

«O'ZBEKISTON TEMIR YO'LLARI» AJ

TOSHKENT TEMIR YO'L MUHANDISLARI INSTITUTI

Oliy matematika kafedrası

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Oliy matematika fanidan loyiha-hisob ishlari

(Sirtqi ta'lim shakli uchun)

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Ushbu uslubiy qo'llanma "Oliy matematika" fani bo'yicha loyiha-hisob ishlarini bajarishda foydalanishga mo'ljallangan. Uslubiy qo'llanmada oliy matematikaning kompleks sonlar, boshlang'ich funksiya, aniqmas integral, aniq integral, ko'p o'zgaruvchili funksiyalar, karrali integrallar kabi bo'limlari bo'yicha misol va masalalar yechib ko'rsatilgan hamda qisqacha nazariy ma'lumotlar keltirilgan.

Uslubiy qo'llanma Toshkent temir yo'l muhandislari institutining professor-o'qituvchilari, hamda barcha texnik ta'lim yo'nalishlarining sirtqi ta'lim shaklida ta'lim olayotgan talabalar foydalanishlari uchun mo'ljallangan.

Institut Ilmiy-uslubiy Kengashi tomonidan nashrga tavsiya etilgan.

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KIRISH

“Oliy matematika” fanini (sirtqi bo’lim talabalari uchun) o’qitishdan asosiy maqsad talabalarda mantiqiy, algoritmik, abstrakt fikrlashlarni va matematik tafakkurni shakllantirish hamda rivojlantirishdan iboratdir. Shuningdek o’zining fikr-muloxazalarini, xulosalarini asosli tarzda bayon eta olishni o’rgatish, iqtisodning nazariy va amaliy masalalarini yecha olishga yetarli matematik apparatni egallashni va uni qo’llashni, hamda iqtisodiy masalalarning matematik modelini tuzish va tahlil qilishni o’rgatishdan iboratdir. Shu maqsadlarni amalga oshirishda mustaqil ish topshiriqlari, loyiha-hisob ishlari va ularni bajarish muhim o’rin egallaydi. Ushbu uslubiy qo’llanmada har bir mavzu bo’yicha nazorat topshiriqlari berilgan bo’lib, berilgan topshiriqlarni bajarish usullari tushunarli qilib bayon etilgan.

Mazkur uslubiy qo’llanma oliy matematika fanining kompleks sonlar, boshlang’ich funksiya, aniqmas integral, aniq integral, ko’p o’zgaruvchili funksiyalar, karrali integrallar kabi bo’limlarini o’z ichiga olgan. Har bir bobning boshida zarur nazariy tushunchalar va kerakli formulalar beriladi. Asosiy e’tibor nazorat topshiriqlaridagi misol va masalalarning yechilishiga qaratiladi.

1-LOYIHA-HISOB ISHI

1 – topshiriq

Mavzu: Kompleks sonlar va ular ustida amallar

Nazariy tushunchalar. Haqiqiy x, y sonlar yordamida tuzilgan $z = x + iy$ son kompleks son $i = \sqrt{-1}$ esa mavhum birlik deyiladi. Bunda x kompleks sonning haqiqiy qismi, y esa mavhum qismi deyiladi.

$z = x + iy$ yozuv kompleks sonning algebraik ifodasi deyiladi.

Agar $z_1 = x_1 + iy_1$ va $z_2 = x_2 + iy_2$ kompleks sonlar berilgan bo'lsa, ular ustida algebraik amallar quyidagicha kiritiladi:

$$z_1 \pm z_2 = (x_1 \pm x_2) + i(y_1 \pm y_2),$$

$$z_1 \cdot z_2 = (x_1x_2 - y_1y_2) + i(x_1y_2 + x_2y_1),$$

$$\frac{z_1}{z_2} = \frac{x_1 + iy_1}{x_2 + iy_2} = \frac{x_1x_2 + y_1y_2}{x_2^2 + y_2^2} + i \frac{x_2y_1 - x_1y_2}{x_2^2 + y_2^2}.$$

$z = x + iy$ uchun $\bar{z} = x - iy$ qo'shma kompleks son deyiladi. $z = x + iy$ songa tekislikdagi $A(x, y)$ nuqtani mos qo'yish mumkin, $x = r \cos \varphi$, $y = r \sin \varphi$.

$|OA| = r = \sqrt{x^2 + y^2}$ haqiqiy son kompleks sonning moduli, OA vektorni Ox o'qining musbat yo'nalishi bilan hosil qilgan φ burchagi esa kompleks sonning argumenti deyiladi.

$x = r \cos \varphi$, $y = r \sin \varphi$ ekanligidan $z = x + yi = r(\cos \varphi + i \sin \varphi)$ kelib chiqadi.

Bu yozuv kompleks sonning trigonometrik shakli deyiladi.

Trigonometrik shakldagi kompleks ustidagi amallar quyidagi xossalarga ega:

$$z_1 \cdot z_2 = r_1(\cos \varphi_1 + i \sin \varphi_1) \cdot r_2(\cos \varphi_2 + i \sin \varphi_2) = r_1 r_2 [\cos(\varphi_1 + \varphi_2) + i \sin(\varphi_1 + \varphi_2)],$$

$$\frac{z_1}{z_2} = \frac{r_1}{r_2} [\cos(\varphi_1 - \varphi_2) + i \sin(\varphi_1 - \varphi_2)],$$

$$z^n = [r(\cos \varphi + i \sin \varphi)]^n = r^n (\cos n\varphi + i \sin n\varphi),$$

$$\sqrt[n]{r(\cos \varphi + i \sin \varphi)} = \sqrt[n]{r} \left(\cos \frac{\varphi + 2k\pi}{n} + i \sin \frac{\varphi + 2k\pi}{n} \right), k = 0, 1, \dots, (n-1)$$

Oxirgi ikki formula Muavr formulalari deyiladi.

$e^{i\phi} = \cos\phi + i \sin\phi$ tenglik o'rinli bo'lib, Eyler formulasi deyiladi.

Mavzuga doir misollarni yechishga namunalar

1-misol. $2+i$ ni $2-i$ ga bo'ling.

Yechish. $\frac{2+i}{2-i} = \frac{(2+i)^2}{(2-i)(2+i)} = \frac{4+4i+i^2}{4-i^2} = \frac{3+4i}{5} = \frac{3}{5} + \frac{4}{5}i$, bunda $i^2 = -1$.

2-misol. $\alpha = (2+3i)$, $\beta = (4+i)$ sonlarni ko'paytiring:

Yechish. $(2+3i)(4+i) = 8+2i+12i+3i^2 = 5+14i$.

3-misol. $3+3i$ kompleks sonni trigonometrik shaklga keltiring.

Yechish. $a=3$, $b=3$, $r = \sqrt{a^2+b^2} = \sqrt{3^2+3^2} = \sqrt{18} = 3\sqrt{2}$, $\operatorname{tg}\varphi = \frac{b}{a} = \frac{3}{3} = 1$, (1,1)

nuqta birinchi chorakka tegishli bo'lganligi sababli $\varphi = \frac{\pi}{4}$ bo'ladi. Demak,

$$3+3i = 3\sqrt{2} \left(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4} \right),$$

4-misol. Berilgan $\alpha = -1+i\sqrt{3}$ sonni trigonometrik shaklga keltiring.

Yechish. $r = \sqrt{1+3} = 2$, $\operatorname{tg}\varphi = \frac{b}{a} = -\sqrt{3}$, berilgan α kompleks songa mos

$(-1; \sqrt{3})$ nuqta ikkinchi chorakka tegishli bo'lganligi sababli $\varphi = \frac{2\pi}{3}$, demak,

$$-1+i\sqrt{3} = 2 \left(\cos \frac{2\pi}{3} + i \sin \frac{2\pi}{3} \right).$$

5-misol. Berilgan kompleks sonlarni ko'paytiring:

$\alpha = 3(\cos 15^\circ + i \sin 15^\circ)$, $\beta = 2(\cos 25^\circ + i \sin 25^\circ)$ va $\gamma = 5(\cos 20^\circ + i \sin 20^\circ)$.

Yechish.

$$\begin{aligned} \alpha \cdot \beta \cdot \gamma &= 3 \cdot 2 \cdot 5 (\cos(15^\circ + 25^\circ + 20^\circ) + i \sin(15^\circ + 25^\circ + 20^\circ)) = 30 (\cos 60^\circ + i \sin 60^\circ) = \\ &= 30 \left(\frac{1}{2} + i \frac{\sqrt{3}}{2} \right) = 15 + i15\sqrt{3}. \end{aligned}$$

6-misol. Quyidagi ildizni hisoblang: $\sqrt[3]{i}$.

Yechish. 1) Avval i ni trigonometrik shaklga keltiramiz. $r=1$ va $\varphi = \frac{\pi}{2}$.

Demak, $i = \cos \frac{\pi}{2} + i \sin \frac{\pi}{2}$. Endi Muavr formulaga ko'ra

$$\beta_k = \sqrt[3]{i} = \cos \frac{\frac{\pi}{2} + 2k\pi}{3} + i \sin \frac{\frac{\pi}{2} + 2k\pi}{3}.$$

Bu misolda $k=0,1,2$. Bundan,

$$\beta_0 = \cos \frac{\frac{\pi}{2} + 2 \cdot 0 \cdot \pi}{3} + i \sin \frac{\frac{\pi}{2} + 2 \cdot 0 \cdot \pi}{3} = \cos \frac{\pi}{6} + i \sin \frac{\pi}{6} = \frac{\sqrt{3}}{2} + \frac{1}{2}i,$$

$$\beta_1 = \cos \frac{\frac{\pi}{2} + 2 \cdot 1 \cdot \pi}{3} + i \sin \frac{\frac{\pi}{2} + 2 \cdot 1 \cdot \pi}{3} = \cos \frac{5\pi}{6} + i \sin \frac{5\pi}{6} = -\frac{\sqrt{3}}{2} + \frac{1}{2}i,$$

$$\beta_2 = \cos \frac{\frac{\pi}{2} + 2 \cdot 2 \cdot \pi}{3} + i \sin \frac{\frac{\pi}{2} + 2 \cdot 2 \cdot \pi}{3} = \cos \frac{3\pi}{2} + i \sin \frac{3\pi}{2} = -i.$$

Nazariy savollar va mashqlar.

1. Algebraik ko'rinishdagi kompleks sonning ta'rifini bering.
2. Algebraik ko'rinishda berilgan kompleks sonlar qanday qilib qo'shiladi, ayriladi, ko'paytiriladi va bo'linadi?
3. Kompleks sonning geometrik tasviri qanday topiladi?
4. Ayniyatni isbotlang: $|z_1 + z_2|^2 + |z_1 - z_2|^2 = 2(|z_1|^2 + |z_2|^2)$
5. Yuqoridagi ayniyat qanday geometrik talqinga ega?
6. Kompleks sonning moduli hamda argumenti deb nimaga aytiladi?
7. Kompleks sonni trigonometrik ko'rinishda yozing. Haqiqiy sonni qanday qilib trigonometrik ko'rinishda yozsa bo'ladi?
8. Muavr formulasini trigonometrik va ko'rsatkichli ko'rinishlarda yozing.
9. Kompleks sonning trigonometrik va ko'rsatkichli ko'rinishlari orasidagi bog'lanishni yozing.
10. Trigonometrik ko'rinishda berilgan kompleks sonlar ustida ko'paytirish, bo'lish, darajaga ko'tarish va ildiz chiqarish amallari qanday bajariladi?

11. Qo'shma kompleks son deb nimaga aytiladi? Quydagilarni isbotlang:

$$1) \overline{z_1 + z_2} = \overline{z_1} + \overline{z_2}$$

$$2) \overline{z_1 \cdot z_2} = \overline{z_1} \cdot \overline{z_2}$$

12. $\sin 4\alpha$ va $\cos 4\alpha$ larni Muavr formulasidan foydalanib α burchak funksiyalari orqali ifodalang.

Loyiha-hisob ishi topshiriqlari

1 – masala. Berilgan kompleks son z ga ko'ra quydagilarni toping:

- a) uning algebraik ko'rinishini;
- b) uning trigonometrik ko'rinishini;
- v) uning ko'rsatkichli ko'rinishini;
- g) uning geometrik tasvirini;
- d) $\sqrt[3]{z}$ ni.

$$1.1.1. \quad z = \frac{16}{\sqrt{3} + i}$$

$$1.1.2. \quad z = \frac{-2\sqrt{2}}{1+i}$$

$$1.1.30. \quad z = \frac{-54}{1+\sqrt{3}i}$$

$$1.1.3. \quad z = \frac{16}{1+\sqrt{3}i}$$

$$1.1.4. \quad z = \frac{-\sqrt{2}}{-1+i}$$

$$1.1.5. \quad z = \frac{16}{-\sqrt{3}+i}$$

$$1.1.6. \quad z = \frac{2\sqrt{2}}{1-i}$$

$$1.1.7. \quad z = \frac{54}{-1+\sqrt{3}i}$$

$$1.1.8. \quad z = \frac{16}{-1-\sqrt{3}i}$$

$$1.1.9. \quad z = \frac{16}{\sqrt{3}-i}$$

$$1.1.10. \quad z = \frac{2\sqrt{2}}{-1-i}$$

$$1.1.11. \quad z = \frac{-\sqrt{2}}{1-i}$$

$$1.1.12. \quad z = \frac{16}{-1+i}$$

$$1.1.13. \quad z = \frac{16}{-\sqrt{3}-i}$$

$$1.1.14. \quad z = \frac{-54}{1-\sqrt{3}i}$$

$$1.1.15. \quad z = \frac{4}{\sqrt{3}+i}$$

$$1.1.16. \quad z = \frac{\sqrt{2}}{1-i}$$

$$1.17. \quad z = \frac{2\sqrt{2}}{1+i}$$

$$1.1.18. \quad z = \frac{54}{1+\sqrt{3}i}$$

$$1.1.19. \quad z = \frac{54}{1-\sqrt{3}i}$$

$$1.1.20. \quad z = \frac{-16}{\sqrt{3}+i}$$

$$1.1.21. \quad z = \frac{2\sqrt{2}}{1-i}$$

$$1.1.22. \quad z = \frac{4}{-\sqrt{3}+i}$$

$$1.1.23. \quad z = \frac{\sqrt{2}}{1+i}$$

$$1.1.24. \quad z = \frac{4}{\sqrt{3}-i}$$

$$1.1.25. \quad z = \frac{-16}{\sqrt{3}-i}$$

$$1.1.26. \quad z = \frac{-54}{-1+\sqrt{3}i}$$

$$1.1.27. \quad z = \frac{-16}{-\sqrt{3}+i}$$

$$1.1.28. \quad z = \frac{16}{-1-\sqrt{3}i}$$

$$1.1.29. \quad z = \frac{2\sqrt{2}}{1+i}$$

2 – topshiriq

Mavzu: Aniqmas integral

Nazariy tushunchalar. Agar $(a; b)$ oraliqning ixtiyoriy nuqtasida

$$F'(x) = f(x)$$

tenglik o'rinli bo'lsa, u holda $F(x)$ funksiya $f(x)$ funksiyaning qaralayotgan oraliqdagi boshlang'ich funksiyasi deyiladi.

Agar $F(x)$ funksiya $f(x)$ funksiyaning boshlang'ich funksiyasi bo'lsa, u holda ixtiyoriy o'zgarmas son C uchun $F(x) + C$ ifoda qaralayotgan funksiyaning aniqmas integrali deyiladi va quyidagicha yoziladi:

$$\int f(x)dx = F(x) + C.$$

Bu yerda \int integral belgisi, $f(x)$ integral ostidagi funksiya, $f(x)dx$ integral ostidagi ifoda, x integrallash o'zgaruvchisi deb yuritiladi.

Sodda formulalar quyidagilardan iborat:

1. $\int dx = x + C;$

7. $\int \cos x = \sin x + C;$

2. $\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1} + C, (\alpha \neq -1);$

8. $\int \frac{dx}{\cos^2 x} = \operatorname{tg}x + C;$

3. $\int \frac{dx}{x} = \ln|x| + C;$

9. $\int \frac{dx}{\sin^2 x} = -\operatorname{ctg}x + C;$

4. $\int e^x dx = e^x + C;$

10. $\int \frac{dx}{\sqrt{1-x^2}} = \arcsin x + C;$

5. $\int a^x dx = \frac{a^x}{\ln a} + C;$

11. $\int \frac{dx}{1+x^2} = \operatorname{arctg}x + C.$

6. $\int \sin x = -\cos x + C;$

Integrallash qoidalari quyidagilardan iborat:

$$\int C \cdot u(x)dx = C \cdot \int u(x)dx,$$

$$\int (u(x) \pm v(x))dx = \int u(x)dx \pm \int v(x)dx.$$

Agar $\int f(x)dx = F(x) + C$ ma'lum bo'lsa, u holda

$$\int f(ax+b)dx = \frac{1}{a} F(ax+b) + C$$

o'rinli bo'ladi.

O'zgaruvchini almashtirib integrallash usuli. Ko'pincha $\int f(x)dx$ integralni bevosita integrallab bo'lmaydi. Bunday hollarda $x = \varphi(t)$ almashtirish yordamida uni $\int f(\varphi(t))\varphi'(t)dt$ ko'rinishda yozib, so'ngra integrallash maqsadga muvofiq bo'ladi.

Ushbu

$$\int u dv = u \cdot v - \int v du$$

ifodaga aniqmas integralni bo'laklab integrallash formulasi deyiladi.

Ushbu

I. $\frac{A}{x-a},$

II. $\frac{A}{(x-a)^k} \quad (k \geq 2),$

III. $\frac{Ax+B}{x^2+px+q} \quad (D < 0),$

IV. $\frac{Ax+B}{(x^2+px+q)^k} \quad (D < 0, k \geq 2)$

ko'rinishdagi funksiyalar eng sodda ratsional kasrlar deyiladi. Ular to'g'ridan-to'g'ri yoki maxrajdagi kvadrat uchhaddan to'la kvadrat ajratish bilan yoki rekkurent formulalardan foydalanib, integrallanadi.

Ushbu $\frac{P_m(x)}{Q_n(x)} \quad (m < n)$ ko'rinishdagi to'g'ri ratsional kasrni integrallash

uchun uning maxrajidagi ko'phadni ko'paytuvchilarga ajratib, so'ngra kasrni sodda ratsional kasrlarga yoyib, hosil bo'lgan koeffitsiyentlarni no'malum koeffitsiyentlar usuli bilan topib, hosil qilingan eng sodda ratsional kasrlarni integrallash kifoya.

Ba'zi trigonometrik funksiyalar qatnashgan ifodalarni va irratsional funksiyalarni integrallashning bir necha hollari mavjud bo'lib, ularga misollar yechishda to'xtalib o'tamiz.

Mavzuga doir misollarni yechishga namunalari

1-misol. $F(x) = \arcsin x$ funksiya $f(x) = \frac{1}{\sqrt{1-x^2}}$ funksiyaning $(-1;1)$

oralisdagi boshlang'ich funksiyasi bo'ladi, chunki

$$F'(x) = (\arcsin x)' = \frac{1}{\sqrt{1-x^2}}, \quad \forall x \in (-1;1)$$

2-misol. $F(x) = \frac{a^x}{\ln a}$, $a \in (0;1) \cup (1;\infty)$ funksiya $f(x) = a^x$ funksiyaning

oralisdagi boshlang'ich funksiyasi bo'ladi, chunki

$$F'(x) = \left(\frac{a^x}{\ln a} \right)' = \frac{a^x \ln a}{\ln a} = a^x, \quad \forall x \in (-\infty;\infty)$$

3-misol. $\int (2x^3 - 5x^2 + 7x - 3) dx$ hisoblansin.

Yechish. Bu integral, integral xossalaridan foydalanib quyidagicha integrallanadi:

$$\int (2x^3 - 5x^2 + 7x - 3) dx = 2 \int x^3 dx - 5 \int x^2 dx + 7 \int x dx - 3 \int dx$$

Dastlabki 3ta integral, integrallash jadvalidagi $\int x^n dx = \frac{x^{n+1}}{n+1} + C$ va so'ngi

integral $\int dx = x + C$ formulalar bilan hisoblanadi.

$$\int (2x^3 - 5x^2 + 7x - 3) dx = 2 \cdot \frac{x^4}{4} - 5 \cdot \frac{x^3}{3} + 7 \cdot \frac{x^2}{2} - 3x + C = \frac{x^4}{2} - \frac{5x^3}{3} + \frac{7x^2}{2} - 3x + C.$$

4-misol. $\int \frac{\sqrt{x} + \sqrt[3]{x^2} + 1}{x^3} dx$ hisoblansin.

Yechish. Bu integral quyidagicha hisoblanadi:

$$\int \frac{\sqrt{x} + \sqrt[3]{x^2} + 1}{x^3} dx = \int \left(\frac{\sqrt{x}}{x^3} + \frac{\sqrt[3]{x^2}}{x^3} + \frac{1}{x^3} \right) dx = \int \left(x^{-\frac{5}{2}} + x^{-\frac{7}{3}} + x^{-3} \right) dx =$$

$$= \int x^{-\frac{5}{2}} dx + \int x^{-\frac{7}{3}} dx + \int x^{-3} dx = \frac{x^{-\frac{5}{2}+1}}{-\frac{5}{2}+1} + \frac{x^{-\frac{7}{3}+1}}{-\frac{7}{3}+1} + \frac{x^{-3+1}}{-3+1} + C =$$

$$= -\frac{2}{3\sqrt{x^3}} - \frac{3}{4\sqrt[3]{x^4}} - \frac{1}{2x^2} + C.$$

5-misol. $\int \left(\sqrt{x^3} - \frac{1}{\sqrt{x^3}} \right)^2 dx$ aniqmas integralni toping.

Yechish. Bu integral quyidagicha hisoblanadi:

$$\int \left(\sqrt{x^3} - \frac{1}{\sqrt{x^3}} \right)^2 dx = \int \left(x^3 - 2 + \frac{1}{x^3} \right) dx = \int x^3 dx - 2 \int dx + \int \frac{dx}{x^3} = \frac{x^4}{4} - 2x - \frac{1}{2x^2} + C.$$

6-misol. $\int \frac{(1+x^2)}{x^3+x} dx$ integralni hisoblang.

Yechish. Bu integral quyidagicha hisoblanadi:

$$\int \frac{(1+x)^2}{x^3+x} dx = \int \frac{1+2x+x^2}{x(1+x^2)} dx = \int \frac{(1+x^2)+2x}{x(1+x^2)} dx = \int \left(\frac{1}{x} + \frac{2}{1+x^2} \right) dx =$$

$$= \int \frac{dx}{x} + 2 \int \frac{dx}{1+x^2} = \ln|x| + 2 \arctg x + C$$

Nazariy savollar va mashqlar.

1. Boshlang'ich funksiya deb nimaga aytiladi? Funktsiyaning ixtiyoriy ikkita boshlang'ich funktsiyalari bir-biridan o'zgarmas songa farq qilinishini isbotlang.
2. Aniqmas integral deb nimaga aytiladi? Aniqmas integral qanday hossalarga ega?
3. $y = \int x dx$ chiziqlar oilasidan $A(2;1)$, $B(2;2)$, $C(2;3)$ nuqtalardan o'tuvchilarini yasang.

4. Quydagi integrallarni hisoblash uchun qulay belgilashni ko'rsating:

$$\int \frac{\sqrt{1+\ln x}}{x} dx, \quad \int e^{\cos^2 x} \cdot \sin 2x dx, \quad \int \frac{e^{\arctg x}}{1+x^2} dx,$$

$$\int x^3 \sqrt{1+4x^6} dx, \quad \int \cos x \cdot \cos(\sin x) dx, \quad \int \frac{dx}{x\sqrt{x^2+1}}.$$

5. Bo'laklab integrallash formulasini keltirib chiqaring.

6. Quydagi integrallarni hisoblash uchun qulay belgilashni ko'rsating:

$$\int x \arctg x dx, \quad \int \frac{dx}{x \ln x}, \quad \int \frac{x^3}{\sqrt{4-x^2}} dx,$$

$$\int \frac{\arcsin x}{x^2} dx, \quad \int \sqrt[7]{\sin^3 x} \cdot \cos^3 x dx, \quad \int \cos x \cdot \ln(\sin x) dx, \quad \int x^2 e^x dx.$$

7. Ratsional kasrlarni integrallash usullari nimadan iborat?

8. Hisoblang va tushuntiring:

$$\int \arcsin x dx + \int \arccos x dx.$$

9. Ushbu $\int R(\sin x, \cos x) dx$ ko'rinishdagi integrallarni hisoblash usulini bayon qiling. Bu erda R – ratsional operator.

10. Ushbu

$$\text{a) } \int R(x, \sqrt{a^2 - x^2}) dx \quad \text{b) } \int R(x, \sqrt{a^2 + x^2}) dx \quad \text{v) } \int R(x, \sqrt{x^2 - a^2}) dx$$

ko'rinishdagi integrallarda qanday almashtirishlar bajarilsa maqsadga muvofiq bo'ladi?

