b=16$, $c=40$.

6.2.4. Trapetsiyaning diagonallari uni to'rtta uchburchakka ajratadi. Trapetsiyaning yuzi S uning asoslariga yopishgan uchburchaklarning yuzlari S_1 va S_2 bo'lsa, $\sqrt{S}=\sqrt{S_1}+\sqrt{S_2}$ tenglikni isbotlang.

Yechish. AD kesma ABCD trapetsiyaning ostki asosi bo'lsin. AC va BD diagonallari O nuqtada kesishadi. C uchidan BD diagonaliga o'tkazilgan parallel to'g'ri chiziq AD to'g'ri chiziq bilan K nuqtada kesishadi.

$S_{DBC}=S_{DCK}$, chunki, ularning asoslari BC= DK va balandliklari trapetsiya balandligidan iborat. U holda:

$S_{ABCD}=S_{ACK}=S$, $S_{SOB}=S_1$ va $S_{AOD}=S_2$ deb olib: SOB, AOD va ACK uchburchaklarning o'xshashligidan: $\frac{BC}{AK}=\frac{\sqrt{S_1}}{\sqrt{S}}$ va $\frac{AD}{AK}=\frac{\sqrt{S_2}}{\sqrt{S}}$ hamda BCKD – parallelogrammda $DK=BC$ va $AK=BC+AD$, u holda $\frac{\sqrt{S_1}+\sqrt{S_2}}{\sqrt{S}}=1$ yoki $\sqrt{S}=\sqrt{S_1}+\sqrt{S_2}$.

6.2.5. Tomonlari a, b, c bo'lgan ABC uchburchak uchun $\left(\frac{b}{c}+\frac{c}{b}\right)\cdot\cos A+\left(\frac{c}{a}+\frac{a}{c}\right)\cdot\cos B+\left(\frac{a}{b}+\frac{b}{a}\right)\cdot\cos C=3$ tenglik o'rinli ekanligini isbotlang.

Yechish. Sinuslar teoremasiga asosan: $\frac{b}{c}=\frac{\sin B}{\sin C}$, $\frac{c}{a}=\frac{\sin C}{\sin A}$ va $\frac{a}{b}=\frac{\sin A}{\sin B}$ bo'lib, u holda: $\left(\frac{b}{c}+\frac{c}{b}\right)\cdot\cos A+\left(\frac{c}{a}+\frac{a}{c}\right)\cdot\cos B+$ $+\left(\frac{a}{b}+\frac{b}{a}\right)\cdot cosC=\left(\frac{\sin B}{\sin C}+\frac{\sin C}{\sin B}\right)\cdot\cos A+\left(\frac{\sin C}{\sin A}+\frac{\sin A}{\sin C}\right)\cdot$ $\cos B+\left(\frac{\sin A}{\sin B}+\frac{\sin B}{\sin A}\right)\cdot\cos C=\frac{\sin B\cdot\cos C+\cos B\cdot\sin C}{\sin A}+$ $+\frac{\sin A\cdot\cos C+\cos A\cdot\sin C}{\sin B}+\frac{\sin A\cdot\cos B+\cos A\cdot\sin B}{\sin C}=\frac{\sin(B+C)}{\sin A}+\frac{\sin(A+C)}{\sin B}+$ $+\frac{\sin(A+B)}{\sin C}=\frac{\sin(\pi-A)}{\sin A}+\frac{\sin(\pi-B)}{\sin B}+\frac{\sin(\pi-C)}{\sin C}=3$

Demak, berilgan tengsizlik isbotlandi.

6.2.6. $y=5\sqrt{x}$ va $y=-\frac{40}{x}$ funksiya grafiklariga o'tkazilgan umumiy urinma tenglamasini yozing

Yechish. Berilgan funksiyalarning umumiy urinma tenglamasi

$y = kx + b$ bo'lsin. Bu to'g'ri chiziq $y = 5\sqrt{x}$ va $y = -\frac{40}{x}$ chiziqlarga urinadi, ya'ni $5\sqrt{x} = kx + b$ va $-\frac{40}{x} = kx + b$ bo'lib birinchisi $\sqrt{x}$ ga nisbatan iikinchisi esa x ga nisbatan $kx - 5\sqrt{x} + b = 0$ va $kx^2 + bx + 40 = 0$ kvadrat tenglama bo'ladi.

Egri chiziqlarga o'tkazilgan urinmalar $y = 5\sqrt{x}$ va $y = -\frac{40}{x}$ bilan bitta umumiy nuqtaga ega bo'ladi.

Demak, har ikkala tenglamaning ham diskriminantlari 0 ga teng. $\begin{cases} 25 - 4kb = 0, \\ b^2 - 160k = 0 \end{cases}$ bo'lib, bu tenglamalar sistemasining yechimi $k_1 = \frac{5}{8}$ va $b_1 = 10$ bo'lib, $y = 5\sqrt{x}$ va $y = -\frac{40}{x}$ chiziqlarga o'tkazilgan umumiy urinma tenglamasi $y = \frac{5}{8}x + 10$ bo'ladi.

6.2.7. Teng tomoni uchburchakning tomoni a-ga teng. Uchburchak ichidagi M nuqtadan uning uchlarigacha bo'lgan masofalar: MA=$\alpha, MB = \beta, MC = \gamma$ bo'lsin. a, α, β, γ kattaliklar orasidagi bo'g'lanishni toping.

Yechish. $MA_1 \perp BC, MB_1 \perp CA, MC_1 \perp AB$ va $AC_1 = x$, $BA_1 = y$, $CB_1 = z$, $MA_1 = \xi$, $MB_1 = \eta$, $MC_1 = \varphi$ bo'lsin. MAB, MBC, MCA uchburchaklardan:

$$x = \frac{a^2 + \alpha^2 - \beta^2}{2a}, \quad y = \frac{a^2 + \beta^2 - \gamma^2}{2a}, \quad z = \frac{a^2 + \gamma^2 - \alpha^2}{2a}. \tag{1}$$

Bundan:

$$x + y + z = \frac{3}{2}a. \tag{2}$$

Quyidagi tenglikni ham shunga o'xshash hosil qilish mumkin:

$$\xi + \eta + \varphi = \frac{\sqrt{3}}{2}a. \tag{3}$$

U holda:

$$\frac{S_{AC_1B_1}}{S_{ACB}} = \frac{AC_1 \cdot AB_1}{AC \cdot AB} = \frac{x(a-x)}{a^2}; \frac{S_{BA_1C_1}}{S_{ACB}} = \frac{y(a-x)}{a^2}; \frac{S_{CA_1B_1}}{S_{ACB}} = \frac{z(a-y)}{a^2};$$

$$\frac{S_{MC_1B_1}}{S_{ABC}} = \frac{\eta\varphi}{a^2}, \frac{S_{MA_1C_1}}{S_{ABC}} = \frac{\xi\varphi}{a^2}, \frac{S_{MA_1B_1}}{S_{ABC}} = \frac{\eta\xi}{a^2}.$$

So'nggi tengliklarni qo'shib, so'ngra maxrajdan qutqazib

quyidagi tenlikni hosil qilish mumkin:

$$a^2 = a(x + y + z) - (yz + zx + xy) + \varphi\eta + \eta\xi + \xi\eta$$

Bundan (2) tenglikni e'tiborga olsak:

$$2(yz + xz + xy) = 2(\eta\xi + \xi\varphi + \varphi\eta) + a^2. \quad (4)$$

(2) va (3) munosabatlardan quyidagilarni yozish mumkin:

$$x^2 + y^2 + z^2 + 2(yz + xz + xy) = \frac{9}{4}a^2. \quad (5)$$

$$\xi^2 + \eta^2 + \varphi^2 + 2(\eta\varphi + \varphi\xi + \xi\eta) = \frac{3}{4}a^2. \quad (6)$$

(4), (5) va (6) tengliklardan:

$$(x^2 + y^2 + z^2) + (\xi^2 + \eta^2 + \varphi^2) = \frac{a^2}{2}. \quad (7)$$

Bundan Pifagor teoremasiga asosan:

$$(x^2 + y^2 + z^2) + (\varphi^2 + \eta^2 + \xi^2) = \alpha^2 + \beta^2 + \gamma^2. \quad (8)$$

(7) va (8) tengliklardan:

$$x^2 + y^2 + z^2 = \frac{2\alpha^2 + 2\beta^2 + 2\gamma^2 + \alpha}{4}. \quad (9)$$

(1) tengliklarni (9) tenglikka qo'yib, soddalashtirib, quyidagilarni hosil qilish mumkin:

$$(a^2 + \alpha^2 - \beta^2)^2 + (a^2 + \beta^2 - \gamma^2)^2 + (a^2 + \gamma^2 - \alpha^2)^2 = \\ = a^2(2\alpha^2 + 2\beta^2 + 2\gamma^2 + \alpha^2)^2,$$

$$a^4 + \alpha^4 + \beta^4 + \gamma^4 - a^2\gamma^2 - a^2\alpha^2 - a^2\beta^2 - \alpha^2\beta^2 - \beta^2\gamma^2 - \gamma^2\alpha^2 \\ = 0.$$

6.2.8. ABC uchburchakka ichki chizilgan aylananing markazi O nuqta bo'lsin. Quyidagi tengsizlikni isbotlang

$$AO^2 \cdot a + BO^2 \cdot b + CO^2 \cdot c = abc$$

Yechish. r-ABC uchburchakka ichki chizilgan aylananing radiusi, p-uning yarim perimetri bo'lsin. O nuqtadan AB tomonga tushirilgan perpendikulyar:

$$OP = r = AO \cdot \sin\frac{A}{2},$$

$$r^2 = AO^2 \sin^2\frac{A}{2} = AO^2\frac{(p-b)(p-c)}{bc},$$

$$AO^2 = \frac{r^2 bc}{(p-b)(p-c)},$$

$$AO^2a = \frac{abcr^2}{(p-b.)(p-c)}$$

Xuddi shunday, quyidagi tengliklarni yozish mumkin:

$$BO^2 \cdot b = \frac{abcr^2}{(p-a)(p-c)},$$

$$CO^2 \cdot c = \frac{abcr^2}{(p-a)(p-b)}.$$

Oxirgi uchta tenglikni qo'shib, ko'zlangan natijaga kelish mumkin:

$$AO^2a + BO^2b + CO^2c$$

$$= abcr^2\left(\frac{1}{(p-b)(p-c)} + \frac{1}{(p-a)(p-c)}\right.$$

$$\left. + \frac{1}{(p-b)(p-a)}\right) =$$

$$= \frac{abcr^2(p-a+p-b+p-c)}{(p-a)(p-b)(p-c)} = \frac{abcr^2p}{(p-a)(p-b)(p-c)} =$$

$$= \frac{abcr^2p^2}{p(p-a)(p-b)(p-c)} = \frac{abcS^2}{S^2} = abc.$$

Mustaqil yechish uchun masalalar

1. Agar biror uchburchakning a,b,c tomonlari orasida $a^2 = b^2 + bc$ munosabat o'rinli bo'lsa, u holda a va b tomonlar qarshisidagi burchaklar uchun $\angle A = 2\angle B$ tenglikni isbotlang.

2. Agar uchburchak tomonlari uzunliklari geometrik progressiya tashkil qilsa, u holda ushbu uchburchak balandliklaridan yasalgan uchburchak berilgan uchburchakka o'xshash ekanligini isbotlang.

3. Agar uchburchak tomonlari uzunliklari arifmetik progressiya tashkil qilsa, u holda uchburchakka ichki chizilgan aylana radiusi uchburchak balandliklaridan birining uchdan biriga iengligini isbotlang.

Ixtiyoriy uchburchak uchun quyidagi tengliklarni isbotlang (4-6).

4. $\frac{\sin(A-B)}{\sin(A+B)} = \frac{a^2-b^2}{c^2}$.

5. $a\sin(B-C) + b\sin(C-A) + c\sin(A-B) = 0$.

6. $\frac{a^2\sin(B-C)}{\sin A} + \frac{b^2\sin(C-A)}{\sin B} + \frac{c^2\sin(A-B)}{\sin C} = 0$.

7. Agar uchburchak tomonlari orasida $a + b = 3c$ munosabat o'rinli bo'lsa, u holda

$$ctg\frac{A}{2}ctg\frac{B}{2} = 2$$

tenglikni isbotlang.

8. Agar uchburchak burchaklari orasida $\sin C = 2\sin A \sin B$ munosabat o'rinli bo'lsa, u holda $|A-B|=\frac{\pi}{2}$ bo'lishini isbotlang.

9. Aylanaga ichki chizilgan ABCD to'rtburchak tomonlari AB=a, AD=b, CD=c, CB=d uchun

$$\frac{\sin A}{\sin B} = \frac{ad + bc}{ab + cd}$$

tenglikni isbotlang.

10. Agar ABC va $A_1B_1C_1$ uchburchaklar uchun $\angle A + \angle A_1 = 180^2$ va $\angle B=\angle B_1$ munosabatlar o'rinli bo'lsa, quyidagi tenglikni isbotlang:

$$aa_1 = bb_1 + cc_1.$$

7-bob. Ehtimollar nazariyasi

7.1. Matematik induksiya usuli

7.1.1. Ixtiyoriy natural n soni uchun quyidagi yig'indini isbotlang

$$1^2 + 2^2 + \ldots + n^2 = \frac{1}{6}n(n+1)(2n+1)$$

Yechish. Berilgan yig'indi S_n bo'lsin, u holda quyidagi tenglikni isbotlash talab etiladi:

$$S_n = \frac{1}{6}n(n+1)(2n+1). \qquad (\cdot)$$

$n = 1$ bo'lsin, u holda $S_1 = 1^2 = \frac{1}{6}(1+1)(2+1) = 1$ bo'lib, $(\cdot)$ tenglik $n = 1$ bo'lganda to'g'ri ekan.

Ixtiyoriy k natural soni uchun $(\cdot)$ tengliknu to'g'ri deb faraz qilib, ya'ni $S_k = \frac{1}{6}k(k+1)(2k+1)$ $(**)$ tenglikdan foydalanib $n = k+1$ bo'lganda $(*)$ tenglikni isbotlash kerak bo'ladi.

$n = k+1$ bo'lganda $S_{k+1} = \frac{1}{6}(k+1)(k+2)(2k+3)$ $(***)$ tenglikni isbotlash uchun shartga ko'ra, $S_{k+1} = S_k + (k+1)^2$ bo'lib, u holda:

$$S_{k+1} = \frac{1}{6}(k+1)(2k+1) + (k+1)^2 = \frac{1}{6}(k+1)(2k^2 + k + 6k + 6)$$

$$= \frac{1}{6}(k+1)(k+2)(2k+3)$$

Demak, $(\cdot)$ formula ixtiyoriy n – natural soni uchun to'g'ri ekan.

7.1.2. Ixtiyoriy n-natural soni uchun

$$(1 + 2 + \ldots + n)^2 = 1^3 + 2^3 + \ldots n^3 \qquad (*)$$

tenglikni isbotlang

Yechish. $(\cdot)$ tenglikning chap tomoni R_n va o'ng tomoni S_n bo'lsin. Ixtiyoriy n-natural son uchun $R_n = S_n$ tenglikni isbot qilish kerak.

1. $n = 1$ da $R_1 = S_1$ bo'lib, $(\cdot)$ tenglik to'g'ri.

2. $n = k$ bo'lganda $(\cdot)$ tenglikni to'g'ri, ya'ni $R_k = S_k$ deb faraz qilib, n=k+1 bo'lganda $R_{k+1} = S_{k+1}$tenglikning to'g'riligini isbot qilish kerak.

3.

$$R_{k+1} = \left(1 + 2 + \cdots + k + (k+1)\right)^2 =$$
$$= R_k + 2(1 + 2 + \ldots + k)(k+1) + (k+1)^2 =$$
$$= R_k + 2 \cdot \frac{k(k+1)}{2} \cdot (k+1) + (k+1)^2 = R_k + (k+1)^3$$
$$= R_k + (k+1)^3 = S_k + (k+1)^3 = S_{k+1}$$

bo'lib, $R_{k+1} = S_{k+1}$ tenglik isbotlandi. Демак, $(\cdot)$ tenglik ixtiyoriy natural n soni uchun to'g'ri ekan.

7.1.3. Ixtiyoriy n-natural son uchun toq sonlar kublari yig'indisi $n^2 \cdot (2n^2 - 1)$ ga tengligini isbotlang, ya'ni $n \geq 1$ bo'lganda

$$1^3 + 3^3 + 5^3 + \ldots + (2n-1)^3 = n^2 \cdot (2n^2 - 1)$$

tenglikni isbotlang

Yechish. Berilgan yig'indi S_n bo'lsin, u holda,

$S_n = n^2 \cdot (2n^2 - 1)$ $(\cdot)$ tenglikni isbotlash talab etiladi.

$n = 1$ da $S_1 = 1^3 = 1$ va $S_1 = 1^2 \cdot (2 \cdot 1^2 - 1) = 1$ bo'lib, $(\cdot)$ tenglik to'g'ri.

Faraz qilaylik $(\cdot)$ tenglik $n = k$ da to'g'ri bo'lsin, ya'ni,

$S_k = k^2 \cdot (2k^2 - 1)$ bo'lib, u holda $(\cdot)$ formulaning $n = k + 1$ da to'g'riligini isbotlash talab etiladi.

$S_{k+1} = (k+1)^2 \cdot (2(k+1)^2 - 1) = 2k^4 + 8k^3 + 11k^2 + 6k + 1 (**)$

S_n ning aniqlanishidan:

$$S_{k+1} = S_k + (2 \cdot (k+1) - 1)^3 = S_k + (2k+1)^3$$

bo'lib, induksion farazga ko'ra:

$S_k = k^2 \cdot (2k^2 - 1) + (2k^2 + 1)^3 = 2k^4 + 8k^3 + 11k^2 + 6k + 1.$

Shunday qilib, $(**)$ tenglik va demak, $(\cdot)$ tenglik ixtiyoriy n-natural soni uchun to'g'ri ekan.

7.1.4. Yig'indini isbotlang

$$\frac{1}{2} + \cos x + \cos 2x + \cdots + \cos nx = \frac{\sin \frac{2n+1}{2} x}{2 \sin \frac{x}{2}}$$

Yechish. 1. n=1 bo'lsin, u holda

$$\frac{\sin \frac{3x}{2}}{2 \sin \frac{x}{2}} = \frac{\sin \frac{x}{2} + (\sin \frac{3x}{2} - \sin \frac{x}{2})}{2 \sin \frac{x}{2}} = \frac{1}{2} + \cos x$$

bo'lib, berilgan tenglik to'g'ri.

2. n=k bo'lganda

$$\frac{1}{2}+\cos x+\cos 2x+\cdots+\cos kx=\frac{\frac{2k+1}{2}x}{2\sin\frac{x}{2}}$$

tenglikni to'g'ri deb faraz qilib, n=k+1 bo'lganda berilgan tenglikning to'g'riligini ko'rsatamiz:

$$\frac{1}{2}+\cos x+\cos 2x+\cdots+\cos kx+\cos(k+1)x$$
$$=\frac{\sin\frac{2k+1}{2}x}{2\sin\frac{x}{2}}+\cos(k+1)x=$$
$$=\frac{\sin\frac{2k+1}{2}x+\sin\frac{x}{2}\cos(k+1)x}{2\sin\frac{x}{2}}=$$
$$=\frac{\sin\frac{2k+1}{2}x+(\sin\frac{2k+3}{2}x-\sin\frac{1k+1}{2}x)}{2\sin\frac{x}{2}}=\frac{\sin\frac{2k+3}{2}x}{2\sin\frac{x}{2}}$$

Demak, berilgan yig'indining to'g'riligi isbotlandi.

7.1.5. Ixtiyoriy n($n\geq 2$)-natural son uchun quyidagi yig'indini isbotlang:

$$1\cdot 2+2\cdot 3+\ldots+(n-1)\cdot n=\frac{n\cdot(n^2-1)}{3} \qquad (*)$$

Yechish. $(\cdot)$ tenglikning chap tomonidagi yig'indi S_n bo'lsin:

$n=2$ da $S_2=1\cdot 2=\frac{2(2^2-1)}{3}=2$ bo'lib, $(\cdot)$ tenglik to'g'ri.

$n=k$ bo'lganda $(\cdot)$ tenglikni to'g'ri deb faraz qilib, ya'ni,

$S_k=\frac{k(k^2-1)}{3}$.$(**)$

$n=k+1$ bo'lganda $S_{k+1}=\frac{(k+1)\cdot((k+1)^2-1)}{3}=\frac{(k+1)\cdot(k^2+2k)}{3}$ $(***)$ tenglikni isbotlash kerak bo'ladi.

Ma'lumki, $S_{k+1}=S_k+k(k+1)$. U holda $(**)$ tenglikka ko'ra:

$$S_{k+1}=\frac{k(k^2-1)}{3}+k(k+1)=\frac{k+1}{3}\cdot(k(k-1)+3k)=$$
$$=\frac{k+1}{3}\cdot(k^2+2k)$$

Demak, $(\cdot)$ tenglik ixtiyoriy $n(n\geq 2)$ uchun to'g'ri ekan.

7.1.6. $F(n)$ –Fibonchchi sonlari ($F(1)=1,\ F(2)=1$ bo'lganda$F(n+2)=F(n)+F(n+1)$ rekkurent formula bilan berilgan sonlarga fibonachchi sonlari deyiladi) uchun:

$$F(1)+F(2)+\ldots+F(n)=F(n+2)-1 \qquad (\cdot)$$

tenglikni isbotlang.

Yechish. $(\cdot)$ tenglikning chap tomoni S_n bo'lsin:

$n=1$ bo'lsa, Fibonachchi sonlari ta'rifidan $F(1)=1$ ikkinchi tomondan $(\cdot)$ formulaga ko'ra: $S_1=F(1)=1$ va

$S_1=F(3)-1=F(1)+F(2)-1=1$, ya'ni $n=1$ da $(\cdot)$ tenglik o'rinli.

$n=k$ da $(\cdot)$ formulani to'g'ri deb faraz qilamiz, ya'ni,

$S_k=F(k+2)-1(**)$ tenglikni orinli deb; $n=k+1$ bo'lganda $S_{k+1}=F(k+3)-1$ $(***)$ tenglikni isbotlash talab etiladi.

Shartga ko'ra: $S_{k+1}=S_k+F(k+1)$ bo'lib, $(**)$ tenglikka asosan:

$S_{k+1}=S_k+F(k+1)=F(k+2)-1+F(k+1)=F(k+3)-1$

bo'ladi, ya'ni $(***)$ tenglik va demak, berilgan tenglik isbotlandi.

7.1.7. n≥ 1 bo'lganda quyidagi tenglikni isbotlang

$$\underbrace{\sqrt{2+\sqrt{2+\ldots+\sqrt{2}}}}_{n}=2\cos\left(\frac{\pi}{2^{n+1}}\right) \qquad (\cdot)$$

Yechish. $(\cdot)$ tenglikning chap tomoni R_n va o'ng tomoni S_n bo'lsin.

$n=1$ da $R_1=\sqrt{2}$, $S_1=2\cos\frac{\pi}{4}=\sqrt{2}$, ya'ni $R_1=S_1$ bo'lib, $(\cdot)$ tenglik to'g'ri.

$n=k$ da $(\cdot)$ tenglik to'g'ri, ya'ni $R_k=S_k$ yoki $\underbrace{\sqrt{2+\sqrt{2+\ldots+\sqrt{2}}}}_{k}=2\cos\left(\frac{\pi}{2^{k+1}}\right)$ $(**)$ deb faraz qilib,

$n=k+1$ da $R_{k+1}=S_{k+1}$ tenglikni isbotlash kerak bo'ladi. $(**)$ tenglikdan

$$\underbrace{\sqrt{2+\sqrt{2+\cdots+\sqrt{2}}}}_{k}=2\cos\left(\frac{\pi}{2^{k+1}}\right)+2,$$

$$\underbrace{\sqrt{2+\sqrt{2+\cdots+\sqrt{2}}}}_{k} = 2(\cos^2\left(\frac{\pi}{2^{k+2}}\right)-1)+2,$$

$$2+\underbrace{\sqrt{2+\sqrt{2+\ldots+\sqrt{2}}}}_{k} = 4\cos^2\left(\frac{\pi}{2^{k+2}}\right),$$

$$\sqrt{2+\underbrace{\sqrt{2+\sqrt{2+\ldots+\sqrt{2}}}}_{k}} = \sqrt{4\cos^2\left(\frac{\pi}{2^{k+2}}\right)},$$

$$\underbrace{\sqrt{2+\sqrt{2+\ldots+\sqrt{2}}}}_{k+1} = 2\cos\left(\frac{\pi}{2^{k+2}}\right)$$

bo'lib, $R_{k+1} = S_{k+1}$ tenglik va demak, $(\cdot)$ tenglik isbotlandi.

7.1.8. Ixtiyoriy $n(n \geq 0)$ butun son uchun

$$\cos\omega \cdot \cos 2\omega \cdot \cos 4\omega \cdot \ldots \cdot \cos 2^n\omega = \frac{\sin 2^{n+1}\omega}{2^{n+1}\sin\omega} \qquad (*)$$

tenglikni isbotlang.

Yechish. $(\cdot)$ tenglikning chap tomoni R_n bo'lsin.

$n = 0$ da $R_0 = \cos\omega$ bo'lib, ikkinchi tomoni

$R_0 = \frac{\sin 2\omega}{2\sin\omega} = \frac{2\sin\omega\cdot\cos\omega}{2\sin\omega} = \cos\omega$, ya'ni $n = 0$ da $(\cdot)$ tenglik to'g'ri ekan.

$n = k$ da $(\cdot)$ tenglik to'g'ri, ya'ni $R_k = \frac{\sin 2^{k+1}\omega}{2^{k+1}\sin\omega}$ $(**)$ bo'lsin, u holda $n = k+1$ uchun $R_{k+1} = R_k \cos 2^{k+1}\omega$ bo'lib, $(**)$ induksion farazga ko'ra: $R_{k+1} = \frac{\sin 2^{k+1}\omega}{2^{k+1}\sin\omega}\cdot\cos 2^{k+1}\omega = \frac{2\sin 2^{k+1}\omega}{2^{k+2}\sin\omega}\cdot\cos 2^{k+1}\omega =$

$= \frac{\sin 2^{k+2}\omega}{2^{k+2}\sin\omega}$.

Demak, $(\cdot)$ tenglik ixtiyoriy $n(n \geq 0)$ butun sonlar uchun to'g'ri ekan.

7.1.9. Ixtiyoriy n-natural soni uchun quyidagi tenglik to'g'ri ekanligini iabotlang:

$$tgx + 2\,tg\,2x + \ldots + 2^{n-1}tg2^{n-1}x = ctgx - 2^n ctg2^n x \qquad (\cdot)$$

Yechish. $(\cdot)$ tenglikning chap tomoni S_n bo'lsin, u holda berilgan tenglikni

$$ctgx - S_n = 2^n ctg2^n x \qquad (**)$$

ko'rinishda yozish mumkin.

$n = 1$ da $S_1 = tgx$ bo'lib, $(**)$ tenglikdan,

$ctgx - tgx = \frac{\cos x}{\sin x} - \frac{\sin x}{\cos x} = 2ctg\,2x$, ya'ni $(**)$ tenglik o'rinli.

2. Faraz qilayluk, $(**)$ tenglik $n = k$ bo'lganda o'rinli bo'lsin, ya'ni:

$$ctgx - S_k = 2^k ctg2^k x \qquad (***).$$

$n = k + 1$ bo'lganda $(**)$ induksion formulaga asosan:

$$ctgx - S_{k+1} = (ctgx - S_k) - 2^k tg2^k x = 2^k \tilde{n}tg2^k x - 2^k tg2^k x =$$

$$= 2^k \left(\frac{\cos 2^k x}{\sin 2^k x} - \frac{\sin 2^k x}{\cos 2^k x}\right) = 2^{k+1} ctg2^{k+1} x$$

bo'lib, $(**)$ tenglik $n = k + 1$ da ham to'g'ri. Demak, $(\cdot)$ tenglik ixtiyoriy n-natural son uchun isbotlandi.

7.1.10. Yig'indini isbotlang

$$\frac{1}{1+x} + \frac{2}{1+x^2} + \frac{4}{1+x^4} + \frac{8}{1+x^8} + \cdots + \frac{2^n}{1+x^{2^n}} =$$

$$= \frac{1}{1-x} + \frac{2^{n+1}}{1-x^{2^{n+1}}}$$

Yechish. 1. n=0 da

$$\frac{1}{1+x} = \frac{1}{1-x} + \frac{2}{1-x^2}$$

bo'lib, berilgan tenglik to'g'ri.

2. n=k bo'lganda

$$\frac{1}{1+x} + \frac{2}{1+x^2} + \frac{4}{1+x^4} + \frac{8}{1+x^8} + \cdots + \frac{2^k}{1+x^{2^k}} =$$

$$= \frac{1}{1-x} - \frac{2^{k+1}}{1+x^{2^{k+1}}}$$

tenglik o'rinli deb faraz qilib, u holda:

3. n=k+1 bo'lganda

$$\frac{1}{1+x} + \frac{2}{1+x^2} + \frac{4}{1+x^4} + \frac{8}{1+x^8} + \cdots + \frac{2^k}{1+x^{2^k}} + \frac{2^{k+1}}{1+x^{2^{k+1}}} = \frac{1}{1-x} +$$

$$+ \frac{2^{k+1}}{1-x^{2^{k+1}}} + \frac{2^{k+1}}{1+x^{2^{k+1}}} = \frac{1}{1-x} + \frac{2^{k+2}}{1-x^{2^{k+2}}}$$

.Demak, berilgan tenglik to'g'ri ekan.

7.1.11. Agar $a_1 a_2 \ldots a_n a_{n+1}$lar arifmetik progressiyaning hadlari

bo‘lsa,

$$\frac{1}{a_1a_2}+\frac{1}{a_2a_3}+\ldots+\frac{1}{a_na_{n+1}}=\frac{n}{a_1a_{n+1}} \qquad (\cdot)$$

tenglikni isbotlang

Yechish. $(\cdot)$ tenglikning chap tomoni S_n bo‘lsin:

$n=1$ da $S_1=\frac{1}{a_1a_2}$ bo‘lib, $(\cdot)$ tenglik to‘g’ri.

$(\cdot)$ tenglikni $n=k$ bo‘lganda to‘g’ri deb, ya’ni $S_k=\frac{k}{a_1a_{k+1}}(**)$. tenglik o‘rinli deb faraz qilib $n=k+1$ uchun $(\cdot)$ tenglikning, ya’ni $S_{k+1}=\frac{k+1}{a_1a_{k+2}}$ $(***)$ tenglikning bajarilishini isbotlash kerak.

$(\cdot)$ tenglikdan $S_{k+1}=S_k+\frac{1}{a_{k+1}a_{k+2}}$ bo‘lib, $(**)$ formulaga asosan:

$$S_{k+1}=\frac{k}{a_1a_{k+1}}+\frac{1}{a_{k+1}a_{k+2}}=\frac{ka_{k+2}+a_1}{a_1a_{k+1}a_{k+2}}=\frac{k(a_{k+1}+d)+a_1}{a_1a_{k+1}a_{k+2}}$$
$$=\frac{ka_{k+1}+a_1+kd}{a_1a_{k+1}a_{k+2}}==\frac{ka_{k+1}+a_{k+1}}{a_1a_{k+1}a_{k+2}}=\frac{k+1}{a_1a_{k+2}}$$

Natijada, $(***)$ tenglik isbotlandi. Demak, $(\cdot)$ tenglik ixtiyoriy n-natural son uchun o‘rinli ekan. Ushbu tenglikni isbotlash jarayonida arifmetik progressiyaning xossalaridan foydalanildi.

7.1.12. n- ning ixtiyoriy natural qiymatlarida

$$4^n+15n-1$$

ifoda 9 ga bo‘linishini isbotlang.

Yechish. $4^n+15n-1=R_n$ bo‘lsin.

Agar $n=1$ bo‘lsa, u holda $R_1=4+15-1=18$ soni 9 ga karrali va berilgan tasdiq $n=1$ da to‘g’ri ekan.

$n=k$ bo‘lganda R_k ifodaning qiymati 9 ga karrali bo‘lsin, u holda R_{k+1} ning 9 ga karrali ekanligini isbotlash kerak bo‘ladi.

Haqiqatdan,

$$R_{k+1}=4^{k+1}+15(k+1)-1=4(4^k+15k-1)-45k+18=$$
$$=4R_k-45k+18=4R_k-9(k-2)$$

Induksion farazimizga ko‘ra R_k 9 ga karrali demak, oxirgi ifoda 9 ga karrali ekanligidan berilgan ifodaning 9 ga bo‘linishi isbotlandi.

7.1.13. 3^n ta bir xil raqamdan iborat son 3^n ga bo‘linishini isbotlang.