Loyiha-hisob ishi topshiriqlari

1 – masala. Aniqmas integrallarni toping va differensiallash yordamida tekshirib ko'ring.

$$2.1.1. \quad \text{a) } \int \frac{\cos 5x}{\sqrt[3]{\sin^2 5x}} dx; \quad \text{b) } \int (3x+4)e^{3x} dx; \quad \text{v) } \int \frac{x+5}{2x^2+2x+3} dx.$$

$$2.1.2. \quad \text{a) } \int e^{\cos 3x} \sin 3x dx; \quad \text{b) } \int \arctg \sqrt{4x-1} dx; \quad \text{v) } \int \frac{x-7}{x^2+4x+13} dx.$$

$$2.1.3. \quad \text{a) } \int \frac{\sin 7x}{3-5\cos 7x} dx; \quad \text{b) } \int (4-3x)e^{-3x} dx; \quad \text{v) } \int \frac{2x-3}{\sqrt{x^2-4x-7}} dx.$$

2. 1. 4. a) $\int \frac{dx}{\cos^2 x(2\operatorname{tg}x+1)}$;	b) $\int \ln(x^2+4)dx$;	v) $\int \frac{x-2}{\sqrt{x^2+4x-3}} dx$.
2. 1. 5. a) $\int \frac{x+\operatorname{arctg}x}{1+x^2} dx$;	b) $\int (1-6x)e^{2x} dx$;	v) $\int \frac{x-3}{\sqrt{3-2x-x^2}} dx$.
2. 1. 6. a) $\int \sqrt{1+e^{5x}} e^{5x} dx$;	b) $\int (4-16x)\sin 4x dx$;	v) $\int \frac{x+1}{4x^2-12x+13} dx$.
2. 1. 7. a) $\int \frac{(\arccos x)^3-1}{\sqrt{1-x^2}} dx$;	b) $\int \sqrt{x} \ln x dx$;	v) $\int \frac{x+2}{\sqrt{3-x^2+2x}} dx$.
2. 1. 8. a) $\int \operatorname{tg}x \ln \cos x dx$;	b) $\int (4x-3)e^{-2x} dx$;	v) $\int \frac{x}{\sqrt{8-2x-x^2}} dx$.
2. 1. 9. a) $\int \frac{\sin x - \cos x}{(\cos x + \sin x)^5} dx$;	b) $\int \operatorname{arctg} \frac{1}{x} dx$;	v) $\int \frac{3x-2}{5x^2-3x+2} dx$.
2. 1. 10. a) $\int \frac{\operatorname{tg}(x+1)}{\cos^2(x+1)} dx$;	b) $\int (4x-2)\cos 2x dx$;	v) $\int \frac{2-x}{\sqrt{1-2x-x^2}} dx$.
2. 1. 11. a) $\int \frac{1-\cos x}{(x-\sin x)^2} dx$;	b) $\int \ln(4x^2+1)dx$;	v) $\int \frac{6-8x}{2x^2-3x+1} dx$.
2. 1. 12. a) $\int \frac{1-\ln(x-1)}{x-1} dx$;	b) $\int x \cos 5x dx$;	v) $\int \frac{2-5x}{\sqrt{4x^2+9x+1}} dx$.
2. 1. 13. a) $\int \frac{8x-\operatorname{arctg} 2x}{1+4x^2} dx$;	b) $\int (5x-2)e^{3x} dx$;	v) $\int \frac{3x-6}{x^2-7x+12} dx$.
2. 1. 14. a) $\int \frac{x+\cos x}{x^2+2\sin x} dx$;	b) $\int \frac{\arcsin x}{\sqrt{1+x}} dx$;	v) $\int \frac{5x-15}{\sqrt{3-2x-x^2}} dx$.
2. 1. 15. a) $\int \frac{dx}{x\sqrt{4-\ln^2 x}}$;	b) $\int \frac{x \cos x}{\sin^3 x} dx$;	v) $\int \frac{3x-2}{x^2+2x+5} dx$.
2. 1. 16. a) $\int \frac{\sin 4x}{\sqrt[3]{\cos^2 4x}} dx$;	b) $\int \frac{\ln x}{x^2} dx$;	v) $\int \frac{x}{\sqrt{x^2-3x+2}} dx$.
2. 1. 17. a) $\int \frac{x-(\operatorname{arctg}x)^4}{1+x^2} dx$;	b) $\int (2-3x)e^{-3x} dx$;	v) $\int \frac{x+1}{4x^2-4x-11} dx$.
2. 1. 18. a) $\int \frac{\sqrt{1+\ln x}}{x} dx$;	b) $\int \operatorname{arctg} \sqrt{6x-1} dx$;	v) $\int \frac{4x-7}{\sqrt{5+2x-x^2}} dx$.
2. 1. 19. a) $\int \frac{3+x}{\sqrt{2-x^2}} dx$;	b) $\int (x+5)\sin 3x dx$;	v) $\int \frac{3-4x}{4x^2-6x+2} dx$.

2. 1. 20. a) $\int \frac{(\arcsin x)^2 + 1}{\sqrt{1-x^2}} dx$; b) $\int (x+2)\ln x dx$; v) $\int \frac{x}{\sqrt{5-2x+x^2}} dx$.

2. 1. 21. a) $\int \frac{x + \frac{1}{x}}{\sqrt{x^2+1}} dx$; b) $\int (x+1)e^{5x} dx$; v) $\int \frac{3x-1}{9x^2+6x+5} dx$.

2. 1. 22. a) $\int \frac{x^2+1}{(x^3+3x+1)^5} dx$; b) $\int \arctg \sqrt{3x-1} dx$; v) $\int \frac{1-x}{\sqrt{5+4x-x^2}} dx$.

2. 1. 23. a) $\int (1+e^{2x})^5 e^{2x} dx$; b) $\int (x+1)\cos 3x dx$; v) $\int \frac{5x+2}{x^2-4x+8} dx$.

2. 1. 24. a) $\int \frac{\cos 3x}{3-5\sin 3x} dx$; b) $\int (x-1)\ln x dx$; v) $\int \frac{x}{\sqrt{x^2+4x+5}} dx$.

2. 1. 25. a) $\int \frac{x^2 + \ln x^2}{x} dx$; b) $\int (2x+3)e^{-x} dx$; v) $\int \frac{x+1}{\sqrt{12x-9x^2-3}} dx$.

2. 1. 26. a) $\int \frac{x}{\sqrt{x^4+x^2+1}} dx$; b) $\int \frac{x}{\cos^2 x} dx$; v) $\int \frac{2x+3}{9x^2-6x+5} dx$.

2. 1. 27. a) $\int \frac{2\cos x + 3\sin x}{(2\sin x - 3\cos x)^3} dx$; b) $\int \arctg \sqrt{2x-1} dx$; v) $\int \frac{x}{9x^2-6x-8} dx$.

2. 1. 28. a) $\int \frac{4\arctg 3x-x}{1+9x^2} dx$; b) $\int (2-x)\sin 2x dx$; v) $\int \frac{x+1}{\sqrt{24-x^2-2x}} dx$.

2. 1. 29. a) $\int e^{\sin 2x} \cos 2x dx$; b) $\int \frac{\ln x}{x^3} dx$; v) $\int \frac{x}{25x^2-10x+17} dx$.

2. 1. 30. a) $\int \frac{dx}{\sin^2 x(3\ctg x+2)}$; b) $\int \frac{x}{\sin^2 x} dx$; v) $\int \frac{x-2}{\sqrt{x^2+3x-1}} dx$.

2 – masala. Aniqmas integralni toping.

2. 2. 1. a) $\int \frac{x+2}{x^3-2x^2+2x} dx$; b) $\int \frac{1+\sqrt[3]{x-1}}{\sqrt{x-1}} dx$; v) $\int \frac{dx}{5+3\cos x}$.

2. 2. 2. a) $\int \frac{x^5+x^4-8}{x^3-4x} dx$; b) $\int \frac{dx}{\sqrt{x-2}(1+\sqrt[3]{x-2})}$; v) $\int \sin^3 x \cos^2 x dx$.

2. 2. 3. a) $\int \frac{x+3}{x^3+x^2-2x} dx$; b) $\int \frac{\sqrt[4]{x+1}}{(\sqrt{x+4})\sqrt[4]{x^3}} dx$; v) $\int \cos^3 x \sin^4 x dx$.

2. 2. 4. a) $\int \frac{x+5}{x^4+2x^3+x^2} dx$; b) $\int \frac{dx}{4\sqrt{x}+\sqrt{x^3}}$; v) $\int \tg^4 3x dx$.

2. 2. 5. a) $\int \frac{3x^2 + 2x + 1}{(x+1)^2(x^2+1)} dx$; b) $\int \frac{dx}{\sqrt{x+3} + \sqrt[3]{x+3}}$; v) $\int \operatorname{ctg}^4 5x dx$.

2. 2. 6. a) $\int \frac{2x^3 - 6x^2 + 8x - 1}{(x-1)^2(x^2+2)} dx$; b) $\int \frac{x+1}{\sqrt[3]{2x+1}} dx$; v) $\int \frac{\cos x}{1 + \cos x} dx$.

2. 2. 7. a) $\int \frac{x^4}{x^4 + 5x^2 + 4} dx$; b) $\int \frac{dx}{\sqrt{x}(\sqrt[3]{x+1})}$; v) $\int \sin^5 x \cos^2 x dx$.

2. 2. 8. a) $\int \frac{x^2 + 1}{x^3 - x^2 + 4x - 4} dx$; b) $\int \frac{dx}{\sqrt[4]{x} + \sqrt{x}}$; v) $\int \cos^2 3x \sin^2 3x dx$.

2. 2. 9. a) $\int \frac{x+1}{x^3 - 2x^2 + 3x - 6} dx$; b) $\int \frac{dx}{\sqrt[4]{x} + \sqrt{x}}$; v) $\int \frac{dx}{(1 + \cos x)^2}$.

2. 2. 10. a) $\int \frac{x^3 + 1}{x^4 + 3x^2 + 2} dx$; b) $\int \frac{dx}{\sqrt{x+1}(1 + \sqrt[3]{x+1})}$; v) $\int \frac{dx}{\sin x + \cos x}$.

2. 2. 11. a) $\int \frac{2x^4 + 1}{x^3 + x^2 + 2x + 2} dx$; b) $\int \frac{dx}{1 + \sqrt{3x+1}}$; v) $\int \frac{dx}{5 + 4 \sin x}$.

2. 2. 12. a) $\int \frac{x^3 - 6}{x^4 - 6x^2 + 8} dx$; b) $\int \frac{dx}{\sqrt{x+2} + \sqrt[3]{x+2}}$; v) $\int \cos^3 x \sin^2 x dx$.

2. 2. 13. a) $\int \frac{dx}{x^3 + 8}$; b) $\int \frac{1 + \sqrt[3]{x+1}}{\sqrt{x+1}} dx$; v) $\int \frac{2 - \sin x}{2 + \cos x} dx$.

2. 2. 14. a) $\int \frac{x^2 + 2x + 4}{x^4 - 5x^2 + 4} dx$; b) $\int \frac{dx}{\sqrt{x-1}(1 + \sqrt[3]{x-1})}$; v) $\int \operatorname{tg}^5 3x dx$.

2. 2. 15. a) $\int \frac{x^2 - x + 1}{x^4 + 2x^2 - 3} dx$; b) $\int \frac{x}{1 + \sqrt{3x+1}} dx$; v) $\int \operatorname{ctg}^6 2x dx$.

2. 2. 16. a) $\int \frac{x^3 - 2x + 5}{x^4 - 1} dx$; b) $\int \frac{e^{2x}}{\sqrt[3]{e^x + 1}} dx$; v) $\int \frac{dx}{3 - 6 \sin x}$.

2. 2. 17. a) $\int \frac{x^6 + 2x^5 - 1}{x^3 + 2x^2 + 2x} dx$; b) $\int \frac{dx}{\sqrt[4]{x+1} + \sqrt{x+1}}$; v) $\int \sin^3 x \cos^3 x dx$.

2. 2. 18. a) $\int \frac{x^2}{x^4 - 81} dx$; b) $\int \frac{dx}{1 + \sqrt[3]{2x+1}}$; v) $\int \operatorname{ctg}^5 3x dx$.

2. 2. 19. a) $\int \frac{2x^2 - 3x + 1}{x^3 + 1} dx$; b) $\int \frac{4x + 5}{(x+2)^3} dx$; v) $\int \frac{dx}{5 - 4 \cos x}$.

2. 2. 20. a) $\int \frac{x^3 - x - 8}{x^4 - 4x^2} dx$; b) $\int \frac{e^{2x}}{\sqrt[4]{e^x + 1}} dx$; v) $\int \sin^4 x \cos^3 x dx$.

2. 2. 21. a) $\int \frac{3x-7}{x^3+4x^2+4x+16} dx$; b) $\int \frac{dx}{\sqrt[3]{(x+1)^2} - \sqrt[3]{x+1}}$; v) $\int \operatorname{tg}^6 2x dx$.

2. 2. 22. a) $\int \frac{x^5-x^3+4}{x^3+2x^2+3x+6} dx$; b) $\int \frac{3x+2}{x\sqrt{x-2}} dx$; v) $\int \sin^3 x \cos^4 x dx$.

2. 2. 23. a) $\int \frac{dx}{x^3+x^2+2x+2}$; b) $\int \frac{dx}{\sqrt{3x+5}+2}$; v) $\int \frac{dx}{2\sin x - \cos x}$.

2. 2. 24. a) $\int \frac{x^2}{x^4-16} dx$; b) $\int \frac{\sqrt{x}}{\sqrt[3]{x^2} - \sqrt{x}} dx$; v) $\int \frac{dx}{2\cos x + \sin x}$.

2. 2. 25. a) $\int \frac{2x^3-4x^2+16x-12}{(x-1)^2(x^2+4x+5)} dx$; b) $\int \frac{x^3}{\sqrt{x-2}} dx$; v) $\int \operatorname{tg}^7 3x dx$.

2. 2. 26. a) $\int \frac{x^3-6x^2+13x-6}{(x^2-4)(x-2)^2} dx$; b) $\int \frac{dx}{\sqrt{x+1} + \sqrt[5]{(x+1)^2}}$; v) $\int \cos^5 x \sin^2 x dx$.

2. 2. 27. a) $\int \frac{dx}{x^3+x^2+4x+4}$; b) $\int \frac{4x+2}{(x-3)^2} dx$; v) $\int \frac{dx}{\sin^3 2x}$.

2. 2. 28. a) $\int \frac{x^3+x+2}{(x+2)x^3} dx$; b) $\int \frac{\sqrt{x+1}}{x(x+2)} dx$; v) $\int \frac{dx}{3+\cos x+2\sin x}$.

2. 2. 29. a) $\int \frac{-3x^3+13x^2+13x+1}{(x-2)^2(x^2-x+1)} dx$; b) $\int \frac{dx}{\sqrt[3]{x+1}(\sqrt[3]{x+1}-1)}$; v) $\int \frac{dx}{2+\operatorname{tg} x}$.

2. 2. 30. a) $\int \frac{x^2}{x^3+5x^2+8x+4} dx$; b) $\int \frac{\sqrt{x+1}}{\sqrt[3]{(x+1)^2} + \sqrt{x+1}} dx$; v) $\int \frac{dx}{\cos^3 2x}$.

3 – topshiriq

Mavzu: Aniq integral

Nazariy tushunchalar. Agar $[a; b]$ kesmada aniqlangan $f(x)$ funksiya uchun ushbu $\lim_{\max \Delta x_i \rightarrow 0} \sum_{i=1}^n f(\xi_i) \cdot \Delta x_i = I$ ($\xi_i \in (x_i; x_{i+1})$ $\sum_{i=1}^n \Delta x_i = b - a$) chekli limit mavjud bo'lsa, u holda I soniga qaralayotgan funksiyaning qaralayotgan kesmadagi aniq integrali deyiladi va quyidagicha yoziladi:

$$I = \int_a^b f(x) dx,$$

bu yerda:

a – integralning quyi chegarasi;

b – integralning yuqori chegarasi.

Aniq integral quyidagi xossalarga ega:

1°. Aniq integral integrallash o'zgaruvchisiga bog'liq emas:

$$\int_a^b f(x)dx = \int_a^b f(y)dy = \int_a^b f(t)dt = \dots = \int_a^b f(z)dz.$$

2°. Agar intergallash chegaralarini almashtirsak, u holda integralning qiymati qarama-qarshisiga o'zgaradi.

$$\int_a^b f(x)dx = -\int_b^a f(x)dx.$$

3°. Agar integrallash chegaralari ustma-ust tushib qolsa, u holda integralning qiymati nolga teng bo'ladi.

$$\int_a^a f(x)dx = 0.$$

4°. O'zgarmas sonni aniq integral ishorasidan tashqariga chiqarish mumkin.

$$\int_a^b C \cdot f(x)dx = C \cdot \int_a^b f(x)dx, \quad C = const.$$

5°. Yig'indining aniq integrali qo'shiluvchilarning aniq integrallari yig'indisiga tengdir.

$$\int_a^b (f(x) \pm \varphi(x))dx = \int_a^b f(x)dx \pm \int_a^b \varphi(x)dx.$$

6°. Agar $[a;b]$ kesmada $f(x) \leq \varphi(x)$ shart bajarilsa, u holda

$$\int_a^b f(x)dx \leq \int_a^b \varphi(x)dx$$

o'rinli.

7°. Agar m va M sonlar $f(x)$ funksiyaning $[a;b]$ kesmadagi eng kichik va eng katta qiymatlari bo'lsa, u holda

$$m(b-a) \leq \int_a^b f(x)dx \leq M(b-a)$$

o'rinli.

8°. Agar $f(x)$ funksiya $[a;b]$ kesmada uzluksiz bo'lsa, u holda $(a;b)$ oraliqda shunday $x=c$ nuqta topiladiki, ushbu

$$\int_a^b f(x)dx = f(c) \cdot (b-a)$$

o'rinli bo'ladi.

9°. Uchta ixtiyoriy a, b, c sonlar uchun ushbu

$$\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$$

tenglik o'rinli.

Nyuton-Leybnits formulasi. Agar $F(x)$ funksiya biror uzluksiz $f(x)$ funksiyaning biror boshlang'ich funksiyasi bo'lsa, u holda

$$\int_a^b f(x)dx = F(b) - F(a)$$

formula o'rinlidir.

Aniq integralda o'zgaruvchini almashtirish usuli bilan hisoblash huddi aniqmas integralda o'zgaruvchini almashtirish usuli kabi bo'ladi, faqat o'zgaruvchini almashtirib, boshlang'ich funksiya topilgandan so'ng eski o'zgaruvchiga qaytmasdan, ya'ni integrallash chegaralari yordamida to'g'ridan-to'g'ri hisoblashlar bajarilaveradi.

Agar $u = u(x)$ va $v = v(x)$ differensiallanuvchi funksiyalar bo'lsa, u holda

$$\int_a^b u dv = (u \cdot v)|_a^b - \int_a^b v du$$

ifodaga aniq integralni bo'laklab integrallash formulasi deyiladi.

Ushbu $\int_a^\infty f(x)dx$, $\int_{-\infty}^b f(x)dx$, $\int_{-\infty}^\infty f(x)dx$ ko'rinishdagi integrallarni, cheksiz

oraliqdagi xosmas integrallar deyiladi. Ular quyidagicha aniqlanadi:

$$\int_a^{\infty} f(x)dx = \lim_{b \rightarrow \infty} \int_a^b f(x)dx,$$

$$\int_{-\infty}^b f(x)dx = \lim_{a \rightarrow -\infty} \int_a^b f(x)dx,$$

$$\int_{-\infty}^{\infty} f(x)dx = \lim_{a \rightarrow \infty} \int_{-a}^a f(x)dx.$$

Agar mos chekli limitlar mavjud bo'lsa, u holda mos xosmas integrallar yaqinlashuvchi deyiladi.

Agar $f(x)$ funksiya $[a;b)$ oraliqda aniqlangan va uzluksiz bo'lib, $x=b$ nuqtada aniqlanmagan yoki uzulishga ega bo'lsa, u holda quyidagi integralni

$$\int_a^b f(x)dx$$

chekli oraliqdagi xosmas integral deyiladi va u quyidagicha aniqlanadi:

$$\int_a^b f(x)dx = \lim_{\varepsilon \rightarrow 0} \int_a^{b-\varepsilon} f(x)dx.$$

Agar chekli limit mavjud bo'lsa, u holda xosmas integral yaqinlashuvchi deyiladi.

Mavzuga doir misollarni yechishga namunalar

1-misol. Quyida keltirilgan aniq integrallar Nyuton-Leybnits formulasi yordamida bevosita hisoblangan:

$$1) \int_0^1 \sqrt[3]{x^2} dx = \int_0^1 x^{\frac{2}{3}} dx = \frac{x^{\frac{5}{3}}}{\frac{5}{3}} \Big|_0^1 = \frac{3}{5}(1-0) = \frac{3}{5},$$

$$2) \int_{\ln 2}^{\ln 3} e^{2x} dx = \frac{1}{2} e^{2x} \Big|_{\ln 2}^{\ln 3} = \frac{1}{2} (e^{2\ln 3} - e^{2\ln 2}) = \frac{1}{2} (9 - 4) = 2,5,$$

$$3) \int_0^{\frac{\pi}{2}} \sin x dx = -\cos x \Big|_0^{\frac{\pi}{2}} = -\left(\cos \frac{\pi}{2} - \cos 0 \right) = 1.$$

2-misol. $\int_0^1 \sqrt{1-x^2} dx$ integralni hisoblang.

Yechish. Bu integralni hisoblash uchun o'zgaruvchini almashtirish usulidan foydalanamiz.

$$\int_0^1 \sqrt{1-x^2} dx \Rightarrow \left. \begin{array}{l} x = \sin t \\ dx = \cos t dt \\ x=0, t=0 \\ x=1, t=\frac{\pi}{2} \end{array} \right\} \Rightarrow \int_0^{\frac{\pi}{2}} \sqrt{1-\sin^2 t} \cos t dt = \int_0^{\frac{\pi}{2}} \cos^2 t dt = \frac{1}{2} \int_0^{\frac{\pi}{2}} (1 + \cos 2t) dt =$$

$$= \frac{1}{2} \left(t + \frac{1}{2} \sin 2t \right) \Big|_0^{\frac{\pi}{2}} = \frac{\pi}{4}.$$

3-misol. $\int_1^{\sqrt{3}} \operatorname{arctg} x dx$ integralni hisoblang.

Hisoblash. Bu integralni hisoblash uchun bo'laklab integrallash usulidan foydalanamiz.

$$\int_1^{\sqrt{3}} \operatorname{arctg} x dx \Rightarrow \left. \begin{array}{l} u = \operatorname{arctg} x, \quad du = \frac{dx}{1+x^2} \\ dv = dx, \quad v = x. \end{array} \right\} \Rightarrow (x \cdot \operatorname{arctg} x) \Big|_1^{\sqrt{3}} - \int_1^{\sqrt{3}} x \cdot \frac{dx}{1+x^2} = (x \cdot \operatorname{arctg} x) \Big|_1^{\sqrt{3}} -$$

$$- \frac{1}{2} \int_1^{\sqrt{3}} \frac{d(1+x^2)}{1+x^2} = (x \cdot \operatorname{arctg} x) \Big|_1^{\sqrt{3}} - \frac{1}{2} \ln |1+x^2| \Big|_1^{\sqrt{3}} = \sqrt{3} \operatorname{arctg} \sqrt{3} - \operatorname{arctg} 1 - \frac{1}{2} \ln 4 + \frac{1}{2} \ln 2 =$$

$$= \frac{\sqrt{3}\pi}{3} - \frac{\pi}{4} - \frac{1}{2} \ln 2.$$

4-misol. $\int_0^2 \frac{dx}{(x-1)^2}$ integralni hisoblang.

Hisoblash. Agar integral ostidagi funksiyani $x=1$ nuqtada uzilishga ega ekanligi e'tiborga olmasdan, to'g'ridan-to'g'ri hisoblashga o'tilsa, quyidagi hosil bo'ladi:

$$\int_0^2 \frac{dx}{(x-1)^2} = -\frac{1}{x-1} \Big|_0^2 = -1 - 1 = -2.$$

Ammo, bu mumkin emas, chunki integralning chekli oraliqdagi xosmas integralligini e'tiborga olinsa, quyidagiga ega bo'lamiz:

$$\int_0^2 \frac{dx}{(x-1)^2} = \lim_{\varepsilon \rightarrow 0} \int_0^{1-\varepsilon} \frac{dx}{(x-1)^2} + \lim_{\varepsilon \rightarrow 0} \int_{1+\varepsilon}^2 \frac{dx}{(x-1)^2} = \lim_{\varepsilon \rightarrow 0} \frac{1}{1-x} \Big|_0^{1-\varepsilon} + \lim_{\varepsilon \rightarrow 0} \frac{1}{1-x} \Big|_{1+\varepsilon}^2 =$$

$$= \lim_{\varepsilon \rightarrow 0} \left(\frac{1}{\varepsilon} - 1 \right) + \lim_{\varepsilon \rightarrow 0} \left(-1 + \frac{1}{\varepsilon} \right) = \infty.$$

Bundan ko'rinadiki, qaralayotgan integral $[0;2]$ kesmada uzoqlashuvchi.

Nazariy savollar va mashqlar.

1. Aniq integralga ta'rif bering. Aniq integral qanday geometrik va mehanik ma'nolarga ega?
2. Agar $\int_a^b f(x)dx = 0$, $f(x) \neq 0$ bo'lsa, uni qanday izohlash mumkin?
3. Aniq integralning hossalari bayon qiling va isbotlang.
4. $\int_{-a}^a \Phi(x^2)dx = 2 \int_0^a \Phi(x^2)dx$ ekanligini isbotlang.
5. Nyuton-Leybnis formulasini keltirib chiqaring.
6. Agar $f(x)$ funksiya toq bo'lsa, u holda $\int_{-a}^0 f(x)dx = - \int_0^a f(x)dx$ va $\int_{-a}^a f(x)dx = 0$ ekanligini isbotlang.
7. $\int_{-1}^1 \sin^2 x \ln \left(\frac{2+x}{2-x} \right) dx$ ni hisoblang.
8. Aniq integral uchun bo'laklab integrallash formulasini keltirib chiqaring. Hosmas integral deb nimaga aytiladi?
9. Aniq integralda qachon va qanday qilib o'zgaruvchini almashtirsa bo'ladi? Misollar keltiring.

Loyiha-hisob ishi topshiriqlari

1 – masala. Aniq integralni hisoblang.

3. 1. 1. a) $\int_{\pi}^{2\pi} \frac{x + \cos x}{x^2 + 2 \sin x} dx$; b) $\int_1^{e^2} \frac{\ln^2 x}{\sqrt{x}} dx$; v) $\int_0^5 \frac{dx}{(25 + x^2)\sqrt{25 + x^2}}$.

3. 1. 2. a) $\int_{e+1}^{e^2+1} \frac{1+\ln(x-1)}{x-1} dx$; b) $\int_{\pi/4}^3 (3x-x^2)\sin 2x dx$; v) $\int_0^{\pi/2} \frac{\cos x}{2+\cos x} dx$.

3. 1. 3. a) $\int_0^1 \frac{4\arctan x - x}{1+x^2} dx$; b) $\int_1^e \sqrt{x} \ln^2 x dx$; v) $\int_0^{2\sqrt{2}} \frac{x^4}{(16-x^2)\sqrt{16-x^2}} dx$.

3. 1. 4. a) $\int_0^1 \frac{x^2+1}{(x^3+3x+1)^2} dx$; b) $\int_{-2}^0 (x^2+2)e^{\frac{x}{2}} dx$; v) $\int_0^{\pi/2} \frac{\cos x - \sin x}{(1-\cos x)^2} dx$.

3. 1. 5. a) $\int_0^{\pi/4} \frac{2\cos x + 3\sin x}{(2\sin x - 3\cos x)^3} dx$; b) $\int_0^{2\pi} (1-8x^2)\cos 4x dx$; v) $\int_0^5 x^2\sqrt{25-x^2} dx$.

3. 1. 6. a) $\int_0^{1/2} \frac{8x - \arctan 2x}{1+4x^2} dx$; b) $\int_0^1 (x^2+1)e^{3x} dx$; v) $\int_0^{\pi/2} \frac{\cos x}{5+4\cos x} dx$.

3. 1. 7. a) $\int_0^2 \frac{x^3}{x^2+4} dx$; b) $\int_{-3}^0 (x^2+6x+9)\sin 2x dx$; v) $\int_0^{1/\sqrt{2}} \frac{dx}{(1-x^2)\sqrt{1-x^2}}$.

3. 1. 8. a) $\int \frac{\sqrt{8}x + \frac{1}{\sqrt{3}}}{\sqrt{x^2+1}} dx$; b) $\int_0^1 (x+1)\ln^2(x+1) dx$; v) $\int_0^{\pi/2} \frac{\sin x}{(1+\sin x)^2} dx$.

3. 1. 9. a) $\int_0^{\sqrt{3}} \frac{\arctan x + x}{1+x^2} dx$; b) $\int_{-1}^1 x^2 e^{-\frac{x}{2}} dx$; v) $\int_0^{5/2} \frac{x^2}{\sqrt{25-x^2}} dx$.

3. 1. 10. a) $\int \frac{\sqrt{8}x - \frac{1}{\sqrt{3}}}{\sqrt{x^2+1}} dx$; b) $\int_0^{\pi/2} (1-5x^2)\sin x dx$; v) $\int_0^{\pi/2} \frac{\sin x}{2+\sin x} dx$.

3. 1. 11. a) $\int_0^{\sqrt{3}} \frac{x - (\arctan x)^4}{1+x^2} dx$; b) $\int_0^{2\pi} (3x^2+5)\cos 2x dx$; v) $\int_0^2 \frac{dx}{(4+x^2)\sqrt{4+x^2}}$.

3. 1. 12. a) $\int_1^3 \frac{1-\sqrt{x}}{\sqrt{x}(x+1)} dx$; b) $\int_{-1}^0 (x+2)^3 \ln^2(x+2) dx$; v) $\int_0^{\pi/2} \frac{\sin x}{5+3\sin x} dx$.

3. 1. 13. a) $\int_1^e \frac{1+\ln x}{x} dx$; b) $\int_0^{\pi} (x^2-3x+2)\sin x dx$; v) $\int_{-3}^3 x^2\sqrt{9-x^2} dx$.

3. 1. 14. a) $\int \frac{dx}{\sqrt{2}x\sqrt{x^2-1}}$; b) $\int_{-3}^0 (x^2+2)e^{\frac{x}{3}} dx$; v) $\int_0^{\pi/4} \frac{dx}{\cos x(1+\cos x)}$.

3. 1. 15. a) $\int_0^1 \frac{x}{\sqrt{x^4+x^2+1}} dx$; b) $\int_1^8 \frac{\ln^2 x}{\sqrt[3]{x^2}} dx$; v) $\int_0^{\sqrt{2}/2} \frac{x^4}{\sqrt{(1-x^2)^3}} dx$.

3. 1. 16. a) $\int_1^e \frac{x^2 + \ln x^2}{x} dx$; b) $\int_0^{2\pi} (2x^2 - 15)\cos 3x dx$; v) $\int_0^{\pi/2} \frac{dx}{(1 + \sin x + \cos x)^2}$.

3. 1. 17. a) $\int_0^1 \frac{x^3}{(x^2 + 1)^2} dx$; b) $\int_0^2 (x+1)^2 \ln^2(x+1) dx$; v) $\int_0^2 \frac{x^2}{\sqrt{16-x^2}} dx$.

3. 1. 18. a) $\int_0^1 \frac{x}{x^4 + 1} dx$; b) $\int_{-1}^0 (x^2 + 2x + 1)\sin 3x dx$; v) $\int_0^{2\pi/3} \frac{\cos^2 x}{(1 + \cos x + \sin x)^2} dx$.

3. 1. 19. a) $\int_0^{2\pi} \frac{1 - \cos x}{(x - \sin x)^2} dx$; b) $\int_{-2}^0 (x^2 + 1)e^{\frac{x}{2}} dx$; v) $\int_0^2 \frac{dx}{\sqrt{(16-x^2)^3}}$.

3. 1. 20. a) $\int_0^{\pi/4} \operatorname{tg} x \ln \cos x dx$; b) $\int_{-2}^0 (x^2 + 5x + 6)\cos 2x dx$; v) $\int_0^{2\pi/3} \frac{1 + \sin x}{1 + \cos x + \sin x} dx$.

3. 1. 21. a) $\int_{-1}^0 \frac{\operatorname{tg}(x+1)}{\cos^2(x+1)} dx$; b) $\int_2^3 (x-1)^3 \ln^2(x-1) dx$; v) $\int_0^1 \frac{x^2}{\sqrt{4-x^2}} dx$.

3. 1. 22. a) $\int_0^1 \frac{x^3 + x}{x^4 + 1} dx$; b) $\int_0^3 (x^2 - 3x)\sin 2x dx$; v) $\int_0^{\pi/2} \frac{1 + \cos x}{1 + \cos x + \sin x} dx$.

3. 1. 23. a) $\int_{\pi/4}^{\pi/2} \frac{x \cos x + \sin x}{(x \sin x)^2} dx$; b) $\int_{-1/2}^0 (x^2 + 2)e^{-2x} dx$; v) $\int_0^4 x^2 \sqrt{16-x^2} dx$.

3. 1. 24. a) $\int_2^9 \frac{x}{\sqrt[3]{x-1}} dx$; b) $\int_0^{\pi} (2x^2 + 4x + 7)\cos 2x dx$; v) $\int_{\pi/3}^{\pi/2} \frac{\cos x}{1 + \sin x - \cos x} dx$.

3. 1. 25. a) $\int_0^1 \frac{x^3}{x^2 + 1} dx$; b) $\int_{-2}^0 (x+3)\ln^2(x+3) dx$; v) $\int_0^1 \frac{x^4}{\sqrt{(2-x^2)^3}} dx$.

3. 1. 26. a) $\int_{\sqrt{2}}^{\sqrt{3}} \frac{x}{\sqrt{x^4 - x^2 - 1}} dx$; b) $\int_0^{\pi} x^2 \sin \frac{x}{2} dx$; v) $\int_0^{\pi/2} \frac{\sin x}{1 + \cos x + \sin x} dx$.

3. 1. 27. a) $\int_0^{\pi/4} \frac{\sin x - \cos x}{(\cos x + \sin x)^5} dx$; b) $\int_0^1 (x^2 - 1)e^{5x} dx$; v) $\int_1^{\sqrt{3}} \frac{dx}{\sqrt{(1+x^2)^3}}$.

3. 1. 28. a) $\int_0^{1/\sqrt{2}} \frac{(\arccos x)^3 - 1}{\sqrt{1-x^2}} dx$; b) $\int_0^1 \sqrt{x+1} \ln^2(x+1) dx$; v) $\int_0^2 \frac{\cos x}{1 + \cos x + \sin x} dx$.

3. 1. 29. a) $\int_0^{\sin 1} \frac{(\arcsin x)^2 + 1}{\sqrt{1-x^2}} dx$; b) $\int_0^{\pi} x^2 \cos \frac{x}{2} dx$; v) $\int_0^{\sqrt{2}} \frac{x^4}{\sqrt{(4-x^2)^3}} dx$.

$$3.1.30. \text{ a) } \int \frac{\sqrt{8}}{\sqrt{3}x\sqrt{x^2+1}} dx; \quad \text{b) } \int_1^4 \frac{\ln^2 x}{\sqrt{x}} dx; \quad \text{v) } \int_0^{\pi/2} \frac{\sin^2 x}{(1+\cos x+\sin x)^2} dx.$$

2 – masala. Hosmas integrallarni yaqinlashishga tekshiring.

$$3.2.1. \text{ a) } \int_{-\infty}^{+\infty} \frac{dx}{x^2+2x+2};$$

$$\text{b) } \int_0^1 \frac{dx}{\sqrt{1-x}}.$$

$$3.2.2. \text{ a) } \int_1^{+\infty} \frac{dx}{(1+x)\sqrt{x}};$$

$$\text{b) } \int_0^1 x \ln x dx.$$

$$3.2.3. \text{ a) } \int_9^{+\infty} \frac{dx}{x \ln^3 x};$$

$$\text{b) } \int_{-1}^0 \frac{dx}{\sqrt[3]{1+x}}.$$

$$3.2.4. \text{ a) } \int_{-\infty}^{+\infty} \frac{dx}{x^2+2x+1};$$

$$\text{b) } \int_0^{1/e} \frac{dx}{x \ln^2 x}.$$

$$3.2.5. \text{ a) } \int_4^{+\infty} \frac{dx}{x\sqrt{x}};$$

$$\text{b) } \int_{-1}^0 \frac{dx}{(x+1)^2}.$$

$$3.2.6. \text{ a) } \int_1^{+\infty} \frac{\ln(x+1)}{x+1} dx;$$

$$\text{b) } \int_1^2 \frac{x}{\sqrt{x-1}} dx.$$

$$3.2.7. \text{ a) } \int_1^{+\infty} \frac{dx}{x^2+4x};$$

$$\text{b) } \int_0^e \frac{dx}{x\sqrt{\ln x}}.$$

$$3.2.8. \text{ a) } \int_3^{+\infty} \frac{x}{x^2-4} dx;$$

$$\text{b) } \int_0^{1/2} \frac{dx}{(2x-1)^3}.$$

$$3.2.9. \text{ a) } \int_2^{+\infty} \frac{dx}{x \ln x};$$

$$\text{b) } \int_0^4 \frac{dx}{x\sqrt{x}}.$$

$$3.2.10. \text{ a) } \int_2^{+\infty} \frac{1+x^2}{x^3} dx;$$

$$\text{b) } \int_1^2 \frac{dx}{x \ln x}.$$

$$3.2.11. \text{ a) } \int_{-\infty}^0 \frac{dx}{1+x+x^2};$$

$$\text{b) } \int_0^4 \frac{dx}{\sqrt{4-x}}.$$

$$3.2.12. \text{ a) } \int_{\sqrt{2}}^{+\infty} \frac{dx}{x\sqrt{x^2-1}};$$

$$\text{b) } \int_0^2 \frac{dx}{(x-1)^2}.$$

$$3.2.13. \text{ a) } \int_0^{+\infty} e^{-3x} dx;$$

$$\text{b) } \int_0^1 \frac{x^4}{\sqrt{1-x^5}} dx.$$

$$3.2.14. \text{ a) } \int_0^{+\infty} \frac{x}{(1+x)^2} dx;$$

$$\text{b) } \int_0^1 \frac{dx}{x^3\sqrt{x}}.$$

$$3.2.15. a) \int_0^{+\infty} x e^{-x^2} dx;$$

$$b) \int_0^1 \frac{dx}{x^2 - 4x + 3}.$$

$$3.2.16. a) \int_e^{+\infty} \frac{\sqrt{\ln x}}{x} dx;$$

$$b) \int_{-3}^2 \frac{dx}{(x+3)^2}.$$

$$3.2.17. a) \int_0^{+\infty} e^{-\sqrt{x}} dx;$$

$$b) \int_0^4 \frac{dx}{\sqrt[3]{(x-3)^2}}.$$

$$3.2.18. a) \int_0^{+\infty} \frac{dx}{(2x+1)^2};$$

$$b) \int_0^1 x^2 \ln x dx.$$

$$3.2.19. a) \int_0^{+\infty} e^{-5x} dx;$$

$$b) \int_0^1 \frac{x^3}{\sqrt{1-x^4}} dx.$$

$$3.2.20. a) \int_1^{+\infty} \frac{dx}{x^2(x+1)};$$

$$b) \int_0^{e-1} \frac{dx}{(x+1)\ln(x+1)}.$$

$$3.2.21. a) \int_2^{+\infty} \frac{dx}{x\sqrt{\ln x}};$$

$$b) \int_2^3 \frac{dx}{(x-2)^3}.$$

$$3.2.22. a) \int_0^{+\infty} \frac{x}{(1+x)^3} dx;$$

$$b) \int_0^1 \frac{x^5}{\sqrt{1-x^6}} dx.$$

$$3.2.23. a) \int_1^{+\infty} \frac{x^2}{1+x^6} dx;$$

$$b) \int_0^{\pi/2} \operatorname{ctg} x dx.$$

$$3.2.24. a) \int_1^{+\infty} \frac{dx}{(x+1)\ln(x+1)};$$

$$b) \int_{-1}^3 \frac{dx}{(x+1)\sqrt{x+1}}.$$

$$3.2.25. a) \int_1^{+\infty} \frac{dx}{x^2+3x};$$

$$b) \int_0^1 x^3 \ln x dx.$$

$$3.2.26. a) \int_0^{+\infty} e^{-2x} dx;$$

$$b) \int_2^3 \frac{x-1}{\sqrt{x-2}} dx.$$

$$3.2.27. a) \int_1^{+\infty} \frac{x}{1+x^4} dx;$$

$$b) \int_1^e \frac{dx}{x^3 \sqrt{\ln x}}.$$

$$3.2.28. a) \int_1^{+\infty} \frac{dx}{x\sqrt{1+x^2}};$$

$$b) \int_{-\infty}^{\pi/2} \operatorname{tg} x dx.$$

$$3.2.29. a) \int_0^{+\infty} x^2 e^{-x^3} dx;$$

$$b) \int_0^1 \frac{x^2}{\sqrt{1-x^3}} dx.$$

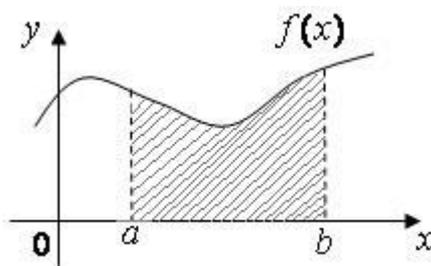
$$3. 2. 30. a) \int_{-\infty}^3 \frac{x}{(x^2+1)^3} dx;$$

$$b) \int_0^2 \frac{x^5}{\sqrt{4-x^2}} dx.$$

4 – topshiriq

Mavzu: Aniq integralning tadbiqu

Nazariy tushunchalar. Agar $[a;b]$ kesmada funksiya manfiy bo'lmasa, u holda $I = \int_a^b f(x)dx$ aniq integral $y = f(x)$ funksiya grafigi, Ox o'qi, $x=a$ va $x=b$ chiziqlar bilan chegaralangan egri chiziqli trapetsiyaning yuzini ifodalaydi.



Ushbu

$$S = \int_a^b \sqrt{1 + (f'(x))^2} dx$$

formula yordamida $f(x)$ funksiya grafigining $x=a$ va $x=b$ chiziqlar orasiga joylashgan qismining uzunligini hisoblash mumkin.

Ushbu

$$V_{Ox} = \pi \int_a^b (f(x))^2 dx,$$

$$V_{Oy} = \pi \int_c^d (\varphi(y))^2 dy$$

formulalar yordamida $y = f(x)$ funksiya grafigining $[a;b]$ kesmadagi qismini Ox o'qi atrofida aylanishidan, $x = \varphi(y)$ funksiya grafigining $[c;d]$ kesmaga mos qismini Oy o'qi atrofida aylantirishdan hosil bo'lgan aylanma jismning hajmini hisoblash mumkin.

Agar $y = f(x)$ funksiya $[a, b]$ intervalda silliq egri chiziq bo'lsa, u holda bu chiziqning $[a, b]$ intervaldagi L uzunligi

$$L = \int_a^b \sqrt{1 + [f'(x)]^2} dx$$

bilan aniqlanadi. Ushbu natija yoy uzunligini ham ta'riflaydi, ham hisoblash formulasi bo'lib hizmat qiladi, ya'ni

$$L = \int_a^b \sqrt{1 + [f'(x)]^2} dx = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx$$

Bundan tashqari, agar egri chiziq $x = g(y)$ ko'rinishda berilgan bo'lib, bunda g' $[c, d]$ intervalda uzluksiz bo'lsa, u holda L uzunligi $y = c$ dan $y = d$ gacha quyidagicha formula orqali topiladi:

$$L = \int_c^d \sqrt{1 + [g'(y)]^2} dy = \int_c^d \sqrt{1 + \left(\frac{dx}{dy}\right)^2} dy$$

Faraz qilaylik,

$$x = \varphi(t),$$

$$y = \psi(t)$$

funksiyalar $[\alpha, \beta]$ da uzluksiz va uzluksiz hosilalarga ega bo'lsin. Bu funksiyalarning $t_0 \in [\alpha, \beta]$ nuqtadagi qiymatlari

$$x_0 = \varphi(t_0), \quad y_0 = \psi(t_0)$$

dan tashkil topgan (x_0, y_0) juftlik, tekislikda koordinatalari x_0 va y_0 bo'lgan $(x_0; y_0)$ nuqtani ifodalaydi. t o'zgaruvchi $[\alpha, \beta]$ da o'zgarganida unga mos

$$x = \varphi(t), \quad y = \psi(t)$$

funksiya qiymatlaridan tashkil topgan (x, y) juftliklar to'plami tekislikda biror egri chiziqni tasvirlaydi.

Berilgan sistema egri chiziqning parametrik tenglamasi deyiladi, t esa parametr deyiladi.

Aytaylik, $\overset{\sim}{AB}$ egri chiziq yuqoridagi sistema bilan berilgan bo'lsin. Uning uzunligi ushbu

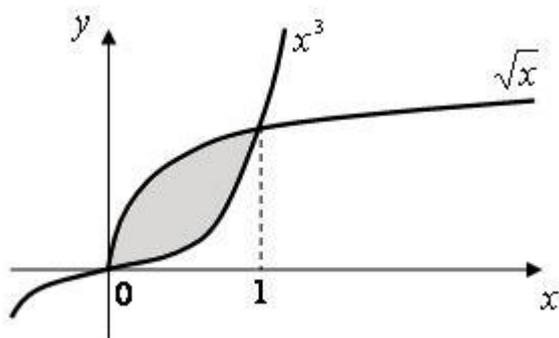
$$l = \int_{\alpha}^{\beta} \sqrt{\varphi'^2(t) + \psi'^2(t)} dt$$

integral yordamida topiladi.

Mavzuga doir misollarni yechishga namunalar

1-misol. Ushbu $y = x^3$ va $y = \sqrt{x}$ funksiyalarning grafiklari bilan chegaralangan tekis figuraning yuzini hisoblang.

Yechish. 1) Shaklni yasaymiz.



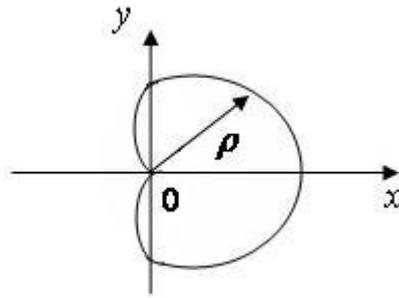
2) shakldan $a = 0, b = 1, f(x) = \sqrt{x}, \varphi(x) = x^3$ ekanligini hosil qilamiz.

3) Ushbu $S = \int_a^b (f(x) - \varphi(x)) dx$ formuladan foydalanib, yuzani hisoblaymiz.

4) Hisoblash. $S = \int_0^1 (\sqrt{x} - x^3) dx = \left(\frac{2\sqrt{x^3}}{3} - \frac{x^4}{4} \right) \Big|_0^1 = \frac{2}{3} - \frac{1}{4} = \frac{8-3}{12} = \frac{5}{12}$ yuza birligi.

2-misol. Qutb koordinata sistemasida berilgan ushbu $\rho = r(1 + \cos\varphi)$ egri chiziqning (kardioida) uzunligini hisoblang.

Yechish. 1) Shakl yasaymiz.



2) Shaklning simmetrikligidan foydalanib, uning yarmining uzunligini hisoblaymiz. Bunda $\varphi_1 = 0$, $\varphi_2 = \pi$, $\rho = r(1 + \cos\theta)$, $\rho' = -r\sin\theta$ ekanligini hosil qilamiz.

3) Ushbu $S = \int_{\varphi_1}^{\varphi_2} \sqrt{(\rho')^2 + \rho^2} d\varphi$ formuladan foydalanib, egri chiziqning uzunligini hisoblaymiz.

4) Hisoblash.

$$S_1 = \int_0^{\pi} \sqrt{r^2 \sin^2 \varphi + r^2 (1 + \cos \varphi)^2} d\varphi = \int_0^{\pi} \sqrt{r^2 \sin^2 \varphi + r^2 (1 + 2 \cos \varphi + \cos^2 \varphi)} d\varphi =$$

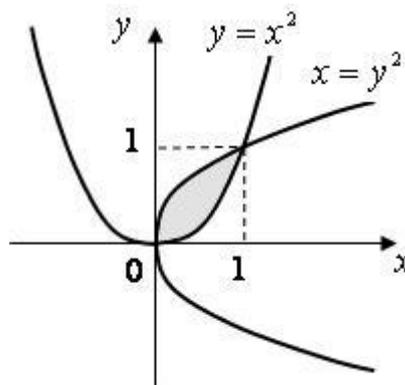
$$= \int_0^{\pi} \sqrt{2r^2 (1 + \cos \varphi)} d\varphi = \int_0^{\pi} 2r \cos \frac{\varphi}{2} d\varphi = 4r \sin \frac{\varphi}{2} \Big|_0^{\pi} = 4r \left(\sin \frac{\pi}{2} - \sin 0 \right) = 4r$$

uzunlik birligi.

Javob: $S = 2S_1 = 8r$.

3-misol. Ushbu $y = x^2$ va $x = y^2$ funksiyalarning grafiklari bilan chegaralangan tekis figuraning Ox o'qi atrofida aylantirishdan hosil bo'lgan aylanma jismning hosilasini hisoblang.

Yechish. 1) Shakl yasaymiz.



2) Shaklda $a=0$, $b=1$, $f(x)=\sqrt{x}$, $\varphi(x)=x^2$ ekanligini hosil qilamiz.

3) Ushbu $V_{ox} = \pi \int_a^b ((f(x))^2 - (\varphi(x))^2) dx$ formuladan foydalanib, hisoblaymiz.