Yechish. $R_n = \underbrace{aaa\ldots a}$ bu yerda a (0;9] oraliqdagi ixtiyoriy raqan bo'lib, R_n soni 3^n ga karrali ekanligini isbotlash kerak.

$n = 1$ da $R_1 = \overline{aaa} = 3a \cdot 37$ soni 3 ga karrali.

Faraz qilaylik R_k soni 3^k ga karrali bo'lsin, u holda R_{k+1} sonining 3^{k+1} ga karrali ekanligini isbotlash kerak.

Shartga ko'ra:

$$R_{k+1} = \underbrace{\overline{aa\cdots a}}_{3^{k+1}} = \underbrace{\overline{aa\cdots a}}_{3^k}\,\underbrace{\overline{aa\cdots a}}_{3^k}\,\underbrace{\overline{aa\cdots a}}_{3^k}$$

$$= \underbrace{\overline{aa\cdots a}}_{3^k} \cdot (10)^{2\cdot 3^k} + \underbrace{\overline{aa\cdots a}}_{3^k} \cdot (10)^{3^k} + \underbrace{\overline{aa\cdots a}}_{3^k} =$$

$$= \underbrace{\overline{aa\cdots a}}_{3^k} \cdot \left((10)^{2\cdot 3^k} + (10)^{3^k} + 1\right) = R_k \cdot (\underbrace{99\ldots 9}_{2\cdot 3^k} + \underbrace{99\ldots 9}_{3^k} + 3).$$

Induksion farazimizga ko'ra R_k soni 3^k ga karrali bo'lib, qavs ichidagi $\underbrace{99\ldots 9}_{2\cdot 3^k} + \underbrace{99\ldots 9}_{3^k} + 3$ soni 3 ga karrali (har bir qo'shiluvchi 3 ga bo'linadi).

Demak, R_{k+1} soni 3^{k+1} ga karrali bo'lib, ixtiyoriy n-natural son uchun R_n soni 3^n ga bo'linadi.

7.1.14. Ixtiyoriy 2 dan kichik bo'lmagan n-natural sonlari uchun

$$\frac{1}{n+1} + \frac{1}{n+2} + \ldots + \frac{1}{2n} > \frac{13}{24} \qquad (\cdot)$$

tengsizlikni isbotlang

Yechish. Tengsizlikning chap tomoni S_n bo'lsin:

$n = 2$ da $S_2 = \frac{1}{3} + \frac{1}{4} = \frac{7}{12} > \frac{13}{24}$ bo'lib, berilgan tengsizlik to'g'ri.

Faraz qilaylik berilgan $(\cdot)$ tengsizlik $n = k$ bo'lganda to'g'ri bo'lsin, ya'ni $S_k > \frac{13}{24}$ tengsizlikdan foydalanib $S_{k+1} > \frac{13}{24}$ tengsizlikni isbotlaymiz.

Shartga ko'raa: $S_k = \frac{1}{k+1} + \frac{1}{k+2} + \ldots + \frac{1}{2k}$ va

$S_{k+1} = \frac{1}{k+2} + \frac{1}{k+3} + \ldots + \frac{1}{2k} + \frac{1}{2k+1} + \frac{1}{2k+2}$ bo'lib, ularning ayirmasi

$$S_{k+1} - S_k = \frac{1}{2k+1} + \frac{1}{2k+2} - \frac{1}{k+1} = \frac{1}{2(k+1)(2k+1)}$$

Ko'rinib turibdiki, k $(k \geq 2)$-natural son uchun oxirgi tenglikning o'ng tomoni musbat.

Demak, $S_{k+1} > S_k > \frac{13}{24}$ bo'lib, $S_{k+1} > \frac{13}{24}$.

7.1.15. Ixtiyoriy musbat x(x> 0) va n-natural son uchun quyidagi tengsizlikni isbotlang

$$x^n + x^{n-2} + x^{n-4} + \cdots + \frac{1}{x^{n-4}} + \frac{1}{x^{n-2}} + \frac{1}{x^n} \geq n + 1 \quad (*)$$

Yechish. 1. a) n=1 bo'lsa, $x+\frac{1}{x} \geq 2$ bo'lib, berilgan tengsizlik o'rinli bo'ladi.

b) n=2 bo'lsa,

$$x^2 + 1 + \frac{1}{x^2} = x^2 + \frac{1}{x^2} + 1 \geq 2 + 1 = 3$$

bo'lib, berilgan tengsizlik bajariladi.

2. Faraz qilaylik n=k bo'lganda (*) tengsizlik, ya'ni

$$x^k + x^{k-2} + \cdots + \frac{1}{x^{k-2}} + \frac{1}{x^k} \geq k + 1 \qquad (**)$$

tengsizlik bajarilganda n=k+2 bo'lganda

$$x^{k+2} + x^k + x^{k-2} + \cdots + \frac{1}{x^{k-2}} + \frac{1}{x^k} + \frac{1}{x^{k+2}} \geq k + 3 \ (***)$$

tengsizlikning to'g'riligini isbotlash kerak bo'ladi. Ma'lum bo'lgan:

$$x^{k+2} + \frac{1}{x^{k+2}} \geq 2$$

tengsizlikni (**) tengsizlikka hadma-had qo'shib:

$$x^{k+2} + x^k + x^{k-2} + \cdots + \frac{1}{x^{k-2}} + \frac{1}{x^k} + \frac{1}{x^{k+2}} \geq k + 1 + 2 = k + 3$$

tengsizlikni, ya'ni talab qilingan tengsizlikni hosil qilish mumkin.

7.1.16. Agar $x > -1$ bo'lsa, n-natural son uchun quyidagi tengsizlikni isbotlang.

$$(1 + x)^n \geq 1 + nx \ (\cdot) \text{ (Bernulli (6) tengsizligi)}$$

Yechish. Agar $n = 1$ bo'lsa, u holda (·) tengsizlik ushbu $1 + x = 1 + x$ tenglikka aylanib, berilgan shart bajariladi.

Faraz qilaylik, berilgan tengsizlik $n = k$ da to'g'ri, ya'ni $(1 + x)^k \geq 1 + kx$ (**) tengsizlik to'g'ri bo'lsin, u holda $n = k + 1$ bo'lganda $(1 + x)^{k+1} \geq 1 + kx + x$ (***) tengsizlikni isbotlash kerak bo'ladi.

Induktiv farazga ko'ra:

$(1 + x)^{k+1} = (1 + x)(1 + x)^k \geq (1 + x)(1 + kx) = 1 + kx + x + kx^2 \geq 1 + kx + x = 1 + (k + 1)x$

bo'lib, (***) tengsizlik va demak, Bernulli (6) tengsizligi ixtiyoriy n-natural sonlar uchun to'g'ri ekan.

7.1.17. Ixtiyoriy 3 dan kichik bo'lmagan n-natural sonlari uchun

$$\sqrt[n]{n} > \sqrt[n+1]{n+1} \qquad (\cdot)$$

tengsizlikni isbotlang

Yechish. Dastlab (·) tengsizlik ikkala tomonini $n(n+1)$ darajaga ko'tarib:

$$n^{n+1} > (n+1)^n \quad \text{yoki} \quad \left(1+\frac{1}{n}\right)^n < n \qquad (**).$$

Demak, (·) va (**) tengsizliklar teng kuchlidir.

$n = 3$ da $\left(1+\frac{1}{3}\right)^3 < 3$ yoki $64 < 81$, ya'ni (**) tengsizlik to'g'ri.

Faraz qilaylik, n=k bo'lganda (**), ya'ni: $\left(1+\frac{1}{k}\right)^k < k$ (***) tengsizlik to'g'ri bo'lsin, u holda (**) tengsizlik $n = k+1$ da bajarilishini, ya'ni $\left(1+\frac{1}{k+1}\right)^{k+1} < k+1$ (****) tengsizlikning to'g'riligini ko'rsatamiz.

(****) tengsizlikda (***) tengsizlikni hisobga olgan holda quyidagicha shakl almashtirishlar bajarish mumkin:

$$\left(1+\frac{1}{k+1}\right)^{k+1} = \left(1+\frac{1}{k+1}\right)\left(1+\frac{1}{k+1}\right)^k < \left(1+\frac{1}{k+1}\right)\left(1+\frac{1}{k}\right)^k$$

$$< \left(1+\frac{1}{k+1}\right)k = \frac{k^2+2k}{k+1} < \frac{(k+1)^2}{k+1} = k+1$$

Shunday qilib, (****) tengsizlik demak, (**) va (·) tengsizliklar o'rinli ekan.

7.1.18. Nomanfiy a, b $(a \geq 0,\ b \geq 0)$ va n-natural soni uchun

$$(a+b)^n \leq 2^{n-1}(a^n + b^n) \qquad (\cdot)$$

tengsizlik o'rinli ekanligini isbotlang

Isbot. $n = 1$ da $a+b = a+b$ tenglik hosil bo'ladi. $n = 2$ da $(a+b)^2 \leq 2(a^2+b^2)$ tengsizlikning to'g'riligi Koshi-Bunyakovskiy tengsizligidan kelib chiqadi.

Faraz qilaylik, (·) tengsizlik $n < k$ uchun to'g'ri bo'lsin, u holda: $n = k$ bo'lganda to'g'riligini ko'rsatish kerak bo'ladi.

k-juft son bo'lsin, u holda: $\frac{k}{2} < k$ bo'lib, induksion faraz va

Koshi-Bunyakovskiy tengsizligiga asosan:

$$\left(a^{\frac{k}{2}}+b^{\frac{k}{2}}\right)^2 \le (1^2+1^2)(a^k+b^k) = 2(a^k+b^k),\text{ya'ni}$$

$$(a+b)^k \le \left((a+b)^{\frac{k}{2}}\right)^2 \le$$

$$\le \left(2^{\frac{k}{2}-1}\cdot\left(a^{\frac{k}{2}}+b^{\frac{k}{2}}\right)\right)^2 = 2^{k-2}\cdot\left(a^{\frac{k}{2}}+b^{\frac{k}{2}}\right)^2 \le 2^{k-1}\cdot(a^k+b^k).$$

k-toq son bo'lsin, u holda: $\frac{k+1}{2} < k$ bo'lib, induksion farazdan foydalanib Koshi-Bunyakovskiy tengsizligiga ko'ra:

$$\left(a^{\frac{1}{2}}a^{\frac{k}{2}}+b^{\frac{1}{2}}b^{\frac{k}{2}}\right)^2 \le (a+b)(a^k+b^k) \text{ yoki}$$

$$(a+b)^{k+1} = \left((a+b)^{\frac{(k+1)}{2}}\right)^2 \le \left(2^{\frac{(k+1)}{2-1}}\cdot\left(a^{\frac{(k+1)}{2}}+b^{\frac{(k+1)}{2}}\right)\right)^2$$

$$= 2^{k-1}\cdot\left(a^{\frac{1}{2}}a^{\frac{k}{2}}+b^{\frac{1}{2}}b^{\frac{k}{2}}\right)^2 \le\le 2^{k-1}(a+b)(a^k+b^k)$$

Shunday qilib, $(\cdot)$ tengsizlik $n=k$ bo'lganda to'g'ri ekanligi va demak, $(\cdot)$ tengsizlik ixtiyoriy n-natural son uchun isbotlandi.

Eslatma: $(\cdot)$ tengsizlik faqat va faqat $a=b$ hamda $n=1$ bo'lgandagina tenglikka aylanadi.

7.1.19. Qavariq n-burchak diagonallari soni $D_n = \frac{n(n-3)}{2}$ $(\cdot)$ $(n\ge 3)$ formula bilan aniqlanishini isbotlang

Yechish. Uchburchakning diagonallari bo'lmaganligi sababli n=3 bo'lganda berilgan formula o'rinli.

$n=k$ bo'lganda qavariq k-burchak diagonallari soni $D_k = \frac{k(k-3)}{2}$ bo'lsin, u holda: $(\cdot)$ formulaga ko'ra, qavariq $(k+1)$-burchak diagonallari soni $D_{k+1} = \frac{(k+1)(k-2)}{2}$ $(\cdot)$ formula bilan hisoblanishini isbotlaymiz.

$A_1A_2 \ldots\ldots A_kA_{k+1}$ qavariq (k+1)-burchakning A_1A_k diagonalini o'tkazib, $(k+1)$ uchidan yana $k-2$ ta jami (k-2+1=(k-1) ta diagonal o'ktazish mumkin. Demak, qavariq (k+1) burchak diagonallari soni: $D_{k+1} = D_k + (k-2) + 1 = \frac{k(k-3)}{2} + k - 1 = \frac{(k+1)(k-2)}{2}$.

Demak, $(**)$ formula va demak, $(\cdot)$ formula ixtiyoriy $(n \ge 3)$ n-

natural son uchun to‘g’ri bo‘ladi.

(·) formulaning to‘g’riligini boshqacha usulda ham ko‘rsatish mumkin:

$n(n \geq 3)$ qavariq n-burchakning uchlari hech bir uchtasi bir to‘g’ri chiziqda yotmaydigan n-ta nuqtadan iborat uchlaridan jami (har ikkitasidan bitta) $C_n^2 = \frac{n(n-1)}{2}$ ta to‘g’ri chiziq o‘tkazish mumkin, ammo ularning n-tasi berilgan ko‘pburchak tomonlari bo‘lib, diagonallari soni:

$$D_n = \frac{n \cdot (n-1)}{2} - n = \frac{n \cdot (n-3)}{3}$$

formula bilan hisoblanadi.

7.1.20. Ixtiyoriy qavariq ko‘p yoqlining uchlari va yoqlari sonlari yig’indisi uning qirralari sonidan 2 ga ko‘p ekanligini isbotlang, ya’ni U+Y-Q=2 (U-uchlari soni, Y-yoqlari soni, Q-qirralari soni)

Yechish. 1. Masalani gipoteza deb olib, uning to‘g’riligini tekshirib ko‘ramiz. Gipoteza qavariq to‘rt yoqli va besh yoqlilar uchun to‘g’ri.

2. Berilgan munosabat qavariq n-yoqli uchun to‘g’ri bo‘lsin, deb faraz qilaylik. Uning qavariq (n+1) yoqli ko‘pyoq uchun ham to‘g’ri ekanligini isbotlaymiz. n-yoqli ko‘pyoqdan (n+1)-yoqli ko‘yoqni hosil qilish uchun birinchi ko‘pyoqning bir qismini bir uchini o’z ichiga oluvchi hech bir uchidan o’tmaydigan tekislik bilan kesish kerak. U holda yoqlar soni bittaga ortadi. Qirralar soni k-taga, uchlari soni (k-1)-taga (k-ta uch hosil qilib, bitta uchni yo‘qotamiz) ortadi.

Yangi hosil bo‘lgan ko‘pyoq uchlari va yoqlari sonlari yig’indisidan qirralari soni ning ayirmasi:

$$(U + K - 1) + (Y + 1) - (Q + K) =$$
$$= U + K - 1 + Y + 1 - Q - K = U + Y - Q = 2$$

Demak, berilgan munosabat ixtiyoriy qavariq ko‘pyoq uchun o‘rinli bo‘ladi.

7.1.21. 2 dan kichik bo‘lmagan n-natural son va ixtiyoriy $a_1 a_2 \ldots a_n$ sonlari uchun

$$|a_1 + a_2 + \ldots + a_n| \leq |a_1| + |a_2| + \ldots + |a_n| \qquad (\cdot)$$

tengsizlikni isbotlang

Yechish. $n = 2$ da (·) tengsizlik: $|a_1 + a_2| \leq |a_1| + |a_2|$ o‘rinli. Faraz qilaylik, $n = k$ bo‘lganda (·) tengsizlik to‘g’ri bo‘lsin, ya’ni

$$|a_1 + a_2 + \ldots + a_k| \le |a_1| + |a_2| + \ldots + |a_k|. \quad (**)$$

U holda, $n = k + 1$ bo'lganda $(\cdot)$ tengsizlik to'g'riligini, ya'ni:

$|a_1 + a_2 + \ldots + a_k + a_{k+1}| \le |a_1| + |a_2| + \ldots + |a_k| + |a_{k+1}|$ $(***)$

tengsizlikni isbotlaymiz.

Haqiqatdan,

$$|a_1 + a_2 + \ldots + a_k + a_{k+1}| =$$
$$= |(a_1 + a_2 + \ldots + a_k) + a_{k+1}| |a_1 + a_2 + \ldots + a_k| + |a_{k+1}|$$

bo'lib, $(**)$ induktiv farazga asosan: $(***)$ tengsizlik va demak, $(\cdot)$ tengsizlik natural $n(n \ge 2)$ sonlari uchun to'g'ri ekan.