4) Hisoblash.

$$V_{ox} = \pi \int_0^1 ((\sqrt{x})^2 - (x^2)^2) dx = \pi \int_0^1 (x - x^4) dx = \pi \left(\frac{x^2}{2} - \frac{x^5}{5} \right) \Big|_0^1 = \pi \left(\frac{1}{2} - \frac{1}{5} \right) = \frac{3\pi}{10} = 0,3\pi$$

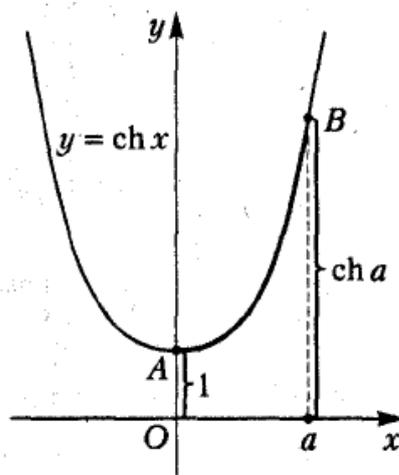
hajm birligi.

4-misol. $y = \frac{e^x + e^{-x}}{2} = chx$ funksiya grafigining $A(0;1)$ nuqtasidan

$B(a, cha)$ nuqtasigacha bo'lgan qismning uzunligini toping.

Yechish. $y = \frac{e^x + e^{-x}}{2} = chx$ funksiya grafigini koordinatalar sistemasida

chizsak 1-rasm hosil bo'ladi.



1-rasm.

$y' = (chx)' = shx$ va $ch^2x - sh^2x = 1$ ekanligini inobatga olsak,

$$\sqrt{1 + (y')^2} = \sqrt{1 + sh^2x} = \sqrt{ch^2x} = chx \quad (ch > 0)$$

kelib chiqadi. Shunga ko'ra

$$S = \int_0^a chx dx = shx \Big|_0^a = sha$$

hosil bo'ladi.

5-misol. $x = a(t - \sin t), y = a(t - \cos t)$ siklorida bitta arki uzunligini hisoblang.

Yechish. Sikloida arklari bir xil bo'lgani uchun uning bitta arkini olamiz.

Bunda t parametr 0 dan 2π gacha o'zgaradi, $x_t^1 = a(1 - \cos t), y_t^1 = a \sin t$ bo'lgani uchun fo'rmulaga ko'ra egri chiziqning uzunligi quyidagicha aniqlanadi:

$$l = \int_0^{2\pi} \sqrt{a^2(1 - \cos t)^2 + a^2 \sin^2 t} dt = a \int_0^{2\pi} \sqrt{2(1 - \cos t)} dt = a \int_0^{2\pi} 2 \sin \frac{t}{2} dt = -4a \cos \frac{t}{2} \Big|_0^{2\pi} = 8a$$

6-misol. Ushbu

$$f(x) = x^{\frac{3}{2}} \quad (0 \leq x \leq 4)$$

funksiya grafigi ifodalovchi yoyning uzunligi topilsin.

Yechish. $a = 0, b = 4,$

$$f'(x) = \left(x^{\frac{3}{2}}\right)' = \frac{3}{2}x^{\frac{1}{2}}, \quad 1 + f'^2(x) = 1 + \frac{9}{4}x$$

bo'lishini e'tiborga olib, (5) formuladan foydalanib topamiz:

$$\begin{aligned} \ell &= \int_0^4 \sqrt{1 + \frac{9}{4}x} dx = \left[1 + \frac{9}{4}x = t, \quad x = 0 \text{ da } t = 1, \quad x = 4 \text{ da } t = 10, \quad dx = \frac{4}{9} dt \right] = \\ &= \int_1^{10} \frac{4}{9} t^{\frac{1}{2}} dt = \frac{4}{9} \cdot \frac{2}{3} t^{\frac{3}{2}} \Big|_1^{10} = \frac{8}{27} (10\sqrt{10} - 1). \end{aligned}$$

7-misol. Ushbu

$$\begin{aligned} \varphi(t) &= a(t - \sin t), \\ \psi(t) &= a(1 - \cos t) \end{aligned} \quad (0 \leq t \leq \pi)$$

tenglamalar sistemasi bilan aniqlangan yoyning (sikloidaning) uzunligi topilsin.

Yechish. Ravshanki,

$$\begin{aligned} \varphi'(t) &= a(1 - \cos t), \\ \psi'(t) &= a \sin t \end{aligned}$$

bo'lib,

$$\sqrt{\varphi'^2(t) + \psi'^2(t)} = \sqrt{a^2(1 - \cos t)^2 + a^2 \sin^2 t} = a\sqrt{2(1 - \cos t)}$$

bo‘ladi. Endi $\alpha = 0$, $\beta = 2\pi$ deb, formuladan foydalanib, egri chiziqning uzunligini topamiz:

$$\begin{aligned} \ell &= \int_{\alpha}^{\beta} \sqrt{\varphi'^2(t) + \psi'^2(t)} dt = \int_0^{2\pi} a\sqrt{2(1 - \cos t)} dt = a \int_0^{2\pi} \sqrt{4 \cdot \sin^2 \frac{t}{2}} dt = \\ &= 2a \int_0^{2\pi} \sin \frac{t}{2} \cdot d\left(\frac{t}{2}\right) \cdot 2 = -4a \cos \frac{t}{2} \Big|_0^{2\pi} = -4a(\cos \pi - \cos 0) = 8a. \end{aligned}$$

Nazariy savollar va mashqlar.

1. Har hil koordinata sistemalarida berilgan tekis figuralarning yuzalari qanday topiladi? Formulalarini yozing.
2. Egri chiziq yoyining uzunligi qanday topiladi?
3. Dekart va qutb koordinata sistemalarida hamda parametrik ko‘rinishda berilgan egri chiziq yoyining uzunligini hisoblash formulalarini yozing.
4. Ushbu $S = \int_c^d f(y) dy$ formuladan qanday hollarda foydalanadi?
5. Ko‘ndalang kesimining yuzasi ma‘lum bo‘lsa jism hajmini hisoblash uchun formula yozing.
6. Aylanma jisimlarning hajmlarini hisoblash formulalarini keltirib chiqaring. Ox va Oy o‘qlari atrofida aylanishdan hosil bo‘lgan jismlarning hajmlari qanday topiladi?
7. Cheksiz cho‘zilgan egri chiziqni biror o‘q atrofida aylanishdan chekli hajmdagi jism hosil bo‘lishi mumkinmi? Javobingizni $y = e^{-x}$ ($0 \leq x < \infty$) funksiyani Ox o‘qi atrofida aylantirib izohlang.
8. Ox va Oy o‘qlari atrofida aylanishdan hosil bo‘lgan jismning sirt yuzlari qanday topiladi?
9. Ushbu $x^2 + y^2 = R^2$ va $y^2 + z^2 = R^2$ silindrlarning umumiy qismining hajmi $\frac{16}{3}\pi R^3$ ekanligini hisoblang.

Loyiha-hisob ishi topshiriqlari

1 – masala. Funksiya grafiklari bilan chegaralangan tekis figuraning yuzini toping.

4. 1. 1. $y = (x-2)^3$, $y = 4x - 8$.

4. 1. 2. $y = x\sqrt{9-x^2}$, $y = 0$ ($0 \leq x \leq 3$).

4. 1. 3. $y = 4 - x^2$, $y = x^2 - 2x$.

4. 1. 4. $y = \sin x \cos^2 x$, $y = 0$ ($0 \leq x \leq \frac{\pi}{2}$).

4. 1. 5. $y = \sqrt{4-x^2}$, $y = 0$, $x = 0$, $x = 1$.

4. 1. 6. $y = x^2\sqrt{4-x^2}$, $y = 0$ ($0 \leq x \leq 2$).

4. 1. 7. $y = \cos x \sin^2 x$, $y = 0$ ($0 \leq x \leq \frac{\pi}{2}$).

4. 1. 8. $y = \sqrt{e^x - 1}$, $y = 0$, $x = \ln 2$.

4. 1. 9. $y = \frac{1}{x\sqrt{1+\ln x}}$, $y = 0$, $x = 1$, $x = e^3$.

4. 1. 10. $y = \arccos x$, $y = 0$, $x = 0$.

4. 1. 11. $y = (x+1)^2$, $y^2 = x+1$.

4. 1. 12. $y = 2x - x^2 + 3$, $y = x^2 - 4x + 3$.

4. 1. 13. $y = x\sqrt{36-x^2}$, $y = 0$ ($0 \leq x \leq 6$).

4. 1. 14. $x = \arccos y$, $x = 0$, $y = 0$.

4. 1. 15. $y = x \operatorname{arctg} x$, $y = 0$, $x = \sqrt{3}$.

4. 1. 16. $y = x^2\sqrt{8-x^2}$, $y = 0$ ($0 \leq x \leq 2\sqrt{2}$).

4. 1. 17. $x = \sqrt{e^y - 1}$, $x = 0$, $y = \ln 2$.

4. 1. 18. $y = x\sqrt{4-x^2}$, $y = 0$ ($0 \leq x \leq 2$).

4. 1. 19. $y = \frac{x}{1+\sqrt{x}}$, $y = 0$, $x = 1$.

4. 1. 20. $y = \frac{1}{1+\cos x}$, $y = 0$, $x = \frac{\pi}{2}$, $x = -\frac{\pi}{2}$.

4. 1. 21. $x = (y-2)^3$, $x = 4y - 8$.

$$4.1.22. y = \cos^5 x \sin 2x, y = 0 \left(0 \leq x \leq \frac{\pi}{2} \right).$$

$$4.1.23. y = \frac{x}{(x^2 + 1)^2}, y = 0, x = 1.$$

$$4.1.24. x = 4 - y^2, y = y^2 - 2y.$$

$$4.1.25. x = \frac{1}{y\sqrt{1 + \ln y}}, x = 0, y = 1, y = e^3.$$

$$4.1.26. y = \frac{e^x}{x^2}, y = 0, x = 2, x = 1.$$

$$4.1.27. y = x^2 \sqrt{16 - x^2}, y = 0 \ (0 \leq x \leq 4).$$

$$4.1.28. x = \sqrt{4 - y^2}, x = 0, y = 0, y = 1.$$

$$4.1.29. y = (x - 1)^2, y^2 = x - 1.$$

$$4.1.30. y = x^2 \cos x, y = 0 \left(0 \leq x \leq \frac{\pi}{2} \right).$$

2 – masala. Tenglamalari parametrik ko'rinishda bo'lgan chiziqlar bilan chegaralangan tekis figuraning yuzasini hisoblang.

$$4.2.1. \begin{cases} x = 4\sqrt{2} \cos^3 t \\ y = 2\sqrt{2} \sin^3 t \end{cases} . \\ x = 2 \ (x \geq 2)$$

$$4.2.2. \begin{cases} x = \sqrt{2} \cos t \\ y = 2\sqrt{2} \sin t \end{cases} . \\ y = 2 \ (y \geq 2)$$

$$4.2.3. \begin{cases} x = 4(t - \sin t) \\ y = 4(1 - \cos t) \end{cases} . \\ y = 4 \ (0 < x < 8\pi, y \geq 4)$$

$$4.2.4. \begin{cases} x = 16 \cos^3 t \\ y = 2 \sin^3 t \end{cases} . \\ x = 2 \ (x \geq 2)$$

$$4.2.5. \begin{cases} x = 2 \cos t \\ y = 6 \sin t \end{cases} . \\ y = 3 \ (y \geq 3)$$

$$4.2.6. \begin{cases} x = 2(t - \sin t) \\ y = 2(1 - \cos t) \end{cases} . \\ y = 3 \ (0 < x < 4\pi, y \geq 3)$$

$$4.2.7. \begin{cases} x = 16 \cos^3 t \\ y = \sin^3 t \end{cases} . \\ x = 6\sqrt{3} \ (x \geq 6\sqrt{3})$$

$$4.2.8. \begin{cases} x = 6 \cos t \\ y = 2 \sin t \end{cases} . \\ y = \sqrt{3} \ (y \geq \sqrt{3})$$

$$4.2.9. \begin{cases} x = 3(t - \sin t) \\ y = 3(1 - \cos t) \end{cases} . \\ y = 3 \ (0 < x < 6\pi, y \geq 3)$$

$$4.2.10. \begin{cases} x = 8\sqrt{2} \cos^3 t \\ y = \sqrt{2} \sin^3 t \end{cases} . \\ x = 4 \ (x \geq 4)$$

$$4.2.11. \begin{cases} x = 2\sqrt{2} \cos t \\ y = 3\sqrt{2} \sin t \\ y = 3 (y \geq 3) \end{cases}$$

$$4.2.12. \begin{cases} x = 6(t - \sin t) \\ y = 6(1 - \cos t) \\ y = 9 (0 < x < 12\pi, y \geq 9) \end{cases}$$

$$4.2.13. \begin{cases} x = 32 \cos^3 t \\ y = \sin^3 t \\ x = 4 (x \geq 4) \end{cases}$$

$$4.2.14. \begin{cases} x = 3 \cos t \\ y = 8 \sin t \\ y = 4 (y \geq 4) \end{cases}$$

$$4.2.15. \begin{cases} x = 6(t - \sin t) \\ y = 6(1 - \cos t) \\ y = 6 (0 < x < 12\pi, y \geq 6) \end{cases}$$

$$4.2.16. \begin{cases} x = 8 \cos^3 t \\ y = 4 \sin^3 t \\ x = 3\sqrt{3} (x \geq 3\sqrt{3}) \end{cases}$$

$$4.2.17. \begin{cases} x = 6 \cos t \\ y = 4 \sin t \\ y = 2\sqrt{3} (y \geq 2\sqrt{3}) \end{cases}$$

$$4.2.18. \begin{cases} x = 10(t - \sin t) \\ y = 10(1 - \cos t) \\ y = 15 (0 < x < 20\pi, y \geq 15) \end{cases}$$

$$4.2.19. \begin{cases} x = 2\sqrt{2} \cos^3 t \\ y = \sqrt{2} \sin^3 t \\ x = 1 (x \geq 1) \end{cases}$$

$$4.2.20. \begin{cases} x = \sqrt{2} \cos t \\ y = 4\sqrt{2} \sin t \\ y = 4 (y \geq 4) \end{cases}$$

$$4.2.21. \begin{cases} x = t - \sin t \\ y = 1 - \cos t \\ y = 1 (0 < x < 2\pi, y \geq 1) \end{cases}$$

$$4.2.22. \begin{cases} x = 8 \cos^3 t \\ y = 8 \sin^3 t \\ x = 1 (x \geq 1) \end{cases}$$

$$4.2.23. \begin{cases} x = 9 \cos t \\ y = 4 \sin t \\ y = 2 (y \geq 2) \end{cases}$$

$$4.2.24. \begin{cases} x = 8(t - \sin t) \\ y = 8(1 - \cos t) \\ y = 12 (0 < x < 16\pi, y \geq 12) \end{cases}$$

$$4.2.25. \begin{cases} x = 24 \cos^3 t \\ y = 2 \sin^3 t \\ x = 9\sqrt{3} (x \geq 9\sqrt{3}) \end{cases}$$

$$4.2.26. \begin{cases} x = 3 \cos t \\ y = 8 \sin t \\ y = 4\sqrt{3} (y \geq 4\sqrt{3}) \end{cases}$$

$$4.2.27. \begin{cases} x = 2(t - \sin t) \\ y = 2(1 - \cos t) \\ y = 2 (0 < x < 4\pi, y \geq 2) \end{cases}$$

$$4.2.28. \begin{cases} x = 4\sqrt{2} \cos^3 t \\ y = \sqrt{2} \sin^3 t \\ x = 2 (x \geq 2) \end{cases}$$

$$4.2.29. \begin{cases} x = 2\sqrt{2} \cos t \\ y = 5\sqrt{2} \sin t \\ y = 5 (y \geq 5) \end{cases}$$

$$4.2.30. \begin{cases} x = 4(t - \sin t) \\ y = 4(1 - \cos t) \\ y = 6 (0 < x < 8\pi, y \geq 6) \end{cases}$$

3 – masala. Qutb koordinata sistemasida berilgan chiziqlar bilan chegaralangan tekis figuraning yuzini toping.

4. 3. 1. $r = 4 \cos 3\phi, r = 2 (r \geq 2)$.
4. 3. 2. $r = \cos 2\phi$.
4. 3. 3. $r = \sqrt{3} \cos \phi, r = \sin \phi \left(0 \leq \phi \leq \frac{\pi}{2} \right)$.
4. 3. 4. $r = 4 \sin 3\phi, r = 2 (r \geq 2)$.
4. 3. 5. $r = 2 \cos \phi, r = 2\sqrt{3} \sin \phi \left(0 \leq \phi \leq \frac{\pi}{2} \right)$.
4. 3. 6. $r = \sin 3\phi$.
4. 3. 7. $r = 6 \sin 3\phi, r = 3 (r \geq 3)$.
4. 3. 8. $r = \cos 3\phi$.
4. 3. 9. $r = \cos \phi, r = \sqrt{2} \cos \left(\phi - \frac{\pi}{4} \right) \left(-\frac{\pi}{4} \leq \phi \leq \frac{\pi}{4} \right)$.
4. 3. 10. $r = \sin \phi, r = \sqrt{2} \cos \left(\phi - \frac{\pi}{4} \right) \left(0 \leq \phi \leq \frac{3\pi}{4} \right)$.
4. 3. 11. $r = 6 \cos 3\phi, r = 3 (r \geq 3)$.
4. 3. 12. $r = \frac{1}{2} + \sin \phi$.
4. 3. 13. $r = \cos \phi, r = \sin \phi \left(0 \leq \phi \leq \frac{\pi}{2} \right)$.
4. 3. 14. $r = \sqrt{2} \cos \left(\phi - \frac{\pi}{4} \right), r = \sqrt{2} \sin \left(\phi - \frac{\pi}{4} \right) \left(\frac{\pi}{4} \leq \phi \leq \frac{3\pi}{4} \right)$.
4. 3. 15. $r = \cos \phi, r = 2 \cos \phi$.
4. 3. 16. $r = \sin \phi, r = 2 \sin \phi$.
4. 3. 17. $r = 1 + \sqrt{2} \cos \phi$.
4. 3. 18. $r = \frac{1}{2} + \cos \phi$.
4. 3. 19. $r = 1 + \sqrt{2} \sin \phi$.
4. 3. 20. $r = \frac{5}{2} \sin \phi, r = \frac{3}{2} \sin \phi$.
4. 3. 21. $r = \frac{3}{2} \cos \phi, r = \frac{5}{2} \cos \phi$.
4. 3. 22. $r = 4 \cos 4\phi$.

$$4.3.23. r = \sin 6\phi.$$

$$4.3.24. r = 2\cos\phi, r = 3\cos\phi.$$

$$4.3.25. r = \cos\phi + \sin\phi.$$

$$4.3.26. r = 2\sin 4\phi.$$

$$4.3.27. r = 2\cos 6\phi.$$

$$4.3.29. r = \cos\phi - \sin\phi.$$

$$4.3.29. r = 3\sin\phi, r = 5\sin\phi.$$

$$4.3.30. r = 2\sin\phi, r = 4\sin\phi.$$

4 – masala. To'g'ri burchakli koordinata sistemasida berilgan egri chiziq yoyining uzunligini hisoblang.

$$4.4.1. y = \ln x, \sqrt{3} \leq x \leq \sqrt{15}.$$

$$4.4.2. y = \frac{x^2}{4} - \frac{\ln 2}{2}, 1 \leq x \leq 2.$$

$$4.4.3. y = \sqrt{1-x^2} + \arcsin x, 0 \leq x \leq \frac{7}{9}.$$

$$4.4.4. y = \ln \frac{5}{2x}, \sqrt{3} \leq x \leq \sqrt{8}.$$

$$4.4.5. y = -\ln \cos x, 0 \leq x \leq \frac{\pi}{6}.$$

$$4.4.6. y = e^x + 6, \ln \sqrt{8} \leq x \leq \ln \sqrt{15}.$$

$$4.4.7. y = 2 + \arcsin \sqrt{x} + \sqrt{x-x^2}, \frac{1}{4} \leq x \leq 1.$$

$$4.4.8. y = \ln(x^2 - 1), 2 \leq x \leq 3.$$

$$4.4.9. y = \ln(1-x^2), 0 \leq x \leq \frac{1}{4}.$$

$$4.4.10. y = \sqrt{1-x^2} + \arccos x, 0 \leq x \leq \frac{8}{9}.$$

$$4.4.11. y = \frac{1-e^x - e^{-x}}{2}, 0 \leq x \leq 3.$$

$$4.4.12. y = 1 - \ln \cos x, 0 \leq x \leq \frac{\pi}{6}.$$

$$4.4.13. y = e^x + 13, \ln \sqrt{15} \leq x \leq \ln \sqrt{24}.$$

$$4.4.14. y = -\arccos \sqrt{x} + \sqrt{x-x^2}, 0 \leq x \leq \frac{1}{4}.$$

$$4.4.15. y = \arcsin x - \sqrt{1-x^2}, 0 \leq x \leq \frac{15}{16}.$$

$$4.4.16. y = 2 - e^x, \ln \sqrt{3} \leq x \leq \ln \sqrt{8}.$$

$$4.4.17. y = 1 - \ln \sin x, \frac{\pi}{3} \leq x \leq \frac{\pi}{2}.$$

$$4.4.18. y = 1 - \ln(x^2 - 1), 3 \leq x \leq 4.$$

$$4.4.19. y = \sqrt{x-x^2} - \arccos \sqrt{x} + 5, \frac{1}{9} \leq x \leq 1.$$

$$4.4.20. y = -\arccos x + \sqrt{1-x^2} + 1, 0 \leq x \leq \frac{9}{16}.$$

$$4.4.21. y = \ln \sin x, \frac{\pi}{3} \leq x \leq \frac{\pi}{2}.$$

$$4.4.22. y = \ln 7 - \ln x, \sqrt{3} \leq x \leq \sqrt{8}.$$

$$4.4.23. y = 1 + \arcsin x - \sqrt{1-x^2}, 0 \leq x \leq \frac{3}{4}.$$

$$4.4.24. y = \frac{e^x + e^{-x}}{2} + 3, 0 \leq x \leq 2.$$

$$4.4.25. y = \ln \cos x + 2, 0 \leq x \leq \frac{\pi}{6}.$$

$$4.4.26. y = e^x + 26, \ln \sqrt{8} \leq x \leq \ln \sqrt{24}.$$

$$4.4.27. y = \arccos \sqrt{x} - \sqrt{x-x^2} + 4, 0 \leq x \leq \frac{1}{2}.$$

$$4.4.28. y = e^x + e, \ln \sqrt{3} \leq x \leq \ln \sqrt{15}.$$

$$4.4.29. y = \frac{e^{2x} + e^{-2x} + 3}{4}, 0 \leq x \leq 2.$$

$$4.4.30. y = \ln \cos x + 3, 0 \leq x \leq \frac{\pi}{3}.$$

5 – masala. Tenglamasi parametrik ko'rinishda berilgan egri chiziq yoyining uzunligini toping.