7.1.22. Ixtiyoriy musbat $x_1 x_2 \ldots x_n$ va n (n≥2)-natural sonlari uchun $x_1 x_2 \ldots x_n = 1$ tenglik o'rinli bo'lsa, u holda:

$$x_1 + x_2 + \ldots x_n \ge n \quad (\cdot)$$

tengsizlikni isbotlang

Yechish. $n = 2$ da $x_1 x_2 = 1$ shartdan $0 < x_1 \le 1$, u holda $x_2 \ge 1$ (yoki, aksincha $x_1 \ge 1$, u holda $0 < x_2 \le 1$) $(x_1 - 1)(x_2 - 1) \le 0$ va $x_1 x_2 = 1$ tenglikdan $x_1 x_2 \ge x_1 x_2 + 1 = 2$, ya'ni $(\cdot)$ tengsizlik bajariladi.

Demak, $n = 2$ bo'lganda $(\cdot)$ tengsizlik to'g'ri ekan.

$n = k$ bo'lganda $(\cdot)$ tengsizlikni to'g'ri deb faraz qilib, $n = k + 1$ va $x_1 x_2 x_3 \ldots . x_k \cdot x_{k+1} = 1$ bo'lganda $(\cdot)$ tengsizlik bajarilishini ko'rsatamiz. $x_{k+1} \le 1$ bo'lsin, u holda: $x_1 x_2 \ldots . x_k x_{k+1} = 1$ shartdan $x_1, x_2, \ldots, x_k$ lar ichida hech bo'lmaganda bitta 1 sonidan katta yoki 1 ga teng bo'lgan son topiladi. Mulohazalarning umumiyligiga putur yetkazmagan holda ketma-ket mulohazalar yuritib, bu son x_k ekanligini aniqlash mumkin, ya'ni $x_k \ge 1$ bo'lib, u holda:

$(x_k - 1)(x_{k+1} - 1) \le 0$ yoki $x_k + x_{k+1} \ge x_k x_{k+1} + 1$. $(**)$

Bundan: $x_{k+1} \le 1$ va $x_k \ge 1$ bo'lib $(**)$ tengsizlik o'rinli bo'ladi.

Masala shartiga ko'ra: $x_1 x_2 \ldots . x_{k-1}(x_k x_{k+1}) = 1$ va induksion farazga ko'ra:

$$x_1 + x_2 + \ldots + x_{k-1} + x_k x_{k+1} \ge k \quad (***)$$

$(**)$ va $(***)$ tengsizliklarni ketma-ket qo'llab:

$x_1 + x_2 + \ldots + x_k + x_{k+1} \ge x_1 + x_2 + \ldots + x_{k-1} + x_k x_{k+1} + 1 \ge k + 1$

tengsizlikni hosil qilish mumkin.

Shunday qilib, berilgan tengsizlik $n = k + 1$ da to'g'ri va demak, $(\cdot)$ tengsizlik ixtiyoriy $n(n \ge 2)$ lar uchun to'g'ri ekan.

7.1.23. Ixtiyoriy $a_1 + a_2 + \ldots + a_n$ musbat sonlar uchun Koshi (1) tengsizligi, ya'ni:

$$\frac{a_1+a_2+\ldots+a_n}{n} \geq \sqrt[n]{a_1 + a_2 + \ldots + a_n} \qquad (\cdot)$$

tengsizlikni isbotlang.

Yechish. Koshi tengsizligini isbotlash uchun oldingi 22 misoldanfoydalanamiz,ya'ni: $x_1, x_2, \ldots, x_n$ larni quyidagicha belgilab olamiz:

$$x_1 = \frac{a_1}{\sqrt[n]{a_1+a_2+\ldots+a_n}},\ x_2 = \frac{a_2}{\sqrt[n]{a_1+a_2+\ldots+a_n}},\ \ldots..\ x_n = \frac{a_n}{\sqrt[n]{a_1+a_2+\ldots+a_n}} \quad (**)$$

va berilgan tengsizlikni tahlil qilamiz.

Haqiqatdan, $x_1 x_2 \ldots x_n = 1$ bo'lib, oldingi misolga ko'ra:

$x + x_2 + \ldots + x_n \geq n$ bo'lib, $(**)$ tengliklardan:

$$\frac{a_1}{\sqrt[n]{a_1 a_2 \ldots a_n}} + \frac{a_2}{\sqrt[n]{a_1 a_2 \ldots a_n}} + \frac{a_3}{\sqrt[n]{a_1 a_2 \ldots a_n}} \geq n$$

Demak, Koshi (1) tengsizligi isbotlandi.

Eslatma. $(\cdot)$ tengsizlik $a_1 \geq 0, a_2 \geq 0, \ldots. a_n \geq 0$ shartda ham o'rinli boladi. Haqiqatdan, $a_1 a_2 \ldots a_n$ lardan biri 0 ga teng bo'lsa,

$a_1 + a_2 + \ldots + a_n > 0$ bo'ladi.

7.1.24. Ixtiyoriy n ($n \geq 8$) so'mlik to'lovni 3 va 5 so'mlik banknotlar yordamida qaytimsiz amalga oshirish mumkinligini isbotlang.

Yechish. $n = 8$ bo'lsin, u holda: $3 + 5 = 8$.

Berilgan masala $k (k \geq 8)$ so'm uchun to'g'ri bo'lsin, u holda:

a) k so'mlik to'lov faqat 3 so'mlik banknotlar yordamida amalga oshirilgan. U holda: $\kappa > 8$ bo'lganligi uchun 3 so'mliklar 3 tadan kam emas, $\kappa+1$ so'mlik to'lovni amalga oshirish uchun 3 ta 3 somlikni 2 ta 5 co'mlikka amashtirish kifoya.

b) k so'mlik to'lovda hech bo'lmaganda bitta 5 so'mlik qatnashgan. U holda: $k + 1$ so'mlik to'lovni amalga oshirish uchun bitta 5 so'mlikni 2 ta 3 so'mlik banknot bilan almashtirish kifoya.

Demak, $k + 1$ so'mlik to'lovni 3 va 5 so'mlik banknotlar yordamida qaytimsiz to'lash mumkin ekan, ya'ni ixtiyoriy 8 so'mdan kam bo'lmagan to'lovni har doim 3 va 5 so'mlik banknotlar yordamida qaytimsiz bajarish mumkin ekan.

Eslatma. Berilgan masalani boshqacha ham yechish mumkin:

ixtiyoriy n sonini $n = 3k - 1$, $n = 3k$ va $n = 3k + 1$ ko'rinishda yozish mumkin (bunda $k = 3, 4, 5, \ldots$) $k \geq 3$ bo'lganligidan:
$n = 3k - 1 = 3(k - 2) + 5$, $n = 3k + 1 = 3(k - 3) + 10$ bo'lib, berilgan mulohaza o'rinli ekan.

7.1.25. Tenglikni isbotlang:

$$(\sqrt{3} - i)^n = 2^n(\cos\frac{\pi n}{6} - i\sin\frac{\pi n}{6})$$

Yechish. 1. n=1 bo'lganda,berilgan tenglik to'g'ri, ya'ni:

$$\sqrt{3} - i = 2(\cos\frac{\pi}{6} - i\sin\frac{\pi}{6})$$

2. n=k bo'lsa, quyidagi tenglikni to'g'ri deb faraz qilamiz:

$$(\sqrt{3} - i)^k = 2^k(\cos\frac{\pi k}{6} - i\sin\frac{\pi k}{6})$$

U holda:

$$(\sqrt{3} - i)^{k+1} = 2^k\left(\cos\frac{\pi k}{6} - i\sin\frac{\pi k}{6}\right)2\left(\cos\frac{\pi}{6} - i\sin\frac{\pi}{6}\right) =$$
$$= 2^{k+1}\left(\cos\frac{(k+1)\pi}{6} - i\sin\frac{(k+1)\pi}{6}\right).$$

7.1.26. Ixtiyoriy n-natural son uchun quyidagi tenglikni isbotlang:

$$\frac{3}{1! + 2! + 3!} + \frac{4}{2! + 3! + 4!} + \cdots + \frac{n+2}{n! + (n+1)! + (n+2)!}$$
$$= \frac{1}{2!} - \frac{1}{(n+2)!}$$

Yechish. n=1 bo'lsin, u holda: $\frac{3}{1!+2!+3!} = \frac{1}{2!} - \frac{1}{3!}$ bo'lib, berilgan tenglik bajariladi.

n=2 bo'lsin, u holda: $\frac{3}{1!+2!+3!} + \frac{4}{2!+3!+4!} = \frac{1}{2!} - \frac{1}{4!}$ bo'lib, berilgan tenglik bajariladi.

n=k bo'lganda

$$\frac{3}{1! + 2! + 3!} + \frac{4}{2! + 3! + 4!} + \cdots + \frac{k+2}{k! + (k+1)! + (k+2)!} =$$
$$= \frac{1}{2!} - \frac{1}{(k+2)!}$$

Tenglikni to'g'ri deb, faraz qilib, n=k+1 bo'lganda quyidagi tenglikni isbotlash talab etiladi:

$$\frac{3}{1!+2!+3!}+\frac{4}{2!+3!+4!}+\cdots+\frac{k+2}{k!+(k+1)!+(k+2)!}$$
$$+\frac{k+3}{(k+1)!+(k+2)!+(k+3)!}=\frac{1}{2!}-\frac{1}{(k+3)!}.$$
$$\frac{3}{1!+2!+3!}+\frac{4}{2!+3!+4!}+\cdots+\frac{k+2}{k!+(k+1)!+(k+2)!}$$
$$+\frac{k+3}{(k+1)!+(k+2)!+(k+3)!}=$$
$$=\frac{1}{2!}-\frac{1}{(k+2)!}+\frac{k+3}{(k+1)!+(k+2)!+(k+3)!}$$
$$=\frac{1}{2!}-\frac{1}{(k+1)!}\left(\frac{1}{k+2}-\frac{1}{k+3}\right)=\frac{1}{2!}-\frac{1}{(k+3)!}.$$

Demak, berilgan tenglik ixtiyoriy n-natural son uchun o‘rinli ekan.

Mustaqil yechish uchun misollar

Quyidagi yig’indilarni isbotlang

1. $1^2+3^2+5^2+\cdots+(2n-1)^2=\frac{n(4n^2-1)}{3}$
2. $3+33+333+\cdots+\underbrace{33\cdots3}_{n}=\frac{10^{n+1}-9n-10}{27}$
3. $1^2-2^2+3^2-4^2+\cdots+(-1)^{n-1}n^2=(-1)^{n-1}\frac{n(n+1)}{2}$
4. $2^1\cdot2+2^2\cdot3+2^3\cdot4+\cdots+2^n(n+1)=n2^{n+1}$
5. $1\cdot2+2\cdot3+\cdots+n(n+1)=\frac{n(n+1)(n+2)}{3}$
6. $1\cdot2\cdot3+2\cdot3\cdot4+3\cdot4\cdot5+\cdots+n(n+1)(n+2)=\frac{n(n+1)(n+2)(n+3)}{4}$
7. $1\cdot2\cdot3\cdot\cdots\cdot k+2\cdot3\cdot\cdots\cdot k\cdot(k+1)+\cdots+n(n+1)(n+2)\cdots(n+k+1)=\frac{n(n+1)\cdots(n+k)}{k+1}$
8. $\frac{1}{1\cdot3}+\frac{2^2}{3\cdot5}+\frac{3^2}{5\cdot7}+\cdots+\frac{n^2}{(2n-1)(2n+1)}=\frac{n(n+1)}{2(2n+1)}$
9. $1\cdot1!+2\cdot2!+3\cdot3!+\cdots+n\cdot n!=(n+1)!-1$
10. $\frac{1}{2}+\frac{3}{2^2}+\frac{5}{2^3}+\cdots+\frac{2n-1}{2^n}=3-\frac{2n-3}{2^n}$
11. $\frac{4}{1\cdot2\cdot3}+\frac{5}{2\cdot3\cdot4}+\frac{6}{3\cdot4\cdot5}+\cdots+\frac{n+3}{n(n+1)(n+2)}=\frac{5}{4}-\frac{2n+5}{2(n+1)(n+2)}$

12. $3+20+168+\cdots+(2n+1)\cdot 2^{n-1}\cdot n! = 2^n\cdot(n+1)!-1$

13. $1\cdot 2^2+2\cdot 3^2+\cdots+(n-1)\cdot n^2 = \frac{n(n^2-1)(3n+2)}{12}$

11. Berilgan 49 sonining raqamlari orasiga 4 va 8 raqamlarini ko'rsatilgan tartibda (yoziladigan raqanlar soni bir xil) yozisa, har gal biror sonning kvadrati hosil bo'lishini isbotlang

49;4489;444889;···

12. $\forall n\epsilon N$ uchun quyidagi tenglikni isbotlang

$$lg\frac{2}{1}+2lg\frac{3}{2}+3lg\frac{4}{3}+\cdots+nlg\frac{n+1}{n} = \lg[\frac{(n+1)^n}{n!}]$$

n-natural sonning qanday qiymatlarida quyidagi tengsizliklar o'rinli bo'ladi

13. $2^n \geq n^2$

14. $3^n \geq 1+4n$

15. $n^3 > n^2+3$

16. $\frac{1}{n!} < \frac{1}{2^n}$

n-natural sonlar uchun quyidagi tengsizliklarni isbotlang

17. $2^n \geq 1+n$

18. $2^n > 2n+1$

19. $3^n \geq n\cdot 2^n$

20. $4^n > n^2$

21. $5^n \geq 1+4n$

22. $(2n)!\geq 2^n(n!)^2$

23. $2!\cdot 4!\cdot 6!\cdots\cdot(2n!) \geq [(n+1)!]^n$

24. $n!> (\frac{n}{3})^n$

25. $1+\frac{1}{2^2}+\frac{1}{3^2}+\cdots+\frac{1}{(n+1)^2} < 2-\frac{1}{n+1}$

26. Tenglikni isbotlang $\sqrt{\underbrace{44\cdots4}_{2n}-\underbrace{88\cdots8}_{n}} = \underbrace{66\cdots6}_{n}$

27. Har uchtasi kesishadigan va hech qanday to'rttasi umumiy nuqtaga ega bo'lmagan n-ta tekislik fazoni $\frac{(n-1)n(n+1)}{6}+n+1$ bo'lakka ajratishini isbotlang

28. Tekislikda n-ta aylana shunday chizilganki, ulardan har ikkitasi o'zaro kesishadi va hech bir uchtasi umumiy nuqtaga ega emas. Tekislik n^2-n+2 qismga ajralishini isbotlang.

$\forall n \epsilon N$ uchun berilgan ifodalarning berilgan sonlarga bo'linishini isbotlang:

29. $n^3 - n \vdots 3$

30. $13^{2n} + 6 \vdots 7$

31. $17^{4n+1} + 3 \cdot 9^{2n} \vdots 5$

32. $20^{2n} + 4^{4n} - 9^n - 1 \vdots 323$

7.2. Kombinatorika elementlari va N'yuton binomi

7.2.1. $C_{n+1}^{m+1} : C_{n+1}^{m} : C_{n+1}^{m-1} = 5:5:3$ munosabat o'rinli bo'lsa, m va n-larni toping.

Yechish. Berilgan proporsiyadan quyidagi tenglamalar sistemasini yozish mumkin:

$$\begin{cases} \dfrac{C_{n+1}^{m+1}}{C_{n+1}^{m}} = \dfrac{5}{5}, \\ \dfrac{C_{n+1}^{m}}{C_{n+1}^{m-1}} = \dfrac{5}{3}; \end{cases}$$

$$\begin{cases} \dfrac{(n+1)!\,m!\,(n-m+1)!}{(m+1)!\,(n-m)!\,(n+1)!} = 1, \\ \dfrac{(n+1)!\,(m-1)!\,(n=m+2)!}{m!\,(n-m+1)!\,(n+1)!} = \dfrac{5}{3}; \end{cases}$$

$$\begin{cases} n-m+1 = m+1, \\ 3(n-m+2) = 5m. \end{cases}$$

Oxirgi sistemaning yechimi m=3 va n=6 bo'ladi.