$$4.5.1. \begin{cases} x = 5(t - \sin t) \\ y = 5(1 - \cos t) \end{cases}, 0 \leq t \leq \pi$$

$$4.5.2. \begin{cases} x = 3(2 \cos t - \cos 2t) \\ y = 3(2 \sin t - \sin 2t) \end{cases}, 0 \leq t \leq 2\pi$$

$$4.5.3. \begin{cases} x = 10 \cos^3 t \\ y = 10 \sin^3 t \end{cases} . \\ 0 \leq t \leq \frac{\pi}{2}$$

$$4.5.4. \begin{cases} x = (t^2 - 2) \sin t + 2t \cos t \\ y = (2 - t^2) \cos t + 2t \sin t \end{cases} . \\ 0 \leq t \leq \pi$$

$$4.5.5. \begin{cases} x = 4(\cos t + t \sin t) \\ y = 4(\sin t - t \cos t) \end{cases} . \\ 0 \leq t \leq 2\pi$$

$$4.5.6. \begin{cases} x = e^t (\cos t + \sin t) \\ y = e^t (\cos t - \sin t) \end{cases} . \\ 0 \leq t \leq \pi$$

$$4.5.7. \begin{cases} x = 3(t - \sin t) \\ y = 3(1 - \cos t) \end{cases} . \\ \pi \leq t \leq 2\pi$$

$$4.5.8. \begin{cases} x = \frac{1}{2} \cos t - \frac{1}{4} \cos 2t \\ y = \frac{1}{2} \sin t - \frac{1}{4} \sin 2t \end{cases} . \\ \frac{\pi}{2} \leq t \leq \frac{2\pi}{3}$$

$$4.5.9. \begin{cases} x = 6 \cos^3 t \\ y = 6 \sin^3 t \end{cases} . \\ 0 \leq t \leq \frac{\pi}{3}$$

$$4.5.10. \begin{cases} x = (t^2 - 2) \sin t + 2t \cos t \\ y = (2 - t^2) \cos t + 2t \sin t \end{cases} . \\ 0 \leq t \leq \frac{\pi}{3}$$

$$4.5.11. \begin{cases} x = 3(\cos t + t \sin t) \\ y = 3(\sin t - t \cos t) \end{cases} . \\ 0 \leq t \leq \frac{\pi}{3}$$

$$4.5.12. \begin{cases} x = e^t (\cos t + \sin t) \\ y = e^t (\cos t - \sin t) \end{cases} . \\ \frac{\pi}{2} \leq t \leq \pi$$

$$4.5.13. \begin{cases} x = 2,5(t - \sin t) \\ y = 2,5(1 - \cos t) \end{cases} . \\ \frac{\pi}{2} \leq t \leq \pi$$

$$4.5.14. \begin{cases} x = 3,5(2 \cos t - \cos 2t) \\ y = 3,5(2 \sin t - \sin 2t) \end{cases} . \\ 0 \leq t \leq \frac{\pi}{2}$$

$$4.5.15. \begin{cases} x = 8 \cos^3 t \\ y = 8 \sin^3 t \end{cases} . \\ 0 \leq t \leq \frac{\pi}{6}$$

$$4.5.16. \begin{cases} x = (t^2 - 2) \sin t + 2t \cos t \\ y = (2 - t^2) \cos t + 2t \sin t \end{cases} . \\ 0 \leq t \leq \frac{\pi}{2}$$

$$4.5.17. \begin{cases} x = 3(\cos t + t \sin t) \\ y = 3(\sin t - t \cos t) \end{cases} . \\ 0 \leq t \leq \pi$$

$$4.5.18. \begin{cases} x = e^t (\cos t + \sin t) \\ y = e^t (\cos t - \sin t) \end{cases} . \\ 0 \leq t \leq 2\pi$$

$$4.5.19. \begin{cases} x = 4(t - \sin t) \\ y = 4(1 - \cos t) \end{cases} . \\ \frac{\pi}{2} \leq t \leq \frac{2\pi}{3}$$

$$4.5.20. \begin{cases} x = 2(2 \cos t - \cos 2t) \\ y = 2(2 \sin t - \sin 2t) \end{cases} . \\ 0 \leq t \leq \frac{\pi}{3}$$

$$4.5.21. \begin{cases} x = 8(\cos t + t \sin t) \\ y = 8(\sin t - t \cos t) \end{cases} \\ 0 \leq t \leq \frac{\pi}{4}$$

$$4.5.22. \begin{cases} x = (t^2 - 2)\sin t + 2t \cos t \\ y = (2 - t^2)\cos t + 2t \sin t \end{cases} \\ 0 \leq t \leq 2\pi$$

$$4.5.23. \begin{cases} x = 2(t - \sin t) \\ y = 2(1 - \cos t) \end{cases} \\ 0 \leq t \leq \frac{\pi}{2}$$

$$4.5.24. \begin{cases} x = e^t(\cos t + \sin t) \\ y = e^t(\cos t - \sin t) \end{cases} \\ 0 \leq t \leq \frac{3\pi}{2}$$

$$4.5.25. \begin{cases} x = (t^2 - 2)\sin t + 2t \cos t \\ y = (2 - t^2)\cos t + 2t \sin t \end{cases} \\ 0 \leq t \leq \pi$$

$$4.5.26. \begin{cases} x = 4(2 \cos t - \cos 2t) \\ y = 4(2 \sin t - \sin 2t) \end{cases} \\ 0 \leq t \leq \pi$$

$$4.5.27. \begin{cases} x = 4 \cos^3 t \\ y = 4 \sin^3 t \end{cases} \\ \frac{\pi}{6} \leq t \leq \frac{\pi}{4}$$

$$4.5.28. \begin{cases} x = (t^2 - 2)\sin t + 2t \cos t \\ y = (2 - t^2)\cos t + 2t \sin t \end{cases} \\ 0 \leq t \leq 3\pi$$

$$4.5.29. \begin{cases} x = 2(\cos t + t \sin t) \\ y = 2(\sin t - t \cos t) \end{cases} \\ 0 \leq t \leq \frac{\pi}{2}$$

$$4.5.30. \begin{cases} x = e^t(\cos t + \sin t) \\ y = e^t(\cos t - \sin t) \end{cases} \\ \frac{\pi}{6} \leq t \leq \frac{\pi}{4}$$

6 – masala. Qutb koordinata sistemasida berilgan egri chiziq yoyining uzunligini toping.

$$4.6.1. \quad r = 3e^{\frac{3}{4}\phi}, -\frac{\pi}{2} \leq \phi \leq \frac{\pi}{2}.$$

$$4.6.2. \quad r = 2e^{\frac{4}{3}\phi}, -\frac{\pi}{2} \leq \phi \leq \frac{\pi}{2}.$$

$$4.6.3. \quad r = \sqrt{2}e^{\phi}, -\frac{\pi}{2} \leq \phi \leq \frac{\pi}{2}.$$

$$4.6.4. \quad r = 5e^{\frac{5}{2}\phi}, -\frac{\pi}{2} \leq \phi \leq \frac{\pi}{2}.$$

$$4.6.5. \quad r = 6e^{\frac{12}{5}\phi}, -\frac{\pi}{2} \leq \phi \leq \frac{\pi}{2}.$$

$$4.6.6. \quad r = 3e^{\frac{3}{4}\phi}, 0 \leq \phi \leq \frac{\pi}{3}.$$

$$4.6.7. \quad r = 4e^{\frac{4}{3}\phi}, 0 \leq \phi \leq \frac{\pi}{3}.$$

$$4.6.8. \quad r = \sqrt{2}e^{\phi}, 0 \leq \phi \leq \frac{\pi}{3}.$$

$$4.6.9. \quad r = 5e^{\frac{5}{12}\phi}, 0 \leq \phi \leq \frac{\pi}{3}.$$

$$4.6.10. \quad r = 12e^{\frac{12}{5}\phi}, 0 \leq \phi \leq \frac{\pi}{3}.$$

$$4.6.11. \quad r = 1 - \sin \phi, -\frac{\pi}{2} \leq \phi \leq -\frac{\pi}{6}.$$

$$4.6.12. \quad r = 2(1 - \cos \phi), -\pi \leq \phi \leq -\frac{\pi}{2}.$$

$$4.6.13. \quad r = 3(1 + \sin \phi), -\frac{\pi}{6} \leq \phi \leq 0.$$

$$4.6.14. \quad r = 4(1 - \sin \phi), 0 \leq \phi \leq \frac{\pi}{6}.$$

4. 6. 15. $r = 5(1 - \cos \phi), -\frac{\pi}{3} \leq \phi \leq 0.$

4. 6. 16. $r = 6(1 + \sin \phi), -\frac{\pi}{2} \leq \phi \leq 0.$

4. 6. 17. $r = 7(1 - \sin \phi), -\frac{\pi}{6} \leq \phi \leq \frac{\pi}{6}.$

4. 6. 18. $r = 8(1 - \cos \phi), -\frac{2\pi}{3} \leq \phi \leq 0.$

4. 6. 19. $r = 2\phi, 0 \leq \phi \leq \frac{3}{4}.$

4. 6. 20. $r = 2\phi, 0 \leq \phi \leq \frac{4}{3}.$

4. 6. 21. $r = 6\sin \phi, 0 \leq \phi \leq \frac{\pi}{3}.$

4. 6. 22. $r = 3\phi, 0 \leq \phi \leq \frac{4}{3}.$

4. 6. 23. $r = 4\phi, 0 \leq \phi \leq \frac{3}{4}.$

4. 6. 24. $r = 2\cos \phi, 0 \leq \phi \leq \frac{\pi}{6}.$

4. 6. 25. $r = 2\sin \phi, 0 \leq \phi \leq \frac{\pi}{6}.$

4. 6. 26. $r = 2\phi, 0 \leq \phi \leq \frac{12}{5}.$

4. 6. 27. $r = 5\phi, 0 \leq \phi \leq \frac{12}{5}.$

4. 6. 28. $r = 6\cos \phi, 0 \leq \phi \leq \frac{\pi}{3}.$

4. 6. 29. $r = 8\cos \phi, 0 \leq \phi \leq \frac{\pi}{4}.$

4. 6. 30. $r = 8\sin \phi, 0 \leq \phi \leq \frac{\pi}{4}.$

7 – masala. Funksiya grafiklari bilan chegaralangan figuralarning Ox o'qi (toq nomerlarda) hamda Oy o'qi (juft nomerlarda) atrofida aylanishdan hosil bo'lgan jismlarning hajmlarini hisoblang.

4. 7. 1. $y = -x^2 + 5x - 6, y = 0.$

4. 7. 2. $y = \arccos \frac{x}{3}, y = \arccos x, y = 0.$

4. 7. 3. $y = 2x - x^2, y = -x + 2, x = 0.$

4. 7. 4. $y = x^2 + 1, y = x, x = 0, x = 1.$

4. 7. 5. $2x - x^2 - y = 0, 2x^2 - 4x + y = 0.$

4. 7. 6. $y = \sqrt{x-1}, y = 0, y = 1, x = 0,5.$

4. 7. 7. $y = 3\sin x, y = \sin x, 0 \leq x \leq \pi.$

4. 7. 8. $y = \ln x, x = 2, y = 0.$

4. 7. 9. $y = 5\cos x, y = \cos x, x = 0, x \geq 0.$

4. 7. 10. $y = (x-1)^2, y = 1.$

4. 7. 11. $y = \sin^2 x, x = \frac{\pi}{2}, y = 0.$

4. 7. 12. $y^2 = x - 2, y = 0, y = x^3, y = 1.$

4. 7. 13. $x = \sqrt[3]{y-2}, x = 1, y = 1.$

4. 7. 14. $y = \arcsin \frac{x}{5}, y = \arcsin x, y = \frac{\pi}{2}.$

4. 7. 15. $y = xe^x, y = 0, x = 1.$

4. 7. 16. $y = x^2 - 2x + 1, x = 2, y = 0.$

4. 7. 17. $y = 2x - x^2, y = -x + 2.$

4. 7. 18. $y = \arccos x, y = \arcsin x, x = 0.$

4. 7. 19. $y = e^{1-x}, y = 0, x = 0, x = 1.$

4. 7. 20. $y = (x-1)^2, x = 0, x = 2, y = 0.$

4. 7. 21. $y = x^2, y^2 - x = 0.$

4. 7. 22. $y = x^3, y = x^2.$

$$4.7.23. y = 1 - x^2, x = 0, x = \sqrt{y-2}, x = 1.$$

$$4.7.24. y = 2x - x^2, y = 0.$$

$$4.7.25. x^2 + (y-2)^2 = 1.$$

$$4.7.26. y = \arccos \frac{x}{5}, y = \arccos \frac{x}{3}, y = 0.$$

$$4.7.27. y = \sin \frac{\pi x}{2}, y = x^2.$$

$$4.7.28. y^2 = 4ax, x = a.$$

$$4.7.29. y = 2\sqrt{x+1}, y = \sqrt{4-2x}, y = 0.$$

$$4.7.30. y^2 = x^3, y = 0, x = 1.$$

2-LOYIHA-HISOB ISHI

1 – topshiriq

Mavzu: Ikki o'zgaruvchili funksiya

Nazariy tushunchalar. Agar ikkita erkli x va y o'zgaruvchilarning bir G sohadan olingan har bir juft (x, y) qiymatlar biror qonuniyat yordamida boshqa z o'zgaruvchining ma'lum bir qiymati mos qo'yilgan bo'lsa, shu qonuniyatga (moslikga) ikki o'zgaruvchili funksiya deyiladi va quyidagicha yoziladi:

$$z = f(x; y).$$

Argumentlarning funksiyaga ma'no beradigan qiymatlar to'plami G sohaga qaralayotgan funksiyaning aniqlanish sohasi deyiladi.

Agar ixtiyoriy kichik musbat ε soni uchun, shunday kichik musbat δ soni topiladiki, ushbu

$$\sqrt{(x - x_0)^2 + (y - y_0)^2} < \delta$$

shartni bajaruvchi barcha nuqtalar uchun

$$|f(x; y) - A| < \varepsilon$$

tengsizlik o'rinli bo'lsa, u holda A o'zgarmas sonni qaralayotgan ikki o'zgaruvchili funksiya $f(x; y)$ ning $(x; y)$ o'zgaruvchilar $(x_0; y_0)$ nuqtaga intilgandagi limiti deyiladi va quyidagicha yoziladi:

$$\lim_{\substack{x \rightarrow x_0 \\ y \rightarrow y_0}} f(x; y) = A.$$

Qaralayotgan $z = f(x; y)$ ikki o'zgaruvchili funksiyaning xususiy hosilalari quyidagicha topiladi:

$$z'_x = \frac{\partial z}{\partial x} = \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x; y) - f(x; y)}{\Delta x},$$

$$z'_y = \frac{\partial z}{\partial y} = \lim_{\Delta y \rightarrow 0} \frac{f(x; y + \Delta y) - f(x; y)}{\Delta y}.$$

Bulardan ko'rinadiki, ko'p o'zgaruvchili funksiyaning biror o'zgaruvchi bo'yicha xususiy hosilasini topish uchun, qolgan o'zgaruvchilariga o'zgarmas son sifatida qarab, xuddi bir o'zgaruvchili funksiyaning hosilasi kabi hosila hisoblash kifoya.

Biror $(x_0; y_0)$ nuqtani o'z ichiga olgan sohada $f(x; y)$ funksiyaning uchunchi tartibgacha uzluksiz xususiy hosilalari mavjud bo'lsin. Hamda $(x_0; y_0)$ nuqtada uning birinchi tartibli xususiy hosilalari nolga aylansin.

Agar $\frac{\partial^2 f(x_0; y_0)}{\partial x^2} \cdot \frac{\partial^2 f(x_0; y_0)}{\partial y^2} - \left(\frac{\partial^2 f(x_0; y_0)}{\partial x \partial y} \right)^2 > 0$ bo'lib, $\frac{\partial^2 f(x_0; y_0)}{\partial x^2} < 0$ yoki

$\frac{\partial^2 f(x_0; y_0)}{\partial y^2} < 0$ bo'lsa, u holda qaralayotgan nuqtada qaralayotgan funksiya

maksimumga erishadi.

Agar $\frac{\partial^2 f(x_0; y_0)}{\partial x^2} \cdot \frac{\partial^2 f(x_0; y_0)}{\partial y^2} - \left(\frac{\partial^2 f(x_0; y_0)}{\partial x \partial y} \right)^2 < 0$ bo'lib, $\frac{\partial^2 f(x_0; y_0)}{\partial x^2} > 0$

yoki $\frac{\partial^2 f(x_0; y_0)}{\partial y^2} > 0$ bo'lsa, u holda qaralayotgan nuqtada qaralayotgan funksiya

minimumga erishadi.

Agar $\frac{\partial^2 f(x_0; y_0)}{\partial x^2} \cdot \frac{\partial^2 f(x_0; y_0)}{\partial y^2} - \left(\frac{\partial^2 f(x_0; y_0)}{\partial x \partial y} \right)^2 < 0$ bo'lsa, u holda

qaralayotgan nuqtada qaralayotgan funksiyaning ekstrimumi mavjud bo'lmaydi.

Agar $\frac{\partial^2 f(x_0; y_0)}{\partial x^2} \cdot \frac{\partial^2 f(x_0; y_0)}{\partial y^2} - \left(\frac{\partial^2 f(x_0; y_0)}{\partial x \partial y} \right)^2 = 0$ bo'lsa, u holda

qaralayotgan nuqtada qaralayotgan funksiyaning ekstrimumga erishish masalasi ochiq qoladi.

Mavzuga doir misollarni yechishga namunalar

1-misol. $z = \sqrt{4 - x^2 - y^2}$ funksiyaning aniqlanish sohasini toping.

Yechish. Qaralayotgan funksiya ma'noga ega bo'lishi uchun ushbu

$$4 - x^2 - y^2 \geq 0$$

shartning bajarilishi talab etiladi. Bundan $x^2 + y^2 \leq 2^2$ o'rinli bo'ladi. Bu esa qaralayotgan funksiyaning aniqlanish sohasi markazi koordinata boshida, radiusi 2 birlikga teng bo'lgan doiradan iborat ekanligini bildiradi.

2-misol. $\lim_{\substack{x \rightarrow 1 \\ y \rightarrow 0}} \frac{\sin(x+y-1)}{x+y-1}$ ni hisoblang.

Hisoblash. Agar $\alpha(x; y) = x + y - 1$ desak, u holda $x \rightarrow 1, y \rightarrow 0$ bo'lganda $\alpha(x; y) \rightarrow 0$ bo'ladi. Shunga ko'ra,

$$\lim_{\substack{x \rightarrow 1 \\ y \rightarrow 0}} \frac{\sin(x+y-1)}{x+y-1} = \lim_{\alpha(x; y) \rightarrow 0} \frac{\sin \alpha(x; y)}{\alpha(x; y)} = 1$$

o'rinli bo'ladi.

Javob: 1.

3-misol. Quyidagi funksiylarning xususiy hosilalarini toping:

$$z = x^2 \sin^2 y,$$

$$u = e^{x^2+y^2+z^2},$$

$$w = \arctg(xy).$$

Yechish.

$$z'_x = (x^2 \cdot \sin^2 y)'_x = 2x \sin^2 y,$$

$$z'_y = (x^2 \cdot \sin^2 y)'_y = x^2 2 \sin y \cdot \cos y = x^2 \sin 2y,$$

$$u'_x = (e^{x^2+y^2+z^2})'_x = 2xe^{x^2+y^2+z^2},$$

$$u'_y = (e^{x^2+y^2+z^2})'_y = 2ye^{x^2+y^2+z^2},$$

$$u'_z = (e^{x^2+y^2+z^2})'_z = 2ze^{x^2+y^2+z^2},$$

$$w'_x = (\arctg(xy))'_x = \frac{y}{1+x^2y^2},$$

$$w'_y = (\arctg(xy))'_y = \frac{x}{1+x^2y^2}.$$

4-misol. Ushbu

$$z = x^2 + xy + y^2 - 3x - 3y + 1$$

funksiya ekstrimumga tekshirilsin.

Yechish. 1) Funksiyaning xususiy hosilalarini topamiz:

$$z'_x = (x^2 + xy + y^2 - 3x - 3y + 1)'_x = 2x + y - 3,$$

$$z'_y = (x^2 + xy + y^2 - 3x - 3y + 1)'_y = x + 2y - 3.$$

2) Topilgan xususiy hosilalarni nolga tenglab, quyidagi sistemani hosil qilamiz:

$$\begin{cases} 2x + y - 3 = 0 \\ x + 2y - 3 = 0 \end{cases}$$

Uni yechib, $x_0 = 1$, $y_0 = 1$ ni topamiz. Bundan ko'rinadiki, (1;1) nuqtada funksiyaning birinchi tartibli xususiy hosilalari nolga aylanadi.

3) Funksiyaning ikkinchi tartibli xususiy hosilalarini topamiz:

$$z''_{xx} = (z'_x)'_x = (2x + y - 3)'_x = 2,$$

$$z''_{yy} = (z'_y)'_y = (x + 2y - 3)'_y = 2,$$

$$z''_{xy} = (z'_x)'_y = (2x + y - 3)'_y = 1.$$

Bulardan ko'rinadiki, $z''_{xx}(1;1) = 2$, $z''_{yy}(1;1) = 2$, $z''_{xy}(1;1) = 1$ bo'ladi.

4) Ikki o'zgaruvchili funksiyaning ekstremumga erishish shartidan foydalanib qaralayotgan funksiyaning ekstremumga tekshiramiz.

$$\frac{\partial^2 z(1;1)}{\partial x^2} \cdot \frac{\partial^2 z(1;1)}{\partial y^2} - \left(\frac{\partial^2 z(1;1)}{\partial x \partial y} \right)^2 = 2 \cdot 2 - 1^2 = 3 > 0$$

$$\frac{\partial^2 z(1;1)}{\partial x^2} = 2 > 0 \text{ yoki } \frac{\partial^2 z(1;1)}{\partial y^2} = 2 > 0.$$

Bulardan ko'rinadiki, qaralayotgan funksiya qaralayotgan (1;1) nuqtada minimumga erishadi. $z_{\min} = z(1;1) = 1^2 + 1 \cdot 1 + 1^2 - 3 \cdot 1 - 3 \cdot 1 + 1 = -2$.

Nazariy savollar va mashqlar.

1. Ikki o'zgaruvchili funksiya deb nimaga aytiladi?
2. Ikki o'zgaruvchili funksiyaning aniqlanish sohasi qanday topiladi? Bu tushunchalar qanday geometrik talqinga ega?
3. Ikki o'zgaruvchili funksiyaning limiti deb nimaga aytiladi?
4. Qachon ikki o'zgaruvchili funksiyaning nuqtada uzluksiz desa bo'ladi? Sohada-chi?
5. Ikki o'zgaruvchili funksiyaning hususiy hosilalari qanday topiladi?
6. Ikki o'zgaruvchili funksiyaning to'la differensial deb nimaga aytiladi?
7. Qanday qilib to'la differensial yordamida funksiyaning qiymatini taqribiy hisoblash mumkin?
8. Yuqori tartibli hususiy hosilalar qanday topiladi?
9. Ikki o'zgaruvchili funksiyaning aralash hususiy hosilalarining tengligi haqidagi teoremani isbotlang.

10. Ushbu $u = \ln \sqrt{x^2 + y^2}$ funksiya $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ tenglamani qanoatlantirishini isbotlang.

11. Ushbu $y = f(x+at) + \phi(x-at)$ ikki marta differensiallanuvchi ixtiyoriy funksiya

$$\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2} \text{ tenglamani qanoatlantirishini ko'rsating. Bu erda } a \text{ o'zgarmas}$$

son.

12. Ushbu
$$z = \begin{cases} \frac{2xy}{x^2 + y^2}, & \text{agar } x^2 + y^2 \neq 0 \\ 0, & \text{agar } x = y = 0 \end{cases}$$

funksiya har bir o'zgaruvchisi bo'yicha alohida-alohida uzluksiz ekanligini, ammo $(0; 0)$ nuqtada esa uzilishga ega ekanligini ko'rsating. Bu funksiya $(0; 0)$ nuqtada hususiy hosilalarga ega bo'la oladimi? Bo'lsa, ular nimaga teng?

13. Ikki o'zgaruvchili funksiyaning ekstremumi deb nimaga aytiladi?
14. Ikki o'zgaruvchili funksiyaning ekstremumga erishishining zaruriy va etarli shartlari nimalardan iborat?

Loyiha-hisob ishi topshiriqlari

1 – masala. Funksiyaning birinchi tartibli hususiy hosilalarini toping.

1. 1. 1. $z = 10\sqrt[5]{3x^4 - 6xy^3}$.

1. 1. 2. $z = \frac{x^2 y^2}{\sqrt{x^2 - y^2}}$.

1. 1. 3. $z = 0,3\sin^4(6x + 5y)$.

1. 1. 4. $z = xtg(2x - 5y)$.

1. 1. 5. $z = 3^{\frac{y-x}{x-y}}$.

1. 1. 6. $z = \sqrt[3]{\cos(13x - 9y)}$.

1. 1. 7. $z = xye^{10y-x}$.

1. 1. 8. $z = \sqrt{1 - 4x^2 + 10y^3}$.

1. 1. 9. $z = 4\ln ctg\left(\frac{x}{4} - \frac{y}{2}\right)$.

1. 1. 10. $z = \frac{1}{10} \arctg \frac{5x}{2y}$.

1. 1. 11. $z = \frac{y^2}{3x} + \arcsin(xy)$.

1. 1. 12. $z = \ln(x + e^{-y})$.

1. 1. 13. $z = e^{-2x} \sqrt{x - y}$.

1. 1. 14. $z = x \sin \frac{y}{x}$.

1. 1. 15. $z = \arccos \frac{y^2}{x}$.

1. 1. 16. $z = xy + \frac{x}{y}$.

1. 1. 17. $z = ytg(2x + y)$.

1. 1. 18. $z = e^{\sin^2 \frac{y}{x}}$.

1. 1. 19. $z = x^3 y^2 (6 - x - y)$.

1. 1. 20. $z = \sqrt{2xy + y^2}$.

1. 1. 21. $z = \ln tg \frac{y}{x}$.

1. 1. 22. $z = y \cos \frac{x}{y}$.

1. 1. 23. $z = \arctg \frac{x}{x^2 + y^2}$.

1. 1. 24. $z = (x - y)e^{-x}$.

1. 1. 25. $z = x^2 e^{x+2y}$.

1. 1. 26. $z = x \ln \frac{y}{x}$.

1. 1. 27. $z = \sqrt{xy + \frac{x}{y}}$.

1. 1. 28. $z = \frac{x}{x^2 + y^2}$.

1. 1. 29. $z = \ln \sin \frac{x+a}{\sqrt{y}}$.

1. 1. 30. $z = ye^{\frac{x^2}{2y^2}}$.

2 – masala. Berilgan funksiya va ikki nuqtaga ko'ra quyidagilarni toping:

a) A nuqtadan B nuqtaga o'tishdagi funksiyaning ortirmasini uning differentsiali bilan almashtirib, so'ngra funksiyaning A nuqtadagi qiymatidan foydalanib, uning B nuqtadagi qiymatini taqribiy hisoblang;

b) funksiyaning B nuqtadagi qiymatini aniq hisoblab, hosil bo'lgan nisbiy hatoni foizlarda baholang.