7.2.2. Quyidagi yoyilma nechta o'xshash bo'lmagan hadlardan iborat bo'ladi

$$(x_1 + x_2 + x_3 + \cdots + x_n)^3$$

Yechish. Yoyilmada n–ta x_i^3 ($i \epsilon N$) ko'rinishdagi, n(n-1)-ta $x_i^2 x_j$ ($i, j \epsilon N, i \neq j$) ko'rinishdagi va C_n^3–ta $x_i x_j x_k$ ko'rinishdagi (i,j,k- turli natural sonlar). Shunday qilib, berilgan yoyilmadagi o'xshash bo'lmagan hadlar soni:

$$n + n(n-1) + \frac{n(n-1)(n-2)}{6} = \frac{n(n+1)(n+2)}{6}$$

7.2.3. Natural n-soni uchun quyidagi tenglikni isbotlang.

$$C_n^1 x(1-x)^{n-1} + 2C_n^2 x^2(1-x)^{n-2} + \cdots + kC_n^k x^k(1-x)^{n-k} + \cdots + nC_n^n x^n = nx$$

Yechish. Chap tomondagi yig'indining umumiy hadini quyidagicha shakl almashtirish mumkin:

$$kC_n^k x^k(1-x)^{n-k} = k\frac{n!}{k!\,(n-k)!}x^k(1-x)^{n-k} =$$

$$= nx\frac{(n-1)!}{(k-1)!\,(n-k)!}x^{k-1}(1-x)^{n-1-(k-1)} =$$

$$= nxC_{n-1}^{k-1}x^{k-1}(1-x)^{n-1-(k-1)}.$$

U holda berilgan tenglikning chap tomoni quyidagi ko'rinishga keladi:

$$nx(C_{n-1}^0(1-x)^{n-1} + C_{n-1}^1 x(1-x)^{n-2} + \cdots + C_{n-1}^{n-1}x^{n-1}) = nx(x+1-x) = nx$$

7.2.4. Hech qaysi uchtasi bir nuqtada kesishmaydigan va hech biri boshqasiga parallel bo'lmagan n-ta to'g'ri chiziq tekislikni

$$1 + \frac{n(n+1)}{2}$$

qismga ajratishini isbotlang.

Yechish. Masalani matematik induksiya usulidan foydalanib isbotlash mumkin.

n=1 bo'lganda tasdiq to'g'ri bo'lib,tekislik $1+\frac{1(1+1)}{2} = 2$ qismga ajraladi.

n=k+1 bo'lganda tekislik:

$$1 + \frac{k(k+1)}{2}$$

qismga ajraladi deb faraz qilib, n=k+1 bo'lganda berilgan tekislik

$$1 + \frac{(n+1)(n+2)}{2}$$

qismga ajralishini isbotlash kerak bo'ladi.

(k+1)-chi to'g'ri chiziq oldingi k ta to'g'ri chiziqning hech biriga parallel emas va avvalgi kesishish nuqtalaridan farqli k-ta kesishish nuqtasi hosil qiladi, hamda har birini ikki qismga ajratib k+1 ta tekislikdan o'tadi, ya'ni tekislik qismlari soni k+1 taga ortadi.

Shunday qilib, k+1 ta to'g'ri chiziq tekislikni

$$(k+1)+\frac{k(k+1)}{2}=1+\frac{(k+1)(k+2)}{2}$$

qismga ajratadi.

7.2.5. Berilgan 1, 2, 3,⋯,100 sonlaridan tuzish mumkin bo‘lgan ikkitalik(jo‘ftlik) ko‘paytmalardan nechtasi 3 soniga karrali bo‘ladi.

Yechish. 1) Ikki sonning ko‘paytmasi 3 ga karrali bo‘lishi uchun ularning hech bo‘lmaganda bittasi 3 ga karrali bo‘lishi kerak. Bunday sonlar 1 dan 100 gacha 33 ta bo‘lib qolgan 67 tasi 3 ga karrali emas.

3 ga karrali sonlardan iborat ko‘paytmalar C_{33}^{2} ta. Biri 3 ga karrali ikkinchisi esa karrali bo‘lmagan ko‘paytmalar $67\cdot 33$ ta va jami 3 ga bo‘linadigan ko‘paytmalar soni

$$C_{33}^{2}+67\cdot 33=2739 \text{ ta}$$

7.2.6. Quyidagi yoyilmaning x-o‘zgaruvchi qatnashmaydigan hadini toping

$$\left(1+x+\frac{6}{x}\right)^{10}$$

Yechish. $x+\frac{6}{x}=y$ bo‘lsin, u holda:

$$(1+x+\frac{6}{x})^{10}=(1+y)^{10}=1+C_{10}^{1}y+Cy^{2}+\cdots+C_{10}^{10}y^{10}$$

bu yerda:

$$y^{k}=(x+\frac{6}{x})^{k}=x^{k}+C_{k}^{1}x^{k-2}\cdot 6+\cdots+C_{k}^{p}x^{k-2p}\cdot 6^{p}+\cdots+\frac{6^{k}}{x^{k}}$$

bo‘lib, x qatnashmaydigan hadda uning darajasi 0-ga teng, ya’ni k-2p=(bo‘ladi. Ushbu qo‘shiluvchi $C_{2p}^{p}6^{p}$ bo‘lib, x qatnashmaydigan barcha qo‘shiluvchilar yig‘indisi quyidagiga teng:

$$1+C_{10}^{2}C_{2}^{1}6+C_{10}^{4}C_{4}^{2}6^{2}+C_{10}^{6}C_{6}^{3}6^{3}+C_{10}^{8}C_{8}^{4}6^{4}+C_{10}^{10}C_{10}^{5}6^{5}$$

7.2.7. Bir to‘g‘ri chiziqda yotuvchi 8 ta katakka 4 ta unli va 4 ta undosh harflar shunday joylashtirilganki, ikkita unli, xuddi shunday, ikkita undosh harflar yonma-yon turmaydi. Buni necha usul bilan bajarish mumkin

Yechish. Dastlab toq o‘rindagi kataklarga unli harflarni joylashtirib 4! ta holatni va har bir holatga jo‘ft o‘rinlarga joylashtirilgan undosh harflarning 4! ta holati mos keladi hamda 4!·4! holat hosil bo‘ladi. Xuddi shunday, unli harflarni jo‘ft kataklarga joylashtirib yana 4!·4! holatni hosil qilish mumkin. Demak, jami $2\cdot 4!\cdot 4!=1152$

holat(usul)da joylashtirish mumkin.

7.2.8. 40 kishilik yig'ilishda rais, kotib va 5 ta a'zodan iborat hay'atni necha usul bilan saylash mumkin

Yechish. 40 kishining har biri rais bo'la oladi. Qolgan 39 kishining har biri kotib bo'la oladi. Demak, yig'ilish raisi va kotibini $40\cdot 39$ usulada saylash mumkin ekan. Hay'at a'zolari sonini C_{38}^{5} usulda saylash mumkin . Demak, hay'atni $40\cdot 39C_{38}^{5}$ usulda saylash mumkin.

7.2.9. Poyezd vagoni kupesida har biri besh o'rindiqli ikkita divan qarana-qarshi qo'yilgan. 10 nafar yo'lovchining 4 nafari poezd harakat yo'nalishi bo'yicha va 3 nafari qarama-qarshi o'tirishni xohlashadi. Qolgan 3 nafar yo'lovchi uchun qanday o'tirishning ahamiyati yo'q ekanligini hisobga olgan holda, yo'lovchilarni necha usulda joylashtirish mumkin.

Yechish. Qanday o'tirishining ahamiyati yo'q 3 nafar yo'lovchini 3 xil usulda 4 nafar yo'nalish bo'yicha o'tirishni xohlovchilarga qo'shib, o'rinlashtirishlar bajarilsa: $3\cdot 5!=43200$ usul bo'ladi.

Javob. 43200.

7.2.10. Raqamlari yig'indisi 3 ga teng bo'ladigan nechta o'n xonali son yozish mumkin.

Yechish. O'n xonali son 0-raqami bilan boshlanmaydi. Undagi barcha 0-lar tashlab yuborilsa, 3, 21, 12, 111 ko'rinishdagi sonlar qoladi. 3 raqami bilan boshlanadigan o'n xonali sonning qolgan raqamlari 0-lardan iborat bo'lib, bunday son bitta. 21 va 12 lar uchun $C_9^1 = 9$ tadan, 111 uchun $C_9^2 = 36$ ta. Jami:

$$1 + 2\cdot 9 + 36 = 55\ ta.$$

Javob. 55.

7.2.11. Quyidagi ko'paytma standart shakldagi ko'phadga keltirilganda nechta hadi bo'ladi

$$(1 + a^3 + a^6 + \cdots + a^{30})(1 + a^5 + a^{10} + \cdots + a^{30})$$

Yechish. Birinchi qavs 11 ta va ikkinchi qavs 7 ta turli darajali hadlardan iborat. Agar qavslar ochib chiqilsa, quyidagi jadvalda ko'rsatilgan darajalar hosil bo'ladi:

0	3	6	9	12	**15**	**18**	**21**	**24**	**27**	**30**
5	8	11	14	17	**20**	**23**	**26**	**29**	**32**	**35**
10	13	16	19	22	**25**	**28**	**31**	**34**	**37**	**40**

15 18 21 24 27 30 33 36 39 42 45
20 23 26 29 32 35 38 41 44 47 50
25 28 31 34 37 40 43 46 49 52 55
30 33 36 39 42 45 48 51 54 57 60

Ko'rinib turibdiki, jami darajalar 77 ta bo'lib, ularning 24 (qalin yozilgan) tasi takrorlanib kelayapti. Demak hosil bo'lgan ko'phad 53 ta haddan iborat bo'ladi.

Mustaqil yechish uchun misollar

1. Quyidagi yoyilmada nechta ratsional had qatnashgan. (j. 32)

$$(\sqrt{3}+\sqrt[4]{5})^{124}$$

2. Quyidagi yoyilmaning uchinchi hadi koeffisienti ikkinch hadi koeffisientidan 35 ga katta bo'lsa,yoyilmaning x-o'zgaruvchi qatnashmaydigan hadini toping. (j. 252)

$$(x+\frac{1}{x})^n$$

3. Tenglikni isbotlang:

a) $C_{2n}^n = 1 + (C_n^1)^2 + (C_n^2)^2 + \cdots + (C_n^n)^2$

b) $1^2 + 2^2 + \cdots + n^2 = C_{n+1}^2 + 2(C_n^2 + C_{n-1}^2 + \cdots + C_2^2)$

c) $C_n^0 + 2C_n^1 + 3C_n^2 + \cdots + (n+1)C_n^n = (n+2)2^{n-1}$

4. Agar $C_{n+1}^{n-1} = 10$ bo'lsa, $P_n$ni hisoblang l. 24

5. Quyidagi ifodada x^3-ning koeffisientini toping

$(1+x)^3 + (1+x)^4 + (1+x)^5 + \cdots + (1+x)^{15}$ j. 559

6. Birinchi, ikkinchi va uchinchi hadlari koeffisientlari yig'indi. 11-ga teng bo'lgan quyidagi yoyilmaning x^2-qatnashgan hadini toping

$(x^3 + \frac{1}{x^2})^n$ j. $6x^2$

7. $(a+b)^n$ yoyilmaning ikkinchi hadi 240, uchinchi hadi 720 va to'rtinchi hadi 1080 ga teng. a, b va n-larni toping. J.2;3;5.

8. Pianino klaviaturasida 88 ta tugmacha bor. 6 ta nota yordamida:

a) notalar takrorlanishi mumkin bo'lgan;

b)notalar takrorlanishi mumkin bo'lmagan;

nechta musiqiy frazalar tuzish mumkin. J.a) 88^6;

b)$A_{88}^6 = 88 \cdot 87 \cdot 86 \cdot 85 \cdot 84 \cdot 83 = 390190489920$

9. Harbiy bo'limdagi 12 nafar askardan:

a) ulardan biri boshliq;

b) boshliqsiz

necha usul bilan ikki kishilik naryad tuzish mumkin j. a)66; b)132

9. Xonani mavjud 5 ta chiroq yordamida necha usul bilan yoritish mumkin. j. $2^5 = 32$

10. 30 kishi har birida 10 kishi bo'lgan uch guruhga bo'lingan. Har xil tarkibdagi nechta go'ruh bo'lishi mumkin.j. $\frac{30!}{(10!)^3}$

11. Har bir sonda bir xil raqamlar ishtirok etmaslik sharti bilan 0,1,3,5,7 raqamlaridan 5 ga karrali nechta to'rt xonali son tuzish mumkin. J.42

12. Mini futbolda bitta darvozabon, uchta himoyachi va ikkita hujumchi bo'lgan 6 kishilik jamoani: ikkita darvozabon, 8 ta himoyachi va 5 ta hujumchidan necha turli xil usulda shakllantirish mumkin. j.1120

13. Faqat 0 va 8 raqamlari yordamida nechta 11 xonali son yozish mumkin. J.45

14. Alida matematikaga doir 7 ta, Valida esa 9 ta kitob bor (matematikaga oid 16 ta turli xildagi kitoblar). Ular necha usul bilan 3 ta kitobga 3 ta kitob almashishlari mumkin. j.2940

15. 4, 5, 6, 7 raqamlari yordamida xona raqamlari oldingi xona raqamidan kichik bo'lmaydigan nechta yetti xonali son yozish mumkin. J.120

16. Hech bo'lmaganda bitta raqami juft bo'lgan nechta to'rt xonali son yozish mumkin. j.8375

17. Hech bo'lmaganda ikkita raqami bir xil bo'lgan nechta olti xonali son yozish mumkin. j. 763920

18. 30 kishilik ekspert guruhining har bir a'zosi taklif qilingan 5 ta loyihadan faqat bittasiga ovoz berish huquqiga ega bo'lsa, turli ovoz berishlar sonini toping. j. C_{34}^4

19. 1000000 (million) so'mlik xaridni 5000 (5 ming) va 10000 (10 ming) so'mlik banknotlar yordamida necha xil usulda to'lash mumkin. J.10201

20. Nechta to'rt xonali sonning uchinchi xonasidagi raqami to'rtinchi xonasidagi raqamidan 2 ga kichik bo'ladi. j. 720

21. Raqamlari yig'indisi 4 ga teng bo'lgan nechta 23 xonali son bor. J. 2300

22. 377353752-to‘qqiz xonali son raqamlari o‘rinlarini almashtirib nechta 5 ga karrali sonlar hosil qilish mumkin. j. 1120

23. $(1+x)(1+x^2)(1+x^3)\cdots(1+x^{1000})$ ko‘paytma standart ko‘phad shakliga keltirilganda nechta haddan iborat bo‘ladi. j.500501

24. $(1+x^2)^{100}(1+x^3)^{100}$ ko‘paytmani necha hadli standart shakldagi ko‘phadga keltirish mumkin. J.695

25. 84605 sonini yetti marta ketma-ket yozib:

8460584605846058460584605846058460584605

o‘ttiz besh xonali son hosil qilindi. Uning ikkita raqami o‘chirilib 15 ga karrali son hosil qilindi. Shu ikki raqamni necha turli usulda o‘chirish mumkin. J. 216

7.3. Ehtimollar nazariyasiga doir oddiy masalalar.

7.3.1. 20 ta o‘quvchidan iborat sinfda, 15 nafar o‘quvchi matematika tugaragi a’zosi bo‘lsa, ixtiyoriy tanlangan o‘quvchi matematika tugaragi a’zosi bo‘lishi ehtimolini toping.

Yechish. $P(A)=\frac{15}{20}=\frac{3}{4}$.

7.3.2. Qutida 5 dona apelsin va 4 dona olma bor. Ixtiyoriy olingan uchta mevaning barchasi apelsin bo‘lish ehtimolini hisoblang.

Yechish. Birinchi marta olinganda apelsin chiqish ehtimoli $\frac{5}{9}$ ga teng. Ikkinchi olishda 8 mevadan 4 apelsin olinadi va nihoyat oxirgi uchinchi olishda 7 mevadan 3 tasi apelsin. Demak, uch marta ham apelsin olinishi ehtimoli $\frac{5}{9}\cdot\frac{4}{8}\cdot\frac{3}{7}=\frac{5}{42}$.

7.3.3. Qutida 14 dona shar bo‘lib, 8 tasi oq va qolgan 6 tasi qora shrlar. Ketma – ket olingan 5 ta shardan 2 tasi oq shar bo‘lish ehtimolini toping.

Yechish. 5 tadan 2 ta oq shar olish $C_8^2\cdot C_6^3$ ta bo‘lsa, barcha hodisalar C_{14}^5.

Demak, $\frac{C_8^2\cdot C_6^3}{C_{14}^5}=\frac{8!}{6!\cdot 2!}\cdot\frac{6!}{3!\cdot 3!}\cdot\frac{9!\cdot 5!}{14!}=\frac{40}{143}$.

7.3.4. Yashikdagi 10 ta detalning 4 tasi bo‘yalgan. Ketma-ket olingan 3 ta detaldan hech bo‘lmaganda bittasi bo‘yalgan bo‘lish ehtimolini toping.