1. 2. 1. $z = 3x^2 + 2y^2 - xy$, $A(-1; 3)$, $B(-0,98; 2,97)$.

1. 2. 2. $z = x^2 + 2xy - x + 5y$, $A(3; 2)$, $B(2,97; 2,02)$.

1. 2. 3. $z = 2x^2 + y^2 + x - 3y$, $A(2; 1)$, $B(2,02; -0,99)$.

1. 2. 4. $z = x^2 - y^2 + 3xy$, $A(-1; -1)$, $B(-0,97; -1,02)$.

1. 2. 5. $z = xy + 2y^2 - 6x^2$, $A(1; 4)$, $B(1,03; 4,01)$.

1. 2. 6. $z = 6x + 5y - 4xy$, $A(3; -4)$, $B(3,04; -4,02)$.

1. 2. 7. $z = x^2 - 4xy - y^2$, $A(-2; 3)$, $B(-2,02; 2,97)$.

1. 2. 8. $z = y^2 - 2xy + 6x$, $A(-2; 5)$, $B(-1,98; 5,01)$.

1. 2. 9. $z = x^2 - y^2 + 5x + 4y$, $A(3; 2)$, $B(3,02; 1,98)$.

1. 2. 10. $z = xy + 4x - 3y$, $A(4; -3)$, $B(3,98; -3,03)$.

1. 2. 11. $z = x^2 + xy + y^2$, $A(1; 2)$, $B(1,02; 1,96)$.

1. 2. 12. $z = 3x^2 - xy + x + y$, $A(1; 3)$, $B(1,06; 2,92)$.

1. 2. 13. $z = x^2 + 3xy - 6y$, $A(4; 1)$, $B(3,96; 1,03)$.

1. 2. 14. $z = x^2 - y^2 + 6x + 3y$, $A(2; 3)$, $B(2,02; 2,97)$.

1. 2. 15. $z = x^2 + 2xy + 3y^2$, $A(2; 1)$, $B(1,96; 1,04)$.

1. 2. 16. $z = x^2 + y^2 + 2x + y - 1$, $A(2; 4)$, $B(1,98; 3,91)$.

1. 2. 17. $z = 2xy + 3y^2 - 5x$, $A(3; 4)$, $B(3,04; 3,95)$.

1. 2. 18. $z = xy + 2y^2 - 2x$, $A(1; 2)$, $B(0,97; 2,03)$.

1. 2. 19. $z = x^2 - 3xy - y^2$, $A(1; 1)$, $B(0,98; 0,97)$.

1. 2. 20. $z = 2x^2 - 3y^2 - x + y$, $A(-1; 2)$, $B(-1,02; 2,02)$.

1. 2. 21. $z = x^2 - 2xy + y^2 + x$, $A(-2; 3)$, $B(-1,98; 2,97)$.

1. 2. 22. $z = 3x^2 - y^2 + 2x + y$, $A(2; 2)$, $B(2,02; 1,98)$.

1. 2. 23. $z = x^2 + y^2 + 4x - y$, $A(1; 3)$, $B(0,98; .3,01)$.
1. 2. 24. $z = -x^2 + xy + y^2 + y$ $A(-2; 1)$, $B(-1,97; 1,02)$.
1. 2. 25. $z = 5x^2 - 2xy - y^2 + 3x$, $A(-1; -3)$, $B(-1,02; -2,98)$.
1. 2. 26. $z = 4x^2 + 3xy + 4x$, $A(1; -3)$, $B(1,04; -3,03)$.
1. 2. 27. $z = 5x^2 + xy - y^2 + 2x$, $A(0; -2)$, $B(0,03; -1,96)$.
1. 2. 28. $z = 7xy - y^2 - 2x - y$, $A(1; -1)$, $B(0,98; -1,03)$.
1. 2. 29. $z = x^2 + 5xy + 2x - 4y$, $A(3; -3)$, $B(3,04; -2,97)$.
1. 2. 30. $z = -2x^2 + 4xy - y^2 - y$, $A(-3; -3)$, $B(-2,97; -3,04)$.

2 – topshiriq

Mavzu: Differensial tenglamalar

Nazariy tushunchalar. Ushbu

$$F(x; y; y') = 0$$

ko'rinishdagi tenglama birinchi tartibli oddiy differensial tenglama deyiladi. Bu yerda $y = y(x)$ izlanayotgan noma'lum funksiya. Berilgan tenglamani ayniyatga aylantiradigan ixtiyoriy $y = f(x)$ funksiyaga shu tenglamaning yechimi deyiladi.

Birinchi tartibli differensial tenglamaning biror C o'zgarmas songa bog'liq bo'lgan $y = f(x; C)$ ko'rinishdagi yechimiga, uning umumiy yechimi deyiladi.

Agar differensial tenglamaning yechimi $\Phi(x; y; C) = 0$ ko'rinishdagi oshkormas funksiya sifatida topilgan bo'lsa, unga differensial tenglamaning umumiy integrali deyiladi.

Ushbu $y(x_0) = y_0$ boshlang'ich shartdan foydalanib tenglamaning umumiy yechimidan kelib chiqqan $y = \Phi(x; C_0)$ funksiyaga shu tenglamaning xususiy yechimi deyiladi.

Xuddi shuningdek, boshlang'ich shartdan foydalanib, tenglamaning umumiy integralidan kelib chiqadigan $\Phi(x; y; C_0) = 0$ funksiyaga shu tenglamaning xususiy integrali deyiladi.

Ushbu

$$M(x)dx + N(y)dy = 0$$

ko'rinishidagi tenglama birinchi tartibli o'zgaruvchilari ajralgan differensial tenglama deyiladi. Bu tenglamani yechish uchun uni bevosita integrallanadi.

Ushbu

$$M_1(x)N_2(y)dx + M_2(x)N_1(y)dy = 0$$

ko'rinishidagi tenglama birinchi tartibli o'zgaruvchilari ajraladigan differensial tenglama deyiladi. Bu tenglamani yechish uchun uning har ikkala tomonini $N_2(y)M_2(x)$ ga bo'lib, natijani bevosita integrallaymiz. U holda

$$\int \frac{M_1(x)}{M_2(x)} dx + \int \frac{N_1(y)}{N_2(y)} dy = C$$

hosil bo'ladi.

Ushbu $f(x; y)$ ikki o'zgaruvchili funksiya uchun $f(\lambda x; \lambda y) = \lambda^m f(x; y)$ munosabat o'rinli bo'lsa, u holda uni argumentlariga nisbatan m - o'lchovli bir jinsli funksiya deb yuritiladi. Masalan $f(x; y) = \sqrt[3]{x^2 + y^2}$ funksiya o'z argumentlarining $m = \frac{2}{3}$ o'lchovli funksiyasidir, chunki

$$f(\lambda x; \lambda y) = \sqrt[3]{(\lambda x)^2 + (\lambda y)^2} = \lambda^{\frac{2}{3}} \sqrt[3]{x^2 + y^2} = \lambda^{\frac{2}{3}} f(x; y)$$

o'rinli. Bu yerda $m = \frac{2}{3}$.

Agar $f(x; y)$ funksiya o'z argumentlarining nol o'lchovli bir jinsli funksiyasi bo'lsa, u holda

$$y' = f(x; y)$$

ko'rinishdagi tenglama birinchi tartibli bir jinsli differensial tenglama deyiladi.

Bunday tenglama $u(x) = \frac{y(x)}{x}$ almashtirish bajarilib yechiladi.

Ushbu

$$y' = f\left(\frac{a_1x + b_1y + c_1}{a_2x + b_2y + c_2}\right)$$

ko'rinishdagi tenglama birinchi tartibli bir jinsli tenglamaga keladigan differensial tenglama deyiladi. Uni yechish uchun $x = t + t_0$, $y = u(t) + u_0$ belgilashlarni kiritib, t_0 va u_0 ni topish uchun

$$\begin{cases} a_1t_0 + b_1u_0 + c_1 = 0 \\ a_2t_0 + b_2u_0 + c_2 = 0 \end{cases}$$

algebraik tenglamalar sistemasini hamda $u(x)$ funksiyani topish uchun

$$\frac{du}{dt} = f\left(\frac{a_1t + b_1u}{a_2t + b_2u}\right)$$

birinchi tartibli bir jinsli differensial tenglamani hosil qilamiz.

Ushbu

$$y' + P(x)y = Q(x)$$

ko'rinishdagi tenglama birinchi tartibli chiziqli differensial tenglama deyiladi. Bu yerda $P(x)$ va $Q(x)$ funksiyalar o'z argumetlarining uzluksiz funksiyalaridir. Chiziqli differensial tenglamaning yechimi $y(x) = u(x) \cdot v(x)$ ko'rinishda, ya'ni ikkita noma'lum funksiyalarning ko'paytmasi ko'rinishida izlanadi. Buning asosiy sababi, noma'lum funksiyalardan birini tanlash imkoniyati borligida. Izlanayotgan yechimni berilgan tenglamaga qo'yib,

$$u'v + uv' + P(x)uv = Q(x)$$

yoki

$$u'v + u(v' + P(x)v) = Q(x)$$

tenglamani hosil qilamiz. $u(x)$ va $v(x)$ funksiyalarni ixtiyoriy tanlash imkoniyatidan foydalanib, qavs ichidagi ifodani nolga tenglaymiz. Natijada

$$v'(x) + P(x)v(x) = 0$$

ko'rinishdagi birinchi tartibli o'zgaruvchilari ajraladigan differensial tenglamaga kelamiz. Uni yechib, topilgan yechimni joyiga qo'yib, ushbu

$$u'(x)v(x) = Q(x)$$

ko'rinishdagi birinchi tartibli o'zgaruvchilari ajraladigan differensial tenglamani hosil qilamiz. Uni yechib, $u = u(x)$ funksiya topiladi. Topilgan funksiyalarni $y = u(x) \cdot v(x)$ belgilashga qo'yib, berilgan birinchi tartibli chiziqli differensial tenglamaning umumiy yechimiga ega bo'lamiz.

Agar birinchi tartibli chiziqli differensial tenglama boshlang'ich shart bilan berilgan bo'lsa, u holda uni Koshi masalasi deb yuritiladi.

Ushbu

$$y' + P(x)y = Q(x)y^\alpha$$

ko'rinishdagi tenglama birinchi tartibli Bernulli differensial tenglamasi deyiladi. Bu yerda $P(x)$ va $Q(x)$ funksiyalar qaralayotgan sohada uzluksiz funksiyalar bo'lib, $\alpha \neq 0$ va $\alpha \neq 1$ ekanligi ma'lum. Aks holda Bernulli differensial tenglamasi birinchi tartibli chiziqli differensial tenglama bo'lib qoladi. Tenglamaning har ikkala tomonini y^α ga bo'lib

$$y^{-\alpha} y' + P(x)y^{-\alpha+1} = Q(x)$$

tenglamani hosil qilamiz. Bunda

$$w(x) = y^{-\alpha+1}(x)$$

almashtirishni kiritsak,

$$w'(x) = (-\alpha + 1)y^{-\alpha} y'(x)$$

ga ega bo'lamiz. Ularni qaralayotgan tenglamaga qo'yib

$$\frac{w'(x)}{1-\alpha} + P(x)w(x) = Q(x)$$

yoki

$$w'(x) + (1-\alpha)P(x)w(x) = (1-\alpha)Q(x)$$

ko'rinishdagi birinchi tartibli chiziqli differensial tenglamaga kelamiz.

Eslatib o'tamizki, Bernulli differensial tenglamasini bevosita Bernulli usuli bilan ham ya'ni, yechimni $y(x) = u(x) \cdot v(x)$ ko'rinishda izlab yechish mumkin.

Agar

$$\frac{\partial M(x; y)}{\partial y} = \frac{\partial N(x; y)}{\partial x}$$

shart bajarilgan bo'lsa, u holda

$$M(x; y)dx + N(x; y)dy = 0$$

tenglamani to'liq differensialli tenglama deyiladi. Bu tenglama uchun shunday $u(x; y)$ funksiya topiladiki, uni

$$du(x; y) = 0$$

ko'rinishda yozish mumkin bo'ladi. Natijada tenglamaning umumiy integrali

$$u(x; y) = C$$

ko'rinishda bo'ladi. Bundan ko'rinadiki, to'liq differensialli tenglamani yechishdagi asosiy muammo $u(x; y)$ funksiyani topishdir.

Faraz qilaylik, ushbu

$$M(x; y)dx + N(x; y)dy = 0$$

tenglama uchun to'liq differensiallilik sharti bajarilmasin. Ammo, shunday $\lambda(x; y)$ funksiya topilsinki, tenglamani shu funksiya ko'paytirilgandan so'ng u to'liq differensialli tenglamaga aylanib qolsin. $\lambda(x; y)$ funksiyani integrallovchi ko'paytuvchi deb yuritiladi. Uni topish uchun

$$\lambda M dx + \lambda N dy = 0$$

tenglamaning to'liq differensialli tenglama bo'lib qolganlik shartidan foydalaniladi.

$$\frac{\partial(\lambda M)}{\partial y} = \frac{\partial(\lambda N)}{\partial x}$$

bundan

$$\lambda \frac{\partial M}{\partial y} + M \frac{\partial \lambda}{\partial y} = \lambda \frac{\partial N}{\partial x} + N \frac{\partial \lambda}{\partial x}$$

bundan

$$M \frac{\partial \lambda}{\partial y} - N \frac{\partial \lambda}{\partial x} = \lambda \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} \right)$$

bundan

$$M \frac{\partial \ln \lambda}{\partial y} - N \frac{\partial \ln \lambda}{\partial x} = \frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}$$

hosil bo'ladi.

Agar integrallovchi ko'paytuvchi faqat x o'zgaruvchining funksiyasi bo'lsa, u holda

$$\frac{\partial \ln \lambda}{\partial x} = \frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N}$$

kelib chiqadi va o'ng tomondagi ifoda faqat x o'zgaruvchining funksiyasi bo'ladi.

Agar integrallovchi ko'paytuvchi faqat y o'zgaruvchining funksiyasi bo'lsa, u holda

$$\frac{\partial \ln \lambda}{\partial y} = \frac{\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}}{M}$$

kelib chiqadi va o'ng tomondagi ifoda faqat y o'zgaruvchining funksiyasi bo'ladi.

Bu hollarda integrallovchi ko'paytuvchini topish qulaylashadi.

Tarkibiga y o'zgaruvchi oshkor kirmaydigan $F(x; y'; y'') = 0$ ikkinchi tartibli differensial tenglama $y' = p(x)$ almashtirish yordamida $F(x; p; p') = 0$ ko'rinishidagi birinchi tartibli differensial tenglamaga keltirilib yechiladi.

Tarkibiga x o'zgaruvchi oshkor kirmaydigan $F(y; y'; y'') = 0$ ikkinchi tartibli differensial tenglama $y' = p(y)$ almashtirish yordamida $F(y; p(y); p'(y)p(y)) = 0$ ko'rinishidagi birinchi tartibli differensial tenglamaga keltirilib yechiladi.

Ushbu

$$y^{(n)} + a_1 y^{(n-1)} + \dots + a_n y = f(x)$$

ko'rinishdagi tenglamaga n -tartibli o'zgarmas koeffitsiyentli chiziqli bir jinsli bo'lmagan differensial tenglama deyiladi. Bu yerda a_1, a_2, \dots, a_n - berilgan o'zgarmas sonlar, $f(x)$ - tenglamaning o'ng tomoni deb ataluvchi uzluksiz

funksiya. Qaralayotgan tenglamaning umumiy yechimi uning o'ng tomoniga mos bo'lgan biror xususiy yechimi $y^*(x)$ bilan bir jinsli qismi

$$y^{(n)} + a_1 y^{(n-1)} + \dots + a_n y = 0$$

n -tartibli o'zgarmas koeffitsiyentli chiziqli bir jinsli differensial tenglamaning umumiy yechimi $\tilde{y}(x)$ larning yig'indisidan iboratdir.

$\tilde{y}(x)$ yechimni topish uchun quyidagi ishlarni bajarish lozim bo'ladi:

- 1) $k^n + a_1 k^{n-1} + a_2 k^{n-2} + \dots + a_n = 0$ xarakteristik tenglama tuziladi.
- 2) xarakteristik tenglamaning ildizlari k_1, k_2, \dots, k_n topiladi.
- 3) har bir karrali bo'lmagan k haqiqiy ildiz uchun e^{kx} ko'rinishdagi xususiy yechim yoziladi.
- 4) har bir juft karrali bo'lmagan $k = \alpha + i\beta$ kompleks ildiz uchun $e^{\alpha x} \cos \beta x$ va $e^{\alpha x} \sin \beta x$ xususiy yechimlar yoziladi.
- 5) har bir m karrali haqiqiy k ildiz uchun $e^{kx}, x e^{kx}, x^2 e^{kx}, \dots, x^{m-1} e^{kx}$ chiziqli erkli xususiy yechimlar yoziladi.
- 6) har bir juft p karrali $k = \alpha + i\beta$ kompleks ildizlar uchun

$$e^{\alpha x} \cos \beta x, x e^{\alpha x} \cos \beta x, x^2 e^{\alpha x} \cos \beta x, \dots, x^{p-1} e^{\alpha x} \cos \beta x,$$

$$e^{\alpha x} \sin \beta x, x e^{\alpha x} \sin \beta x, x^2 e^{\alpha x} \sin \beta x, \dots, x^{p-1} e^{\alpha x} \sin \beta x$$

chiziqli erkli xususiy yechimlar yoziladi.

- 7) Topilgan chiziqli erkli xususiy y_1, y_2, \dots, y_n yechimlardan foydalanib

$$\tilde{y}(x) = C_1 y_1 + C_2 y_2 + \dots + C_n y_n$$

ko'rinishdagi umumiy yechim yoziladi.

Tenglamaning o'ng tomonidagi funksiyaning ba'zi ko'rinishlari uchun $y^*(x)$ xususiy yechimni topish usulini qisqacha bayon qilamiz:

- 1) $f(x) = e^{\alpha x} P_n(x)$ ko'rinishda bo'lsin. Bu yerda $P_n(x)$ n -tartibli ko'phad.

Agar α soni xarakteristik tenglamaning ildizlarining birortasi bilan ham ustma-ust tushmasa, u holda xususiy yechim $y^*(x) = e^{\alpha x} Q_n(x)$ ko'rinishda

qidiriladi. Bu yerda $Q_n(x)$ n -tartibli noma'lum koeffitsiyentli ko'phad bo'lib, uning koeffitsiyentlari noma'lum koeffitsiyentlar usuli bilan topiladi.

Agar α soni xarakteristik tenglamaning biror m karrali ildizi bilan ustma-ust tushib qolsa, u holda xususiy yechim $y^*(x) = x^m e^{\alpha x} Q_n(x)$ ko'rinishda izlanadi.

2) $f(x) = e^{\alpha x} (U_p(x) \cos \beta x + W_q(x) \sin \beta x)$ ko'rinishida bo'lsin. Bu yerda $U_p(x)$, $W_q(x)$ - mos ravishda p va q -tartibli ko'phadlar.

Agar $\alpha + i\beta$ kompleks soni xarakteristik tenglamaning hech qaysi ildizi bilan ustma-ust tushmasa, u holda xususiy yechim

$$y^*(x) = e^{\alpha x} (N_s(x) \cos \beta x + M_s(x) \sin \beta x)$$

ko'rinishida izlanadi. Bu yerda $N_s(x)$, $M_s(x)$ - mos ravishda noma'lum koeffitsiyentli s -tartibli ko'phadlar ($s = \max\{p; q\}$) bo'lib, ularning koeffitsiyentlari noma'lum koeffitsiyentlar usuli bilan topiladi.

Agar $\alpha + i\beta$ kompleks soni xarakteristik tenglamaning biror l karrali kompleks ildizi bilan ustma-ust tushib qolsa, u holda xususiy yechim

$$y^*(x) = x^l e^{\alpha x} (N_s(x) \cos \beta x + M_s(x) \sin \beta x)$$

ko'rinishida izlanadi.

Mavzuga doir misollarni yechishga namunalar

1-misol. $52x dx - 5y dy = 5x^2 y dy - 13xy^2 dx$ tenglamaning umumiy yechimini toping.

Yechish. Berilgan tenglama birinchi tartibli o'zgaruvchilari ajraladigan differensial tenglamadir. O'zgaruvchilarini ajratish uchun quyidagi almashtirishni bajaramiz:

$$52x dx + 13xy^2 dx - 5y dy - 5x^2 y dy = 0,$$

$$13x(4 + y^2) dx - 5y(1 + x^2) dy = 0,$$

$$\frac{13xdx}{1+x^2} - \frac{5ydy}{4+y^2} = 0.$$

Hosil bo'lgan ifodani bevosita integrallab:

$$\int \frac{13xdx}{1+x^2} - \int \frac{5ydy}{4+y^2} = C$$

yoki

$$\frac{13}{2} \ln(1+x^2) - \frac{5}{2} \ln(4+y^2) = \frac{1}{2} \ln C_1$$

yoki

$$\ln \frac{(1+x^2)^{13}}{(4+y^2)^5} = \ln C_1$$

yoki

$$\frac{(1+x^2)^{13}}{(4+y^2)^5} = C_1$$

ko'rinishdagi umumiy integralni topamiz.

2-misol. $y' = \frac{y^2}{x^2} + \frac{7y}{x} + 9$ differensial tenglamani yeching.

Yechish. Berilgan tenglama birinchi tartibli bir jinsli differensial tenglamadir. Chunki, $f(x; y) = \frac{y^2}{x^2} + \frac{7y}{x} + 9$, $f(\lambda x; \lambda y) = \frac{\lambda^2 y^2}{\lambda^2 x^2} + \frac{7\lambda y}{\lambda x} + 9 = \frac{y^2}{x^2} + \frac{7y}{x} + 9 = \lambda^0 \left(\frac{y^2}{x^2} + \frac{7y}{x} + 9 \right) = \lambda^0 f(x; y)$, $m = 0$.

Tenglamaning yechimini $y = x \cdot u(x)$ ko'rinishda qidiramiz. U holda $y' = u(x) + xu'(x)$ bo'ladi. Bularni berilgan tenglamaga qo'yib $u(x) + xu'(x) = u^2(x) + 7u(x) + 9$ yoki $xu'(x) = u^2(x) + 6u(x) + 9$ ko'rinishdagi birinchi tartibli o'zgaruvchilari ajraladigan differensial tenglamani hosil qilamiz. Uning o'zgaruvchilarini ajratib:

$$\frac{du}{u^2 + 6u + 9} = \frac{dx}{x}$$

yoki

$$\frac{du}{(u+3)^2} = \frac{dx}{x}$$

va bevosita integrallab,

$$-\frac{1}{u+3} = \ln|x| + \ln C$$

yoki

$$-\frac{1}{u+3} = \ln C|x|$$

yoki

$$(u(x)+3) \ln C|x| + 1 = 0$$

ko'rinishdagi yechimni topamiz. Unga $u(x) = \frac{y(x)}{x}$ ni qo'yib, berilgan tenglamaning

$$(y(x)+3x) \ln C|x| + x = 0$$

ko'rinishdagi umumiy integralini hosil qilamiz.

3-misol. $y' = \frac{x+y-4}{x-y-5}$ tenglamani yeching.

Yechish. Berilgan tenglama birinchi tartibli bir jincli tenglamaga keladigan differensial tenglamadir. Haqiqatdan ham, $x = t + t_0$, $y = u(t) + u_0$ almashtirishni kiritsak:

$$\frac{du}{dx} = \frac{t+u+t_0+u_0-4}{t-u+t_0-u_0-5}$$

hosil bo'ladi. Agar

$$\begin{cases} t_0 + u_0 - 4 = 0 \\ t_0 - u_0 - 5 = 0 \end{cases}$$

deb, hosil qilingan algebraik tenglamalar sistemasini yechsak $t_0 = 4,5$; $u_0 = -0,5$ ekanligini topamiz. U holda

$$\frac{du}{dx} = \frac{t+u}{t-u}$$

ko'rinishdagi birinchi tartibli bir jinsli differensial tenglama hosil bo'ladi. Chunki,

$$f(t; u) = \frac{t+u}{t-u}, \quad f(\lambda t; \lambda u) = \frac{\lambda t + \lambda u}{\lambda t - \lambda u} = \frac{\lambda(t+u)}{\lambda(t-u)} = \lambda^0 \frac{t+u}{t-u} = \lambda^0 f(t; u), \quad m=0.$$

Tenglamaning yechimini $u = t \cdot w(t)$ ko'rinishda izlaymiz. U holda $u' = w + tw'$ hosil bo'ladi. Ularni yechayotgan tenglamamizga qo'ysak:

$$w + tw' = \frac{t+tw}{t-tw}$$

yoki

$$w + tw' = \frac{1+w}{1-w}$$

yoki

$$tw' = \frac{1+w}{1-w} - w$$

yoki

$$tw' = \frac{1+w^2}{1-w}$$

ko'rinishdagi o'zgaruvchilari ajraladigan birinchi tartibli differensial tenglama hosil bo'ladi. Uning o'zgaruvchilarini ajratib

$$\frac{(1-w)dw}{1+w^2} = \frac{dt}{t}$$

yoki

$$\frac{dw}{1+w^2} - \frac{wdw}{1+w^2} = \frac{dt}{t}$$

ifodani hosil qilamiz. Uni bevosita integrallab,

$$\arctg w - \frac{1}{2} \ln(1+w^2) = \ln|t| + \ln C$$

yoki

$$\arctg w = \ln C |t| \sqrt{1+w^2}$$

ko'rinishdagi umumiy integralni topamiz. Bunda $w(x) = y(x) + 0,5$, $t = x - 4,5$ almashtirishni bajarib, ushbu

$$\operatorname{arctg}(y + 0,5) = \ln C|x - 4,5|\sqrt{1 + (y + 0,5)^2}$$

ko'rinishda umumiy integralni hosil qilamiz.

4-misol. $y' - y \cos x = \sin 2x$, $y(0) = -1$ Koshi masalasini yeching.

Yechish. Berilgan tenglama birinchi tartibli chiziqli differensial tenglamadir.

Bu yerda $P(x) = -\cos x$, $Q(x) = \sin 2x$ uzluksiz funksiyalardir. Yechimni

$$y(x) = u(x) \cdot v(x)$$

ko'rinishda izlaymiz. U holda

$$y'(x) = u'(x) \cdot v(x) + u(x) \cdot v'(x)$$

bo'ladi. Bularni berilgan tenglamaga qo'yib,

$$u' \cdot v + u \cdot v' - u \cdot v \cos x = \sin 2x$$

yoki

$$u' \cdot v + u(v' - v \cos x) = \sin 2x$$

tenglamani hosil qilamiz. $u(x)$ va $v(x)$ funksiyalarni ixtiyoriy tanlash imkoniyatidan foydalanib qavs ichidagi ifodani nolga tenglaymiz va ushbu

$$v' - v \cos x = 0$$

ko'rinishdagi birinchi tartibli o'zgaruvchilari ajraladigan differensial tenglamani hosil qilamiz. Uni yechamiz.

$$\frac{dv}{dx} = v \cos x,$$

$$\frac{dv}{v} = \cos x dx,$$

$$\int \frac{dv}{v} = \int \cos x dx,$$

Shuni ta'kidlash lozimki, yechilayotgan differensial tenglamaning xususiy yechimini topish yetarli, shuning uchun yechimni hosil qilishda ixtiyoriy o'zgarmas sonni yozish shart emas.

$$\ln|v| = \sin x,$$

$$v(x) = e^{\sin x}.$$

Topilgan yechimni $u' \cdot v = \sin 2x$ tenglamaga qo'yib

$$u'(x) \cdot e^{\sin x} = \sin 2x$$

ko'rinishda yana o'zgaruvchilari ajraladigan differensial tenglamani hosil qilamiz. Uni yechamiz.

$$\frac{du}{dx} = \sin 2x \cdot e^{-\sin x}$$

yoki

$$du = \sin 2x \cdot e^{-\sin x} dx$$

yoki

$$u(x) = \int \sin 2x \cdot e^{-\sin x} dx + C$$

bundan

$$\begin{aligned} \int \sin 2x e^{-\sin x} dx &= 2 \int \sin x \cos x e^{-\sin x} dx \Rightarrow \left| \begin{array}{l} \sin x = t \\ \cos x dx = dt \end{array} \right| \Rightarrow 2 \int t e^{-t} dt \Rightarrow \\ \Rightarrow \left| \begin{array}{l} u = t, \quad du = dt \\ dv = e^{-t} dt, \quad v = -e^{-t} \end{array} \right| &\Rightarrow 2(-te^{-t} + \int e^{-t} dt) = 2(-te^{-t} - e^{-t}) = -2e^{-t}(t+1) = \\ &= -2e^{-\sin x}(\sin x + 1) \end{aligned}$$

hosil bo'ladi. Natijada,

$$u(x) = -2e^{-\sin x}(\sin x + 1) + C$$

yechimni topamiz. U holda berilgan chiziqli tenglamaning yechimi

$$y = u(x) \cdot v(x) = (-2e^{-\sin x}(\sin x + 1) + C) \cdot e^{\sin x} = -2(\sin x + 1) + C \cdot e^{\sin x}$$

ko'rinishda bo'ladi. Endi boshlang'ich shartdan foydalansak,

$$-1 = -2 \cdot (\sin 0 + 1) + C \cdot e^{\sin 0},$$

$$-1 = -2 \cdot 1 + C \cdot 1,$$

$$C = 1$$

ni topamiz. Uni o'rniga qo'yib, berilgan Koshi masalasining

$$y(x) = -2(\sin x + 1) + e^{\sin x}$$

ko'rinishdagi xususiy yechimini topamiz.

5-misol. $y' + \frac{y}{2x} = -x^2 y^3$ tenglamaning umumiy integralini toping.

Yechish. Berilgan tenglama birinchi tartibli Bernulli differensial tenglamasidir. Chunki $P(x) = \frac{1}{2x}$, $Q(x) = -x^2$ va $\alpha = 3$.

Tenglamaning har ikkala tomonini y^3 ga bo'lib, so'ngra $w(x) = y^{-2}$ almashtirishni kiritib, $w'(x) = -2y^{-3}$ ni topib, quyidagi

$$w' - \frac{1}{x}w = 2x^2$$

ko'rinishdagi birinchi tartibli chiziqli differensial tenglamaga kelamiz. Uni yechamiz:

$$w(x) = u(x) \cdot v(x),$$

$$w'(x) = u'(x) \cdot v(x) + u(x) \cdot v'(x),$$

$$u'(x) \cdot v(x) + u(x) \cdot v'(x) - \frac{1}{x}u(x) \cdot v(x) = 2x^2,$$

$$u'v + u\left(v' - \frac{1}{x}v\right) = 2x^2,$$

$$v' - \frac{1}{x}v = 0,$$

$$\frac{dv}{dx} = \frac{1}{x}v,$$

$$\frac{dv}{v} = \frac{dx}{x},$$

$$\ln|v| = \ln|x|,$$

$$v(x) = x,$$

$$u'x = 2x^2,$$

$$du = 2xdx,$$

$$u(x) = x^2 + C,$$

$$w(x) = u(x) \cdot v(x) = x^3 + Cx.$$

Natijada berilgan tenglamaning $y^{-2}(x) = w(x) = x^3 + Cx$ yoki $(x^3 + Cx) \cdot y^2 = 1$ ko'rinishdagi yechimi topiladi.

6-misol. $(\cos(x + y^2) + \sin x)dx + 2y \cos(x + y^2)dy = 0$ differensial tenglamani yeching.

Yechish. Berilgan tenglama to'liq differensialli tenglamadir. Chunki,

$$M(x; y) = \cos(x + y^2) + \sin x,$$

$$N(x; y) = 2y \cos(x + y^2)$$

ekanligini e'tiborga olsak,

$$\frac{\partial M}{\partial y} = -2y \sin(x + y^2),$$

$$\frac{\partial N}{\partial x} = -2y \sin(x + y^2)$$

bo'lib, to'liq differensiallik sharti bajariladi. Natijada shunday $u(x; y)$ funksiya topiladiki

$$du(x; y) = (\cos(x + y^2) + \sin x)dx + 2y \cos(x + y^2)dy$$

tenglik o'rinli bo'ladi.

$$du(x; y) = \frac{\partial u}{\partial x} dx + \frac{\partial u}{\partial y} dy$$

ekanligini e'tiborga olsak,

$$\begin{cases} \frac{\partial u}{\partial x} = \cos(x + y^2) + \sin x \\ \frac{\partial u}{\partial y} = 2y \cos(x + y^2) \end{cases}$$

tenglamalar sistemasi hosil bo'ladi. Sistemaning birinchi tenglamasini x o'zgaruvchi bo'yicha integrallab

$$u(x; y) = \sin(x + y^2) - \cos x + \varphi(y)$$

ifodani topamiz. Integrallanayotgan funksiya ikki o'zgaruvchili bo'lganligi tufayli, umumiylikni buzmasdan, integrallashda paydo bo'ladigan ixtiyoriy o'zgaruvchi y o'zgaruvchining funksiyasi deb, uni $C = \varphi(y)$ ko'rinishda olindi. Bu funktsiyani topish uchun yechimni sistemaning ikkinchi tenglamasiga qo'yamiz. U holda

$$2y \cos(x + y^2) + \varphi'(y) = 2y \cos(x + y^2),$$

$$\varphi'(y) = 0,$$

$$\varphi(y) = \text{const}$$

hosil bo'ladi. To'liq differensialli tenglamaning umumiy integrali $u(x; y) = C$ ko'rinishda ekanligini hisobga olib

$$\sin(x + y^2) - \cos x = C$$

ni topamiz.

7-misol. $(y + xy^2)dx - xdy = 0$ differensial tenglamani yeching.

Yechish. Berilgan tenglama to'liq differensialli tenglama emas. Chunki,

$$M(x; y) = y + xy^2, N(x; y) = -x, \frac{\partial M}{\partial y} = 1 + 2xy, \frac{\partial N}{\partial x} = -1, \frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x},$$

ya'ni to'liq differensiallik sharti bajarilmadi.

Integrallovchi ko'paytuvchi topishga harakat qilib ko'ramiz. Buning uchun yuqorida keltirilgan ifodalarning x yoki y o'zgaruvchilardan bog'liqligini tekshirib ko'ramiz.

$$\frac{\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}}{M} = \frac{-1 - 1 - 2xy}{y + xy^2} = -\frac{2}{y}.$$

Natijada $\frac{\partial \ln \lambda}{\partial y} = -\frac{2}{y}$ kelib chiqadi. Bundan $\ln \lambda = -2 \ln y$ yoki $\lambda(y) = \frac{1}{y^2}$ ni

topamiz. Berilgan tenglamani $\lambda(y) = \frac{1}{y^2}$ ga ko'paytirib

$$\left(\frac{1}{y} + x \right) dx - \frac{x}{y^2} dy = 0$$

ko'rinishdagi tenglamani hosil qilamiz. U to'liq differensialli tenglamadir. Chunki,

$$M(x; y) = \frac{1}{y} + x, N(x; y) = -\frac{x}{y^2}, \frac{\partial M}{\partial y} = -\frac{1}{y^2}, \frac{\partial N}{\partial x} = -\frac{1}{y^2} \text{ va } \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \text{ to'liq}$$

differensiallik sharti bajariladi. Endi $u(x; y)$ funksiyani qidirsa bo'ladi.

$$\begin{cases} \frac{\partial u}{\partial x} = \frac{1}{y} + x \\ \frac{\partial u}{\partial y} = -\frac{x}{y^2} \end{cases} \Rightarrow u(x; y) = \frac{x}{y} + \frac{x^2}{2} + \varphi(y) \Rightarrow -\frac{x}{y^2} + \varphi'(y) = -\frac{x}{y^2} \Rightarrow \varphi'(y) = 0 \Rightarrow \varphi(y) = C \quad \text{N}$$

atijada, yechilishi talab etilayotgan tenglamaning $\frac{x}{y} + \frac{x^2}{2} = C$ ko'rinishda umumiy

integralini topamiz.

8-misol. $xy'' - y' = 0$ tenglamani yeching.

Yechish. Tenglamaning tarkibiga y o'zgaruvchi oshkor kirmaydi. Shuning uchun $y' = p(x)$ almashtirish yordamida berilgan tenglamani

$$xp' - p = 0$$

ko'rinishiga keltiramiz. Uni yechib, $p(x) = C_1x$ ni topamiz. Buni kiritilgan

$y' = p(x)$ almashtirishga qo'yib $y'(x) = C_1x$ yoki $y(x) = \frac{1}{2}C_1x^2 + C_2$ umumiy

yechimni hosil qilamiz.

9-misol. $y'' = 2 \sin^3 y \cos y$, $y(1) = \frac{\pi}{2}$, $y'(1) = 1$ differensial tenglamani

yeching.

Yechish. Tenglama tarkibiga x o'zgaruvchi oshkor kirmaydigan ikkinchi tartibli differensial tenglamadir. Uni $y' = p(y)$ almashtirish yordamida darajasini

pasaytiramiz. $y'' = \frac{dp}{dy} \cdot \frac{dy}{dx} = \frac{dp}{dy} \cdot y' = p \cdot p'_y$ ekanligini e'tiborga olib,

$$p \cdot p'_y = 2 \sin^3 y \cos y$$

o'zgaruvchilari ajraladigan birinchi tartibli tenglamani hosil qilamiz. Uning o'zgaruvchilarini ajratamiz va integrallaymiz.

$$pdp = 2 \sin^3 y \cos y dy$$

yoki

$$\int pdp = \int 2 \sin^3 y \cos y dy + \frac{C_1^2}{2}$$

yoki

$$\frac{p^2}{2} = \frac{\sin^4 y}{2} + \frac{C_1^2}{2}.$$

Boshlang'ich shartdan foydalansak $C_1 = 0$ ekanligi kelib chiqadi. Natijada, $p(y) = \sin^2 y$ kelib chiqadi. Topilgan funksiyani $y'(x) = p(y)$ belgilashga qo'ysak,

$$y' = \sin^2 y$$

ko'rinishdagi birinchi tartibli o'zgaruvchilari ajraladigan differensial tenglama hosil bo'ladi. Uni yechib,

$$ctgy + x = C$$

ko'rinishdagi umumiy integralni topamiz. Yana boshlang'ich shartdan foydalansak $C = 1$ ekanligi kelib chiqadi. U holda berilgan tenglamaning hususiy integrali

$$ctgy + x = 1$$

ko'rinishda bo'ladi.

10-misol. $y''' + 5y'' = 7e^{3x}$ tenglamani yeching.

Yechish. Berilgan tenglama uchinchi tartibli o'zgarmas koeffitsiyentli chiziqli bir jinsli bo'lmagan differensial tenglamadir.

1) uning bir jinsli qismiga mos bo'lgan $\tilde{y}(x)$ umumiy yechimini topamiz.

Xarakteristik tenglama

$$k^3 + 5k^2 = 0$$

$k_1 = 0$, $k_2 = 0$, $k_3 = -5$ ko'rinishda haqiqiy ildizlarga ega. $k = 0$ ikki karrali ildiz. Shuning uchun umumiy yechim

$$\tilde{y}(x) = C_1 + C_2 x + C_3 e^{-5x}$$

ko'rinishida bo'ladi.

2) $f(x) = 7e^{3x}$ ko'rinishga ega. Bundan ko'rinadiki $\alpha = 3$, $U_0(x) = 7$, $\beta = 0$. $\alpha + i\beta = 3 + 0 \cdot i$ kompleks ildiz xarakteristik tenglamaning birorta ham ildizi bilan ustma-ust tushmaydi. Shuning uchun xususiy yechim

$$y^*(x) = e^{3x}(A \cos 0 \cdot x + B \sin 0 \cdot x)$$

ko'rinishda qidiriladi. Endi noma'lum koeffitsiyentlari A va B larni topamiz. Buning uchun $y^*(x)$ yechimni berilgan tenglamaga qo'yib bir xil ifodalar oldidagi koeffitsiyentlarni tenglashtiramiz. Natijada ushbu

$$y' = 3Ae^{3x},$$

$$y'' = 9Ae^{3x},$$

$$y''' = 27Ae^{3x},$$

$$27Ae^{3x} + 45Ae^{3x} = 7e^{3x},$$

$$72A = 7,$$

$$A = \frac{7}{72}.$$

Berilgan tenglamaning $y^*(x)$ xususiy yechimi $y^*(x) = \frac{7}{72}e^{3x}$ ko'rinishda bo'ladi.

2) Topilgan umumiy $\tilde{y}(x)$ va xususiy $y^*(x)$ yechimlarini qo'shib, berilgan tenglamaning umumiy yechimini topamiz:

$$y = \tilde{y}(x) + y^*(x) = C_1 + C_2x + C_3e^{-5x} + \frac{7}{72}e^{3x}.$$

11-misol. $y'' - 2y' + 10y = 2\sin 3x + 5\cos x$ tenglamani yeching.

Yechish. Avval $y'' - 2y' + 10y = 0$ bir jinsli tenglamaning umumiy yechimni topamiz. Uning $k^2 - 2k + 10 = 0$ xarakteristik tenglamasi ildizlari $k_{1,2} = 1 \pm 3i$. $\alpha = 1$ va $\beta = 3$ bo'lgani uchun umumiy yechim $y = e^x(C_1 \cos 3x + C_2 \sin 3x)$ ko'rinishda bo'ladi. Berilgan tenglamaning o'ng tomoni turli argumentli trigonometrik funksiyalardan iborat bo'lgani uchun xususiy yechimni ikki xil argumentli trigonometrik funksiyalar yig'indisi ko'rinishda izlaymiz: $y_1 = A \cos 3x + B \sin 3x + C \cos x + D \sin x$.

Uning hosilalarini topib:

$$y_1' = -3A \cos 3x + 3B \sin 3x - C \sin x + D \cos x,$$

$$y_1'' = -9A \cos 3x - 9B \sin 3x - C \cos x - D \sin x$$

berilgan tenglamaga qo'yamiz va bir hil argumentli trigonometrik funksiyalar oldidagi koeffitsientlarni tenglashtiramiz:

$$-9A \cos 3x - 9B \sin 3x - C \cos x - D \sin x + 6A \sin 3x - 6B \cos 3x + 2C \sin x - 2D \cos x + 10A \cos 3x + 10B \sin 3x + 10C \cos x + 10D \sin x = 2 \sin 3x + 5 \cos x;$$

$$\cos 3x: \quad 0 = -9A - 6B + 10A,$$

$$\sin 3x: \quad 2 = -9B + 6A + 10C,$$

$$\cos x: \quad 5 = -C - 2D + 10C,$$

$$\sin x: \quad 0 = -D + 2C + 10D.$$

Hosil bo'lgan sistemani yechib, $A = \frac{12}{37}$, $B = \frac{2}{37}$, $C = \frac{9}{17}$ va $D = -\frac{2}{17}$ larni

topamiz. Qaralayotgan bir jinsli bo'lmagan tenglamaning umumiy yechimi

$$y = e^x (C_1 \cos 3x + C_2 \sin 3x) + \frac{2}{37} (6 \cos 3x + \sin 3x) + \frac{1}{17} (9 \cos x - 2 \sin x)$$

ko'rinishda bo'ladi.

12-misol. $y'' - 2y' + y = \frac{e^x}{x^2 + 1}$ tenglamani yeching.

Yechish. $y'' - 2y' + y = 0$ tenglama uchun xarakteristik tenglamani tuzamiz:

$k^2 - 2k + 1 = 0$. Bu tenglama ildizlari $k_1 = k_2 = 1$ bo'lgani uchun bir jinsli tenglamaning umumiy yechimi $\tilde{y} = (C_1 + C_2 x)e^x$ bo'ladi.

O'zgarmaslarni variatsiyalash usulidan foydalanib, bir jinsli bo'lmagan tenglamaning yechimini $y = C_1(x)e^x + C_2(x)xe^x$ ko'rinishda izlaymiz, bu yerda $C_1(x)$ va $C_2(x)$ sistemadan topiladi. $y_1 = e^x$, $y_2 = xe^x$ belgilanib,

$$\begin{cases} C_1' e^x + C_2' x e^x = 0, \\ C_1' e^x + C_2' (1+x) e^x = \frac{e^x}{x^2 + 1} \end{cases} \text{ sistemani hosil qilamiz. Uni yechib } C_1' = -\frac{x}{x^2 + 1},$$

$C_2' = \frac{1}{x^2 + 1}$ larni topamiz. Oxirgi ifodalarni integrallab $C_1 = -\frac{1}{2} \ln(x^2 + 1) + \bar{C}_1$,

$C_2 = \arctg x + \bar{C}_2$ larni topamiz.

Ularni umumiy yechimga qo'yib,

$$y = \left(C_1 + C_2 x - \frac{1}{2} \ln(x^2 + 1) + x \arctg x \right) e^x$$

ni hosil qilamiz.

Nazariy savollar va mashqlar.

1. Differensial tenglama deb nimaga aytiladi? Differensial tenglamaning tartibi deb-chi?
2. Birinchi tartibli differensial tenglamaning umumiy va hususiy yechimlari deb nimaga aytiladi?
3. Umumiy va hususiy yechimlar qanday geometrik talqinga ega?
4. Birinchi tartibli differensial tenglamaning yechimini mavjud va yagonaligi haqidagi teoremani aytib bering.
5. Nimaga asosan birinchi tartibli differensial tenglamalar o'zgaruvchilari ajraladigan, bir jinsli, chiziqli, Bernulli, to'la differensial tenglamalar deb yuritiladi? Ular qanday integrallanadi?
6. Quydagi differensial tenglamalarning turini aniqlang: a) $(x^2 - y^2)y' = 2xy$;
b) $x \operatorname{tg} y \cdot y' = \ln x$; c) $(1 + x^2)y' - 2xy = (1 + x^2)^2$; g) $y' + \frac{y'}{x^2 - 1} = 0$.
7. Ikkinchi tartibli differensial tenglamaning umumiy va hususiy yechimlari deb nimaga aytiladi?
8. Darajalari pasaytirilib yechiladigan ikkinchi tartibli differensial tenglamalarga misollar keltiring.
9. Harakteristik tenglama deb nimaga aytiladi? U qanday topiladi?
10. Ikkinchi tartibli bir jinsli bo'lmagan chiziqli o'zgarmas koeffitsientli differensial tenglamaning umumiy yechimi qanday topiladi?
11. Ushbu $y'' + py' + qy = f_1(x)$ va $y'' + py' + qy = f_2(x)$ tenglamalarining hususiy yechimlarini yig'indisi $y'' + py' + qy = f_1(x) + f_2(x)$ tenglamaning ham yechimi bo'lishini ko'rsating.

12. Ikkinchi tartibli bir jinsli bo'lmagan chiziqli o'zgarmas koeffitsientli differensial tenglamaning o'ng tomoni $f(x) = P_n(x)$, $f(x) = P_n(x)e^{\alpha x}$, $f(x) = (A \cos \alpha x + B \sin \alpha x)e^{\beta x}$ bo'lgan hollarda uning hususiy yechimni topish usullarini bayon qiling. Bu yerda $P_n(x)$ – ko'phad.

13. Agar y_1 funktsiya $y'' + P(x)y' + Q(x)y = 0$ tenglamaning hususiy yechimi bo'lsa, u holda $y_2 = cy_1 \int e^{-\int P(x)dx} \frac{dx}{y_1^2}$ ($c = const$) ham qaralayotgan tenglamaning yechimi ekanligini ko'rsating.

14. Ixtiyoriy o'zgarmasni variatsiyalash usulini mohiyati nimadan iborat?

Loyiha-hisob ishi topshiriqlari

1 – masala. Differensial tenglamaning umumiy integralini toping.