Yechish. $P(A)=1-P(\bar{A})=1-\frac{6}{10}\cdot\frac{5}{9}\cdot\frac{4}{8}=1-\frac{1}{6}=\frac{5}{6}$.

7.3.5. Harbiy bo'linmada ikkita radist bo'lib, birinchi radistning uzatilgan ma'lumotni qabul qilish ehtimoli 60%. Ikkinchi radistning uzatilgan ma'lumotlni qabul qilish ehtimoli 80%. Bu ikkala radistdan hech bo'lmaganda bittasining uzatilgan ma'lumotni qabul qilish ehtimolini toping.

Yechish.

$$P(A)=(1-P(\overline{A_1})\left(1-P(\overline{A_2})\right)=(1-0{,}4)(1-0{,}2)=0{,}92.$$

7.3.6. Ikkita bir xil qutilardan birida 7 ta qora va 3 ta oq shar bor. Ikkinchi qutida esa 4 ta qora va 6 ta oq shar bor. Qutilarning biridan olingan shar oq bo'lsin. Ushbu sharning birinchi qutidan olingan bo'lish ehtimolini toping.

Yechish. Birinch olingan sharning oq bo'lish ehtimoli:

$$P(A)=P(B_1)\cdot P\left(A/B_1\right)+P(B_2)\cdot P\left(A/B_2\right)=\frac{1}{2}\cdot\frac{3}{10}+\frac{1}{2}\cdot\frac{6}{10}$$

$$=0{,}45.$$

$$P_A(B_1)=\frac{P(B_1)\cdot P(A/B_1)}{P(A)}=\frac{0{,}5\cdot 0{,}3}{0{,}45}=\frac{1}{3}.$$

Mustaqil yechish uchun misollar

1. Ikki sinfdosh 16^{00} va 17^{00} vaqt oralig'ida uchrashishga kelishib olishdi. Uchrashuvga birinchi kelgan o'quvchi 20 min kutsa va ketib qolsa, ularning uchrashish ehtimolini toping. 0,31.

2. Qutida 20 ta oq va 5 ta qora shar bor. Qaytarib qo'ymaslik sharti bilan oq shar olish uchun uchinchi urinishni amalga oshirish kerak bo'ladigan ehtimollikni toping. $\frac{1}{30}$.

3. Ixtiyoriy olingan natural to'rt xonali son 5 ga karrali bo'lish ehtimolini toping. 0,2.

4. O'quvchi o'qituvchisining telefon nomerini oxirgi ikkita raqamini yoddan chiqarib qo'ydi, lekin eslashga urinib ikkala raqamlardan bittasi 0 va narigi raqam juft ekanligini esladi. O'quvchi birinchi urinishda o'qituvchisiga to'g'ri tushish ehtimolini toping. 0,1.

5. Ikkita mergan nishonga o'q uzishmoqda. Birinchisining

nishonga tekkizish ehtimoli 70%. Ikkinchisining nishonga tekkizish ehtimoli 50%. Ikkala mergan ham nishonga tekkizish ehtimolini toping. 0,35.

6. Zavodda ishlab chiqarilgan elektr chiroqlarning o'rtacha 4% I yaroqsiz bo'lsa, ombordan olingan ikkala chiroqning ham yaroqsiz bo'lish ehtimolini toping. 0,0784.

7. Qutida 3 ta oq va 4 ta qora shar bor. Ixtiyoriy 3ta shar olinganda ular orasida hech bo'lmaganda bittasi oq shar chiqish ehtimolini toping. $\frac{31}{35}$.

8. Raqamli o'yin kubikchasi 6 marta tashlanganda hech bo'lmaganda bitta 6 tushish ehtimolini toping. 0,6651.

9. Karta kodi to'rtta raqamdan iborat. Raqamlar qat'iy o'sish tartibida yozilganini bilgan holda, birinchi urinishda to'g'ri kod yozish ehtimolini toping. 0,5.

10. Ehtimollar nazariyasi fanidan yakuniy imtixonni muvaffaqiyatli topshira olmaslik ehtimoli Ali uchun 0,03, Vali uchun 0,01 va Ahmad uchun 0,1 bo'lsa, shu uch o'quvchidan faqat ikkitasi imtixondan muvaffaqiyatli o'tish ehtimolini toping. 0,131.

11. Matematikadan olimpiadada bir guruhdan ikkita o'quvchi ishtirok etmoqda. Birinchi o'quvchining hamma misollarni to'liq ishlash ehtimoli 0,89 ga, ikkinchi o'quvchining hamma misollarni to'liq ishlash ehtimoli 0,92 ga teng bo'lsa, shu ikki o'quvchining faqat bittasi g'olib bo'lish ehtimolini toping. 0,1724.

12. Qutida 4 oq va 5 ta qora shar bor. Ixtiyoriy olingan uchta shardan uchalasi ham bir xil rangda bo'lish ehtimolini toping. $\frac{1}{6}$.

Mavzuga oid testlar.

1. 5 ta darsni necha xil usulda dars jadvaliga bir kunda qo'ysa bo'ladi?

A) 30 B) 100 C) 120 D) 5

2. Guruhda 32 o'quvchi bo'lsa, necha usulda 4 kishilik kichik guruhlar tashkil qilish mumkin bo'ladi?

A) 128 B) 35960 C) 36 D) 46788

3. 1, 2, 3, 4, 5, 6 raqamlaridan nechta turli raqamli ikki xonali sonlar yozish mumkin?

A) 10 B) 60 C) 20 D) 30

4. Hisoblang. $6! - 5!$.

A) 600 B) 300 C) 1 D) 1000

5. Qutida 45 ta shar bo'lib, 17 tasi oqsharlar. Oq bo'lmagan ikkita shar tushib qoldi. Qutidan olingan shar oq bo'lish ehtimolini toping.

A) $\frac{17}{45}$ B) $\frac{17}{43}$ C) $\frac{43}{45}$ D) $\frac{17}{45}$

6. Uchta tanga tashlandi. 2 ta gerb va 1 tasi raqam tushishi ehtimolini toping.

A) $\frac{3}{2}$ B) $\frac{1}{2}$ C) $\frac{1}{8}$ D) $\frac{1}{3}$

7. 1, 2, 3, 4, 5 raqamlaridan nechta besh xonali son yozib bo'ladi?

A) 100 B) 30 C) 5 D) 120

8. Olma, nok va uzum mevalari berilgan bo'lib, har bir likopchaga faqat ikki xil meva qo'yish mumkin. Dasturxonga necha xil turli ko'rinishda qo'yish mumkin?

A) 3 B) 6 C) 2 D) 1

9. Dars jadvali bo'yicha bir kunda 9 ta fandan necha xil usulda 6 ta fan qo'yish mumkin?

A) 1000 B) 60480 C) 56 D) 39450

10. O'yin kartalar jamlanmasida 36 ta karta bor. Ixtiyoriy olingan karta tuz bo'lishi ehtimolini toping.

A) $\frac{1}{36}$ B) $\frac{1}{9}$ C) $\frac{1}{35}$ D) $\frac{1}{8}$

11. Savollar bankida savollarning 40% matematika fanidan, 10% informatika fanidan va qolgan savollar boshqa fanlardan. Ixtiyoriy olingan savol matematika yoki informatika fanidan bo'lishi ehtimolini toping.

A) 0,5 B) 0,4 C) 0,04 D) 0,8

12. Ikkita o'yin kubikchalari tashlanganda ikkalasi ham juft raqam bo'lishi ehtimolini toping.

A) 0,25 B) $\frac{1}{3}$ C) 0,5 D) 0,125

13. 4 ta har xil matematika kitobini bitta kitob javoniga necha xil usulda qo'yish mumkin?

A) 24 B) 4 C) 16 D) 20

14. Futbol jamo'asida 11 futbolchi bo'lsa, necha xil usulga futbolchilardan jamo'a sardori va uning o'rinbosarini tanlash mumkin?

A) 22 B) 11 C) 150 D) 110

15. Ali va Vali insho yozganda, Alining xato qilish ehtimoli 60%, Valining xato qilish ehtimoli 40% bo‘lsa, Ali va Valining ikkalasi ham inshoni bexato yozish ehtimolini toping.

A) 0,25 B) 0,4 C) 0,48 D) 0,2

16. Zavod 15% oliy navli, 25% birinchi navli, 40% ikkinchi navli va qolgani uchinchi navli mahsulot ishlab chiqaradi. Ixtiyoriy olingan shu zavod mahsulotining uchinchi navli bo‘lmasligi ehtimolini toping.

A) 0,8 B) 0,1 C) 0,015 D) 0,35

17. Raqamlari turlicha va toq bo‘lgan nechta uch xonali son mavjud?

A) 120 B) 30 C) 50 D) 60

18. Uchta mergan nishonga o‘q uzayapti. Birinchi merganning nishonga tekkizish ehtimoli 90%, ikkinchi merganning nishonga tekkizish ehtimoli 80%, uchinchi merganning nishonga tekkizish ehtimoli 70% bo‘lsa, Uchala merganning ham nishonga tekkizish ehtimolini toping.

A) 0,504 B) 0,006 C) 0,5 D) 0,3

19. Sport sinfda 30 ta o‘quvchi o‘qiydi. O‘quvchilarning 11 tasi futbol seksiyasiga, 6 tasi voleybol seksiyasiga, 8 tasi yugurish seksiyasiga, qolganlari balandlikka sakrash seksiyasiga qatnashadi. Shu sinfdan ixtiyoriy tanlangan o‘quvchi sport o‘yinlari bilan shug’ullanish ehtimolini toping.

A) 0,5 B) $\frac{17}{30}$ C) $\frac{28}{30}$ D) $\frac{14}{30}$

20. 12 qavatli uyda liftga 1 qavatda biryo‘la 9 kishi chiqdi. Yo‘lovchilar 2, 3, 4 kishilik guruhlar ko‘rinishida tushib qolishi ma’lum bo‘lsa va lift 2 qavatda to‘xtamasa bu 9 yulovchi necha xil usulda tushib qolishi mumkin?

A) 100 B) 300 C) 720 D) 60

8-bob. Tenglama tuzishga doir masalalar.

8.1. Harakatga doir masalalar.

1. A shahardan B shahar tomon velosipedchi yo'lga chiqdi va uch soatdan keyin B shahardan A shahar tomon tezligi velosipedchining tezligidan uch marta katta bo'lgan motosiklchi yo'lga chiqdi va yo'lning o'rtasida uchrashishdi. Agar motosiklchi uch soat emas, ikki soatdan keyin yo'lga chiqqanida ular A shaharga B shaharga qaraganda 15 km yaqinroqda uchrashishi ma'lum bo'lsa, A va B shaharlar orasidagi masofani toping.

Yechish. A va B shaharlar orasidagi masofani S deb olib, masalani shartini yozamiz.

Masala sharti.	Tenglamalar.
Motosikl tezligi velosiped tezligidan uch marta katta.	$v_m = 3v_v$.
Velosiped yo'lda motosiklga qaraganda 3 soat ko'p vaqt bo'lgan. Ular yo'lning o'rtasida uchrashishgan.	$\frac{\frac{S}{2}}{v_v} = \frac{\frac{S}{2}}{v_m} + 3.$
Agar motosikl velosipeddan 2 soatdan keyin yo'lga chiqqanda A shaharga 15 km yaqinroqda uchrashar edilar.	$\frac{\frac{S}{2} - 15}{v_v} = \frac{\frac{S}{2} + 15}{v_m}.$

Yuqoridagi tengliklardan foydalanib, quyidagi sistemani hosil qilish mumkin:

$$\begin{cases} \frac{S}{2v_v} = \frac{S}{6v_v} + 3, \\ \frac{S-30}{2v_v} = \frac{S+30}{6v_v}. \end{cases} \begin{cases} v_v = \frac{S}{9}, \\ \frac{3S-180}{S} = 2. \end{cases}$$

Demak, $S = 180km$.

Javob. $S = 180km$.

2. Aylana shaklidagi uzunligi 19 km bo'lgan trassaning diametrial qarama-qarshi nuqtalaridan bir vaqtda bir yo'nalishda tezliklari orasidagi farq 15 km/s. bo'lgan ikki motosikchi harakatlana boshladi. Qancha vaqtdan keyin ulardan biri ikkinchisini qo'vib o'tadi.

Yechish. Motosiklchilar t-soatdan keyin uchrashishgan. Agar birinchi motosiklchining tezligi x km/s. bo'lsa, ikkinchisining tezligi

(x+15) km/s bo‘ladi. U holda:

S (km)	t (soat)	v (km/s)	tenglamalar
tx	t	x	$t(x+15)-tx=9,5$
t(x+15)	t	x+15	

$$t(x+15)-tx=9,5;$$

$$15t=9,5;t=\frac{19}{30}(soat)=38\ (minut).$$

3. Soat 6^{45} ni ko‘rsatib turibdi. Yana qancha vaqtdan keyin soat millari beshinchi marta ustma-ust tushadi.

Yechish. Ma’lumki, soatning minut mili soat milidan 12 marta tezroq aylanadi, ya’ ni soat mili x masofaga aylanganda minut mili 12x masofani o‘tadi. Demak,

$$x+48+9,75=12x,$$

$$11x=57,75;x=5,25\ (soat)=325\ (minut).$$

4. Vali harakatlanayotgan eskalatorda yurib pastga tushguncha 45 ta zina sanadi. Xuddi o‘sha tezlik bilan harakatlanayotgan yurib (eskalatorga teskari) yuqoriga chiqquncha 105 ta zina sanagan bo‘lsa, Vali o‘sha tezligi bilan to‘xtab turgan eskalatorda pastga tushguncha nechta zina sanaydi.

Yechish. Valining tezligi- x, eskalatorning tezligi- y $(x>y)$ ۷ S=1 (uzunlik birligi) bo‘lsin.

	S	v	t	tenglamalar
Pastga $y\neq 0$	1	x+y	$\frac{1}{x+y}$	$\frac{1}{x+y}=45$
Yuqoriga $y\neq 0$	1	x-y	$\frac{1}{x-y}$	$\frac{1}{x-y}=105$
Pastga y=0	1	x	$\frac{1}{x}$	$\frac{1}{x}=?$

Demak, $\begin{cases}\frac{1}{x+y}=45,\\ \frac{1}{x-y}=105.\end{cases}$ $\frac{1}{x}=63.$

5. Basseynga ikkita quvur o‘tkazilgan. Birinchi quvurning basseynni to‘ldirishga ketgan vaqti, ikkinchi quvurning uni bo‘shatishga

ketadigan vaqtidan 2 soat ko'p. Agar uchdan bir qismi to'ldirilgan basseyn ikkala quvur birdaniga ochilgandan so'ng 8 soatda bo'shasa, birinchi quvur bo'sh basseynni necha soatda to'ldiradi.

Yechish. Birinchi quvur bo'sh basseynni n soatda to'ldiradi. Ikkinchi quvur to'la basseynni m soatda bo'shatadi. Qulaylik uchun basseynning hajmi- V=1 bo'lsin, u holda $n-m=2$ bo'ladi.

	V	t	1-soatda to'ldirilgan qismi	
1-quvur (to'ldiruvchi)	1	n	$\frac{1}{n}$	
2-quvur (bo'shatuvchi)	1	m	$\frac{1}{m}$	

Masalaning shartiga ko'ra:

$$\begin{cases} n-m=2, \\ \frac{1}{3}+8\left(\frac{1}{n}-\frac{1}{m}\right)=0; \end{cases}$$

$$\begin{cases} n-m=2, \\ nm=48; \end{cases} \quad \begin{cases} n=8, \\ m=6. \end{cases}$$

6. Basseynga uchta quvur o'tkazilgan. Agar birinchi va ikkinchi quvurlar birga bo'sh basseynni 9 soatda, ikkinchi va uchinchi quvurlar 18 soatda, birinch va uchinchi quvurlar 12 soatda to'ldirishi ma'lum bo'lsa, uchala quvur birgalikda bo'sh basseynni necha soatda to'ldiradi.

Yechish. Birinchi ikkinchi va uchinchi quvurlar bo'sh basseynni mos ravishda n, m va k soatda to'ldirsin.

	V	t	1 s.da to'ldirilgan qismi
1-quvur	1	n	$\frac{1}{n}$
2-quvur	1	m	$\frac{1}{m}$
3-quvur	1	k	$\frac{1}{k}$

Masalaning shartiga ko'ra:

$$\begin{cases} 9 \cdot \left(\frac{1}{n} + \frac{1}{m}\right) = 1, \\ 18 \cdot \left(\frac{1}{m} + \frac{1}{k}\right) = 1, \\ 12 \cdot \left(\frac{1}{n} + \frac{1}{k}\right) = 1. \end{cases}$$

$$\frac{1}{n} + \frac{1}{m} + \frac{1}{k} = \frac{1}{8}.$$

Javob. 8 soat.