- | | |
|---|---|
| 2. 1. 1. $4xdx - 3ydy = 3x^2 ydy - 2xy^2 dx$. | 2. 1. 2. $\sqrt{4 - y^2} dx - ydy = x^2 ydy$. |
| 2. 1. 3. $6xdx - 6ydy = 2x^2 ydy - 3xy^2 dx$. | 2. 1. 4. $x\sqrt{1 + y^2} + yy'\sqrt{1 + x^2} = 0$. |
| 2. 1. 5. $x\sqrt{3 + y^2} dx + y\sqrt{2 + x^2} dy = 0$. | 2. 1. 6. $(e^{2x} + 5)dy + ye^{2x} dx = 0$. |
| 2. 1. 7. $6xdx - 6ydy = 3x^2 ydy - 2xy^2 dx$. | 2. 1. 8. $\sqrt{3 + y^2} dx - ydy = x^2 ydy$. |
| 2. 1. 9. $x\sqrt{5 + y^2} dx + y\sqrt{4 + x^2} dy = 0$. | 2. 1. 10. $y'y\sqrt{\frac{1 - x^2}{1 - y^2}} + 1 = 0$. |
| 2. 1. 11. $2xdx - 2ydy = x^2 ydy - 2xy^2 dx$. | 2. 1. 12. $y(4 + e^x)dy - e^x dx = 0$. |
| 2. 1. 13. $x\sqrt{4 + y^2} dx + y\sqrt{1 + x^2} dy = 0$. | 2. 1. 14. $\sqrt{4 - x^2} y' + xy^2 + x = 0$. |
| 2. 1. 15. $6xdx - ydy = yx^2 dy - 3xy^2 dx$. | 2. 1. 16. $(e^x + 8)dy - ye^x dx = 0$. |
| 2. 1. 17. $\sqrt{5 + y^2} dx + 4(x^2 y + y)dy = 0$. | 2. 1. 18. $y \ln y + xy' = 0$. |
| 2. 1. 19. $x dx - y dy = yx^2 dy - xy^2 dx$. | 2. 1. 20. $\sqrt{5 + y^2} + y'y\sqrt{1 - x^2} = 0$. |
| 2. 1. 21. $6xdx - 2ydy = 2yx^2 dy - 3xy^2 dx$. | 2. 1. 22. $(1 + e^x)y' = ye^x$. |
| 2. 1. 23. $3(x^2 y + y)dy + \sqrt{2 + y^2} dx = 0$. | 2. 1. 24. $\sqrt{1 - x^2} y' + xy^2 + x = 0$. |
| 2. 1. 25. $2xdx - ydy = yx^2 dy - xy^2 dx$. | 2. 1. 26. $y(1 + \ln y) + xy' = 0$. |
| 2. 1. 27. $20xdx - 3ydy = 3x^2 ydy - 5xy^2 dx$. | 2. 1. 28. $(3 + e^x)yy' = e^x$. |

$$2.1.29. 2x + 2xy^2 + \sqrt{2-x^2}y' = 0.$$

$$2.1.30. \sqrt{4+y^2} + \sqrt{4-x^2}yy' = 0.$$

2 – masala. Differensial tenglamaning umumiy integralini toping.

$$2.2.1. y' = \frac{y^2}{x^2} + 4\frac{y}{x} + 2.$$

$$2.2.2. xy' = \frac{3y^3 + 2yx^2}{2y^2 + x^2}.$$

$$2.2.3. y' = \frac{x^2 + xy - y^2}{x^2 - 2xy}.$$

$$2.2.4. xy' = \sqrt{x^2 + y^2} + y.$$

$$2.2.5. 2y' = \frac{y^2}{x^2} + 6\frac{y}{x} + 3.$$

$$2.2.6. xy' = \frac{3y^3 + 4yx^2}{2y^2 + 2x^2}.$$

$$2.2.7. y' = \frac{x^2 + 2xy - y^2}{2x^2 - 2xy}.$$

$$2.2.8. xy' = 2\sqrt{x^2 + y^2} + y.$$

$$2.2.9. 3y' = \frac{y^2}{x^2} + 8\frac{y}{x} + 4.$$

$$2.2.10. xy' = \frac{3y^3 + 6yx^2}{2y^2 + 3x^2}.$$

$$2.2.11. y' = \frac{x^2 + 3xy - y^2}{3x^2 - 2xy}.$$

$$2.2.12. xy' = \sqrt{2x^2 + y^2} + y.$$

$$2.2.13. y' = \frac{y^2}{x^2} + 6\frac{y}{x} + 6.$$

$$2.2.14. xy' = \frac{3y^3 + 8yx^2}{2y^2 + 4x^2}.$$

$$2.2.15. y' = \frac{x^2 + xy - 3y^2}{x^2 - 4xy}.$$

$$2.2.16. xy' = 3\sqrt{x^2 + y^2} + y.$$

$$2.2.17. 2y' = \frac{y^2}{x^2} + 8\frac{y}{x} + 8.$$

$$2.2.18. xy' = \frac{3y^3 + 10yx^2}{2y^2 + 5x^2}.$$

$$2.2.19. y' = \frac{x^2 + xy - 5y^2}{x^2 - 6xy}.$$

$$2.2.20. xy' = 3\sqrt{2x^2 + y^2} + y.$$

$$2.2.21. y' = \frac{y^2}{x^2} + 8\frac{y}{x} + 12.$$

$$2.2.22. xy' = \frac{3y^3 + 12yx^2}{2y^2 + 6x^2}.$$

$$2.2.23. y' = \frac{x^2 + 2xy - 5y^2}{2x^2 - 6xy}.$$

$$2.2.24. xy' = 2\sqrt{3x^2 + y^2} + y.$$

$$2.2.25. 4y' = \frac{y^2}{x^2} + 10\frac{y}{x} + 5.$$

$$2.2.26. xy' = \frac{3y^3 + 14yx^2}{2y^2 + 7x^2}.$$

$$2.2.27. y' = \frac{4x^2 + xy - 2y^2}{x^2 - 3xy}.$$

$$2.2.28. xy' = 4\sqrt{x^2 + y^2} + y.$$

$$2.2.29. 3y' = \frac{y^2}{x^2} + 10\frac{y}{x} + 10.$$

$$2.2.30. xy' = 4\sqrt{2x^2 + y^2} + y.$$

3 – masala. Koshi masalasini yeching.

$$2.3.1. \quad y' - \frac{y}{x+2} = x^2 + 2x, \quad y(-1) = \frac{3}{2}.$$

$$2.3.2. \quad y' + \frac{y}{x} = \sin x, \quad y(\pi) = \frac{1}{\pi}.$$

$$2.3.3. \quad y' + y \cos x = \frac{1}{2} \sin 2x, \quad y(0) = 0.$$

$$2.3.4. \quad y' + y \operatorname{tg} x = \cos^2 x, \quad y\left(\frac{\pi}{4}\right) = \frac{1}{2}.$$

$$2.3.5. \quad y' - y \operatorname{ctg} x = 2x \sin x, \quad y\left(\frac{\pi}{2}\right) = 0.$$

$$2.3.6. \quad y' - \frac{y}{x} = x^2, \quad y(1) = 0.$$

$$2.3.7. \quad y' - \frac{1}{x+1} y = e^x(x+1), \quad y(0) = 1.$$

$$2.3.8. \quad y' - \frac{y}{x} = x \sin x, \quad y\left(\frac{\pi}{2}\right) = 1.$$

$$2.3.9. \quad y' + \frac{2x}{1+x^2} y = \frac{2x^2}{1+x^2}, \quad y(0) = \frac{2}{3}.$$

$$2.3.10. \quad y' + \frac{y}{2x} = x^2, \quad y(1) = 1.$$

$$2.3.11. \quad y' - \frac{2x-5}{x^2} y = 5, \quad y(2) = 4.$$

$$2.3.12. \quad y' - \frac{y}{x} = -\frac{12}{x^2}, \quad y(1) = 4.$$

$$2.3.13. \quad y' - \frac{2y}{x+1} = (x+1)^3, \quad y(0) = \frac{1}{2}.$$

$$2.3.14. \quad y' + \frac{y}{x} = 3x, \quad y(1) = 1.$$

$$2.3.15. \quad y' - y \cos x = -\sin 2x, \quad y(0) = 3.$$

$$2.3.16. \quad y' + \frac{1-2x}{x^2} y = 1, \quad y(1) = 1.$$

$$2.3.17. \quad y' - \frac{2}{x+1} y = e^x(x+1)^2, \quad y(0) = 1.$$

$$2.3.18. \quad y' + xy = -x^3, \quad y(0) = 3.$$

$$2.3.19. \quad y' + 2xy = xe^{-x^2} \sin x, \quad y(0) = 1.$$

$$2.3.20. \quad y' + \frac{3y}{x} = \frac{2}{x^3}, \quad y(1) = 1.$$

$$2.3.21. \quad y' + \frac{xy}{2(1-x^2)} = \frac{x}{2}, \quad y(0) = \frac{2}{3}.$$

$$2.3.22. \quad y' - \frac{y}{x} = -\frac{\ln x}{x}, \quad y(1) = 1.$$

$$2.3.23. \quad y' - 3x^2 y = \frac{1}{3} x^2(1+x^3), \quad y(0) = 0.$$

$$2.3.24. \quad y' - y \cos x = \sin 2x, \quad y(0) = -1.$$

$$2.3.25. \quad y' - \frac{2xy}{1+x^2} = 1+x^2, \quad y(1) = 3.$$

$$2.3.26. \quad y' + \frac{2}{x} y = x^3, \quad y(1) = -\frac{5}{6}.$$

$$2.3.27. \quad y' + 2xy = -2x^3, \quad y(1) = e^{-1}.$$

$$2.3.28. \quad y' + \frac{y}{x} = \frac{x+1}{x} e^x, \quad y(1) = e.$$

$$2.3.29. \quad y' - \frac{y}{x} = -2 \frac{\ln x}{x}, \quad y(1) = 1.$$

$$2.3.30. \quad y' - 4xy = -4x^3, \quad y(0) = -\frac{1}{2}.$$

4 – masala. Tenglamaning umumiy yechimini toping.

$$2.4.1. \quad y' + xy = (1+x)e^{-x}y^2.$$

$$2.4.2. \quad xy' + y = 2y^2 \ln x.$$

$$2.4.3. \quad 2(xy' + y) = xy^2.$$

$$2.4.4. \quad y' + 4x^3 y = 4(x^2 + 1)e^{-4x}y^2.$$

$$2.4.5. \quad xy' - y = -y^2(\ln x + 2) \ln x.$$

$$2.4.6. \quad 2(y' + xy) = (1+x)e^{-x}y^2.$$

$$2.5.16. \left(xy^2 + \frac{x}{y^2}\right)dx + \left(x^2y - \frac{x^2}{y^2}\right)dy = 0. \quad 2.5.17. \left(2x - 1 - \frac{y}{x^2}\right)dx - \left(2y - \frac{1}{x}\right)dy = 0.$$

$$2.5.18. \left(\frac{y}{x^2} \sin \frac{y}{x}\right)dx - \left(\frac{1}{x} \sin \frac{y}{x} + 2y\right)dy = 0. \quad 2.5.19. \left(y^2 + \frac{y}{\cos^2 x}\right)dx + (2xy + \operatorname{tg} x)dy = 0.$$

$$2.5.20. (3x^2y + 2y + 3)dx + (x^3 + 2x + 3y^2)dy = 0. \quad 2.5.21. \left(\frac{1}{x^2} + \frac{3y^2}{x^4}\right)dx - \frac{2y}{x^3}dy = 0.$$

$$2.5.22. \left(\frac{x}{\sqrt{x^2 + y^2}} + \frac{1}{x} + \frac{1}{y}\right)dx + \left(\frac{y}{\sqrt{x^2 + y^2}} + \frac{1}{y} - \frac{x}{y^2}\right)dy = 0.$$

$$2.5.23. \frac{y}{x^2} \cos \frac{y}{x} dx - \left(\frac{1}{x} \cos \frac{y}{x} + 2y\right)dy = 0.$$

$$2.5.24. \left(\frac{y}{x^2 + y^2} + \cos 2x\right)dx - \frac{x}{x^2 + y^2} dy = 0.$$

$$2.5.25. \frac{1+xy}{x^2y} dx + \frac{1-xy}{xy^2} dy = 0. \quad 2.5.26. \frac{y}{x^2} dx - \frac{xy+1}{x} dy = 0.$$

$$2.5.27. \left(\frac{x}{\sqrt{x^2 + y^2}} + y\right)dx + \left(x + \frac{y}{\sqrt{x^2 + y^2}}\right)dy = 0.$$

$$2.5.28. \left(xe^x + \frac{y}{x^2}\right)dx - \frac{dy}{x} = 0.$$

$$2.5.29. (\cos(x+y^2) + \sin x)dx + 2y \cos(x+y^2)dy = 0. \quad 2.5.30. \frac{dx}{y} - \frac{x+y^2}{y^2} dy = 0.$$

6 – masala. Koshi masalasini yeching.

$$2.6.1. \quad y''x \ln x = y', \quad y(e) = e, \quad y'(e) = 1.$$

$$2.6.2. \quad 4y^3 y'' = y^4 - 1, \quad y(0) = \sqrt{2}, \quad y'(0) = \frac{1}{2\sqrt{2}}.$$

$$2.6.3. \quad \operatorname{tg} x \cdot y'' - y' + \frac{1}{\sin x} = 0, \quad y\left(\frac{\pi}{2}\right) = \frac{1}{2}, \quad y'\left(\frac{\pi}{2}\right) = 0.$$

$$2.6.4. \quad y'' = 128y^3, \quad y(0) = 1, \quad y'(0) = 8.$$

$$2.6.5. \quad y'' = \frac{y'}{x} - (y')^2, \quad y(2) = 2, \quad y'(2) = 1.$$

$$2.6.6. \quad y'' + 2 \sin y \cos^3 y = 0, \quad y(0) = 0, \quad y'(0) = 1.$$

$$2.6.7. \quad y'' = \frac{y'}{x} + \frac{x^2}{y'}, \quad y(2) = 0, \quad y'(2) = 4.$$

2. 6. 8. $y''y^3 + 64 = 0, y(0) = 4, y'(0) = 2.$
2. 6. 9. $x^4y'' + x^3y' = 1, y(1) = \frac{1}{4}, y'(1) = -\frac{1}{2}.$
2. 6. 10. $y'' = 32\sin^3 y \cos y, y(1) = \frac{\pi}{2}, y'(1) = 4.$
2. 6. 11. $(1 + x^2)y'' + 2xy' = x^3, y(0) = 0, y'(0) = 0.$
2. 6. 12. $4y^3y'' = 16y^4 - 1, y(0) = \frac{\sqrt{2}}{2}, y'(0) = \frac{1}{\sqrt{2}}.$
2. 6. 13. $x^2y'' + xy' = 1, y(1) = \frac{1}{2}, y'(1) = 0.$
2. 6. 14. $y'' + 18\sin y \cos^3 y = 0, y(0) = 0, y'(0) = 3.$
2. 6. 15. $y''(x^2 + 1) = 2xy', y(0) = 1, y'(0) = 3.$
2. 6. 16. $y'' = 32y^3, y(4) = 1, y'(4) = 4.$
2. 6. 17. $y'' = \frac{y'}{x} \left(1 + \ln \frac{y'}{x} \right), y(1) = \frac{1}{2}, y'(1) = 1.$
2. 6. 18. $y'' = 8\sin^3 y \cos y, y(1) = \frac{\pi}{2}, y'(1) = 2.$
2. 6. 19. $y''(x^2 + 1) = xy', y(0) = 1, y'(0) = 1.$
2. 6. 20. $y''y^3 + 16 = 0, y(1) = 2, y'(1) = 2.$
2. 6. 21. $x^4y'' + x^3y' = 4, y(1) = -2, y'(1) = 2.$
2. 6. 22. $y'' = 2\sin^3 y \cos y, y(1) = \frac{\pi}{2}, y'(1) = 1.$
2. 6. 23. $y''(x^2 + 1) = 4xy', y(0) = 1, y'(0) = 5.$
2. 6. 24. $y^3y'' = y^4 - 16, y(0) = 2\sqrt{2}, y'(0) = \sqrt{2}.$
2. 6. 25. $y''(x^2 + 1) + 2xy' = 2x(x^2 + 1), y(0) = 0, y'(0) = 0.$
2. 6. 26. $y'' = 2y^3, y(-1) = 1, y'(-1) = 1.$
2. 6. 27. $(1 + x^2)y'' + 2xy' = 12x^3, y(0) = 0, y'(0) = 0.$
2. 6. 28. $y'' = 50\sin^3 y \cos y, y(1) = \frac{\pi}{2}, y'(1) = 5.$
2. 6. 29. $y''(1 + \ln x) + \frac{1}{x}y' = 2 + \ln x, y(1) = \frac{1}{2}, y'(1) = 1.$

2. 6. 30. $y''+32\sin y \cos^3 y = 0, y(0) = 0, y'(0) = 4.$

7 – masala. Differensial tenglamaning berilgan boshlang'ich shartlarni qanoatlantiruvchi hususiy yechimini toping.

2. 7. 1. $y''+2y'+5y = 0, y(0) = 0, y'(0) = 2.$

2. 7. 2. $y''+4y'+4y = 0, y(0) = 0, y'(0) = 1.$

2. 7. 3. $y''+y'-2y = 0, y(0) = 0, y'(0) = 1.$

2. 7. 4. $y''+6y'+13y = 0, y(0) = 0, y'(0) = 2.$

2. 7. 5. $y''-6y'+9y = 0, y(0) = 0, y'(0) = 1.$

2. 7. 6. $y''-2y'-3y = 0, y(0) = 0, y'(0) = 1.$

2. 7. 7. $y''-4y'+8y = 0, y(0) = 0, y'(0) = 2.$

2. 7. 8. $y''-4y'+4y = 0, y(0) = 0, y'(0) = 2.$

2. 7. 9. $y''+2y'-3y = 0, y(0) = 0, y'(0) = 1.$

2. 7. 10. $y''-2y'+5y = 0, y(0) = 0, y'(0) = 1.$

2. 7. 11. $y''-10y'+25y = 0, y(0) = 0, y'(0) = 1.$

2. 7. 12. $y''-7y'+6y = 0, y(0) = 0, y'(0) = 1.$

2. 7. 13. $y''-1,5y'+0,5y = 0, y(0) = 0, y'(0) = 1.$

2. 7. 14. $y''-2y'+5y = 0, y(0) = 0, y'(0) = 3.$

2. 7. 15. $y''+y'+0,25y = 0, y(0) = 0, y'(0) = 2.$

2. 7. 16. $y''-\frac{2}{3}y'+\frac{1}{9}y = 0, y(0) = 0, y'(0) = \frac{2}{3}.$

2. 7. 17. $y''-3y'+4y = 0, y(0) = 0, y'(0) = 5.$

2. 7. 18. $y''+6y'+10y = 0, y(0) = 0, y'(0) = 6.$

2. 7. 19. $y''-8y'+16y = 0, y(0) = 1, y'(0) = 4.$

2. 7. 20. $y''-7y'+10y = 0, y(0) = 0,5, y'(0) = 4.$

2. 7. 21. $4y''-4y'+y = 0, y(0) = 0, y'(0) = 4.$

2. 7. 22. $y''-6y'+10y = 0, y(0) = 0, y'(0) = 3.$

2. 7. 23. $y''+25y = 0, y(0) = 0, y'(0) = -5.$

2. 7. 24. $y''+3y'+2y = 0, y(0) = 0, y'(0) = -1.$

$$2.7.25. y'' - y' + \frac{1}{4}y = 0, y(0) = 0, y'(0) = 2.$$

$$2.7.26. y'' + 7y' + 10y = 0, y(0) = 0, y'(0) = -7.$$

$$2.7.27. y'' - 25y = 0, y(0) = 1, y'(0) = 5.$$

$$2.7.28. y'' - 3y' + 2y = 0, y(0) = 1, y'(0) = 3.$$

$$2.7.29. y'' + 2y' = 0, y(0) = 2, y'(0) = -4.$$

$$2.7.30. y'' + 5y' - 14y = 0, y(0) = 1, y'(0) = -7.$$

8 – masala. Differensial tenglamaning umumiy yechimini toping.

$$2.8.1. y'' - 5y' + 6y = x + e^{-3x}.$$

$$2.8.2. y'' + 4y = e^{-2x} + \sin x.$$

$$2.8.3. y'' - 2y' + y = e^{2x} + 2\cos x.$$

$$2.8.4. y'' + 2y' = 2x - 1 + e^x.$$

$$2.8.5. y'' + 9y = 5e^{5x} + x.$$

$$2.8.6. y'' - 6y' + 9y = 5\sin x + e^x.$$

$$2.8.7. y'' - 4y' + 3y = 3x - e^{-x}.$$

$$2.8.8. y'' - 9y = 2x - 1 + \cos x.$$

$$2.8.9. y'' + y' = 3x + 1 - \sin 2x.$$

$$2.8.10. y'' - 2y' + y = 5x - 2 + e^x.$$

$$2.8.11. y'' + 3y' - 10y = 9e^{-2x} + \cos x.$$

$$2.8.12. y'' + 3y' = \sin 5x - x.$$

$$2.8.13. y'' - 3y' = 3x + e^{-3x}.$$

$$2.8.14. y'' + y = \sin 2x - e^x.$$

$$2.8.15. y'' - 4y' + 4y = x^2 + 2 + e^{2x}.$$

$$2.8.16. y'' - 2y' = x^2 + 3e^{2x}.$$

$$2.8.17. y'' + 6y' + 13y = 2x + \sin 3x.$$

$$2.8.18. y'' + 25y = 10e^{-x} + \cos 2x.$$

$$2.8.19. y'' - 3y' + 2y = (2x - 3)e^{5x} + \cos x.$$

$$2.8.20. y'' - 5y' + 4y = 3e^{4x} - \sin x.$$

$$2.8.21. y'' - 7y' + 12y = (x + 2)e^{2x} - \cos x.$$

$$2.8.22. y'' - 25y = x^2 - 2 + e^{5x}.$$

$$2.8.23. y'' - 4y' + 3y = x^2 e^{-x} + \sin 2x.$$

$$2.8.24. y'' - 7y' + 10y = x^2 - 5x + 3 - e^{2x}.$$

$$2.8.25. y'' - 5y' - 14y = e^{-2x} - \cos x.$$

$$2.8.26. y'' + 3y' = xe^x + 3x^2 - 6x - 2.$$

$$2.8.27. y'' + 5y' + 4y = xe^{-x} + \sin x.$$

$$2.8.28. y'' + 49y = 3x + 1 - \sin 2x.$$

$$2.8.29. y'' - 6y' + 10y = x^2 - 3x + \cos x.$$

$$2.8.30. y'' + 6y' + 9y = (x + 2)e^{-3x} + x^2.$$

9 – masala. Differensial tenglamalar sistemasini yeching.

$$2.9.1. \begin{cases} \frac{dx}{dt} = 4x + 6y \\ \frac{dy}{dt} = 4x + 2y \end{cases}.$$

$$2.9.2. \begin{cases} \frac{dx}{dt} = -5x - 4y \\ \frac{dy}{dt} = -2x - 3y \end{cases}.$$

$$2.9.3. \begin{cases} \frac{dx}{dt} = 3x + y \\ \frac{dy}{dt} = 8x + y \end{cases}.$$

$$2.9.4. \begin{cases} \frac{dx}{dt} = 6x + 3y \\ \frac{dy}{dt} = -8x - 5y \end{cases}.$$

$$2.9.5. \begin{cases} \frac{dx}{dt} = -x + 5y \\ \frac{dy}{dt} = x + 3y \end{cases}.$$

$$2.9.6. \begin{cases} \frac{dx}{dt} = 3x - 2y \\ \frac{dy}{dt} = 2x + 8y \end{cases}.$$

$$2.9.7. \begin{cases} \frac{dx}{dt} = -4x - 6y \\ \frac{dy}{dt} = -4x - 2y \end{cases}.$$

$$2.9.8. \begin{cases} \frac{dx}{dt} = -5x - 8y \\ \frac{dy}{dt} = -3x - 3y \end{cases}.$$

$$2.9.9. \begin{cases} \frac{dx}{dt} = -x - 5y \\ \frac{dy}{dt} = -7x - 3y \end{cases}.$$

$$2.9.10. \begin{cases} \frac{dx}{dt} = -7x + 5y \\ \frac{dy}{dt} = 4x - 8y \end{cases}.$$

$$2.9.11. \begin{cases} \frac{dx}{dt} = x - 2y \\ \frac{dy}{dt} = x + 4y \end{cases}.$$

$$2.9.12. \begin{cases} \frac{dx}{dt} = x - 3y \\ \frac{dy}{dt} = x + 5y \end{cases}.$$

$$2.9.13. \begin{cases} \frac{dx}{dt} = 3x + y \\ \frac{dy}{dt} = y + 3x \end{cases}.$$

$$2.9.14. \begin{cases} \frac{dx}{dt} = 9x + 6y \\ \frac{dy}{dt} = 2x + 8y \end{cases}.$$

$$2.9.15. \begin{cases} \frac{dx}{dt} = 7x - 5y \\ \frac{dy}{dt} = -4x + 8y \end{cases}.$$

$$2.9.16. \begin{cases} \frac{dx}{dt} = x + 7y \\ \frac{dy}{dt} = x + 9y \end{cases}.$$

$$2.9.17. \begin{cases} \frac{dx}{dt} = 2x + 2y \\ \frac{dy}{dt} = x + 3y \end{cases}.$$

$$2.9.18. \begin{cases} \frac{dx}{dt} = 8x + 2y \\ \frac{dy}{dt} = -3x + 15y \end{cases}.$$

$$2.9.19. \begin{cases} \frac{dx}{dt} = 5x + 4y \\ \frac{dy}{dt} = -2x + 11y \end{cases}.$$

$$2.9.20. \begin{cases} \frac{dx}{dt} = x - 2y \\ \frac{dy}{dt} = 3x + 6y \end{cases}.$$

$$2.9.21. \begin{cases} \frac{dx}{dt} = 2x - y \\ \frac{dy}{dt} = 3x - 2y \end{cases}.$$

$$2.9.22. \begin{cases} \frac{dx}{dt} = 4x + 2y \\ \frac{dy}{dt} = -x + y \end{cases}.$$

$$2.9.23. \begin{cases} \frac{dx}{dt} = 7x + 2y \\ \frac{dy}{dt} = 3x + 2y \end{cases} .$$

$$2.9.24. \begin{cases} \frac{dx}{dt} = -6x + 5y \\ \frac{dy}{dt} = 2x \end{cases} .$$

$$2.9.25. \begin{cases} \frac{dx}{dt} = 2x + y \\ \frac{dy}{dt} = 3x + 4y \end{cases} .$$

$$2.9.26. \begin{cases} \frac{dx}{dt} = x + y \\ \frac{dy}{dt} = 3x - y \end{cases} .$$

$$2.9.27. \begin{cases} \frac{dx}{dt} = x + 4y \\ \frac{dy}{dt} = 2x + 3y \end{cases} .$$

$$2.9.28. \begin{cases} \frac{dx}{dt} = 2x + 8y \\ \frac{dy}{dt} = x + 4y \end{cases} .$$

$$2.9.29. \begin{cases} \frac{dx}{dt} = x + 5y \\ \frac{dy}{dt} = 7x + 3y \end{cases} .$$

$$2.9.30. \begin{cases} \frac{dx}{dt} = -x - y \\ \frac{dy}{dt} = -3x + y \end{cases} .$$

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