7. Birinchi idishda 20 kg va ikkinchi idishda 30 kg turli konsentratsiyali kislota aralashmalari bor. Agar birinchi idishdagi 20 kg, ikkinchi idishdagi 30 kg suyuqlik to‘liq aralashtirilsa 68% lik eritma hosil bo‘ladi. Agar har ikkala idishdan bir xil miqdordagi eritmalar aralashtirilsa, u holda 70% lik eritma hosil bo‘ladi. Birinchi idishdagi kislota miqdorini toping.

Yechish. Birinchi idishdagi kislota miqdori x kg, ikkinchi idishdagi kislota miqdori y kg bo‘lsin. U holda masalaning shartiga ko‘ra:

$$(20 \cdot \frac{x}{20} \cdot 100\% + 30 \cdot \frac{y}{30} \cdot 100\%) : 50 = 68\%$$

va

$$(t \cdot \frac{x}{20} \cdot 100\% + t \cdot \frac{y}{30} \cdot 100\%) : (2t) = 70\%$$

tenglamalarni hosil qilish mumkin. Yoki:

$$\begin{cases} x + y = 34, \\ 3x + 2y = 84. \end{cases} \begin{cases} x = 16, \\ y = 18. \end{cases}$$

Javob. 16 kg.

8. Birinchi qotishmadagi mis miqdori 20% ni, ikkinchi qotishmadagi mis miqdori 60% ni tashkil qiladi.Tarkibidagi mis miqdori 50% bo‘ladigan 400 kg qotishma tayyorlash uchun har bir qotishmadan qanchadan olish kerak.

Yechish. Birinchi qotishmadan x kg olinsa, u holda ikkinchi qotishmadan $(400-x)$ kg olishga to‘g’ri keladi. Demak,

$$x \cdot 20 + (400 - x) \cdot 60 = 400 \cdot 50,$$

$$x = 100\ (kg); 400 - 100 = 300\ (kg).$$

Javob. Birinchi qotishmadan 100 kg va ikkinchi qotishmadan 300 kg olish kerak.

9. Mahsulotning narxi avval 15% ga, so‘ngra yana 25% ga arzonlashdi. Savdo mavsumi oldidan 20% ga qimmatlashdi. Natijada mahsulotning dastlabki narxi necha foizga o‘zgargan.

Yechish. Mahsulotning daslabki narxi A so‘m bo‘sin. 15% ga arzonlashgach, 0,85A so‘m bo‘ladi. Yana 25% arzonlashsa 0,75·0,85A so‘m bo‘lib, 20% qimmatlashsa $1{,}2\cdot 0{,}75\cdot 0{,}85A = 0{,}765A$ bo‘ladi, ya’ni mahsulotning oxirgi narxi dastlabki narxining 76,5% ni tashkil qiladi. $100-76{,}5 = 23{,}5$.

Javob. Mahsulot 23,5% arzonlashgan.

10. Temir koni o‘zlashtirilib, birinchi yili 100 ming tonna temir rudasi qazib olindi. Keyingi bir necha yil ruda qazib olish har yili 25% dan o‘sib bordi va oxirgi 4 yilda bir xil miqdorda o‘zgarishsiz qoldi. Agar kondan jami 850 ming tonna ruda qazib olingan bo‘lsa, konda necha yil qazish ishlari olib borilgan.

Yechish. n-yil davomida konda ruda qazib olish miqdori o‘sgan bo‘lsin. Oxirgi 4 yil qazib olingan ruda miqdori o‘zgarmagan:

$$100\cdot\left(1+\frac{5}{4}+\left(\frac{5}{4}\right)^2+\cdots+\left(\frac{5}{4}\right)^{n-1}+3\left(\frac{5}{4}\right)^{n-1}\right)=850,$$

$$4\cdot\left(\left(\frac{5}{4}\right)^n-1\right)+3\cdot\left(\frac{5}{4}\right)^{n-1}=\frac{17}{2},$$

$$4\cdot\left(\frac{5}{4}\right)^n-4+\frac{12}{5}\cdot\left(\frac{5}{4}\right)^n=\frac{17}{2},$$

$$\frac{32}{5}\cdot\left(\frac{5}{4}\right)^n=\frac{25}{2},\qquad n=3.$$

Javob. 6 yil.

11. Vali 3-kurs oxirida keyingi o‘quv yili uchun shartnoma bo‘yich to‘lovni amalga oshira olmasligini tushundi va 4 yil muddatga 9760000 so‘m kredit oldi. Kredit shatri juda oddiy bo‘lib, har yilning dekabr oyining birinchi yarmida mijozning mavjud qarziga p% qo‘shimcha qarz yuklaydi. Vali keyingi yildan boshlab dekabr oyining ikkinchi yarmida 3 marta bir xil to‘lovni amalga oshirib, olgan kreditini to‘lab bo‘ldi. Agar Vali bankka jami 18750000 so‘m to‘lagan bo‘lsa, bank necha foizlik kredit bergan.

Yechish. A=9760000 so‘m, $t=\frac{100+p}{100}$, x-Valining har safar

to‘lagan puli.

yillar	Qarz hisobi (oldingi)	Qarz hisobi (keyingi)	To‘lovdan keying qarz	To‘lov miqdori
1	A	tA	tA	–
2	tA	t^2A	t^2A-x	x
3	t^2A-x	$t(t^2A-x)$	$t(t^2A-x)-x$	x
4	$t(t^2a-x)-x$	t(t($t^2A-x)-x)$	t(t($t^2A-x)-x)-x$	x

Maslaning shartiga ko‘ra:

$$t(t(t^2A-x)-x)-x=x.$$

$$t^4A-x(t^2+t+1)=0,$$

$$976t^4-625(t^2+t+1)=0, t=1{,}25.$$

Demak, p=25.

12. Konteynerda uch xil butlovchi qismlar (detal) bor. Birinchi detalning donasi 12 kg va bahosi 400 ming so‘m. Ikkinchi detalning donasi 16 kg va bahosi 500 ming so‘m. Uchinchi detalning donasi 15 kg va bahosi 600 ming so‘m. Agar konteynerda uch xil detaldan jami 326 kg yuk bo‘lsa, konteynerdagi yuk bahosining eng katta va eng kichik qiymatlarini toping.

Yechish. Konteynerdagi birinchi, ikkinchi, uchinchi detallarning soni mos ravishda x, y, z va bu yuklarning jami bahosi 100t ming so‘m bo‘lsin, u holda quyidagi tenglamalar sistemasini yozish mumkin:

$$\begin{cases} 12x+16y+15z=326, \\ 4x+5y+6z=t. \end{cases}$$

$3t=326+3z-y$. Oxirgi tenglikdagi t-ning eng katta va eng kichik qiymatlarini topish uchun y-o‘zgaruvchi qabul qilishi mumkin bo‘lgan eng katta va eng kichik qiymatlarini topish kerak bo‘ladi. Sistemaning birinchi tenglamasi va oxirgi tenglikdan z-juft ($z\leq 20$) son va $y\leq 19$ ekanligi ma’lum. z=20, x=y=1 bo‘lsa, konteynerdagi yukning og’irligi 326 kg dan oshib ketadi. z=18 bo‘lsin, u holda 3t=380−y bo‘lib, x=y=2 va t-ning maksimal qiymati t=126 bo‘ladi. y=18 bo‘lsin, u holda 3t=3z+308 tenglama butun yechimlarga ega emas. y=17 bo‘lsin, u holda x=z=2 bo‘lib, t-ning eng kichik qiymati t=105 bo‘ladi.

Javob. 10500 ming va 12600 ming so‘m.

13. Avtomobilning old tomonidagi davlat belgisi 6 yilda, orqa

tomondagi davlat belgisi 12 yilda yaroqsiz holga keladi. Agar ular o'z vaqtida alamshtib turilsa eng ko'pi bilan necha yil xizmat qilishi mumkin.

Yechish. Old tomondagi belgining eskirishi $y_1(t)=\frac{t}{6}$ va $y_1(t)=1$ bo'lsa u yaroqsiz holga keladi. Orqa tomondagi belgining eskirishi $y_2(t)=\frac{t}{12}$. (t_1,t_2) vaqtlar oralig'i bo'lib, t_2 vaqt boshlanishi bilan belgilar o'rinlari almashadi.

$$\begin{cases} y_1(t_1)+y_2(t_2)\le 1, \\ y_2(t_1)+y_1(t_2)\le 1. \end{cases} \begin{cases} \frac{t_1}{6}+\frac{t_2}{12}\le 1, \\ \frac{t_1}{12}+\frac{t_2}{6}\le 1. \end{cases} \begin{cases} 2t_1+t_2\le 12, \\ t_1+2t_2\le 12. \end{cases} t_1+t_2\le 8.$$

Demak, t_1+t_2 yig'indining eng katta qiymati 8 ga teng.

Javob. 8 yil.

14. Bankka qo'yilgan pul summasiga bir muddat (bir necha butun oylar davomida) 5%, ikkinchi muddat 12%, keying muddat $11\frac{1}{9}\%$ va oxirgi muddat davomida 12,5% foyda to'landi. Omonat muddati tugagandan keyin qo'yilgan mablag'ning $104\frac{1}{6}\%$ ga oshgani ma'lum bo'lsa, omonat necha oyga qo'yilgan.

Yechish. x, y, z, t –oylar soni bo'lsin.

$$\left(1+\frac{5}{100}\right)^x\cdot\left(1+\frac{12}{100}\right)^y\cdot\left(1+\frac{11\frac{1}{9}}{100}\right)^z\cdot\left(1+\frac{12{,}5}{100}\right)^t=1+\frac{104\frac{1}{6}}{100},$$

$$\left(\frac{21}{20}\right)^x\cdot\left(\frac{28}{25}\right)^y\cdot\left(\frac{10}{9}\right)^z\cdot\left(\frac{9}{8}\right)^t=\frac{49}{24},$$

Natural sonning tub ko'paytuvchilarga yoyilmasining yagonaligidan:

$$2^{2y+z+3}\cdot 3^{x+2t+1}\cdot 5^x\cdot 7^{x+y}=2^{2x+3t}\cdot 3^{2z}\cdot 5^{x+2y}\cdot 7^2,$$

$$\begin{cases} 2y+z+3=2x+3t, \\ x+2t+1=2z, \\ z=x+2y, \\ x+y=2. \end{cases} \begin{cases} x=1, \\ y=1, \\ z=3, \\ t=2. \end{cases}$$

Demak, x+y+z+t=7.

Javob. 7 oy.

Mustaqil yechish uchun topshiriqlar.

1. Kater daryoda oqimga qarshi 4 km va oqim boyicha 33 km masofani 1 soatda bosib o'tdi. Agar daryo oqimining tezligi 6,5 km/s bo'lsa, kemaning turg'un suvdagi tezligini toping.

2. Kater va sol bir pristandan bir vaqtda oqim bo'yicha yo'lga chiqishdi. Kater oqim bo'yicha $13\frac{1}{3}$ km suzib, darrov ortga qaytdi. Agar kater oqimga qarshi $9\frac{1}{3}$ km suzgandan keyin solni uchratgan bo'lsa, katerning turg'un suvdagi tezligi oqim tezligidan necha marta katta bo'ladi?

3. Ikkita avtomobil bir paytda bir punktdan bir xil yo'nalishda yo'lga chiqishdi. Birinchi avtomobilning tezligi 40 km/s. Ikkinchi avtomobilning tezligi birinchi avtomobil tezligining 125% ini tashkil etadi. 30 minutdan so'ng usha punktdan avtomobillarni quvib, uchinchi avtomobil yo'lga chiqdi va birinchi avtomobilni quyib o'tganidan 1,5 soat keyin ikkinchi avtomobilni quvib o'tgan bo'lsa, uchinchi avtomobilning tezligini toping.

4. 120 m masofaga yugurish bo'yicha uchta sportchi qatnashayapti. Birinchi sportchining tezligi ikkinchi sportchining tezligidan 1 m/s ga ko'p. Ikkinchi sportchining tezligi, birinchi va uchinchi sportchilar tezliklarining o'rta arifmetigiga teng. Masofani birinchi sportchi uchinchi sportchiga qaraganda 3 sekund tezroq yugurib o'tgan bo'lsa, uchinchi sportchining tezligini toping.

5. Sun'iy suv havzasi to'g'ri to'rtburchak shaklida bo'lib, tomonlari uzunliklari 1 km ga farq qiladi. Hovuzning bir uchidan bir vaqtda bir xil 4 km/s tezlik bilan ikki baliqchi: birinchisi kemada diogonal bo'ylab, ikkinchisi piyoda qirg'oq bo'ylab qarama-qarshi uchga 30 minut farq bilan yetib kelishdi. Hovuzning tomonlari uzunliklarini toping.

6. Ikkita velosipedchi oralaridagi masofa 270 km bo'lgan shaharlardan bir paytda bir-biriga qarab yo'lga chiqishdi. Ikkinchi velosipedchi birinchi velosipedchiga qaraganda soatiga 1,5 km kam yo'l

bosadi va birinchi velosipedchi tezligining son qiymati ular uchrashguncha o'tgan vaqtga teng bo'lsa, ularning tezliklarini toping.

7. Ali va Vali ish yuzasidan A shaharda uchrashishdi. Uchrashuvdan so'ng Vali 120 km uzoqlikdagi B shaharga qarab velosipedda yo'lga chiqdi. Oradan bir soat o'tgach, Ali do'sti Valining telefoni qolib ketganini ko'rib qoldi va shu vaqtning o'zida uning ortidan motosiklda 50 km/s tezlik bilan yo'lga chiqdi. Valiga telefonini berib, yana A shaharga qaytib kelganda Vali ham B shaharga kirib borgan bo'lsa, Valining velosipeddagi tezligini toping.

8. Motosiklchi 1 km masofani velosipedchiga qaraganda 4 minut tezroq bosib o'tadi. Agar 5 soatda motosiklchi velosipedchidan 100 km ko'p yo'lni bosib o'tsa, shu vaqt mobaynida ularning har biri necha kilometrdan yurishgan?

9. Avtomobil haydovchisi yo'lda 40 km/s tezlik bilan ketayotib, unga parallel temir yo'lda qarama-qarshi harakatlanayotgan va uzunligi 75 metr bo'lgan yo'lovchi poezd 3 sekundda o'tib ketganini bilgan bo'lsa, yo'lovchi poezdning tezligini toping.

10. Sport avtorallisida 3 ta turli avtomobillar ishtirok etishmoqda. Birinchi avtomobil startdan biroz keyin texnik nosozliklar sabab yo'lda 3 soat to'xtab qoldi va marraga ikkinchi avtomobildan 1 soat keyin yetib keldi. Agar ikkinchi va uchinchi avtomobillar tezliklari nisbati 5:4 kabi, uchinchi avtomobilning tezligi birinchi avtomobil tezligidan 30 km/s kam hamda marraga ikkinchi va uchinchi avtomobillar 3 soat farq bilan yetib kelishgani ma'lum bo'lsa, ularning tezliklarini toping.

11. Daryodagi soldan ikki suzuvchi bir paytda suvga tushib har biri qarama – qarshi yo'nalisda 5 minut suzib ortga qaytib yana solga chiqishdi. Agar suzuvchilarning turg'un suvdagi tezliklari bir xil bo'lsa, ularning har qaysi necha minutdan suvda suzishgan?

12. Aylana shaklidagi yugurish yo'lakchasining A nuqtasidan birinchi sportchi yugurishni boshladi. Oradan 2 minut o'tgach, ikkinchi sportchi birinchi sportchi ortidan yugurishni boshladi. Oradan yana 2 minut o'tgach uchinchi sportchi oldingilarga teskari yo'nalishda yugurishni boshladi. Agar uchala sportchining tezliklari bir xil va o'zgarmas bo'lib, har safar uchinchi sportchi birinchi va ikkinchi

sportchilarni uchratgan vaqtlar oralig'i ham bir xil o'zgarmas t minut bo'lsa, t ni toping.

13. Aylana shakldagi uzunligi 400 m bo'lgan yugurish yo'lakchasining diametrial qarama-qarshi nuqtalaridan bir paytda qarama-qarshi yo'nalishda ikki sportchi yugurishni boshlashdi. Birinchi sportchi 1 soatda ikkinchi sportchidan 1,4 km ortiq masofani yugurib o'tadi. Agar birinchi sportchining tezligi ikkinchi sportchining tezligidan 1,2 marta katta bo'lsa, ular birinchi uchrashguncha o'tgan vaqtni soatda ifodalang va 1 soatni shu topilgan vaqtga nisbati nechaga teng bo'lishini toping. 77

14. Ali yuqoridan pastga harakatlanayotgan eskalatorda yurib 36 sekundda, to'xtab turgan eskalatorda usha tezlikda yurib 1 minut 3 sekundda pastga tushadi. Ali harakatlanayotgan eskalatorda harakatsiz turgan holda necha sekundda pastga tushadi? 84

15. Motosiklchi mo'ljaldagi tezlikni 15 km/soat ga orttirib, 6 soatda 7 soatda bosib o'tishi kerak bo'lgan masofaga qaraganda 40 km ko'p yo'lni bosib o'tgan bo'lsa, motosiklchining mo'ljaldagi tezligini toping.

16. Yo'l qurilish korxonasi bir necha ish kunida 216 km yo'lr ta'mirlab chiqishi kerak edi. Dastlabki 3 kun reja asosida ishlashdi Keyingi kunlarda rejadagidan kuniga 8 km ko'p yo'lni ta'mirla boshlashdi va rejadagidan bir kun oldin 232 km yo'lni ta'mirlashgani ma'lum bo'ldi. Yo'l qurilish korxonasi 1 kunda necha kilometr yo'lni ta'mirlashni rejalashtirgan?

17. Majlislar zalida ikkita: katta va kichik eshik bo'lib, majlis qatnashchilari majlis tugagandan keyin shu ikkala eshik orqali $3\frac{3}{4}$ minutda chiqib ketishadi. Agar faqat katta eshikdan chiqish uchun ketgan vaqt kichik eshikdan chiqish uchun ketadigan vaqtdan 4 minut kam bo'lsa, har qaysi eshikdan chiqish uchun ketadigan vaqtlarni toping.

18. Ikkita ishchi birga ishlab biror ishni 10 kunda bajarishdi. Keyin ma'lum bo'lishicha, birinchi ishchi oxirgi 2 kun ishlamagan. Agar dastlabki 7 kunda ikkala ishchi birgalikda ishning 80% ini bajarishgan bo'lishsa, birinchi ishchining yolg'iz o'zi butun ishni necha kunda bajaradi?

19. Ikkita ishchiga bir xil miqdarda detal tayyorlash vazifasi topshirildi. Birinchi ishchi butun ishni 8 soatda bajaradi. Ikkinchi ishchi dastlabki 2 soatni uskunani jixozlash hamda qayta sozlashga sarfladi va butun ishni 3 soatda bajardi. Agar ikkinchi ishchi uskunani qayta sozlab 1 soat ishlaganda tayyorlagan detallar soni, birinchi ishchining shu vaqtgacha tayyorlagan detallar soniga teng bo'lsa, ikkinchi ishchining mehnat unumdorligi birinchi ishchining mehnat unumdorligidan necha marta katta bo'ladi?

20. Dalada bug'doy hosilini yig'ib olish uchun bir xil quvvatli bir necha kombayn ajratildi. Ajratilgan kombaynlar bir vaqtda ishga tushishsa 24 soatda butun hosilni yig'ib olishadi. Agar kombaynlar bir xil vaqt oralig'ida ketma-ket ishga tushirilgani va birinchi ishga tushgan kombayn oxirgi ishga tushgan kombayndan 5 marotaba ko'p vaqt ishlaganligi ma'lum bo'lsa, bug'doy yig'imi qancha vaqt davom etgan?

21. 12 ta ishchi ma'lum hajmdagi ishni 4 soatda bajaradi. Xuddi shu hajmdagi ishni 3 soatda bajarish uchun nechta ishchi kerak bo'ladi?

22. Ikkita ishchi birgalikda ishlab, ma'lum ishni 12 kunda tamomlaydi. Agar ishchilarning birinchisi shu ishning yarmini bajargandan keyin, ikkinchi ishchi qolgan yarmini bajarsa butun ishni 25 kunda tugatish mumkin bo'lsa, ishchilarning har biri bu ishni necha kunda bajarishadi?

23. Oltin va misdan iborat ikki xil qotishma bo'lib, birinchi qotishmadagi oltinning misga nisbati 1:2, ikkinchi qotishmada esa 2:3 kabi. Agar birinchi qotishmaning $\frac{1}{3}$ qismini, ikkinchi qotishmaning $\frac{5}{6}$ qismi bilan aralashtirilganda hosil bo'lgan qotishmadagi oltin miqdori, birinchi qotishmadagi mis miqdoriga teng va agar birinchi qotishmaning $\frac{2}{3}$ qismini, ikkinchisining $\frac{1}{2}$ qismi bilan qo'shib eritilsa hosil bo'lgan qotishmadagi mis miqdori, ikkinchi qotishmadagi oltin miqdoridan 1 kg ortiq bo'ladi. Dastlabki qotishmalarning har biridagi oltin miqdorini toping? 1,2 kg, 2,4 kg.

24. Sof misdan iborat bo'lakdan 2 kg olib, o'rniga 2 kg rux qo'shib eritildi. Keyin hosil bo'lgan quymadan 2 kg olib, o'rniga 2 kg rux qo'shib eritildi. Nihoyat hosil bo'lgan quymadan yana 2 kg olib,

o‘rniga 2 kg rux qo‘shib eritilgandan so‘ng hosil bo‘lgan qotishmada rux misdan 3 kg ko‘p bo‘lib qolgan bo‘lsa, dastlab olingan sof mis bo‘lagi necha kilogramm bo‘lgan?4 kg.

25. Idishdagi sof glitsirindan $1l$ olib, $1l$ suv qo‘shib aralashtirildi. Keyin aralashmadan $1l$ olib, $1l$ suv qo‘shib yana aralashtirildi. Nihoyat hosil bo‘lgan aralashmadan $1l$ olib, $1l$ suv qo‘shib yana aralashtirilgandan keyin eritmadagi suv hajmi glitsirin hajmidan 7 marta ko‘p bo‘lsa, oxirgi aralashmadagi suv va glitsirin hajmlarini toping. $0,25l, 1,75l$.

26.Har birining sig’imi $30l$ bo‘lgan ikkita idishda birgalikda $30l$ kislota bor. Birinchi idishni suv bilan liq to‘ldirib, undan ikkinchi idishga to‘lgunicha quyildi. Keyin ikkinchi idishdan birinchi idishga $12\ l$ aralashma quyildi. Natijada ikkinchi idishdagi kislota, birinchi idishdagi kislotaga qaraganda $2l$ kam bo‘lib qoldi. Dastlab idishlarning har birida necha litrdan kislota bo‘lgan?$20l, 10l$.

27. Massasi 400 gr va konsentratsiyasi 8% bo‘lgan eritma massasi 600 gr va konsentratsiyasi 13% bo‘lgan eritma bilan aralashtirildi. Hosil bo‘lgan aralashmaning konsentratsiyasini toping.

28. Yig’ilgan 1 tonna mevaning 82% i suvdan iborat. Ma’lum vaqtdan keyin bu mevadagi suvning miqdori 70% ga tushgan bo‘ls mevaning og’irligini toping.

29. Mahsulotning bahosi 30% ga oshirildi. Ma’lum vaqtdan keyin 20% ga arzonlashtirildi, shundan so‘ng uning narxi 7800 so‘m bo‘ldi. Mahsulotning dastlabki bahosi necha so‘m bo‘lgan?

30. Ikkita musbat sonning birinchisi ikkinchisidan 60% ga katta. Shu sonlarning ko‘paytmasi 1000 ga teng bo‘lsa, ularning yig’indisini toping.

31. Kilosi 24000 so‘mdan baliq sotib olindi. Tozalangandan keyin baliqning og’irligi dastlabki og’irligining 80% ini tashkil etsa, 1 kilogramm tozalangan baliq necha so‘m turadi?

32. Savdo korxonasi ikkita uskunani 22,5 million so‘mga olib, 40% foyda bilan sotdi. Agar birinchi uskunadan 25% foyda, ikkinchisidan 50% foyda ko‘rgan bo‘lsa, korxona bu uskunalarni xar birini necha so‘mdan olgan?9 va 13,5 milliondan.

33. Avtozavod birinchi chorakda yillik ishlab chiqarish rejasini 25% ini bajardi. Ikkinchi, uchinchi va to'rtinchi choraklarda ishlab chiqarilgan mashinalar soni 15, 16 va 18 sonlariga proporsional bo'ldi. Agar ikkinchi chorakdagi ishlab chiqarish birinchi chorakdagidan 8% ga ortiq bo'lsa, avtozavod yillik rejani necha foiz ortig'i bilan bajargan?13,2%.

34. Ishchining oylik maoshi bir yilda ikki marta oshirildi: birinchi marta $p\%$ ga, ikkinchi marta $2p\%$ ga. Shunda ishchining 3500000 so'm bo'lgan dastkabki maoshi ikki marta oshirilgandan keyin 4620000 so'mni tashkil qilgan bo'lsa, birinchi oshirilgan $p\%$ ni toping.

35. Vali bankka har oyda bir xil foiz daromad to'laydigan shart bilan va yiliga 9 million foyda qilishni ko'zda tutib o'z pul mablag'larini omonatga qo'ydi. 6 oydan keyin Vali bank hisobidan 4 million so'm olishga majbur bo'ldi. Yil yakunida Valining bank hisob raqamidagi puli 20 million so'm bo'lgan bo'lsa, Vali dastlab bankka necha so'm qo'ygan?

36. Korxona ishchilarga oylik to'ladi. Ijtimoiy fondlarga va yangi uskunalar sotib olish uchun oylikning 26% iga teng bo'lgan pul sarfladi. Korxona umumiy summadan davlatga 15% soliq to'ladi. Bunda hammasi bo'lib, 20240000 so'm sarflandi. Agar korxona ishchilar oyligini 10% oshirib, oylikning 30% icha ijtimoiy fondlarga va yangi uskunalar sotib olishga sarflanganda va shuningdek, umumiy summadan 15% soliq to'laganda edi, jami 23414000 so'm sarflagan bo'lar edi. Korxona ishchilarga necha so'm oylik to'lagan?

37. Vali o'z pul mablag'larini 3 yil muddatga bankka qo'yib foyda qilmoqchi. Uning e'tiborini ikkita bank jalb qildi:

- birinchi bank: har yil yakunida hisob raqamidagi pulga 10% qo'shimcha foyda to'laydi.
- ikkinchi bank: birinchi yili 4%, ikkinchi yili $y\%$, uchinchi yili $2y\%$ foyda to'laydi.

Vali foydaliroq ikkinchi bankni tanlagan bo'lsa, y ning 5 ga karrali eng kichik butun qiymatini toping.

38. Korxona iyul oyida bankdan nechadir yilga 160 million so'm kredit olishni rejalashtirdi. Kreditni qaytarish shartlari quyidagicha:

- Har yilning yanvarida qarz oldingi yil yakuniga ko'ra 25% ga oshadi.

- Har yilning fevral oyidan iyun oyigacha, korxona qarzining qandaydir qismini to'lashi kerak bo'ladi.

- Har yilning iyul oyidagi qarz, oldingi yil iyul oyidagi qarzdan bir xil farq qilishi kerak.

Agar korxona kreditni yopish uchun bankka 380 million to'lagan bo'lsa, u necha yilga kredit olingan. 10 yil.

39. Vali 2020 yilning boshida 20 million so'mlik qimmatli qog'ozlar sotib oldi. Har yilning yakunida qog'ozlar 3 millionga qimmatlashadi. Vali istalgan yil boshida qimmatli qog'ozlarini sotib, pulni har yilning oxirida 10% foyda to'laydigan bankka o'tkazishi mumkin. Vali 15 yildan keyin eng katta foyda ko'rishi uchun qaysi yilning boshida qimmatli qog'ozlarini pul ko'rinishida bankka o'tkazishi kerak?2014 yil.

40. Korxona iyul oyida bankdan 100 million so'm kredit olishni rejalashtirdi. Kreditni qaytarish shartlari quyidagicha:

- Har yilning yanvarida qarz oldingi yil yakuniga ko'ra $p\%$ g oshadi.

- Har yilning fevral oyidan iyun oyigacha, korxona qarzining qandaydir qismini to'lashi kerak bo'ladi.

Agar korxona birinchi yili 55 million va ikkinchi yili 69 million to'lab, kredit qarzidan qutilgan bo'lsa, olingan kredit foizini toping. 15%.

41. $v\,km/s$ soat tezlik bilan harakatlanayotgan og'ir yuk tortuvchi kranni ekspluatatsiya qilish(foydalanish)ning bir soati $(90 + 0{,}4v^2)$ so'm qiymatga ega. Kranni eng kam sarf bilan bir kilometr ekspluatatsiya qilish uchun qanday tezlik bilan harakatlanish kerak?

42. n –birlik mahsulot ishlab chiqarish narxi $(24 + 4n + n^2)$ bo'lsa, necha birlik mahsulot ishlab chiqarilganda, bir birlik mahsulot narxi eng arzon bo'ladi?

43. Avtomobil oldingi g'ildirak pokrishkasi 24000 km, orqa g'ildirak pokrishkasi 36000 km yo'l yura oladi. Avtomobil oldingi va orqa g'ildiraklarini almashtirish mumkin bo'lsa, shu 2 ta oldingi va 2

orqa g'ildiraklar bilan bosib o'tish mumkin bo'lgan eng uzoq masofani toping.

44. Uch xil nikel, mis, marganets metallaridan tashkil topgan uchta qotishma berilgan. Birinchi qotishmada 20% nikel , 30% mis, 50% marganets bor. Ikkinchi qotishmada 50% nikel, 20% mis, 30% marganets bor. Uchinchi qotishmada 30% nikel, 40% mis, 30% marganets bor. Uchala qotishmalardan qanchadan olinsa,10 kg qotishma tarkibida nikel miqdori 25% va mis miqdori minimal bo'ladi?

1-dan $\frac{25}{3}kg$, 2-dan $\frac{5}{3}kg$.

45. Ikki xil qurilish materiallaridan uch tipdagi ko'p qavatli uy qurish mumkin. 1-tip 12 xonadonli ko'p qavatli bitta uy qurish uchun birinchi qurilish mahsulotidan 70 va ikkinchisidan 100 ta kerak bo'ladi. 2-tip 16 xonadonli ko'p qavatli bitta uy qurish uchun birinchi qurilish mahsulotidan 110 va ikkinchisidan 150 ta kerak bo'ladi. 3-tip 21 xonadonli ko'p qavatli bitta uy qurish uchun birinchi qurilish mahsulotidan 150 va ikkinchisidan 200 ta kerak bo'ladi. Agar qurilish kompaniyasida birinchi qurilish materiallaridan 900 ta va ikkinchisidan 1300 ta bo'lsa, eng ko'p xonadon bo'ladigan qilib, qaysi tip uylardan nechtadan qurish kerak?11 ta 1-tip va 1 ta 2-tip.

46. Zavoddan qurilish maydoniga 24 ta katta va 510 kichik beton bloklarni maxsus mashinada yetkazish kerak. Mashina yuk ko'tarish quvvati 10 tonna. Kichik beton blokning og'irligi 200 kg. Katta beton blokning og'irligi 3,6 tonna va mashinada 14 ta kichik beton o'rnini egallaydi. Hamma beton bloklarni qurilish maydoniga yetkazish uchun eng kamida necha marta qatnash kerak? 20.

47. Brokerlik firmasi donasi 100 dollardan neft kompaniyasi aksiyasini va donasi 65 dollardan gaz kompaniyasi aksiyalarini sotuvga qo'yib, jami 13120 dollarga sotgan bo'lsa, qanchasi gaz kompaniyasi aksiyalarini sotishdan tushgan?

48. Birinchi kuni Valining puli Alinikidan 3000 so'm ko'p edi. Vali bor pulining $\frac{1}{n}$ $(n \in N)$ qismiga, Ali esa bor pulining yarmiga kitob sotib oldi. Alining olgan kitobi Valining olgan kitobidan 2000 so'mga qimmatroq.

Ikkinchi kuni bolalar daftarlar sotib olish uchun chiqishdi. Endi Valining puli Alining pulidan 6000 so‘m ko‘proq. Vali daftarlar sotib olishga bor pulining $\frac{1}{n}$ qismini, Ali esa bor pulining $\frac{1}{4}$ qismini sarfladi. Bunda Vali Ali sarflagandan 4000 so‘m ko‘proq sarflagan bo‘lsa, Ali birinchi va ikkinchi kunlari o‘zi bilan qancha pul olib chiqqanini toping.

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